PURDUE UNIVERSITY

Department of Mathematics

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## Homework 11 (Apr 18 – Apr 25)

**1** (5) Let $G = \mathbb{Z}/p^n\mathbb{Z}$, where $p$ is a prime number. Construct a subnormal series $G_j$ of subgroups of $G$ such that $|G_{j-1}/G_j| = p$.

**2** (5+5) *a*) Let $G$ be a group. Prove that $G'$ is a normal subgroup of $G$ such that $G/G'$ is abelian.
*b*) Prove that if $N$ is any normal subgroup of $G$ such that $G/N$ is abelian, then $G' \leq N$.

**3** (10) Let $\mathbb{F}$ be a field and

$$H := \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \ : \ a, b, c \in \mathbb{F} \right\} \tag{1}$$

be the Heisenberg group. Prove that $H$ is soluble.

**4** (15) Prove that $A_n$, $n \geq 3$ is generated by 3–cycles.

**5** (5+5+5) Let $G$ be a group. Find $G'$ for
*a*) $G = S_3$ *b*) $G = A_4$ *c*) $G = S_4$ (use the previous question).