PURDUE UNIVERSITY

Department of Mathematics

# GALOIS THEORY HONORS, MA 45401

## Homework 4 (Feb 13 – Feb 21)

**1** (5+5+15+20) For each of the following polynomials, construct a splitting field $L$ over $\mathbb{Q}$ and compute the degree $[L : \mathbb{Q}]$.

1) $t^4 + 7t^2 + 12$
2) $t^4 + t^2 - 12$
3) $t^{2n} - 2^n$, where $n = 3, 4$.
4) $t^{14} - 1$.

**2** (15) Let $K - L - M$ be a field extension and $K - L$, $L - M$ are algebraic extensions. Prove that $K - M$ is also an algebraic extension.

**3** (15) Let $\alpha$ be transcendental over a field $K \subset \mathbb{C}$. Show that $K(\alpha)$ is not algebraically closed (hint: consider the polynomial $t^2 - \alpha$).

**4** (15) Let $L : K$ be a splitting field extension for a non–constant polynomial $f \in K[t]$. Prove that $[L : K]$ divides $(\deg f)!$ (hint: at the very end look at some binomial coefficients).