1 Algebraic Conjugates

Lemma 1.1. *Let $\mathbb{F}$ be a field with $f \in \mathbb{F}[t]$ irreducible. Then $\mathbb{F}[t] \,/\, (f)$ is a field.*

Corollary 1.2. *If $L : K$ with $\alpha \in L$ algebraic over K, then $K[t] \,/\, (\mu_\alpha^K)$ is a field.*

Theorem 1.3. *Let K be a field, and suppose that $f \in K[t]$ is irreducible. Then there exists a field extension $L : K$, with associated embedding $\varphi : K[t] \to L[y]$, having the property that L contains a root of $\varphi(f)$.*

Definition 1 (Algebraic conjugate)**.** Suppose α algebraic over K and μ_α^K factors as a product of linear polynomials over a field $L \supseteq K$:

$$\mu_\alpha^K(x) = (x - \alpha_1) \cdots (x - \alpha_n), \quad \alpha_1, \ldots, \alpha_n \in L.$$

Then $\alpha_1, \ldots, \alpha_n$ are <u>algebraic conjugates</u> of α.

Lemma 1.4. *Let $(x - \alpha_1) \cdots (x - \alpha_n) \in K[x]$ and $f(\overline{y}, x_1, \ldots, x_n) \in K[\overline{y}, x_1, \ldots, x_n]$ be symmetric polynomial in $x_1, \ldots, x_n$. Then $f(\overline{y}, x_1, \ldots, x_n) \in K[\overline{y}]$.*

Theorem 1.5. *Let α be algebraic over K with algebraic conjugates $\alpha = \alpha_1, \ldots, \alpha_n$. Then for all $f \in K[x]$, the conjugates of $f(\alpha)$ are exactly $f(\alpha_1), \ldots, f(\alpha_n)$.*