Exercise 11.1. Let $G = \mathbb{Z}/p^n\mathbb{Z}$, where p is a prime number. Construct a subnormal series G_j of subgroups of G such that $|G_{j-1}/G_j| = p$.

Solution. □

Exercise 11.2.1. Let G be a group. Prove that G' is a normal subgroup of G such that G/G' is abelian.

Solution. □

Exercise 11.2.2. Prove that if N is any normal subgroup of G such that G/N is abelian, then $G' \leqslant N$.

Solution. □

Exercise 11.3. Let $\mathbb{F}$ be a field and

$$H := \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} : a, b, c \in \mathbb{F} \right\}$$

be the Heisenberg group. Prove that H is soluble.

Solution. □

Exercise 11.4. Prove that A_n, $n \geq 3$ is generated by 3-cycles.

Solution. □

Exercise 11.5. Let G be a group. Find G' for
a) $G = S_3$ *b)* $G = A_4$ *c)* $G = S_4$ (use the previous question).

Solution. □