

PURDUE UNIVERSITY  
Department of Mathematics

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**Homework 10 (Apr 11 – Apr 18)**

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- 1** (10+10+5+5) Let  $K, E, F \subseteq L$  be fields,  $E : K, F : K$  be finite extensions. Prove:
- a) if  $E : K$  is separable, then  $EF : F$  is separable;
  - b) if  $E : K$  and  $F : K$  are both separable, then  $EF : K$  and  $E \cap F : K$  are both separable;
  - c) if  $E : K$  is Galois, then  $EF : F$  is Galois;
  - d) if  $E : K$  and  $F : K$  are both Galois, then  $EF : K$  and  $E \cap F : K$  are both Galois.
- 2** (5+5+10) a) Find the splitting field  $L$  of the polynomial  $f(t) = t^4 - 4t^2 + 5$ .  
b) Prove that  $[L : \mathbb{Q}]$  is either 4 or 8.  
c) Find 10 intermediate fields of the extension  $L : \mathbb{Q}$  and their degrees.  
d) (for enthusiasts) Draw the lattice of subfields and corresponding lattice of subgroups of  $\text{Gal}_{\mathbb{Q}}(f)$ .
- 3** (30) Draw the lattice of subfields and corresponding lattice of subgroups of  $\text{Gal}_{\mathbb{Q}}(t^6 + 3)$ . *Hint:* Use the calculations (and the notation, if you like) from Lecture 18.