Problem Set # 9

Joshua Petitma

10/11/20

1

1.a

$\frac{a_k}{k} < a_k \therefore$ Must be Convergent bc of Comparison test

1.b $\sum_{k=1}^{\infty} \sqrt{a_k}$

$a_k = \frac{1}{k^2}$ $\qquad \frac{1}{\sqrt{k^2}}$

$\frac{1}{k} =$ Divergence bc p-series

$a_k = \frac{1}{k^3}$ $\qquad \frac{1}{k^3}$

$\frac{1}{k^{3/2}} \;\therefore\;$ Convergent bc p-series

$\boxed{\text{Impossible to decide}}$

1.c $\sum_{k=1}^{\infty} \sin(a_k)$

$sin(a_k)$ comp to a_k

$\lim_{k\to\infty} \frac{\sin(a_k)}{a_k}$

$\lim_{k\to 0} \frac{\sin(x)}{x} \to LH$

$\cos(x) = 1 \;\therefore$ Must be convergent bc LCT

1.d $\cos(a_k)$

$$\lim_{k\to\infty} \cos(a_k) = \cos(0) = 1 \therefore \text{Must diverge bc Test for divergence}$$

1.e $\sum_{k=1}^{\infty} \cos(k) a_k$

$$|\cos(k)a_k| \ \leq |a_k| \therefore \text{Converge bc of Comparison test}$$
$$\text{Must converge bc of Absolute Convergence Test}$$

1.f

$$a_k = \frac{1}{k^2}$$
$$b_k = \frac{1}{k^2}$$
$$\lim_{k\to\infty} \frac{1/k^2}{1/k^2} = 1 \therefore \text{Diverge because of limit test}$$
$$a_k = \frac{1}{k^4}$$
$$b_k = \frac{1}{k^2}$$
$$\lim_{k\to\infty} \frac{1}{k^4} * \frac{k^2}{1}$$
$$\frac{1}{k^2} \therefore \text{Converge because of p-series}$$
$$\boxed{\text{Impossible to decide}}$$

1.g $\arctan(\frac{1}{a_k})$

If a_k converges, $\lim_{k\to\infty} a_k = 0$

$$\lim_{k\to\infty} \arctan(\frac{1}{0}) = \arctan(\infty) = \frac{\pi}{2} \neq 0$$

$\therefore$ Must diverge, test for divergence

2

2.a

$$f(x) = e^{x^2},\ f(0) = 1$$
$$f'(x) = 2x * e^{x^2},\ f'(0) = 0$$
$$f''(x) = 4x^2 * e^{x^2} + 2e^{x^2},\ f''(0) = 2$$
$$f'''(x) = 4x^2 * 2x * e^{x^2} + e^{x^2} * 8x + 2x * 2e^{x^2},\ f'''(0) = 0$$
$$T_3(x) = 1 + \frac{2(x)^2}{2!} = 1 + x^2$$

2.b

$$\begin{aligned} T_3(\frac{1}{10}) &= 1 + (\frac{1}{10})^2 \\ &= 1 + \frac{1}{100} = \boxed{\frac{101}{100}} \end{aligned}$$

2.c

$$\int_0^{\frac{1}{10}} T_3(x)dx$$
$$\int_0^{\frac{1}{10}} 1 + x^2dx$$
$$(x + \frac{x^3}{3})\Big|_0^{\frac{1}{10}}$$
$$(\frac{1}{10} + \frac{1}{3000}) - 0 = \frac{301}{3000}$$

3

3.a

$f(x),\ f(1) = 0$

$f'(x) = \frac{1}{x} * x + \ln(x),\ f'(1) = 1 + 0 = 1$

$f''(x) = \frac{1}{x},\ f''(1) = 1$

$f'''(x) = -\frac{1}{x^2},\ f'''(1) = -1$

$$\boxed{T_3(x) = 0 + 1(x-1) + \frac{1(x-1)^2}{2!} + -\frac{1(x-1)^3}{3!}}$$

3.b

$f(x),\ f(1) = \sqrt{2}$

$f'(x) = \frac{1}{2}(1+x)^{-1/2},\ f'(1) = \frac{1}{2\sqrt{2}}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2},\ f''(1) = -\frac{1}{4\sqrt{8}}$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2},\ f'''(1) = \frac{3}{8\sqrt{32}}$

$$\boxed{T_3(x) = \sqrt{2} + \frac{\frac{1}{2\sqrt{2}}(x-1)}{1!} - \frac{\frac{1}{4\sqrt{8}}(x-1)^2}{2!} + \frac{\frac{3}{8\sqrt{32}}(x-1)^3}{3!}}$$

3.c

$f(x),\ f(\pi/4) = \tan(\pi/4) = 1$

$f'(x) = \ \sec^2(x),\ f''(\pi/4) = \sec^2(\pi/4) = 2$

$f''(x) = 2\sec(x) * sec(x)tan(x),\ f''(\pi/4) = 4$

$T_2(x) = 1 + 2(x - \frac{\pi}{4}) + \frac{4(x-\frac{\pi}{4})^2}{2!}$

$T_2(x) = 1 + 2(x - \frac{\pi}{4}) + 2(x - \frac{\pi}{4})^2$

4

4.a

$f(-1) = T(-1) = 0 \ngtr 0$
$\therefore$ Impossible bc T(-1) is not positive

4.b

$f(-1) = T(-1) = 1 > 0$
$T'(x) = 4 = 4(x+1)$
$T''(x) = 4$
$T''(-1) = 4 > 0$
$\therefore$ Possible because T(-1) and T"(-1) are positive

4.c

$f(-1) = T(-1) = 9 > 0$
$T'(x) = 4(x-1)$
$T''(x) = 4 > 0$
$\therefore$ Possible because T(-1) and T"(-1) are positive

4.d

$T(-1) = -\frac{1}{6} + 2 - 10 = -\# \ngtr 0$
$\therefore$ Impossible because T(-1) is not positive

5

5.a

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^4(x)$
0	X	X	X	X	X
1	1	2	2	0	X
2	7	3	1	0	48

5.b

$u = f''(x)$
$du = f'''(x)$
$\int_2^1 f''(u)du$ $\qquad f''(1) = 2,\ f''(2) = 1$
$-(f(u)\big|_1^2)$
$-(f(2) - f(1))$
$-(7-1) = \boxed{-6}$

5.c

$$\lim_{x\to 0} \frac{f(1)-1}{f'''(2)} = \frac{0}{0} \to LH$$
$$\lim_{x\to 0} \frac{e^x * f'(e^x)}{2 * f^4(2x+2)} = \frac{2}{2*48} = \boxed{\frac{1}{48}}$$

5.d

$g(x) = that,\ g(1) = e^{(f(2)^2)} = e^{49}$
$g'(x) = 2f(x) * f'(x) * e^{f(x)^2},\ g'(2) = 14 * 3 * e^{49} = 42e^{49}$
$T_1(x) = e^{49} + 42e^{49}(x-1)$

5.e

$h(x) = that,\ h(1) = f(1) = 1$
$h'(x) = \pi\cos(\pi x) * f'(1 + sin(\pi x)),\ h'(1) = -\pi * f'(1) = -2\pi$
$h''(x) = \pi\cos(\pi x) * f''(1+\sin(\pi x)) * \pi\cos(\pi x) - f'(1+\sin(\pi x)) * \pi\sin(\pi x)$
$= \pi^2 * f''(1) = \pi^2 * 2 = 2\pi^2$
$$T_2(x) = 1 - 2\pi(x-1) + \frac{2\pi^2(x-1)^2}{2!}$$
$T_2(x) = 1 + 2\pi(x-1) + \pi^2(x-1)^2$