Practice Questions for MA 341 Midterm, Spring 2024

1. (5pts) Let (x_n) be a sequence of real numbers. Give the definition of
$$\lim_{n\to\infty} x_n = L$$
2. (5pts) Give the definition of a Cauchy sequence.
3. (5pts) Give the definition of the supremum and infimum.
4. (5pts) State the Principle of Mathematical Induction.
5. (5pts) State Cantor's Theorem.
6. (5pts) Let A be the set of prime numbers. Is the power set of A countable?
7. (5pts) Give an example of a function $f : \mathbb{N} \to \{n \in \mathbb{N} \mid n \text{ even}, \ n \geq 10\}$ which is surjective but not injective.
8. (5pts) Give an example of a function $f : X \to Y$ and a subset $Z \subseteq Y$ such that there is no bijection between Z and $f^{-1}(Z)$. (The sets X, Y, and Z are up to you.)
9. (5pts) Suppose that a sequence (x_n) has a constant subsequence. Must (x_n) converge? If so, say why. If not, give a counterexample.
10. (5pts) Give an example of unbounded subsets of $\mathbb{R}$, say $A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots$, such that
$$\bigcap_{i=1}^{\infty} A_i = \emptyset.$$
11. (5pts) State the Bolzano-Weierstrass Theorem.
12. (20pts) Prove that $\mathbb{R}$ is uncountable.
13. (20pts) Prove that a bounded increasing sequence converges.
14. (20pts) Prove by induction that $5^{2n} - 1$ is divisible by 8 for every $n \in \mathbb{N}$.
15. (20pts) Let $A \subseteq \mathbb{R}$ be a bounded nonempty set of positive numbers, and let $c > 0$. Define
$$cA = \{y \in \mathbb{R} \mid y = cx \text{ for some } x \in A\}.$$
Prove that
$$\sup(cA) = c\sup(A).$$
Remember to prove first that $\sup(cA)$ exists!

16. (20pts) Prove, using the definition of the limit, that

$$\lim_{n\to\infty} \frac{n^2-1}{n^2+1} = 1$$

17. (20pts) Prove any of questions 1. through 4. in Section 3.3 of the text.