INTRODUCTION II

## 1 Introduction II

**Theorem 1.1** (Lagrange). Let  $\varphi = \varphi(x_1, \ldots, x_n)$  and

$$\operatorname{orb}(\varphi) = \left\{ \varphi^{\omega} = \varphi(x_{\omega(1)}, \dots, x_{\omega(n)}) \mid \omega \in S_n \right\}.$$

Then  $y_1, \ldots, y_k$  are roots of some polynomial with degree  $\leq k$  whose coefficients depend on elementary symmetric polynomials  $\sigma_1, \ldots, \sigma_n$  in a polynomial way.

**Theorem 1.2** (Lagrange). Let  $\varphi, \psi \in K[x_1, \dots, x_n]$  and  $G_{\varphi} = \{\omega \in S_n \mid \varphi^{\omega} = \varphi\} \leq G_{\psi}$ . Then  $\psi = R(\varphi)$  where R is a rational function whose coefficients are symmetric functions on  $x_1, \dots, x_n$ .

**Definition 1** (Group action). Let G be a group and X be a set. The (left) group action of G on X is the map  $\cdot : G \times X \to X$  such that

- 1.  $e_G \cdot x = x$ ,  $\forall x \in X$
- 2.  $g \cdot (h \cdot x) = (g \cdot h) \cdot x$ ,  $\forall x \in X, \forall g, h \in G$

**Definition 2** (Orbit). Let G be a group, X be a set, and  $x \in X$ . Then we define the orbit of x,  $G \cdot x = \operatorname{orb}(x)$ , as  $\{g \cdot x \mid g \in G\}$ . Moreover,  $\operatorname{orb}(x) \subseteq X$ .

**Definition 3** (Stabilizer). Let G be a group, X be a set, and  $x \in X$ . Then we define the stabilizer of x, stab(x), as  $\{g \in G \mid g \cdot x = g\}$ . Moreover, stab $(x) \leq G$ .

**Theorem 1.3.** Let G be a finite group that acts on X. Then for all  $x \in X$ ,  $|\operatorname{orb}(x)| \cdot |\operatorname{stab}(x)| = |G|$ .

**Definition 4** (Polynomial ring). Let R be a commutative ring. Then the ring of polynomials with coefficients in R is

$$R[t] = \left\{ \sum_{i=0}^{n} c_i t^i : n \in \mathbb{Z}_+, c_i \in R \right\}$$