MA 450 Homework 10

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Exercise 10.31 Suppose that ϕ is a homomorphism from $U(30)$ to $U(30)$ and that $\ker\phi = \{1, 11\}$. If $\phi(7) = 7$, find all elements of $U(30)$ that map to 7.

Exercise 10.32 Find a homomorphism ϕ from $U(30)$ to $U(30)$ with kernel 1, 11 and $\phi(7)$ 5 7.

Exercise 10.41 (Second Isomorphism Theorem) If K is a subgroup of G and N is a normal subgroup of G, prove that $K/(K > N)$ is isomorphic to KN/N.

Exercise 10.42 (Third Isomorphism Theorem) If M and N are normal subgroups of G and $N \leq M$, prove that $(G/N)/(M/N) \cong G/M$.

Exercise 11.4 Calculate the number of elements of order 2 in each of $\mathbb{Z}_{16}$, $\mathbb{Z}_8 \oplus \mathbb{Z}_2$, $\mathbb{Z}_4 \oplus \mathbb{Z}_4$, and $\mathbb{Z}_4 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$. Do the same for the elements of order 4.

s **Exercise 11.8** Show that there are two Abelian groups of order 108 that have exactly 13 subgroups of order 3.

Exercise 11.15 How many Abelian groups (up to isomorphism) are there

- **a.** of order 6?
- **b.** of order 15?
- **c.** of order 42?
- **d.** of order pq, where p and q are distinct primes?
- **e.** of order pqr, where p, q, and r are distinct primes?
- **f.** Generalize parts d and e.

Exercise 11.26 Let $G = \{1, 7, 17, 23, 49, 55, 65, 71\}$ under multiplication modulo 96. Express G as an external and an internal direct product of cyclic groups.

Exercise 11.28 The set $G = \{1, 4, 11, 14, 16, 19, 26, 29, 31, 34, 41, 44\}$ is a group under multiplication modulo 45. Write G as an external and an internal direct product of cyclic groups of prime-power order.

Exercise 11.30 Suppose that G is an Abelian group of order 16, and in computing the orders of its elements, you come across an element of order 8 and two elements of order 2. Explain why no further computations are needed to determine the isomorphism class of G.

Exercise 11.36 Suppose that G is a finite Abelian group. Prove that G has order p^n, where p is prime, if and only if the order of every element of G is a power of p.