PURDUE UNIVERSITY

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Homework 6 (Feb 28 – Mar 7)

1 (5+10+10) Find Galois groups for the following polynomials f over $\mathbb{Q}$:
1) $(t^2-3)(t^2+1)$
2) t^4-t^2+1
3) t^4-2

2 (10+10) 1) Find $\mathrm{Gal}_{\mathbb{F}_3(t^2)}(\mathbb{F}_3(t))$.
2) Find $\mathrm{Gal}_{\mathbb{F}_2(t^2)}(\mathbb{F}_2(t))$.

3 (10+5) (a) Let $K-M-L$ be a field extension and $L:K$ is a normal extension. Prove that $L:M$ is also a normal extension.
(b) Give an example of three fields K, M, L such that $[L:K]=4$ and $[M:K]=[L:M]=2$ (hence $K-M$ and $M-L$ are normal extensions) but $L:K$ is not a normal extension.

4 (10) Let $L:K$ be a splitting field extension for a non–constant polynomial $f\in K[t]$. Prove that $|\mathrm{Gal}_L(K)|$ divides $(\deg f)!$.

5 (15+20) a) Let $f=t^3+t+1\in\mathbb{F}_2[t]$. Prove that $\mathrm{Gal}_{\mathbb{F}_2}(f)$ is isomorphic to $\mathbb{Z}_3$.
b) Let $f=t^3+t^2+1\in\mathbb{F}_2[t]$. Prove that $\mathrm{Gal}_{\mathbb{F}_2}(f)$ is isomorphic to S_3.