1 Algebraic Closure II

Theorem 1.1. Let $L : K$ be an algebraic extension with $K \subseteq L$ and $\varphi : K \to \overline{K}$ be a homomorphism. Then there exists an extension of φ to a homomorphism $\psi : L \to \overline{K}$.

Theorem 1.2. If L and M are both algebraic closures of K, then $L \cong M$.

Corollary 1. Let $L : K$ be an extension with $K \subseteq L$. Suppose that $g \in L[t]$ is irreducible over L, and that $g \mid f$ in $L[t]$, where $f \in K[t] \setminus \{0\}$. Then g divides a factor of f that is irreducible over K.

Thus, there exists an irreducible $h \in K[t]$ such that $h \mid f$ in $K[t]$, and $g \mid h$ in $L[t]$.

Definition 1 (Normal extension)**.** The extension $L : K$ is *normal* if it is algebraic, and every irreducible polynomial $f \in K[t]$ either splits over L or has no root in L.

Theorem 1.3. $K(\alpha) : K$ is normal $\iff$all conjugates of α are contained in $K(\alpha)$.

Theorem 1.4. A finite extension $L : K$ is normal $\iff$ L is a splitting field extension for some $f \in K[t] \setminus K$.