1 Galois Groups III

Theorem 1.1 (Kronecker)**.** Let $p \geq 3$ be a prime and $f \in \mathbb{Q}[x]$ be irreducible over $\mathbb{Q}$ with $\deg f = p$. If the equation $f(x) = 0$ is solvable by radicals, then the number of real roots of f is 1 or p.

Lemma 1.2. Let p be prime and $G \leqslant S_p$ such that G acts transitively on $\{1, \ldots, p\}$. Then G contains a cycle of order p.

Theorem 1.3. If $L : K$ is a finite extension, then $|\mathrm{Gal}_K(L)| \leq [L : K]$.