# Problem Set #4

Joshua Petitma

**I tried**

1. $f(0) = 1, f(2) = 5, f(4) = 3,\ and \int_0^2 f(t)dt = 10$

   (a) $\frac{d}{dx}\int_0^x f(t)dt$ at x = 2

   $$f(x) * 1,\ x = 2$$
   $$f(2) = 5$$

   (b) $\frac{d}{dx}\int_0^{\sqrt{x}} f(t)dt$ at x =4

   $$f(\sqrt{x}) * \frac{1}{2\sqrt{x}},\ x = 4$$
   $$f(2) * \frac{1}{2*2} \to 5 * \frac{1}{4} = \frac{5}{4}$$

   (c) $\frac{d}{dx}\int_0^x \sqrt{f(t)}dt$ at x = 2

   $$\sqrt{f(x)} * 1,\ x = 4$$
   $$\sqrt{f(4)} = \sqrt{3}$$

   (d) $\frac{d}{dx}\int_0^2 f(t)dt$

   $$\int_0^2 f(t)dt = 10$$
   $$\frac{d}{dx}(10) = 0$$

(e) $\int_0^4 \frac{d}{dt} f(t)dt$

$$f(t)|_0^4$$
$$f(4) - f(0)$$
$$3 - 1 = 2$$

(f) $\frac{d}{dx}(x^2 \int_0^x f(t)dt)$ at x = 2

$^t dt * 2x$This line glitches idk how to fix it

$$x^2 * f(x) + \int_0^x f(t)dt * 2x, \text{ x} = 2$$
$$4 * f(2) + \int_0^2 f(t)dt * 4$$
$$4 * 5 + 10 * 4 = 60$$

2. (a) $\int_0^{\pi/2} h'(4\sin(x))\cos(x)$

$$\frac{1}{4}\int_0^4 h'(u)du \qquad u = 4\sin(x)$$
$$\frac{1}{4}(h(u)|_0^4) \qquad du = 4\cos(x)$$
$$\frac{1}{4}(h(4) - h(0)) \qquad \frac{1}{4}du = cos(x)$$
$$\frac{1}{4}(5 - 8) = -\frac{3}{4}$$

(b) $\int_2^4 e^{h'(x)} h''(x)dx$

$$\int e^u du \qquad u = h'(x)$$
$$\int_1^3 e^u du \qquad du = h''(x)$$
$$e^u\big|_1^3$$
$$e^3 - e$$

(c) $\int_{\ln(2)}^{\ln(4)} h(e^x)e^x dx$

$$\int_2^4 h(u)du \qquad u = e^x$$
$$du = e^x$$
$$\begin{aligned}\int_2^4 h(u)du &= \int_0^4 h(x)dx - \int_0^2 h(x)dx \\ &= 12 - 6 \\ &= 6\end{aligned}$$

(d) $\int_e^{e^2} \frac{h(\ln(x))}{x} dx$

$$\int_1^2 h(u)du \qquad u = ln(x)$$
$$du = \frac{1}{x}$$
$$\begin{aligned}\int_1^2 h(u)du &= \int_0^2 h(x)dx - \int_0^1 h(x)dx \\ &= 6 - 2 \\ &= 4\end{aligned}$$

3. Let $g(x) = \int_2^{x^2} e^{t^2}$

(a) Find tangent of $g(x)$ at $x = \sqrt{2}$

$$g'(x) = e^{x^4} * 2x, \; x = \sqrt{2}$$

$$g'(\sqrt{2}) = e^4 * 2\sqrt{2} \qquad\qquad g(\sqrt{2}) = \int_2^2 e^{t^2} \to 0$$

$$y + 0 = e^4 * 2\sqrt{2}(x - \sqrt{2})$$

$$y = e^4 * 2\sqrt{2}x - 4e^4$$

(b) Approximate $\int_2^{(\sqrt{2}+0.5)^2} e^{t^2} dt$ with the tangent line over or under?

$$x = \sqrt{2} + 0.5$$

$$y = e^4 * 2\sqrt{2}(\sqrt{2} + 0.5 - \sqrt{2})$$

$$y = e^4 * 2\sqrt{2}(.5)$$

$$y = \sqrt{2} * e^4$$

(c) Is (b) an under or over estimate?

$g''(x) = 2e^{x^4} + 4x^3 e^{x^4}$

$g''(x) > 0 \;\; over \;\; 2 < x < (\sqrt{2} + 0.5)^2$

$\therefore$ Tangent line is an underestimation

(d) $\sum$ vs $g(3)$

$$\frac{1}{2}\sum_{2}^{15} e^{(2+\frac{k-1}{2})^2}$$

$$\triangle x = .5 \qquad k = 2$$

$$f(x_k) = (2 + \frac{k-1}{2})$$

$$f(x_k) = (2 + \frac{2-1}{2}) \qquad k = 2$$

$$= 2.5$$

$$f(x_k) = (2 + \frac{15-1}{2}) \qquad k = 15$$

$$= 9$$

$$2.5 \to 9$$

$$g(3) = \int_2^9 e^{t^2} dt$$

$$\sum = RHS \text{ Because at } k = 2,\ f(x_k) = 2.5$$

$$g'(x) > 0 \ \ over\ 2 < x < 9$$

RHS' are overestimates when increasing

$$\therefore \sum > g(3)$$

(e) g(x) vs h(x)

$$\lim_{x\to\infty}\frac{g(x)}{h(x)}=\frac{\infty}{\infty}\therefore LH$$
$$\lim_{x\to\infty}\frac{g'(x)}{h'(x)}=\lim_{x\to\infty}\frac{e^{x^4}*2x}{e^{x^4}}=2x\to\infty$$
$$g(x)\text{ dominates because }\lim_{x\to\infty}\frac{g(x)}{h(x)}\to\infty$$

4. Evaluate:

(a) $\int_0^1 xf''(x)dx$

$$xf'(x)\Big|_0^1-\int_0^1 f'(x)dx \qquad u=x \quad dv=f''(x)$$
$$(1f'(1)-0)-f(x)\Big|_0^1 \quad du=1 \qquad v=f'(x)$$
$$-2-(f(1)-f(0))$$
$$-2-(4-10)=4$$

(b) $\int_0^1 f''(x)(f'(x))^2$

$$\int_1^{-2}u^2du \qquad u=f'(x)$$
$$-\int_{-2}^1 u^2du \qquad du=f''(x)$$
$$-(\frac{u^3}{3}\Big|_{-2}^1)$$
$$-(\frac{1}{3}+\frac{8}{3})=-3$$

(c) $\int_0^1 x^2 f''(x)$

$$x^2 f'(x)\Big|_0^1 - \int_0^1 2xf'(x) \quad u = x^2 \quad dv = f''(x)$$

$$(1^2 * -2) - 0 - \int_0^1 2xf'(x) \quad du = 2x \quad v = f'(x)$$

$$u = 2x \quad dv = f'(x)$$

$$du = 2 \quad v = f(x)$$

$$-2 - ((2xf(x)\Big|_0^1) - \int_0^1 2f(x))$$

$$-2 - (8 - 2(3)) = -4$$

(d) $\int_0^1 \frac{f''(x)}{f'(x)}$

$$\int_1^{-2} \frac{1}{u} du \qquad u = f'(x)$$

$$-\int_{-2}^1 \frac{1}{u} du \qquad du = f''(x)$$

$$-\ln u\Big|_{-2}^1$$

$$0 + \ln|-2| = \ln(2)$$

5. (a)

$$
\begin{aligned}
f(1) &= \int_0^1 \frac{dt}{(t+1)(t+2)} \\
\frac{A}{(t+1)} + \frac{B}{t+2} &= \frac{1}{\dots} \\
A(t+2) + B(t+1) &= 1 \\
B(-2+1) &= 1 && x = -2 \\
B &= -1 \\
A &= 1 && x = -1 \\
\int_0^1 \frac{1}{t+1} - \frac{1}{t+2} dt \\
\ln(t+1) - ln(t+2)\Big|_0^1 \\
\ln(\frac{t+1}{t+2})\Big|_0^1 \\
\ln(\frac{2}{3}) - \ln(\frac{1}{2}) \\
\ln(\frac{4}{3})
\end{aligned}
$$

(b) h'(1)

$$
\begin{aligned}
h'(x) &= g'(f(x))f'(x) \\
h'(1) &= g'(f(1))f'(1) \quad & f'(x) &= \frac{1}{(x+1)(x+2)} \\
h'(1) &= g'(\ln(\frac{4}{3})) * \frac{1}{6} \quad & f'(1) &= \frac{1}{6} \\
h'(1) &= \sin(\frac{\pi}{2}) * \frac{1}{6} \quad & g'(x) &= \sin(\frac{3}{8}\pi e^x) \\
h'(1) &= \frac{1}{6} \quad & g'(\ln(\frac{4}{3})) &= \sin(\frac{3}{8}\pi * \frac{4}{3}) = \sin(\frac{\pi}{2})
\end{aligned}
$$