# FIRST PRACTICE MIDTERM
# MATH 18.703, MIT, SPRING 13

You have 80 minutes. This test is closed book, closed notes, no calculators.

There are 7 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.* Points will be awarded on the basis of neatness, the use of complete sentences and the correct presentation of a logical argument.

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Name:______________________________

Signature:______________________________

Student ID #:______________________________

| Problem | Points | Score |
|---|---|---|
| 1 | 15 | |
| 2 | 15 | |
| 3 | 20 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 15 | |
| 7 | 10 | |
| Presentation | 5 | |
| Total | 100 | |

1. (15pts) (i) Give the definition of a normal subgroup.

(ii) Give the definition of a group homomorphism.

(iii) Give the definition of $A_n$, the alternating group.

2. (15pts) (i) Exhibit a proper normal subgroup $V$ of $A_4$. To which group is $V$ isomorphic to?

(ii) Give the left cosets of $V$ inside $A_4$.

(iii) To which group is $A_4/V$ isomorphic to?

3. (20pts) Let $G$ be a group and let $H$ be a subgroup. Prove that the following are equivalent.

(1) $H$ is normal in $G$.
(2) For every $g \in G$, $gHg^{-1} = H$.
(3) For every $a \in G$, $aH = Ha$.
(4) The set of left cosets is equal to the set of right cosets.

4. (10pts) Let $G$ be a group and let $N$ be a normal subgroup. Prove that $G/N$ is abelian iff $N$ contains the commutator of every pair of elements of $G$.

5. (10pts) Let $H$ and $K$ be two normal subgroups of a group $G$, whose intersection is the trivial subgroup. Prove that every element of $H$ commutes with every element of $K$.

6. (15pts) (i) State the Sylow Theorems.

(ii) Prove that if $G$ is a group of order $pq$, where $p$ and $q$ are distinct primes, then $G$ is not simple.

7. (10pts) Let $G$ be a group and let $N$ be a normal subgroup. Show that there is a natural bijection between the set of subgroups $H$ of $G$ which contain $N$ and the subgroups of the quotient group $G/N$. Show that this bijection preserves normality, so that normal subgroups of $G$ which contain $N$ correspond to normal subgroups of $G/N$.