

**Exercise 6.1.** Find Galois groups for the following polynomials  $f$  over  $\mathbb{Q}$ :

1.  $(t^2 - 3)(t^2 + 1)$

*Solution.*

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2.  $t^4 - t^2 + 1$

*Solution.*

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3.  $t^4 - 2$

*Solution.*

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**Exercise 6.2.1.** Find  $\text{Gal}_{\mathbb{F}_3(t^2)}(\mathbb{F}_3(t))$ .

*Solution.*

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**Exercise 6.2.2.** Find  $\text{Gal}_{\mathbb{F}_2(t^2)}(\mathbb{F}_2(t))$ .

*Solution.*

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**Exercise 6.3.1.** Let  $K - M - L$  be a field extension and  $L : K$  is a normal extension. Prove that  $L : M$  is also a normal extension.

*Solution.*

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**Exercise 6.3.2.** Give an example of three fields  $K, M, L$  such that  $[L : K] = 4$  and  $[M : K] = [L : M] = 2$  (hence  $K - M$  and  $M - L$  are normal extensions) but  $L : K$  is not a normal extension.

*Solution.*

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**Exercise 6.4.** Let  $L : K$  be a splitting field extension for a non-constant polynomial  $f \in K[t]$ . Prove that  $|\text{Gal}_K(L)|$  divides  $(\deg f)!$ .

*Solution.*

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**Exercise 6.5.1.** Let  $f = t^3 + t + 1 \in \mathbb{F}_2[t]$ . Prove that  $\text{Gal}_{\mathbb{F}_2}(f)$  is isomorphic to  $\mathbb{Z}_3$ .

*Solution.*

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**Exercise 6.5.2.** Let  $f = t^3 + t^2 + 1 \in \mathbb{F}_2[t]$ . Prove that  $\text{Gal}_{\mathbb{F}_2}(f)$  is isomorphic to  $S_3$ .

*Solution.*

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