1 The Primitive Element Theorem

Definition 1 (Simple extension). Suppose L: K is a field extension relative to the embedding $\varphi: K \to L$. We say that L: K is a simple extension if there is some $\gamma \in L$ having the property that $L = \varphi(K)(\gamma)$.

Theorem 1.1 (The Primitive Element Theorem). If L : K be a finite, separable extension with $K \subseteq L$, then L : K is a simple extension.

Corollary 1.2. Suppose that L: K is an algebraic, separable extension, and suppose that for every $\alpha \in L$, the polynomial μ_{α}^{K} has degree at most n over K. Then $[L:K] \leq n$.

Fact: Let L: K be a normal extension and let $\deg(\mu_{\alpha}^K) \leq n$ for all $\alpha \in L$. Then $[L:K] \leq n$.

Corollary 1.3. If $f \in K[t]$ is irreducible over K, then $Gal_K(f)$ acts transitively on the roots of f.