

# 1 Algebraic Conjugates

**Lemma 1.1.** *Let  $\mathbb{F}$  be a field with  $f \in \mathbb{F}[t]$  irreducible. Then  $\mathbb{F}[t] / (f)$  is a field.*

**Corollary 1.2.** *If  $L : K$  with  $\alpha \in L$  algebraic over  $K$ , then  $K[t] / (\mu_\alpha^K)$  is a field.*

**Theorem 1.3.** *Let  $K$  be a field, and suppose that  $f \in K[t]$  is irreducible. Then there exists a field extension  $L : K$ , with associated embedding  $\varphi : K[t] \rightarrow L[y]$ , having the property that  $L$  contains a root of  $\varphi(f)$ .*

**Definition 1** (Algebraic conjugate). *Suppose  $\alpha$  algebraic over  $K$  and  $\mu_\alpha^K$  factors as a product of linear polynomials over a field  $L \supseteq K$ :*

$$\mu_\alpha^K(x) = (x - \alpha_1) \cdots (x - \alpha_n), \quad \alpha_1, \dots, \alpha_n \in L.$$

*Then  $\alpha_1, \dots, \alpha_n$  are algebraic conjugates of  $\alpha$ .*

**Lemma 1.4.** *Let  $(x - \alpha_1) \cdots (x - \alpha_n) \in K[x]$  and  $f(\bar{y}, x_1, \dots, x_n) \in K[\bar{y}, x_1, \dots, x_n]$  be symmetric polynomial in  $x_1, \dots, x_n$ . Then  $f(\bar{y}, x_1, \dots, x_n) \in K[\bar{y}]$ .*

**Theorem 1.5.** *Let  $\alpha$  be algebraic over  $K$  with algebraic conjugates  $\alpha = \alpha_1, \dots, \alpha_n$ . Then for all  $f \in K[x]$ , the conjugates of  $f(\alpha)$  are exactly  $f(\alpha_1), \dots, f(\alpha_n)$ .*