1 Algebraic Closure II

Theorem 1.1. *Let E be an algebraic extension of K with $K \subseteq E$, and let $\overline{K}$ be an algebraic closure of K.*

Given a homomorphism $\varphi : K \to \overline{K}$, the map φ can be extended to a homomorphism from E into $\overline{K}$.

Theorem 1.2. *If L and M are both algebraic closures of K, then $L \cong M$.*

Corollary 1.3. *Let $L : K$ be an extension with $K \subseteq L$. Suppose that $g \in L[t]$ is irreducible over L, and that $g \mid f$ in $L[t]$, where $f \in K[t] \setminus \{0\}$. The g divides a factor of f that is irreducible over K.*

Thus, there exists an irreducible $h \in K[t]$ having the property that $h \mid f$ in $K[t]$, and $g \mid h$ in $L[t]$.

Definition 1 (Normal extension)**.** *The extension $L : K$ is normal if it is algebraic, and every irreducible polynomial $f \in K[t]$ either splits over L or has no root in L.*

Theorem 1.4. *$K(\alpha) : K$ is normal $\iff$ all conjugates of α are contained in $K(\alpha)$.*

Theorem 1.5. *A finite extension $L : K$ is normal $\iff$ L is a splitting field extension for some $f \in K[t] \setminus K$.*