Problem Set # 9

Joshua Petitma

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$\frac{a_n}{n} < a_n$

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CT

~~1~~.a

$$\lim_{k\to\infty}\frac{a_k}{k} = a_k * \lim_{k\to\infty}\frac{1}{k} = a_k * Divergent \therefore \text{ Divergent}$$

1.b $\sum_{k=1}^{\infty}\sqrt{a_k}$

$a_k = \frac{1}{k^2}$

$$\frac{1}{\sqrt{k^2}}$$

$$\frac{1}{k} = Divergnet$$

$a_k = \frac{1}{k^3}$

$$\frac{1}{k^3}$$

$$\frac{1}{k^{3/2}} \therefore \text{ Convergent bc p-series}$$

Impossible to decide

1.c $\sum_{k=1}^{\infty}\sin(a_k)$

$a_k > 0$

$a_k = (\frac{1}{2})^k \therefore \lim_{k\to\infty} a_k = \frac{1}{.5}$

$\lim_{k\to\infty} \sin$

$$\lim_{n\to\infty}\frac{\sin(a_n)}{a_n} = 1$$

$$\lim_{x\to 0}\frac{\sin(x)}{x}$$

$$\lim_{x\to 0}\frac{\cos x}{1} = 1$$

1.d $\cos(a_k)$ Div

1.e $\sum_{k=1}^{\infty} \cos(k) a_k$

$$a_k * \lim_{k\to\infty} \cos(k) = DNE$$

$$\therefore \text{Must Diverge}$$

$$\sum_{k=1}^{\infty} |\cos k \cdot a_n| \leq \sum_{k=1}^{\infty} |a_n|$$

$$\therefore \sum_{k=1}^{\infty} |\cos(k) a_n| \text{ Conv}$$

C.T.

Abs. Conv. :

$\sum \cos(k) a_n$ Conv.

1.f

$a_k = \dfrac{1}{k^2}$

$b_k = \dfrac{1}{k^2}$

$\lim\limits_{k\to\infty} \dfrac{1/k^2}{1/k^2} = 1 \therefore$ Diverge because of limit test

$a_k = \dfrac{1}{k^4}$

$b_k = \dfrac{1}{k^2}$

$\lim\limits_{k\to\infty} \dfrac{1}{k^4} * \dfrac{k^2}{1}$

$\dfrac{1}{k^2} \therefore$ Converge because of p-series

Impossible to tell

1.g $\arctan(\frac{1}{a_k})$

If a_k converges, $\lim\limits_{k\to\infty} a_k = 0$

$\lim\limits_{k\to\infty} \arctan(\dfrac{1}{0}) = \arctan(\infty) = \dfrac{\pi}{2} \neq 0$

$\therefore$ Must diverge, limit test

Test 4 dV.

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2.a

$$f(x) = e^{x^2},\ f(0) = 1$$
$$f'(x) = 2x * e^{x^2},\ f'(0) = 0$$
$$f''(x) = 4x^2 * e^{x^2},\ f''(0) = 0$$
$$T_3(X) = 1$$

2.b

2.c

3

3.a

$$f(x),\ f(1) = 0$$
$$f'(x) = \frac{1}{x} * x + \ln(x),\ f'(1) = 1 + 0 = 1$$
$$f''(x) = \frac{1}{x},\ f''(1) = 1$$
$$f'''(x) = -\frac{1}{x^2},\ f'''(1) = -1$$
$$\boxed{T_3(x) = 0 + 1(x-1) + \frac{1(x-1)^2}{2!} + -\frac{1(x-1)^3}{3!}}$$

~~3.b~~

$f(x),\ f(1) = \sqrt{2}$

$f'(x) = \frac{1}{2}(1+x)^{-1/2},\ f'(1) = \frac{1}{2\sqrt{2}}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2},\ f''(1) = -\frac{1}{4\sqrt{8}}$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2},\ f'''(1) = \frac{3}{8\sqrt{32}}$

$$\boxed{T_3(x) = \sqrt{2} + \frac{\frac{1}{2\sqrt{2}}(x-1)}{1!} - \frac{\frac{1}{4\sqrt{8}}(x-1)^2}{2!} + \frac{\frac{3}{8\sqrt{32}}(x-1)^{\cancel{2}}}{3!}} \quad 3$$

Simplyfy

$T(x) = f(a) + f'(a)(x+1) + \frac{f''(a)(x+1)^2}{2}$

$f'(-1) =$

$T'(-1) = 3 - 2 + 1 = 2$

$f''(-1) = T''(-1) = -4 < 0$

4a $f(-1) = T(-1) = 0$ which is not positive

$\therefore$ can't be

4b) $f(-1) = T(-1) = 1 > 0$ ✓

$T'(x) = 4(x+1)$ $T'(-1) = 0$

$T''(x) = 4$ $T''(-1) = 4 > 0$ ✓

possible!

I

4c) $f(-1) = T(-1) = 9 > 0$

$T'(x) = 4(x-1)$

$T''(x) = 4 > 0$ ✓

4d) $T(-1) = -\frac{1}{6} + 2 - 10 = -\# \not> $ X

5a)

	0	1	2	3	4
0	X	X	X	X	X
1	1	2	2	0	X
2	7	3			

Q5 b)

$$\int_1^2 f'(f''(x))\, f'''(x)\,dx \qquad \begin{array}{l} u = f''(x) \\ du = f'''(x) \end{array}$$

$$= -\int_1^2 f'(u)\,du = -f(u)\Big|_1^2$$

$$(\;)\ \frac{1}{48} \quad -(7-1) = -6$$

$\cancel{1},\ \cancel{2},\ \cancel{2},\ \cancel{9},\ \cancel{x}$

$7 \quad 3 \quad 1 \quad 0 \quad 48$