1 Fundamental Theorem of Galois Theory I

Theorem 1.1. *Let $L : K$ be a Galois extension with $G = \mathrm{Gal}_K L$. Define $\mathcal{I}(K, L)$ and $\mathcal{S}(G)$ as the collection of all intermediate fields of $L : K$ and the family of all subgroups of G, respectively. Then*

$$\begin{aligned} \forall P \in \mathcal{I}(K, L), &\quad L^{G_P} = P \\ \forall H \in \mathcal{S}(G), &\quad G_{L^H} = H. \end{aligned}$$

Also, $P_1 \subseteq P_2 \iff G_{P_1} \geqslant G_{P_2}$ and $H_1 \leqslant H_2 \iff L^{H_1} \subseteq L^{H_2}$ (by Theorem 1 of Lecture 19)