MA 34100 Homework 6

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Exercise 3.3.3

Since $x_k \geq 2$,

$$x_{k+1} = 1 + \sqrt{x_k - 1} \geq 1 + \sqrt{2 - 1} = 2$$

So, $x_n \geq 2$ for all $n \in \mathbb{N}$ by induction.
If $x_{k+1} \leq x_k$,

$$x_{k+2} = 1 + \sqrt{x_{k+1} - 1} \leq 1 + \sqrt{x_k - 1} = x_{k+1}$$

So (x_n) is decreasing. $x = 1$ is not possible, so $\lim(x_n) = 2$

Exercise 3.3.6

Case 1: $z_1 \geq \sqrt{a + z_1}$. Then $z_2 = \sqrt{a + z_1} \geq z_1$, so the sequence is monotonically increasing.

$$z* = \frac{1 + \sqrt{1 + 4a}}{2}$$

Case 2: This case contradicts the initial assumption since $z_1 > 0$ and $a > 0$, leading to a positive square root. So, the sequence must be increasing.

Exercise 3.3.9

Exercise 3.4.1

Exercise 3.4.4

Exercise 3.4.14

Exercise 3.5.2

Exercise 3.5.3

Exercise 3.5.7