

1 Fundamental Theorem of Galois Theory II

Theorem 1.1. 1. Let $L : K$ be a Galois extension with $G = \text{Gal}_K L$. Then

$$\begin{aligned}\forall P \in \mathcal{I}(K, L), \quad L^{G_P} &= P \\ \forall H \in \mathcal{S}(G), \quad G_{L^H} &= H.\end{aligned}$$

Also, $P_1 \subseteq P_2 \iff G_{P_1} \geq G_{P_2}$ and $H_1 \leq H_2 \iff L^{H_1} \supseteq L^{H_2}$ (by thm 1 lec 19)

2. For $P \in \mathcal{I}(K, L)$ suppose $P : K$ is a normal extension. Then $G_P \triangleleft G$ and $\text{Gal}_K P \cong G/G_P$.

Lemma 1.2. Let $K - P - L$ be a tower of fields and $g \in \text{Aut } L$. Then $G_{gP} = gG_Pg^{-1}$.