1 Galois Groups II

Lemma 1.1. *Suppose that $L : K$ is a normal extension with $K \subseteq L \subseteq \overline{K}$. Then for any K-homomorphism $\tau : L \to \overline{K}$, we have $\tau(L) = L$.*

Lemma 1.2. *For $n \geq 2$, S_n is generated by*

1. *transpositions $(i\,j)$;*
2. *transpositions $(1\,i)$;*
3. *adjacent transpositions $(1\,2), (2\,3), \ldots, (n-1,\, n)$;*
4. *$(1\,2)$ and $(1\,2 \ldots n)$;*
5. *$(1\,2)$ and $(2\,3 \ldots n)$;*
6. *$(i\,j)$ and $(i \ldots i_p)$ where p is prime.*

Lemma 1.3. *Let $(i_1 \ldots i_k) \in S_n$. Then for all $\sigma \in S_n$, one has $\sigma(i_1 \ldots i_k)\sigma^{-1} = (\sigma(i_1) \ldots \sigma(i_k))$.*

Note: $|Gal_K(f)| = [L : K]$ where $L : K$ is a splitting field extension for f.