PURDUE UNIVERSITY

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Homework 9 (Apr 4 – Apr 11)

1 (10+5) *a*) Let L be the splitting field of the polynomial $t^{13}-1$. Find all subgroups of $\mathrm{Gal}_{\mathbb{Q}}(L)$.
b) How many intermediate subfields are there in the extension $L:\mathbb{Q}$?

2 (10) Draw the lattice of subfields and corresponding lattice of subgroups of $\mathrm{Gal}_{\mathbb{F}_3}(\mathbb{F}_{3^8})$. Find orders of all subgroups of $\mathrm{Gal}_{\mathbb{F}_3}(\mathbb{F}_{3^8})$.

3 (10) Prove Artin's theorem: let $[L:K]<\infty$, $G:=\mathrm{Gal}_K(L)$. Then $[L:L^G]$ is a Galois extension.

4 (10) Let $L:K$ be a finite Galois extension, $G:=\mathrm{Gal}_K(L)$. For any $\alpha\in L$ define

$$\mathrm{Tr}(\alpha)=\sum_{g\in G} g(\alpha) \qquad \text{and} \qquad \mathrm{Norm}(\alpha)=\prod_{g\in G} g(\alpha)\,.$$

Prove that for an arbitrary $\alpha\in L$ one has $\mathrm{Tr}(\alpha),\mathrm{Norm}(\alpha)\in K$.

5 (15+15) *a*) Find all of the subfields of $\mathbb{Q}(2^{1/3},e^{2\pi i/3})$.
b) Draw the lattice of subfields and corresponding lattice of subgroups of $\mathrm{Gal}_{\mathbb{Q}}(\mathbb{Q}(2^{1/3},e^{2\pi i/3}))$.