MA 450 Homework 10

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Exercise 10.31 Suppose that ϕ is a homomorphism from $U(30)$ to $U(30)$ and that $\ker \phi = \{1, 11\}$. If $\phi(7) = 7$, find all elements of $U(30)$ that map to 7.

Exercise 10.32 Find a homomorphism ϕ from $U(30)$ to $U(30)$ with kernel 1, 11 and $\phi(7)$ 5 7.

Exercise 10.41 (Second Isomorphism Theorem) If K is a subgroup of G and N is a normal subgroup of G, prove that $K/(K > N)$ is isomorphic to KN/N.

Exercise 10.42 (Third Isomorphism Theorem) If M and N are normal subgroups of G and $N \leq M$, prove that $(G/N)/(M/N) \cong G/M$.

Exercise 11.4 Calculate the number of elements of order 2 in each of $\mathbb{Z}_{16},\ \mathbb{Z}_8 \oplus \mathbb{Z}_2,\ \mathbb{Z}_4 \oplus \mathbb{Z}_4$, and $\mathbb{Z}_4 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$. Do the same for the elements of order 4.

s **Exercise 11.8** Show that there are two Abelian groups of order 108 that have exactly 13 subgroups of order 3.

Exercise 11.15 How many Abelian groups (up to isomorphism) are there

a. of order 6?

b. of order 15?

c. of order 42?

d. of order pq, where p and q are distinct primes?

e. of order pqr, where p, q, and r are distinct primes?

f. Generalize parts d and e.

Exercise 11.26 Let $G = \{1, 7, 17, 23, 49, 55, 65, 71\}$ under multiplication modulo 96. Express G as an external and an internal direct product of cyclic groups.

Exercise 11.28 The set $G = \{1, 4, 11, 14, 16, 19, 26, 29, 31, 34, 41, 44\}$ is a group under multiplication modulo 45. Write G as an external and an internal direct product of cyclic groups of prime-power order.

Exercise 11.30 Suppose that G is an Abelian group of order 16, and in computing the orders of its elements, you come across an element of order 8 and two elements of order 2. Explain why no further computations are needed to determine the isomorphism class of G.

Exercise 11.36 Suppose that G is a finite Abelian group. Prove that G has order p^n, where p is prime, if and only if the order of every element of G is a power of p.