PURDUE UNIVERSITY

Department of Mathematics

GALOIS THEORY HONORS, MA 45401

Homework 3 (Jan 31 – Feb 13).

1 (5+10+15) 1) Show that t^3+t+1 is irreducible in $\mathbb{F}_2[t]$.
2) Consider the quotient ring $L := \mathbb{F}_2[t]/(t^3+t+1)$ and compute its size.
3) Take $g = t+1$ and prove that the set $\{0, g, g^2, \ldots, g^7\}$ coincides with L.

2 (15) Let K be a field and $p, q \in K[t]$ be irreducible polynomials over K, $(p) \neq (q)$ (this is equivalent to the statement that p is coprime to q). Consider the field $\mathbb{F} := K(t)$ and the polynomial $g(x) = x^n + px + pq \in \mathbb{F}[x]$. Prove that g is irreducible over $\mathbb{F}$.

3 (10) Prove that $t^2 - 7$ is irreducible over $\mathbb{Q}(\sqrt{5})$.

4 (5+5+5+10+20) 1) Let $\alpha = 2^{1/6}$ and $\varepsilon_3^3 = 1$, $\varepsilon_3 \neq 1$. Find the minimal polynomials of α over
a) $\mathbb{Q}$ b) $\mathbb{Q}(\alpha)$ c) $\mathbb{Q}(\alpha^2)$ d) $\mathbb{Q}(\alpha\varepsilon_3)$.

2) In each case (a—d), find the conjugate elements of all roots of $x^6 - 2$.

5 Midterm exam is next Thursday!