Exercise 5.1. Which of the following field extensions are normal? Justify your answers.

1. $\mathbb{Q}(i) : \mathbb{Q}$

   Solution. □

2. $\mathbb{Q}(2^{1/4}) : \mathbb{Q}$

   Solution. □

3. $\mathbb{Q}(2^{1/4}, i) : \mathbb{Q}$

   Solution. □

4. $\mathbb{Q}(2^{1/4}, i, \sqrt{5}) : \mathbb{Q}$

   Solution. □

5. $\mathbb{Q}(3^{1/3}, i, \sqrt{3}) : \mathbb{Q}$

   Solution. □

Exercise 5.2. Let $\psi : L \to M$ be a homomorphism, suppose that L is algebraically closed. Prove that $\psi(L)$ is algebraically closed.

Solution. □

Exercise 5.3. Let $L : K$ be a field extension. Then $\overline{K}$ is isomorphic to $\overline{L}$. In addition, if $K \subset L \subseteq \overline{L}$, then $\overline{K} = \overline{L}$.

Solution. □

Exercise 5.4. Let $K - L$ be a normal extension, $K \subseteq L \subseteq \overline{K}$. Then for any K-homomorphism $\tau : L \to \overline{K}$ one has $\tau(L) = L$.

Solution. □

Exercise 5.5. Put $K = \mathbb{F}_2(t)$ and consider $L = K(t^{1/3})$. Prove that the extension $L : K$ is algebraic but not normal.