1

1.a Find the value of c

$$\frac{ck+1}{4} + c(\frac{1}{2})^k = 1$$

$$\frac{1}{4} + \frac{c+1}{4} + \sum_{2}^{\infty} c(\frac{1}{2})^k = 1$$

$$\frac{1}{4} + \frac{1}{4} + \frac{c}{4} + \sum_{2}^{\infty} c(\frac{1}{2})^k = 1$$

$$\frac{1}{2} + \frac{c}{4} + \sum_{2}^{\infty} c(\frac{1}{2})^k = 1$$

$$\frac{c}{4} + \sum_{2}^{\infty} c(\frac{1}{2})^k = \frac{1}{2}$$

$$\frac{c}{4} + \frac{c/4}{1-\frac{1}{2}} = \frac{1}{2}$$

$$c + \frac{c}{1/2} = 2$$

$$3c = 2$$

$$c = \frac{2}{3}$$

1.b Find $P(10 \leq X \leq 100)$

$$\sum_{k=m}^{n} r^k = r^m(\frac{1-r^{n-m+1}}{1-r})$$

$$\sum_{k=10}^{100} p(k) = \boxed{\frac{2}{3}(\frac{1}{2})^{10}(\frac{1-(\frac{1}{2})^{91}}{1-\frac{1}{2}})}$$

1.c Find the probability that X is an even number $>$ than 2

$$
\begin{aligned}
P(x=2k) &= \sum_{k=2}^{\infty} \frac{2}{3}(\frac{1}{2})^{2k} \\
&= \frac{2}{3} * \frac{(1/4)^2}{1-\frac{1}{4}} \\
&= \frac{2}{3} * \frac{\frac{1}{16}}{(3/4)} \\
&= \frac{\frac{1}{8}}{\frac{9}{4}} = \boxed{\frac{1}{18}}
\end{aligned}
$$

2

2.a

2.b

2.c Two geometric series that converge but $\sum \frac{a_k}{b_k}$ diverge

This is not possible, because when two series converge, the $\sum_{k=1}^{\infty} \frac{a_k}{b_k}$ must converge

3

3.a Explain why $\sum_{k=1}^{\infty} a_k$ must converge

4

4.a $\sum_{k=1}^{\infty} 2^{-2k+4}$

$$\lim_{k\to\infty} 2^{-2k+4} = 2^{-\infty} = 0$$

$$\sum_{k=1}^{\infty} 2^{-2k} * 2^4$$

$$\sum_{k=1}^{\infty} (\frac{1}{4})^k * 16$$

$$16 \sum_{k=1}^{\infty} (\frac{1}{4})^k = 16 * \frac{1/4}{1-\frac{1}{4}}$$

$$\frac{4}{1-\frac{1}{4}}$$

4.b $\sum_{k=1}^{\infty} \frac{1}{k^2+4k+3}$

$$\lim_{k\to\infty} \frac{1}{k^2+4k+3} = 0$$

$$\frac{1}{(k+3)(k+1)}$$

$$\frac{A}{(k+3)} + \frac{B}{(k+1)}$$

$$A(k+1) + B(k+3) = 1$$

$$A(-3+1) = 1 \qquad k = -3$$

$$-2A = 1$$

$$A = -\frac{1}{2}$$

$$B(-1+3) = 1 \qquad k = -1$$

$$2B = 1$$

$$B = \frac{1}{2}$$

$$\frac{1}{2}\sum_{k=1}^{\infty} \frac{1}{k+1} - \frac{1}{k+3} dk$$

$$S_3 = (\frac{1}{2} - \frac{1}{4}) + (\frac{1}{3} - \frac{1}{5}) + (\frac{1}{4} - \frac{1}{6})$$

$$S_n = \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}$$

$$S_n = \frac{5}{6} - \frac{1}{n+2} - \frac{1}{n+3}$$

$$\lim_{n\to\infty} S_n = \boxed{\frac{5}{6}}$$

$\therefore$ Converges to $\frac{5}{6}$ by telescoping series test

4.c $\sum_{k=1}^{\infty} \frac{(2k)!}{7^k(k!)^2}$

$$\frac{(2(k+1))!}{7^{k+1}(k+1)!^2} * \frac{7^k(k!)^2}{(2k)!}$$
$$\frac{(2k+2)!}{7^{k+1}(k+1)!^2} * \frac{7^k(k!)^2}{(2k)!}$$
$$\frac{(2k+2)!}{7(k+1)(2k)!}$$
$$\frac{(2k+2)(2k+1)}{7(k+1)^2}$$
$$\lim_{k\to\infty} \frac{4k^2+\dots}{7k^2+\dots}$$
$$\frac{4}{7} < 1 \therefore \text{Convergent}$$

4.d $\sum_{k=1}^{\infty} \sin(\frac{1}{2^k})$

$$\sin(\frac{1}{2k}) < \frac{1}{2^k}$$
$$\lim_{k\to\infty} \frac{sin(\frac{1}{2^k})}{1/2^k}$$
$$\lim_{k\to\infty} \frac{sin(\frac{1}{2^k})}{1/2^k} = \frac{0}{0} \to \ LH$$
$$\lim_{k\to\infty} \frac{(\frac{1}{2})^k ln(2)\cos(\frac{1}{2k})}{(1/2)^k ln(2)}$$

What am I doing

4.e $\sum_{k=2}^{\infty} \frac{\ln(k)}{k^2}$

$$\lim_{k\to\infty} \frac{\ln(k)}{k^2} \to \; LH$$
$$\lim_{k\to\infty} \frac{\frac{1}{k}}{2k} = 0$$

$$\frac{\ln(k+1)}{2(k+1)^2} * \frac{2k^2}{ln(k)}$$
$$\lim_{k\to\infty} \frac{2k^2 * ln(k+1)}{2(k+1)^2 * ln(k)}$$
$$\lim_{k\to\infty} \frac{2k^2 * ln(\frac{k+1}{k})}{2(k+1)^2}$$
$$\lim_{k\to\infty} \frac{2k^2 * 0}{2(k+1)^2}$$
$$\lim_{k\to\infty} \frac{0}{2(k+1)^2} = 0 < 1$$

$\therefore$ Converges because of Ratio Test

4.f $\sum_{k=1}^{\infty} k \sin(\frac{1}{k})$

$$\lim_{k\to\infty} \frac{\sin(\frac{1}{k})}{1/k} \to LH$$
$$\lim_{k\to\infty} \frac{-\frac{1}{k^2}\cos(\frac{1}{k})}{-\frac{1}{k^2}}$$
$$\lim_{k\to\infty} cos(\frac{1}{k}) = 1 \neq 0$$

$\therefore$ Diverges because of limit test

4.g $\sum_{k=1}^{\infty} \frac{1}{k+e^k}$

$$\frac{1}{(k+1)+e^{k+1}} * \frac{k+e^k}{1}$$

$$\frac{k+e^k}{k+1+e^{k+1}}$$

$$\lim_{k\to\infty} \frac{k+e^k}{k+1+e^k * e} \to \ LH$$

$$\lim_{k\to\infty} \frac{1+e^k}{1+e^{k+1}}$$

$$\lim_{k\to\infty} \frac{e^k}{e^{k+1}} = \frac{1}{e} < 1$$

$\therefore$ Convergent ecause of ration test