Exercise 6.1. Find Galois groups for the following polynomials f over $\mathbb{Q}$:

1. $(t^2-3)(t^2+1)$

   Solution. □

2. t^4-t^2+1

   Solution. □

3. t^4-2

   Solution. □

Exercise 6.2.1. Find $\mathrm{Gal}_{\mathbb{F}_3(t^2)}(\mathbb{F}_3(t))$.

Solution. □

Exercise 6.2.2. Find $\mathrm{Gal}_{\mathbb{F}_2(t^2)}(\mathbb{F}_2(t))$.

Solution. □

Exercise 6.3.1. Let $K - M - L$ be a field extension and $L : K$ is a normal extension. Prove that $L : M$ is also a normal extension.

Solution. □

Exercise 6.3.2. Give an example of three fields K, M, L such that $[L : K] = 4$ and $[M : K] = [L : M] = 2$ (hence $K - M$ and $M - L$ are normal extensions) but $L : K$ is not a normal extension.

Solution. □

Exercise 6.4. Let $L : K$ be a splitting field extension for a non-constant polynomial $f \in K[t]$. Prove that $|\mathrm{Gal}_K(L)|$ divides $(\deg f)!$.

Solution. □

Exercise 6.5.1. Let $f = t^3 + t + 1 \in \mathbb{F}_2[t]$. Prove that $\mathrm{Gal}_{\mathbb{F}_2}(f)$ is isomorphic to $\mathbb{Z}_3$.

Solution. □

Exercise 6.5.2. Let $f = t^3 + t^2 + 1 \in \mathbb{F}_2[t]$. Prove that $\mathrm{Gal}_{\mathbb{F}_2}(f)$ is isomorphic to S_3.

Solution. □