

Exercise 6.1. Find Galois groups for the following polynomials f over \mathbb{Q} :

1. $(t^2 - 3)(t^2 + 1)$

Solution.

□

2. $t^4 - t^2 + 1$

Solution.

□

3. $t^4 - 2$

Solution.

□

Exercise 6.2.1. Find $\text{Gal}_{\mathbb{F}_3(t^2)}(\mathbb{F}_3(t))$.

Solution.

□

Exercise 6.2.2. Find $\text{Gal}_{\mathbb{F}_2(t^2)}(\mathbb{F}_2(t))$.

Solution.

□

Exercise 6.3.1. Let $K - M - L$ be a field extension and $L : K$ is a normal extension. Prove that $L : M$ is also a normal extension.

Solution.

□

Exercise 6.3.2. Give an example of three fields K, M, L such that $[L : K] = 4$ and $[M : K] = [L : M] = 2$ (hence $K \subset M$ and $M \subset L$ are normal extensions) but $L : K$ is not a normal extension.

Solution.

□

Exercise 6.4. Let $L : K$ be a splitting field extension for a non-constant polynomial $f \in K[t]$. Prove that $|\text{Gal}_K(L)|$ divides $(\deg f)!$.

Solution.

□

Exercise 6.5.1. Let $f = t^3 + t + 1 \in \mathbb{F}_2[t]$. Prove that $\text{Gal}_{\mathbb{F}_2}(f)$ is isomorphic to \mathbb{Z}_3 .

Solution.

□

Exercise 6.5.2. Let $f = t^3 + t^2 + 1 \in \mathbb{F}_2[t]$. Prove that $\text{Gal}_{\mathbb{F}_2}(f)$ is isomorphic to S_3 .

Solution.

□