Problem Set # 8

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1.a $\sum_{k=1}^{\infty} \frac{(-1)^k}{k\ln(k)}$

$\lim_{k \to \infty} \frac{1}{k\ln(k)} = 0$

$S_n < S_{n+1} \therefore \ Converges$

$\frac{1}{kln(k)} < \frac{1}{k}$

$\frac{k}{1} * \frac{1}{kln(k)} = \frac{1}{\ln(k)}$

$\lim_{k \to \infty} \frac{1}{ln(k)} = 0 \therefore$ Divergent because of LCT

Converges Conditionally

1.b $\sum_{k=1}^{\infty} (\frac{-4}{5})^k$

$$\frac{-4}{5} * \frac{1}{1+\frac{4}{5}}$$
$$\frac{4}{5+4}$$
$$\frac{4}{9}$$

Converges Absolutely

1.c $\sum_{k=1}^{\infty} \frac{(-4)^k}{k^2}$

$$\lim_{k\to\infty} \frac{(-4)^k}{k^2} \neq 0 \therefore Divergent$$

1.d $\sum_{k=1}^{\infty} \frac{2+(-1)^k}{k^2}$

$$\lim_{k\to\infty} \frac{2+1}{k^2}$$

$$\frac{3}{k^2}\ 2 > 1 \therefore \text{ Convergent because of p-series}$$

$$\frac{2+(-1)^k}{k^2} \leq \frac{3}{k^2}\ \therefore \text{Converges absolutely bc of CT}$$

1.e $(-1)^k \sin(\frac{1}{k})$

$$\lim_{k\to\infty} a_k = 0$$

$$S_n < S_{n+1} \therefore \text{Convergent b/c of AST}$$

$$\sum_{k=1}^{\infty} \sin(\frac{1}{k})$$

$$\sin(\frac{1}{k}) \text{ Compare too } \frac{1}{k}$$

$$\frac{k}{1} * \sin(\frac{1}{k})$$

$$\lim_{k\to\infty} \frac{sin(\frac{1}{k})}{\frac{1}{k}}$$

$$-\frac{cos(\frac{1}{k}) * -\frac{1}{k^2}}{\frac{1}{k^2}} = -\frac{1}{0} \therefore \text{Converges Conditionally b/c LCT}$$

1.f $\sum_{k=1}^{\infty} (-1)^k k \arctan(\frac{1}{k})$

$$\lim_{k\to\infty} \frac{\tan^{-1}(\frac{1}{k})}{\frac{1}{k}} = \frac{tan^{-1}(0)}{0} = \frac{\pi/2}{0} \therefore \text{ Divergent bc of test for divergence}$$

1.g $(-1)^k \cos(\frac{1}{k})$

$$\lim_{k\to\infty} \cos(\frac{1}{k}) = 1 \ \therefore \text{ Divergent bc of limit test}$$

1.h $\sum_{k=1}^{\infty} \frac{(-1)^k k}{\sqrt{k^3-1}}$

$$\lim_{k\to\infty} \frac{(-1)^k k}{\sqrt{k^3-1}} = 0$$
$$S_n < S_n + 1 \ \therefore \text{ Converges bc of AST}$$
$$\frac{k}{\sqrt{k^3-1}} > \frac{k}{\sqrt{k^3}}$$
$$\frac{1}{\sqrt{k}}$$
$$\frac{\sqrt{k}}{1} * \frac{k}{\sqrt{k^3-1}}$$
$$\lim_{k\to\infty} \frac{\sqrt{k^3}}{\sqrt{k^3-1}} = 1 \ \therefore \text{Converges Absolutely bc of LCT}$$

2 $a_k = \frac{(-1)^k}{k^2+1}$

2.a Show sum converges

$$\lim_{k\to\infty} a_k = 0$$
$$\frac{(-1)^k}{k^2+1} < \frac{1}{k^2} \ \therefore \text{ Converges bc of comparison test}$$

2.b Find error upperbound with 10

$$E_{10} < |S_{11}| = \frac{1}{122}$$

2.c Find smallest value so differences of sum to infty and sum to n is $< \frac{1}{101}$

$$a_{n+1} < \frac{1}{101}$$
$$\frac{1}{(n+1)^2+1} < \frac{1}{101}$$
$$\boxed{n \leq 9}$$

2.d Find an upperbound

$$E_{10} < \lim_{a \to \infty} \int_{10}^{a} \frac{1}{k^2+1}$$
$$\tan^{-1}(k)\Big|_{10}^{a}$$
$$\boxed{\frac{\pi}{2} - \tan^{-1}(10)}$$

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$$a_k = \frac{(-1)^k}{k^2} : \; E_{100} < \frac{1}{10000}$$
$$b_k = \frac{2}{k^3} : \; E_{100} < \frac{2}{1000000}$$
E_{100} for $a_k < E_{100}$ for b_k $\therefore$ a_k aprox. is better

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4.a

$$\int_{11}^{\infty} a_k < E_n < \int_{10}^{\infty} a_k$$

4.b

$$\int_3^\infty f(x) + \frac{5n}{2n+5} < \int_2^\infty f(x)$$

Adding the sum brings the lower bound closer to the actual value

4.c $\quad a_k = ke^{-k^2}$

$$E_n < \int_n^\infty xe^{-x^2}dx \qquad\qquad u = -x^2$$

$$du = -2xdx$$

$$-\frac{1}{2}\int e^u du$$

$$\lim_{a\to\infty} e^{-k^2}\Big|_n^\infty$$

$$-\frac{1}{2}(0 - e^{-n^2})$$

$$\frac{1}{2e^{n^2}} \leq \frac{1}{100}$$

$$\frac{1}{e^{n^2}} \leq \frac{1}{50}$$

$$e^{n^2} \leq 50$$

$$\ln(50) \leq n^2$$

$$\boxed{\sqrt{\ln(50)} \leq n}$$