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1a) $A \cap B \cap C = \{5, 11, 17\}$

b) $(A \cap B) \setminus C = \{2, 8, 14, 20\}$

c) $(A \cap C) \setminus B = \{4, 6, 10, 12, 16, 18\}$

3) Show that $A \subseteq B \iff A \cap B = A$

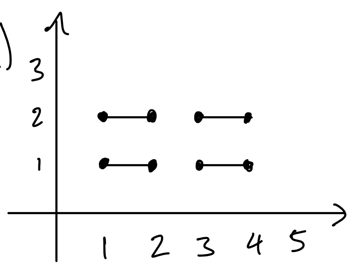
Suppose $A \cap B \neq A$. That is, there exists some $x \in A$ s.t. $x \notin B$
This implies $A \not\subseteq B$, which we know is not true.
Now suppose $A \not\subseteq B$ is true. This implies there exists some $x \in A$ s.t. $x \notin B$
which implies $A \cap B \neq A$. Since the statements rely on each other to be true, $A \subseteq B \iff A \cap B = A$

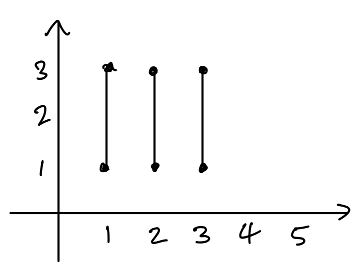
5a)
$$\begin{aligned} x \in A \cap (B \cup C) &\Rightarrow x \in A \land (x \in B \lor x \in C) \\ &\Rightarrow (x \in A \land x \in B) \lor (x \in A \land x \in C) \\ &\Rightarrow x \in A \cap B \lor x \in A \cap C \\ &\Rightarrow x \in (A \cap B) \cup (A \cap C) \end{aligned}$$

7a) $\underbrace{A_1 = \{(1+1)k : k \in \mathbb{N}\}}_{\text{even \#s}}$, $A_2 = \underbrace{\{(2+1)k : k \in \mathbb{N}\}}_{\text{multiples of 3}}$

$\therefore A_1 \cap A_2 = \{n : n \in \mathbb{N}, \text{ n is a multiple of } 6\} = A_5$

b) $\bigcup\{A_n : n \in \mathbb{N}\} = \mathbb{N} \setminus \{1\}$, $\bigcap\{A_n : n \in \mathbb{N}\} = \emptyset$

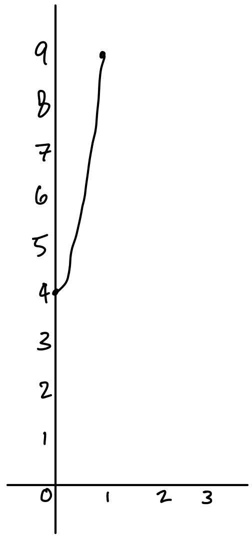
8a)


8b)


11a) $g(x)=x^2 \qquad f(x)=x+2$

$h=g\circ f \Rightarrow h(x)=(x+2)^2$

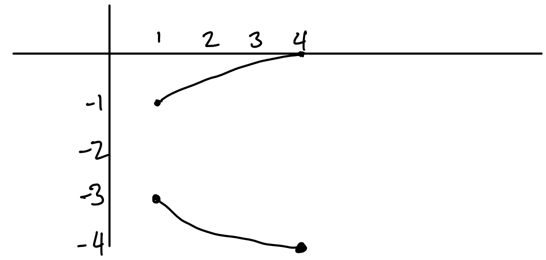
$h(E)$:



b) $x=(y+2)^2$

$\pm\sqrt{x}=y+2$

$y=\pm\sqrt{x}-2 \Rightarrow h^{-1}(x)=\pm\sqrt{x}-2$



15) $f^{-1}(G\cup H)=f^{-1}(G)\cup f^{-1}(H)$

If $x\in f^{-1}(G\cup H)$, $f(x)\in G\cup H$ such that $f(x)\in G \vee f(x)\in H$.

Thus, $x\in f^{-1}(G) \vee x\in f^{-1}(H) \Rightarrow x\in f^{-1}(G)\cup f^{-1}(H)$

and $f^{-1}(G\cup H)=f^{-1}(G)\cup f^{-1}(H)$

$f^{-1}(G\cap H)=f^{-1}(G)\cap f^{-1}(H)$

If $y\in f^{-1}(G\cap H)$, $f(y)=G\cap H$. Thus, we know $f(y)\in G \wedge f(y)\in H$.

It follows that $y\in f^{-1}(G) \wedge y\in f^{-1}(H) \Rightarrow y\in f^{-1}(G)\cap f^{-1}(H)$

and $f^{-1}(G\cap H)=f^{-1}(G)\cap f^{-1}(H)$

17 $f(x)=\frac{x-a}{b-a}$