Integration Practice

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1. $\int xe^x$

$$xe^x - \int e^x + c \qquad u = x \qquad dv = e^x$$
$$xe^x - e^x dx + c \qquad du = 1 \qquad v = e^x$$

2. $\int xe^{x^2}$

$$\frac{1}{2}\int e^u du \qquad u = x^2$$
$$\frac{1}{2}e^u + c \qquad du = 2x$$
$$\frac{1}{2}e^{x^2} + c \qquad \frac{1}{2}du = x$$

3. $\int \frac{x}{\sqrt{1-x^4}}$

$$\int \frac{x}{\sqrt{1-(x^2)^2}}dx$$
$$\frac{1}{2}\arcsin(x^2) + c$$

4. $\int 2x\ln(x)dx$

$$x^2\ln(x) - \int x^2 * \frac{1}{x}dx \qquad u = \ln(x) \qquad dv = 2x$$
$$x^2\ln(x) - \int xdx \qquad du = \frac{1}{x} \qquad v = x^2$$
$$x^2\ln(x) - \frac{x^2}{2} + c$$

5. $\int \frac{x+7}{\sqrt{5-x}}$

$$\int \frac{12-u}{\sqrt{u}} \qquad u = 5-x$$
$$12\int \frac{1}{\sqrt{u}} - \int \frac{u}{\sqrt{u}} \qquad 5-u = x$$
$$12 * -2\sqrt{u} - \int \sqrt{u} \qquad 12-u = x+7$$
$$-24\sqrt{u} - \frac{2}{3}u^{3/2} \qquad -du = 1$$
$$-24\sqrt{5-x} - \frac{2}{3}(5-x)^{3/2} + c$$

6. $\arcsin x$

$$u = \arcsin(x) \qquad dv = 1$$
$$du = \frac{1}{\sqrt{1-x^2}} \qquad v = x$$
$$x\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}$$
$$x\arcsin(x) + \frac{1}{2}\int \frac{1}{\sqrt{u}}du \qquad u = 1-x^2$$
$$x\arcsin(x) + \sqrt{u} + c \qquad du = -2x$$
$$arcsin(x) + \sqrt{1-x^2} + c \qquad -\frac{1}{2}du = x$$

7. $\int x^5\sqrt{x^3+1}$

$$\int x^3 * x^2\sqrt{x^3+1} \qquad u = x^3+1$$
$$\frac{1}{3}\int (u-1)\sqrt{u} \qquad u-1 = x^3$$
$$\frac{1}{3}\int u^{3/2} - u^{1/2} \qquad du = 3x^2$$
$$\frac{1}{3} * (\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2})$$
$$\frac{2}{15}(x^3+a)^{5/2} - \frac{2}{9}(x^3+1)^{3/2} + c$$

8. $\int \frac{1}{x^2+6x+8}$

$$\begin{aligned}
\frac{1}{(x+4)(x+2)} \\
\frac{A}{x+4} + \frac{B}{x+2} = \frac{1}{...} \\
A(x+2) + B(x+4) = 1 \\
B(-2+4) = 1 & \qquad\qquad x = -2 \\
B = \frac{1}{2} \\
A(-4+2) = 1 & \qquad\qquad x = -4 \\
A = -\frac{1}{2} \\
\frac{1}{2}\int -\frac{1}{x+4} + \frac{1}{x+2} \\
\frac{1}{2}(-\ln(x+4) + ln(x+2)) + c
\end{aligned}$$

1. $\int_{-3}^{1} \frac{1}{x^2-x-2} dx$

$$\int_{-3}^{1} \frac{1}{(x-2)(x+1)}$$

$$\frac{A}{(x-2)} + \frac{B}{x+1} = \frac{1}{...}$$
$$A(x+1) + B(x-2)$$
$$B(-1-2) = 1 \qquad x = -1$$
$$B = -\frac{1}{3}$$
$$A(2+1) = 1 \qquad x = 2$$
$$A = \frac{1}{3}$$
$$\frac{1}{3}\int_{-3}^{1} \frac{1}{x-2} - \frac{1}{x+1}$$
$$\frac{1}{3}(\int_{-3}^{-1} \frac{1}{x-2} - \frac{1}{x+1} dx + \int_{-1}^{1} \frac{1}{x-2} - \frac{1}{x+1})$$
$$\frac{1}{3}(\ln(x-2) - ln(x+1))\Big|_{-3}^{-1}$$
$$\frac{1}{3}(\ln(\frac{x-2}{x+1})\Big|_{-3}^{-1})$$
$$ln(\frac{-3}{0}) \to undefined \therefore Divergent$$

2. $\int_{-\infty}^{\infty} \frac{1}{25+x^2}$

$$\int_0^{\infty} \frac{1}{25+x^2}$$
$$\lim_{b\to\infty} \int_0^{b} \frac{1}{25+x^2}$$
$$\lim_{b\to\infty} \int_0^{b} \frac{1}{25(1+\frac{x^2}{25})}$$
$$\lim_{b\to\infty} \frac{1}{25} \int_0^{b} \frac{1}{(1+\frac{x^2}{25})}$$
$$\lim_{b\to\infty} \frac{1}{25} \arctan(\frac{x}{5}) * \frac{1}{5}\Big|_0^b$$
$$\lim_{b\to\infty} \frac{\arctan(x/5)}{25}\Big|_0^b$$
$$\lim_{b\to\infty} \frac{\arctan(b)}{25} + \frac{\arctan(0)}{25}$$
$$\frac{\pi/2}{25}$$

$$\int_{-\infty}^{0} \frac{1}{25+x^2}$$
$$\lim_{a\to-\infty} \int_a^{0} \frac{1}{25+x^2}$$
$$\lim_{a\to-\infty} \int_a^{0} \frac{1}{25(1+\frac{x^2}{25})}$$
$$\lim_{a\to-\infty} \frac{1}{25} \int_a^{0} \frac{1}{(1+\frac{x^2}{25})}$$
$$\lim_{a\to-\infty} \frac{1}{25} \arctan(\frac{x}{5}) * \frac{1}{5}\Big|_a^0$$
$$\lim_{a\to-\infty} \frac{\arctan(x/5)}{25}\Big|_a^0$$
$$\frac{\arctan(0)}{25} + \lim_{a\to-\infty} \frac{\arctan(a)}{25}$$
$$-\frac{\pi/2}{25}$$

$$\frac{\pi/2}{25} - \frac{\pi/2}{25} = 0 \therefore Convergent$$

3. $\int_5^\infty \frac{1}{x^2+x}dx$

$$\lim_{a\to\infty}\int_5^a \frac{1}{x(x+1)}dx$$

$$\begin{aligned} \frac{A}{x}+\frac{B}{x+1} &= \frac{1}{...} \\ A(x+1)+B(x) &= 1 \\ B(-1) &= 1 && x=-1 \\ B &= -1 \\ A &= 1 && x=0 \end{aligned}$$

$$\lim_{a\to\infty}\int_5^a \frac{1}{x}-\frac{1}{x+1}$$

$$\lim_{a\to\infty}\ln(x)-ln(x+1)\Big|_5^a$$

$$\lim_{a\to\infty}\ln\frac{x}{x+1}\Big|_5^a$$

$$L.H\to\lim_{a\to\infty}\ln\frac{a}{a+1}-\ln(\frac{5}{6})$$

$$\ln(1)-\ln(\frac{5}{6})$$

$$ln(\frac{6}{5})$$

4. $\int_0^\pi \tan(x)$

$$\int_0^\pi \frac{\sin x}{\cos x} \qquad\qquad x\neq\frac{\pi}{2}$$

$$\lim_{a\to\pi/2}\int_0^a \frac{\sin x}{\cos x} \qquad\qquad \lim_{b\to\pi/2}\int_b^\pi \frac{\sin x}{\cos x}$$

$$\lim_{a\to\pi/2}-\ln(|\cos x|)\Big|_0^a \qquad\qquad \lim_{b\to\pi/2}-\ln(|\cos x|)\Big|_b^\pi$$

$$\lim_{a\to\pi/2}-\ln(\cos(a))+\ln(cos(0)) \qquad \lim_{b\to\pi/2}-\ln(|\cos(\pi)|)+\ln(|cos(b)|)$$

$$-ln(0)+0 \qquad\qquad -ln(1)+ln(0)$$

$$\ln(0)\to-\infty \therefore Divergent$$

5. $\int_0^{10} x^3 \ln(x)$

$$\frac{x^4}{4}\ln(x)\Big|_0^{10} - \int_0^{10} \frac{x^3}{4} dx \qquad u = \ln(x) \qquad dv = x^3$$

$$du = \frac{1}{x} \qquad v = \frac{x^4}{4}$$

$$\frac{10^4}{4} ln(10) - \frac{x^4}{16}\Big|_0^{10}$$

$$\frac{10^4}{4} ln(10) - \frac{10^4}{16} \qquad \therefore Convergent$$