

1 Fundamental Theorem of Galois Theory II

Theorem 1.1 (Fundamental Theorem of Galois Theory, Part 2). For all $P \in \mathcal{I}(K, L)$, we have $P : K$ is a normal extension $\iff G_P \triangleleft G$. Then, $\text{Gal}_K P \cong G/G_P$.

Lemma 1.2. Let $K - P - L$ be a tower of fields and $g \in \text{Aut } L$. Then $G_{gP} = gG_Pg^{-1}$.

Remark 1. Let $L : P : K$ be a tower of fields, where $[L : K] = [L : P][P : K]$. Then $\text{Id.} : G_P : G$ is a tower of groups, where $[G : G_P] \cdot |G_P|$. That is, for all $P \leq L$ we have $[P : K] = [G : G_P]$ and $[L : P] = |G_P|$.