1 Fundamental Theorem of Galois Theory I

Theorem 1.1 (Fundamental Theorem of Galois Theory, Part 1)**.** Let $L : K$ be a Galois extension with $G = \mathrm{Gal}(L : K)$. Define $\mathcal{I}(K, L)$ and $\mathcal{S}(G)$ as the set of all intermediate fields of $L : K$ and the set of all subgroups of G, respectively. For all $P \in \mathcal{I}(K, L)$, we have $P = L^{G_P}$ where $G_P = \mathrm{Aut}_P(L)$ Then

$$\begin{aligned} \forall P \in \mathcal{I}(K, L), &\quad L^{G_P} = P, \\ \forall H \in \mathcal{S}(G), &\quad G_{L^H} = H, \end{aligned}$$

Also, $P_1 \subseteq P_2 \iff G_{P_1} \geqslant G_{P_2}$ and $H_1 \leqslant H_2 \iff L^{H_1} \supseteq L^{H_2}$.