1 Introduction II

Theorem 1.1 (Lagrange)**.** Let $\varphi = \varphi(x_1, \dots, x_n)$ and

$$\mathrm{orb}(\varphi) = \left\{ \varphi^\omega = \varphi(x_{\omega(1)}, \dots, x_{\omega(n)}) \mid \omega \in S_n \right\}.$$

Then $y_1, \dots, y_k$ are roots of some polynomial with degree $\leq k$ whose coefficients depend on elementary symmetric polynomials $\sigma_1, \dots, \sigma_n$ in a polynomial way.

Theorem 1.2 (Lagrange)**.** Let $\varphi, \psi \in K[x_1, \dots, x_n]$ and $G_\varphi = \{\omega \in S_n \mid \varphi^\omega = \varphi\} \leqslant G_\psi$. Then $\psi = R(\varphi)$ where R is a rational function whose coefficients are symmetric functions on $x_1, \dots, x_n$.

Definition 1 (Group action)**.** Let G be a group and X be a set. The (left) group action of G on X is the map $\cdot : G \times X \to X$ such that

1. $e_G \cdot x = x, \quad \forall x \in X$
2. $g \cdot (h \cdot x) = (g \cdot h) \cdot x, \quad \forall x \in X, \forall g, h \in G$

Definition 2 (Orbit)**.** Let G be a group, X be a set, and $x \in X$. Then we define *the orbit* of x, $G \cdot x = \mathrm{orb}(x)$, as $\{g \cdot x \mid g \in G\}$. Moreover, $\mathrm{orb}(x) \subseteq X$.

Definition 3 (Stabilizer)**.** Let G be a group, X be a set, and $x \in X$. Then we define *the stabilizer* of x, $\mathrm{stab}(x)$, as $\{g \in G \mid g \cdot x = x\}$. Moreover, $\mathrm{stab}(x) \leq G$.

Theorem 1.3. Let G be a finite group that acts on X. Then for all $x \in X$, $|\mathrm{orb}(x)| \cdot |\mathrm{stab}(x)| = |G|$.

Definition 4 (Polynomial ring)**.** Let R be a commutative ring. Then the ring of polynomials with coefficients in R is

$$R[t] = \left\{ \sum_{i=0}^{n} c_i t^i : n \in \mathbb{Z}_+, c_i \in R \right\}$$