1 $\int \frac{x+1}{x^2+2x-3}$

1.a Solve integral with partial Fraction

$$\int \frac{x+1}{(x-2)(x-1)}dx$$
$$\int \frac{A}{x-2} + \frac{B}{x-1}dx = \frac{x+1}{\dots}$$

$$A(x-1) + B(x-2) = x+1$$
$$-1B = 2 \qquad x = 1$$
$$B = -2$$
$$A = 3 \qquad x = 2$$
$$\int \frac{3}{x-2} - \frac{2}{x-1}dx$$

$3\ln(|x-2|) - \ln(|x-1|) + c$

1.b Solve using another Technique

$$\int \frac{x+1}{x^2+2x-3} \qquad u = x^2 + 2x - 3$$
$$du = 2x + 2$$
$$\frac{1}{2}\int \frac{du}{u} \qquad \frac{1}{2}du = x + 1$$
$$\frac{1}{2}\ln(|u|) + c$$
$$\frac{1}{2}\ln(|x^2 + 2x - 3)$$

1.c Evaluate $\int_0^5 dx$ or show that it diverges

$$\int_0^5 \frac{x+1}{x^2{+}2x-3} \qquad x \neq 1$$

$$\lim_{a\to 1}\int_0^a \frac{x+1}{x^2+2x-3} + \lim_{b\to 5}\int_1^b \frac{x+1}{x^2+2x-3} \qquad u = x^2+2x-3$$

$$\lim_{a\to 1}\frac{1}{2}\int\frac{du}{u} + \frac{1}{2}\lim_{b\to 5}\int\frac{du}{u} \qquad \frac{1}{2}du = x+1$$

$$\frac{1}{2}\ln(|u|) + \frac{1}{2}\ln(|u|)$$

$$\lim_{a\to 1}\ln(\sqrt{x^2+2x-3})\Big|_0^a + \dots$$

$$\ln(\sqrt{1^2+2-3})$$

$$\ln(0) \to DNE \therefore \ Diverges$$

2 Find using given values

2.a $\int_0^2 3(f(x)-2)dx$

$$3\int_0^2 f(x)dx - 3\int_0^2 2dx$$

$$3(\frac{1}{2}*6) - 3(2x\Big|_0^2) \qquad \int_0^2 f(x) = \frac{1}{2}\int_{-2}^2 f(x)dx$$

$$9 - 3(2(2)-0)$$

$$9 - 3(4) = -3$$

2.b $\int_{-2}^{-1} f(x)dx$

$$\begin{aligned}
\int_{-2}^{0} f(x) &= 3 \\
\int_{0}^{1} f(x) = \int_{-1}^{0} f(x) &= 1 \\
\int_{-2}^{0} f(x) - \int_{-1}^{0} f(x) &= \int_{-2}^{-1} f(x) \\
3 - 1 &= \int_{-2}^{-1} f(x) \\
\int_{-2}^{-1} f(x) &= 2
\end{aligned}$$

2.c $\int_{-1}^{2} xf(x)$

$$\int_{0}^{1} xf(x)dx = \int_{-1}^{0} xf(x)dx = -\frac{1}{2}$$

$$\begin{aligned}
\int_{-1}^{2} xf(x) &= \int_{-1}^{0} xf(x) + \int_{0}^{2} xf(x)dx \\
&= -\frac{1}{2} + 4 \\
&= \frac{7}{2}
\end{aligned}$$

3 f(x) and f'(x) are continuous. Use the table

3.a $\int_{0}^{1} f'(x)$

$$\begin{aligned}
f(x)\Big|_{0}^{1} \\
f(1) - f(0) \\
5 - 3 = 2
\end{aligned}$$

3.b $\int_0^2 (f'(x)\sin(x) + f(x)\cos(x))$

$$\int_0^2 f'(x)sin(x)dx + \int_0^2 f(x)cos(x)$$

$$\int_0^2 f'(x)sin(x)dx \qquad u = \sin(x) \qquad dv = f'(x)$$

$$du = cos(x) \qquad v = f(x)$$

$$f(x)\sin(x)\big|_0^2 - \int_0^2 f(x)cos(x)dx$$

$$f(2)sin(2) - f(0)sin(0) - \int_0^2 f(x)cos(x)dx$$

$$8\sin(2) - \int_0^2 f(x)cos(x) + \int_0^2 f(x)cos(x)$$

$$8sin(2)$$

3.c $\int_0^3 \frac{f'(x)e^x - f(x)e^x}{(e^x)^2}dx$

$$\int_0^3 \frac{f'(x) - f(x)}{e^x}dx$$

$$\int_0^3 \frac{f'(x)}{e^x} - \int_0^3 \frac{f(x)}{e^x}$$

$$u = e^{-x} \qquad dv = f'(x)$$

$$du = -e^{-x} \qquad v = f(x)$$

$$\frac{f(x)}{e^x}\big|_0^3 + \int_0^3 \frac{f(x)}{e^x}$$

$$\frac{f(3)}{e^3} - \frac{f(0)}{1} + \int_0^3 \frac{f(x)}{e^x} - \int_0^3 \frac{f(x)}{e^x}$$

$$\frac{9}{e^3} - 3$$

$$9e^{-3} - 3$$

3.d $\int_0^1 f(x)f'(x)$

$$\int u du \qquad u = f(x)$$

$$\int_3^5 u du \qquad du = f'(x)$$

$$\frac{1}{2}u^2\Big|_3^5$$

$$\frac{1}{2}(5)^2 - \frac{1}{2}3^3$$

$$\frac{25}{2} - \frac{9}{2} = 8$$

4 Average value $= \frac{1}{b-a}\int_a^b f(x)dx$

4.a Find number(s) b, so a.v of $f(x) = 2+6x-3x^2$ on interval [0,b] equal to 3

$$\frac{1}{b-0}\int_0^b 2+6x-3x^2 dx = 3$$

$$\frac{1}{b}(2x+3x^2-x^3)\Big|_0^b = 3$$

$$\frac{1}{b}(2b+3b^2-b^3-0) = 3$$

$$\frac{1}{b}(2b+3b^2-b^3) = 3$$

$$2+3b-b^2 = 3$$

$$-1+3b-b^2 = 0$$

$$b = \frac{-3 \pm \sqrt{9-4(-1)(-1)}}{-2}$$

$$b = \frac{-3 \pm \sqrt{9-4}}{-2}$$

$$b = \frac{-3 \pm \sqrt{5}}{-2}$$

4.b

$$\frac{1}{8-1}\int_1^8 f(x)dx$$

$$\begin{aligned}
\int_1^6 f(x) &= 5 * A.V(1 < x < 6) \qquad \text{Equals inverse of } \frac{1}{b-a} * value \\
&= 5 * 4 = 20 \\
\int_6^8 f(x) &= 2 * A.V(6 < x < 8) \\
&= 2(5) = 10 \\
\frac{1}{7}\int_1^8 f(x) &= \frac{1}{7}(\int_1^6 f(x) + \int_6^8 f(x)) \\
&= \frac{1}{7} * (20 + 10) \\
&= \frac{30}{7}
\end{aligned}$$

5

5.a $\int_1^\infty \frac{1}{(x+1)(2x+3)}dx$

$$\lim_{a\to\infty}\int_1^a \frac{1}{(x+1)(2x+3)}dx$$
$$\frac{A}{x+1}+\frac{B}{2x+3}$$
$$A(2x+3)+B(x+1)=1$$
$$A(1)=1 \qquad x=-1$$
$$B(-\frac{3}{2}+1)=1 \qquad x=\frac{-3}{2}$$
$$-\frac{1}{2}B=1$$
$$B=-2$$
$$\lim_{a\to\infty}\int_1^a \frac{1}{x+1}-\lim_{a\to\infty}2\int_1^a \frac{1}{2x+3}$$
$$\lim_{a\to\infty}\ln(|x+1|)\Big|_1^a-2\lim_{a\to\infty}\ln(|2x+3|)*\frac{1}{2}$$
$$\lim_{a\to\infty}\ln(|x+1|)\Big|_1^a-\lim_{a\to\infty}\ln(|2x+3|)$$
$$\lim_{a\to\infty}\ln\frac{|x+1|}{|2x+3|}\Big|_1^a$$
$$LH\to\lim_{a\to\infty}\ln\frac{a+1}{2a+3}-\ln\frac{2}{5}$$
$$\lim_{a\to\infty}\ln(\frac{1}{2})+\ln(\frac{5}{2})$$
$$\ln(\frac{5}{4})$$

5.b $\int_0^e \ln(x)dx$

$$\lim_{a\to 0}\int_a^e \ln(x)$$

$$u = ln(x) \qquad dv = 1$$

$$du = \frac{1}{x} \qquad v = x$$

$$\lim_{a\to 0} xln(x)\Big|_a^e - \lim_{a\to 0}\int_a^e 1dx$$

$$\lim_{a\to 0}\frac{ln(x)}{\frac{1}{x}}\Big|_a^e - \lim_{a\to 0} x\Big|_a^e$$

$$LH\frac{1}{1/e} - \frac{\ln(a)}{1/a} - e$$

$$e - \frac{1/a}{-1/a^2} - e$$

$$\lim_{a\to 0}\frac{a^{-1}}{-a^{-2}}$$

$$\lim_{a\to 0} -a^1 = 0$$

5.c $\int_{-\pi/2}^{\pi/2}\frac{x\cos(x^2)}{(\sin(x^2))^2}$

$$\lim_{a\to 0}\int_{-\pi/2}^{a}\frac{x\cos(x^2)}{(\sin(x^2))^2} + \lim_{b\to 0}\int_b^{\pi/2}\frac{x\cos(x^2)}{(\sin(x^2))^2}$$

$$\lim_{b\to 0}\frac{1}{2}\int_b^{\pi/2}\frac{du}{u^2} \qquad u = sin(x^2)$$

$$\lim_{b\to 0}\frac{1}{2}\int_b^{\pi/2} u^{-2} \qquad du = 2xcos(x^2)$$

$$\frac{1}{2}\lim_{b\to 0}(-u^{-1})\Big|_b^{\pi/2} \qquad \frac{1}{2}du = xcos(x^2)$$

$$\frac{1}{2}\lim_{b\to 0}(-\frac{1}{\sin(x^2)})\Big|_b^{\pi/2}$$

$$\frac{1}{2}(-\frac{1}{sin(\pi/4)} + \frac{1}{sin(0)})$$

$$\frac{1}{\sin(0)} = \frac{1}{0} \to \infty \therefore Divergent$$