1 The Primitive Element Theorem

Definition 1 (Simple extension)**.** Suppose $L : K$ is a field extension relative to the embedding $\varphi : K \to L$. We say that $L : K$ is a *simple extension* if there is some $\gamma \in L$ such that $L = \varphi(K)(\gamma)$.

Theorem 1.1 (The Primitive Element Theorem)**.** If $L : K$ is a finite, separable extension with $K \subseteq L$, then $L : K$ is a simple extension.

Corollary 1. Suppose that $L : K$ is an algebraic, separable extension, and suppose that for every $\alpha \in L$, the polynomial μ_α^K has degree at most n over K. Then $[L : K] \leq n$.

Fact: Let $L : K$ be a normal extension and let $\deg(\mu_\alpha^K) \leq n$ for all $\alpha \in L$. Then $[L : K] \leq n$.

Corollary 2. If $f \in K[t]$ is irreducible over K, then $\mathrm{Gal}_K(f)$ acts transitively on the roots of f.