1 Composita

Remark 1. Let A, B be sets. Then $A \cap B$ can be expressed using only the operation $\subseteq$. Notice $A \cap B \subseteq A, B$ and $A \cap B$ is the maximal set with this property:

$$\forall C \text{ such that } C \subseteq A, B \quad \implies \quad C \subseteq A \cap B.$$

Let $H_1, H_2 \leqslant G$. Then $H_1 \cap H_2 \leqslant G$ is the *maximal* subgroup contained in both H_1 and H_2. Hence by the Galois correspondence we have $L^{H_1 \cap H_2}$ is the *minimal* subfield of L containing both L^{H_1} and L^{H_2}.

Definition 1 (Compositum)**.** Let K_1 and K_2 be fields contained in some field L. The *compositum* of K_1 and K_2 in L (or the *composite field*), denoted by K_1K_2, is the smallest subfield of L containing both K_1 and K_2.

Lemma 1.1. Let $K, E, F \subseteq L$. Then

1. $E : K,\ F : K$ finite $\implies$ $EF : K$ finite;
2. $E : K,\ F : K$ normal $\implies$ $E \cap F : K$ normal;
3. $E : K,\ F : K$ finite and $E : K$ normal $\implies$ $EF : F$ normal;
4. $E : K,\ F : K$ finite and normal $\implies$ $EF : K,\ E \cap F : K$ normal;
5. $E : K,\ F : K$ normal $\implies$ $EF : E \cap F$ normal.