1 Galois Groups II

Lemma 1.1. *Suppose that $L : K$ is a normal extension with $K \subseteq L \subseteq \overline{K}$. Then for any K-homomorphism $\tau : L \to \overline{K}$, we have $\tau(L) = L$.*

Lemma 1.2. *For $n \geq 2$, S_n is generated by*

1. *transpositions (ij);*
2. *transpositions $(1i)$;*
3. *adjacent transpositions $(12), (23), \ldots, (n-1, n)$;*
4. *(12) and $(12 \ldots n)$;*
5. *(12) and $(23 \ldots n)$;*
6. *(ij) and $(i \ldots i_p)$ where p is prime.*

Lemma 1.3. *Let $(i_1 \ldots i_k) \in S_n$. Then for all $\sigma \in S_n$, one has $\sigma(i_1 \ldots i_k)\sigma^{-1} = (\sigma(i_1) \ldots \sigma(i_k))$.*

Note: $|Gal_K(f)| = [L : K]$ where $L : K$ is a splitting field extension for f.