1 Final remarks I

Definition 1 (Sylvester matrix).

Definition 2 (Resultant).

Theorem 1.1. Let α_i be roots of f and β_j be roots of g. Then

$$\begin{aligned} R(f,g) &= a_0^m b_0^n \prod_i (\alpha_i - \beta_j) \\ &= a_0^m \prod_i g(\alpha_i) = b_0^n \prod_i f(\beta_i) \end{aligned}$$

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Corollary 1.
1. $R(f,g) = (-1)^{\deg f \cdot \deg g} R(g,f)$
2. If $f = gq + r \implies R(f,g) = b_0^{\deg f - \deg R} R(r,g)$
3. $R(f,gh) = R(f,g)R(f,h)$

Corollary 2. Let $f(t) = a_0 t^n + \cdots + a_n,\ a_0 \neq 0$. Then $R(f,f') = (-1)^{\frac{n(n-1)}{2}} \prod_{i<j} (\alpha_i - \alpha_j)^2$