Problem Set #4

Instructions: Please submit your homework via Gradescope. You can either submit a pdf (by first scanning your homework at the Library or using an app on your phone), or images by taking pictures of your work. All of the following should be solved by hand (no calculators) and you should show detailed work.

1. Suppose $f(0) = 1$, $f(2) = 5$, $f(4) = 3$, and $\displaystyle\int_0^2 f(t)\,dt = 10$. Find:

   (a) $\displaystyle\frac{d}{dx}\int_0^x f(t)\,dt$ at $x = 2$

   (b) $\displaystyle\frac{d}{dx}\int_0^{\sqrt{x}} f(t)\,dt$ at $x = 4$

   (c) $\displaystyle\frac{d}{dx}\int_0^x \sqrt{f(t)}\,dt$ at $x = 4$

   (d) $\displaystyle\frac{d}{dx}\int_0^2 f(t)\,dt$

   (e) $\displaystyle\int_0^4 \frac{d}{dt} f(t)\,dt$

   (f) $\displaystyle\frac{d}{dx}\left(x^2\int_0^x f(t)\,dt\right)$ at $x = 2$

2. Suppose h is a continuous function such that:

   - $h(0) = 8$, $h(2) = 4$, $h(4) = 5$
   - $h'(0) = 2$, $h'(2) = 1$, $h'(4) = 3$
   - $\displaystyle\int_0^1 h(x)\,dx = 2$, $\displaystyle\int_0^2 h(x)\,dx = 6$, $\displaystyle\int_0^4 h(x)\,dx = 12$

   Given this information, find the following (and make sure to show all work):

   (a) $\displaystyle\int_0^{\pi/2} h'\big(4\sin(x)\big)\cos(x)\,dx$

   (b) $\displaystyle\int_2^4 e^{h'(x)} h''(x)\,dx$

   (c) $\displaystyle\int_{\ln(2)}^{\ln(4)} h(e^x)e^x\,dx$

   (d) $\displaystyle\int_e^{e^2} \frac{h(\ln(x))}{x}dx$

3. Let $g(x) = \displaystyle\int_2^{x^2} e^{t^2}\,dt$.

   (a) Find the line tangent to $g(x)$ at $x = \sqrt{2}$.

(b) Use your tangent line from (a) to approximate $\displaystyle\int_2^{(\sqrt{2}+0.5)^2} e^{t^2}\,dt$.

(c) Is your approximation from (b) an over or an underestimate? Explain.

(d) Which is larger: $g(3)$ or $\displaystyle\frac{1}{2}\sum_{k=2}^{15} e^{\left(2+\frac{(k-1)}{2}\right)^2}$? Explain.

(e) Which function is more dominant: $g(x)$ or $h(x) = \displaystyle\int_2^x e^{t^4}\,dt$? Explain.

4. Suppose $\displaystyle\int_0^1 f(x)\,dx = 3,\ f(0) = 10,\ f(1) = 4,\ f'(0) = 1,\ f'(1) = -2,\ f''(0) = 5,\ f''(1) = 8.$
Evaluate the following:

(a) $\displaystyle\int_0^1 x f''(x)\,dx$

(b) $\displaystyle\int_0^1 f''(x)\Big(f'(x)\Big)^2\,dx$

(c) $\displaystyle\int_0^1 x^2 f''(x)\,dx$

(d) $\displaystyle\int_0^1 \frac{f''(x)}{f'(x)}\,dx$

5. Suppose

$$f(x) = \int_0^x \frac{dt}{(t+1)(t+2)} \qquad g(x) = \int_0^x \sin\left(\frac{3}{8}\pi e^t\right)\,dt \qquad h(x) = g\left(f(x)\right)$$

(a) Show that $f(1) = \ln\left(\dfrac{4}{3}\right)$.

(b) Find $h'(1)$.