Problem Set # 9

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1.a

$\frac{a_k}{k} < a_k \therefore$ Must be Convergent bc of Comparison test

1.b $\sum_{k=1}^{\infty} \sqrt{a_k}$

$a_k = \frac{1}{k^2}$ $\qquad \frac{1}{\sqrt{k^2}}$

$\frac{1}{k} =$ Divergence bc p-series

$a_k = \frac{1}{k^3}$ $\qquad \frac{1}{k^3}$

$\frac{1}{k^{3/2}} \therefore$ Convergent bc p-series

$\boxed{\text{Impossible to decide}}$

1.c $\sum_{k=1}^{\infty} \sin(a_k)$

$sin(a_k)$ comp to a_k

$\lim_{k\to\infty} \frac{\sin(a_k)}{a_k}$

$\lim_{k\to 0} \frac{\sin(x)}{x} \to LH$

$\cos(x) = 1 \therefore$ Must be convergent bc LCT

1.d $\cos(a_k)$

$$\lim_{k\to\infty} \cos(a_k) = \cos(0) = 1 \therefore \text{Must diverge bc Test for divergence}$$

1.e $\sum_{k=1}^{\infty} \cos(k) a_k$

$$|\cos(k)a_k| \ \leq |a_k| \therefore \text{Converge bc of Comparison test}$$
$$\text{Must converge bc of Absolute Convergence Test}$$

1.f

$$a_k = \frac{1}{k^2}$$
$$b_k = \frac{1}{k^2}$$
$$\lim_{k\to\infty} \frac{1/k^2}{1/k^2} = 1 \therefore \text{Diverge because of limit test}$$
$$a_k = \frac{1}{k^4}$$
$$b_k = \frac{1}{k^2}$$
$$\lim_{k\to\infty} \frac{1}{k^4} * \frac{k^2}{1}$$
$$\frac{1}{k^2} \therefore \text{Converge because of p-series}$$
$$\boxed{\text{Impossible to decide}}$$

1.g $\arctan(\frac{1}{a_k})$

If a_k converges, $\lim_{k\to\infty} a_k = 0$

$$\lim_{k\to\infty} \arctan(\frac{1}{0}) = \arctan(\infty) = \frac{\pi}{2} \neq 0$$
$\therefore$ Must diverge, test for divergence

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2.a

$$f(x) = e^{x^2},\ f(0) = 1$$
$$f'(x) = 2x * e^{x^2},\ f'(0) = 0$$
$$f''(x) = 4x^2 * e^{x^2} + 2e^{x^2}$$
$$f''(0) = 2$$
$$T_3(x) = 1 + \frac{2(x)^2}{2!} = 1 + x^2$$

2.b

$$\begin{aligned} T_3(\frac{1}{10}) &= 1 + (\frac{1}{10})^2 \\ &= 1 + \frac{1}{100} = \boxed{\frac{101}{100}} \end{aligned}$$

2.c

2.d

$$\int_0^{\frac{1}{10}} T_3(x)dx$$
$$\int_0^{\frac{1}{10}} 1 + x^2 dx$$
$$(x + \frac{x^3}{3})\Big|_0^{\frac{1}{10}}$$
$$(\frac{1}{10} + \frac{1}{3000}) - 0 = \frac{301}{3000}$$

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3.a

$$f(x),\ f(1)=0$$

$$f'(x)=\frac{1}{x}*x+\ln(x),\ f'(1)=1+0=1$$

$$f''(x)=\frac{1}{x},\ f''(1)=1$$

$$f'''(x)=-\frac{1}{x^2},\ f'''(1)=-1$$

$$\boxed{T_3(x)=0+1(x-1)+\frac{1(x-1)^2}{2!}+-\frac{1(x-1)^3}{3!}}$$

3.b

$$f(x),\ f(1)=\sqrt{2}$$

$$f'(x)=\frac{1}{2}(1+x)^{-1/2},\ f'(1)=\frac{1}{2\sqrt{2}}$$

$$f''(x)=-\frac{1}{4}(1+x)^{-3/2},\ f''(1)=-\frac{1}{4\sqrt{8}}$$

$$f'''(x)=\frac{3}{8}(1+x)^{-5/2},\ f'''(1)=\frac{3}{8\sqrt{32}}$$

$$\boxed{T_3(x)=\sqrt{2}+\frac{\frac{1}{2\sqrt{2}}(x-1)}{1!}-\frac{\frac{1}{4\sqrt{8}}(x-1)^2}{2!}+\frac{\frac{3}{8\sqrt{32}}(x-1)^3}{3!}}$$