

## Problem Set # 9

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10/11/20

**1**

**1.a**

$\frac{a_k}{k} < a_k \therefore$  Must be Convergent bc of Comparison test

**1.b**  $\sum_{k=1}^{\infty} \sqrt{a_k}$

$$a_k = \frac{1}{k^2}$$

$$\frac{1}{\sqrt{k^2}}$$

$\frac{1}{k} =$  Divergence bc p-series

$$a_k = \frac{1}{k^3}$$

$$\frac{1}{k^3}$$

$\frac{1}{k^{3/2}} \therefore$  Convergent bc p-series

Impossible to decide
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**1.c**  $\sum_{k=1}^{\infty} \sin(a_k)$

$\sin(a_k)$  comp to  $a_k$

$$\lim_{k \rightarrow \infty} \frac{\sin(a_k)}{a_k}$$

$$\lim_{k \rightarrow 0} \frac{\sin(x)}{x} \rightarrow LH$$

$\cos(x) = 1 \therefore$  Must be convergent bc LCT

**1.d**  $\cos(a_k)$

$$\lim_{k \rightarrow \infty} \cos(a_k) = \cos(0) = 1 \therefore \text{Must diverge bc Test for divergence}$$

**1.e**  $\sum_{k=1}^{\infty} \cos(k)a_k$

$$|\cos(k)a_k| \leq |a_k| \therefore \text{Converge bc of Comparison test}$$

$$\text{Must converge bc of Absolute Convergence Test}$$

**1.f**

$$a_k = \frac{1}{k^2}$$

$$b_k = \frac{1}{k^2}$$

$$\lim_{k \rightarrow \infty} \frac{1/k^2}{1/k^2} = 1 \therefore \text{Diverge because of limit test}$$

$$a_k = \frac{1}{k^4}$$

$$b_k = \frac{1}{k^2}$$

$$\lim_{k \rightarrow \infty} \frac{1}{k^4} * \frac{k^2}{1}$$

$$\frac{1}{k^2} \therefore \text{Converge because of p-series}$$

Impossible to decide

**1.g**  $\arctan(\frac{1}{a_k})$

If  $a_k$  converges,  $\lim_{k \rightarrow \infty} a_k = 0$

$$\lim_{k \rightarrow \infty} \arctan(\frac{1}{0}) = \arctan(\infty) = \frac{\pi}{2} \neq 0$$

$\therefore$  Must diverge, test for divergence

## 2

### 2.a

$$f(x) = e^{x^2}, \quad f(0) = 1$$

$$f'(x) = 2x * e^{x^2}, \quad f'(0) = 0$$

$$f''(x) = 4x^2 * e^{x^2} + 2e^{x^2}, \quad f''(0) = 2$$

$$f'''(x) = 4x^2 * 2x * e^{x^2} + e^{x^2} * 8x + 2x * 2e^{x^2}, \quad f'''(0) = 0$$

$$T_3(x) = 1 + \frac{2(x)^2}{2!} = 1 + x^2$$

### 2.b

$$\begin{aligned} T_3\left(\frac{1}{10}\right) &= 1 + \left(\frac{1}{10}\right)^2 \\ &= 1 + \frac{1}{100} = \boxed{\frac{101}{100}} \end{aligned}$$

### 2.c

### 2.d

$$\int_0^{\frac{1}{10}} T_3(x) dx$$

$$\int_0^{\frac{1}{10}} 1 + x^2 dx$$

$$\left(x + \frac{x^3}{3}\right) \Big|_0^{\frac{1}{10}}$$

$$\left(\frac{1}{10} + \frac{1}{3000}\right) - 0 = \frac{301}{3000}$$

### 3

#### 3.a

$$f(x), f(1) = 0$$

$$f'(x) = \frac{1}{x} * x + \ln(x), f'(1) = 1 + 0 = 1$$

$$f''(x) = \frac{1}{x}, f''(1) = 1$$

$$f'''(x) = -\frac{1}{x^2}, f'''(1) = -1$$

$$T_3(x) = 0 + 1(x-1) + \frac{1(x-1)^2}{2!} + -\frac{1(x-1)^3}{3!}$$

#### 3.b

$$f(x), f(1) = \sqrt{2}$$

$$f'(x) = \frac{1}{2}(1+x)^{-1/2}, f'(1) = \frac{1}{2\sqrt{2}}$$

$$f''(x) = -\frac{1}{4}(1+x)^{-3/2}, f''(1) = -\frac{1}{4\sqrt{8}}$$

$$f'''(x) = \frac{3}{8}(1+x)^{-5/2}, f'''(1) = \frac{3}{8\sqrt{32}}$$

$$T_3(x) = \sqrt{2} + \frac{\frac{1}{2\sqrt{2}}(x-1)}{1!} - \frac{\frac{1}{4\sqrt{8}}(x-1)^2}{2!} + \frac{\frac{3}{8\sqrt{32}}(x-1)^3}{3!}$$

#### 3.c

$$f(x), f(\pi/4) = \tan(\pi/4) = 1$$

$$f'(x) = \sec^2(x), f''(\pi/4) = \sec^2(\pi/4) = 2$$

$$f''(x) = 2\sec(x) * \sec(x)\tan(x), f''(\pi/4) = 4$$

$$T_2(x) = 1 + 2(x - \frac{\pi}{4}) + \frac{4(x - \frac{\pi}{4})^2}{2!}$$

$$T_2(x) = 1 + 2(x - \frac{\pi}{4}) + 2(x - \frac{\pi}{4})^2$$

## 4

### 4.a

$$f(-1) = T(-1) = 0 \not> 0$$

$\therefore$  Impossible bc  $T(-1)$  is not positive

### 4.b

$$f(-1) = T(-1) = 1 > 0$$

$$T'(x) = 4 = 4(x + 1)$$

$$T''(x) = 4$$

$$T''(-1) = 4 > 0$$

$\therefore$  Possible because  $T(-1)$  and  $T''(-1)$  are positive

### 4.c

$$f(-1) = T(-1) = 9 > 0$$

$$T'(x) = 4(x - 1)$$

$$T''(x) = 4 > 0$$

$\therefore$  Possible because  $T(-1)$  and  $T''(-1)$  are positive

### 4.d

$$T(-1) = -\frac{1}{6} + 2 - 10 = -\# \not> 0$$

$\therefore$  Impossible because  $T(-1)$  is not positive

## 5

### 5.a

$x$	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^4(x)$
0	X	X	X	X	X
1	1	2	2	0	X
2	7	3	1	0	48

**5.b**

$$u = f''(x)$$

$$du = f'''(x)$$

$$\int_2^1 f''(u) du$$

$$f''(1) = 2, \quad f''(2) = 1$$

$$- (f(u)|_1^2)$$

$$- (f(2) - f(1))$$

$$- (7 - 1) = \boxed{-6}$$

**5.c**

$$\lim_{x \rightarrow 0} \frac{f(1) - 1}{f'''(2)} = \frac{0}{0} \rightarrow LH$$

$$\lim_{x \rightarrow 0} \frac{e^x * f'(e^x)}{2 * f^4(2x + 2)} = \frac{2}{2 * 48} = \boxed{\frac{1}{48}}$$

**5.d**

$$g(x) = \text{that}, \quad g(1) = e^{(f(2)^2)} = e^{47}$$

$$g'(x) = 2f(x) * f'(x) * e^{f(x)^2}, \quad g'(2) = 14 * 3 * e^{47} = 42e^{47}$$

$$T_1(x) = e^{47} + 42e^{47}(x - 1)$$

**5.e**

$$h(x) = \text{that}, \quad h(1) = f(1) = 1$$

$$h'(x) = \pi \cos(\pi x) * f'(1 + \sin(\pi x)), \quad h'(1) = -\pi * f'(1) = -2\pi$$

$$h''(x) = \pi \cos(\pi x) * f''(1 + \sin(\pi x)) * \pi \cos(\pi x) - f'(1 + \sin(\pi x)) * \pi \sin(\pi x)$$

$$= \pi^2 * f''(1) = \pi^2 * 2 = 2\pi^2$$

$$T_2(x) = 1 - 2\pi(x - 1) + \frac{2\pi^2(x - 1)^2}{2!}$$

$$T_2(x) = 1 + 2\pi(x - 1) + \pi^2(x - 1)^2$$