PURDUE UNIVERSITY

Department of Mathematics

GALOIS THEORY HONORS, MA 45401

Homework 7 (Mar 7 – Mar 14)

1 (10) Let $K = \mathbb{Q}$, $M = \mathbb{Q}(2^{1/3})$ and $L = \mathbb{Q}(2^{1/3}, \sqrt{3}, i)$. Prove that $L : K$ and $L : M$ are normal but $M : K$ is not normal.

2 (10+5) *a)* Let $K - L$ be algebraic, $\alpha \in L$ and $\sigma : K \to \overline{K}$ be a homomorphism. Prove that μ_α^K is separable over K iff $\sigma(\mu_\alpha^K)$ is separable over $\sigma(K)$.

b) Let $L : K$ be a splitting filed for $f \in K[t]$. Prove that if f is separable, then $L : K$ is separable.

3 (10) Let $L : K$ be a splitting field extension for a polynomial $f \in K[t]$. Then $L : K$ is separable iff f is separable over K.

4 (15) Let $K - M - L$ be an algebraic extension. Prove that $K - L$ is separable iff $K - M$ and $M - L$ are separable.