# 1 Galois Groups III

**Theorem 1.1** (Kronecker)**.** Let $p \geq 3$ be a prime and $f \in \mathbb{Q}[x]$ be irreducible over $\mathbb{Q}$ with $\deg f = p$. If the equation $f(x) = 0$ is solvable by radicals, then the number of real roots of $f$ is 1 or $p$.

**Lemma 1.2.** Let $p$ be prime and $G \leqslant S_p$ such that $G$ acts transitively on $\{1, \ldots, p\}$. Then $G$ contains a cycle of order $p$.

**Theorem 1.3.** If $L : K$ is a finite extension, then $|\mathrm{Gal}_K(L)| \leq [L : K]$.