1 Galois Groups II

Lemma 1.1. Suppose that $L : K$ is a normal extension with $K \subseteq L \subseteq \overline{K}$. Then for any K-homomorphism $\tau : L \to \overline{K}$, we have $\tau(L) = L$.

Lemma 1.2. For $n \geq 2$, S_n is generated by

1. transpositions $(i\,j)$;
2. transpositions $(1\,i)$;
3. adjacent transpositions $(1\,2), (2\,3), \ldots, (n-1,\, n)$;
4. $(1\,2)$ and $(1\,2 \ldots n)$;
5. $(1\,2)$ and $(2\,3 \ldots n)$;
6. $(i\,j)$ and $(i \ldots i_p)$ where p is prime.

Lemma 1.3. Let $(i_1 \ldots i_k) \in S_n$. Then for all $\sigma \in S_n$, one has $\sigma(i_1 \ldots i_k)\sigma^{-1} = (\sigma(i_1) \ldots \sigma(i_k))$.

Note: $|Gal_K(f)| = [L : K]$ where $L : K$ is a splitting field extension for f.