# 1 Introduction II

**Theorem 1.1** (Lagrange)**.** *Let* $\varphi = \varphi(x_1, \ldots, x_n)$ *and*

$$\operatorname{orb}(\varphi) = \left\{\varphi^\omega = \varphi(x_{\omega(1)}, \ldots, x_{\omega(n)}) \mid \omega \in S_n\right\}.$$

*Then* $y_1, \ldots, y_k$ *are roots of some polynomial with degree* $\leq k$ *whose coefficients depend on elementary symmetric polynomials* $\sigma_1, \ldots, \sigma_n$ *in a polynomial way.*

**Theorem 1.2** (Lagrange)**.** *Let* $\varphi, \psi \in K[x_1, \ldots, x_n]$ *and* $G_\varphi = \{\omega \in S_n \mid \varphi^\omega = \varphi\} \leqslant G_\psi$*. Then* $\psi = R(\varphi)$ *where* $R$ *is a rational function whose coefficients are symmetric functions on* $x_1, \ldots, x_n$*.*

**Definition 1** (Group action)**.** Let $G$ be a group and $X$ be a set. The (left) group action of $G$ on $X$ is the map $\cdot : G \times X \to X$ such that

1. $e_G \cdot x = x, \quad \forall x \in X$
2. $g \cdot (h \cdot x) = (g \cdot h) \cdot x, \quad \forall x \in X, \forall g, h \in G$

**Definition 2** (Orbit)**.** Let $G$ be a group, $X$ be a set, and $x \in X$. Then we define <u>the orbit</u> of $x$, $G \cdot x = \operatorname{orb}(x)$, as $\{g \cdot x \mid g \in G\}$. Moreover, $\operatorname{orb}(x) \subseteq X$.

**Definition 3** (Stabilizer)**.** Let $G$ be a group, $X$ be a set, and $x \in X$. Then we define <u>the stabilizer</u> of $x$, $\operatorname{stab}(x)$, as $\{g \in G \mid g \cdot x = g\}$. Moreover, $\operatorname{stab}(x) \leq G$.

**Theorem 1.3.** *Let* $G$ *be a finite group that acts on* $X$*. Then for all* $x \in X$*,* $|\operatorname{orb}(x)| \cdot |\operatorname{stab}(x)| = |G|$*.*

**Definition 4** (Polynomial ring)**.** Let $R$ be a commutative ring. Then the ring of polynomials with coefficients in $R$ is

$$R[t] = \left\{\sum_{i=0}^{n} c_i t^i : n \in \mathbb{Z}_+, c_i \in R\right\}$$