

1 Algebraic Conjugates

Lemma 1.1. Let \mathbb{F} be a field with $f \in \mathbb{F}[t]$ irreducible. Then $\mathbb{F}[t] / (f)$ is a field.

Corollary 1. If $L : K$ with $\alpha \in L$ algebraic over K , then $K[t] / (\mu_\alpha^K)$ is a field.

Theorem 1.2. Let K be a field, and suppose that $f \in K[t]$ is irreducible. Then there exists a field extension $L : K$, with associated embedding $\varphi : K[t] \rightarrow L[y]$, such that L contains a root of $\varphi(f)$.

Definition 1 (Algebraic conjugate). Suppose α is algebraic over K and μ_α^K factors as a product of linear polynomials over a field $L \supseteq K$:

$$\mu_\alpha^K(x) = (x - \alpha_1) \cdots (x - \alpha_n), \quad \alpha_1, \dots, \alpha_n \in L.$$

Then $\alpha_1, \dots, \alpha_n$ are *algebraic conjugates* of α .

Lemma 1.3. Let $(x - \alpha_1) \cdots (x - \alpha_n) \in K[x]$ and $f(\bar{y}, x_1, \dots, x_n) \in K[\bar{y}, x_1, \dots, x_n]$ be symmetric polynomial in x_1, \dots, x_n . Then $f(\bar{y}, x_1, \dots, x_n) \in K[\bar{y}]$.

Theorem 1.4. Let α be algebraic over K with algebraic conjugates $\alpha = \alpha_1, \dots, \alpha_n$. Then for all $f \in K[x]$, the conjugates of $f(\alpha)$ are exactly $f(\alpha_1), \dots, f(\alpha_n)$.