1 GALOIS GROUPS II 1

1 Galois Groups II

Lemma 1.1. Suppose that L:K is a normal extension with $K\subseteq L\subseteq \overline{K}$. Then for any K-homomorphism $\tau:L\to \overline{K}$, we have $\tau(L)=L$.

Lemma 1.2. For $n \geq 2$, S_n is generated by

- 1. transpositions(ij);
- 2. transpositions (1i);
- 3. adjacent transpositions $(1\,2),(2\,3),\ldots,(n-1,\,n)$;
- 4. (12) and (12...n);
- 5. (12) and (23...n);
- 6. (i j) and $(i \dots i_p)$ where p is prime.

Lemma 1.3. Let $(i_1 \ldots i_k) \in S_n$. Then for all $\sigma \in S_n$, one has $\sigma(i_1 \ldots i_k) \sigma^{-1} = (\sigma(i_1) \ldots \sigma(i_k))$.

Note: $|Gal_K(f)| = [L:K]$ where L:K is a splitting field extension for f.