Problem Set 0: Math 453 Spring 2019

January 9, 2019

This is an optional problem set and will not be handed in. In what follows below, $\mathbf{N}$ is the set of natural numbers, $\mathbf{Z}$ is the set of integers, $\mathbf{R}$ is the set of real numbers, and $\mathbf{C}$ is the set of complex numbers.

Problem 1. Let $f\colon X \to Y$ be a function, let $A, B \subset X$ and $C, D \subset Y$. Prove the following:

(a)
$$f(A \cup B) = f(A) \cup f(B).$$

(b)
$$f(A \cap B) \subset f(A) \cap f(B).$$

(c)
$$f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D).$$

(d)
$$f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D).$$

Problem 2. Under what conditions on f in Problem 1 do we get equality in (b)?

Problem 3. Construct a bijective function $f\colon \mathbf{N} \to \mathbf{Z}$.

Problem 4. Let $A, B \subset X$. Prove the following:

(a)
$$X - (A \cup B) = (X - A) \cap (X - B).$$

(b)
$$X - (A \cap B) = (X - A) \cup (X - B).$$

Problem 5. Let X be a set with $|X| = n$. Let $\mathscr{P}_k(X)$ denote the set of subsets $A \subset X$ such that $|A| = k$.

(a) Prove that $|\mathscr{P}_k(X)| = |\mathscr{P}_{n-k}(X)|$.

(b) Prove that
$$|\mathscr{P}_k(X)| = \binom{n}{k}$$
where
$$\binom{n}{k}$$
is the kth binomial coefficient.

(c) Prove that
$$|\mathscr{P}(X)| = 2^n.$$