

Problem Set #5

Instructions: Please submit your homework via Gradescope. You can either submit a pdf (by first scanning your homework at the Library or using an app on your phone), or images by taking pictures of your work. All of the following should be solved by hand (no calculators) and you should show detailed work.

1. Consider $\int \frac{x+1}{x^2+2x-3} dx$.
 - (a) Solve this integral using partial fraction decomposition.
 - (b) Solve this integral using a different integration technique we have seen in this class. Show that your answer is equivalent to that found in (a).
 - (c) Evaluate $\int_0^5 \frac{x+1}{x^2+2x-3} dx$, or show that it diverges.

2. Let $f(x)$ be a continuous, **even** function such that:

- $\int_{-2}^2 f(x) dx = 6$
- $\int_0^1 f(x) dx = 1$
- $\int_0^2 xf(x) dx = 4$
- $\int_0^1 xf(x) dx = \frac{1}{2}$

Find

- (a) $\int_0^2 3(f(x) - 2) dx$
- (b) $\int_{-2}^{-1} f(x) dx$
- (c) $\int_{-1}^2 xf(x) dx$

3. Suppose $f(x)$ and $f'(x)$ are continuous functions. Use the table below to answer the questions on this page.

x	0	1	2	3	4
$f(x)$	3	5	8	9	12
$f'(x)$	2	4	3	1	2

Evaluate the following:

- (a) $\int_0^4 f'(x) dx$

(b) $\int_0^2 (f'(x) \sin(x) + f(x) \cos(x)) \, dx$

(c) $\int_0^3 \frac{f'(x)e^x - f(x)e^x}{(e^x)^2} \, dx$

(d) $\int_0^1 f(x)f'(x) \, dx$

4. We define the **average value** of $f(x)$ over $[a, b]$ as $\frac{1}{b-a} \int_a^b f(x) \, dx$.

(a) Find the number(s), b , such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

(b) The average value of a function equals 4 for $1 \leq x \leq 6$, and equals 5 for $6 \leq x \leq 8$. What is the average value of the function for $1 \leq x \leq 8$?

5. For each of the following, either evaluate the integral, or show that it diverges. Make sure to use proper notation.

(a) $\int_1^\infty \frac{1}{(x+1)(2x+3)} \, dx$

(b) $\int_0^e \ln(x) \, dx$

(c) $\int_{-\pi/2}^{\pi/2} \frac{x \cos(x^2)}{(\sin(x^2))^2} \, dx$