Exercise 7.1. Let $K = \mathbb{Q}$, $M = \mathbb{Q}(2^{1/3})$ and $L = \mathbb{Q}(2^{1/3}, \sqrt{3}, i)$. Prove that $L : K$ and $L : M$ are normal but $M : K$ is not normal.

Solution. We know that a field extension $F_1 : F_2$ is normal iff it is a splitting field extension for some $f \in F_2[t]$.

□

Exercise 7.2.1. Let $K - L$ be algebraic, $a \in L$ and $\sigma : K \to \overline{K}$ be a homomorphism. Prove that μ_α^K is separable over K iff $\sigma(\mu_\alpha^K)$ is seperable over $\sigma(K)$.

Solution. □

Exercise 7.2.2. Let $L : K$ be a splitting field for $f \in K[t]$. Prove that if f is separable, then $L : K$ is separable.

Solution. □

Exercise 7.3. Let $L : K$ be a splitting field extension for a polynomial $f \in K[t]$. Then $L : K$ is separable iff f is separable over K.

Solution. □

Exercise 7.4. Let $K - M - L$ be an algebraic extension. Prove that $K - L$ is separable iff $K - M$ and $M - L$ are separable.

Solution. □