

Problem Set #5

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1 $\int \frac{x+1}{x^2+2x-3}$

1.a Solve integral with partial Fraction

$$\int \frac{x+1}{(x-2)(x-1)} dx$$
$$\int \frac{A}{x-2} + \frac{B}{x-1} dx = \frac{x+1}{\dots}$$

$$A(x-1) + B(x-2) = x+1$$

$$-1B = 2 \qquad x = 1$$

$$B = -2$$

$$A = 3 \qquad x = 2$$

$$\int \frac{3}{x-2} - \frac{2}{x-1} dx$$

$$3 \ln(|x-2|) - \ln(|x-1|) + c$$

1.b Solve using another Technique

$$\begin{aligned} \int \frac{x+1}{x^2+2x-3} & \qquad u = x^2 + 2x - 3 \\ & du = 2x + 2 \\ \frac{1}{2} \int \frac{du}{u} & \qquad \frac{1}{2} du = x + 1 \\ \frac{1}{2} \ln(|u|) + c & \\ \frac{1}{2} \ln(|x^2 + 2x - 3|) & \end{aligned}$$

1.c Evaluate $\int_0^5 dx$ or show that it diverges

$$\begin{aligned} & \int_0^5 \frac{x+1}{x^2+2x-3} \qquad x \neq 1 \\ \lim_{a \rightarrow 1} \int_0^a \frac{x+1}{x^2+2x-3} + \lim_{b \rightarrow 5} \int_1^b \frac{x+1}{x^2+2x-3} & \qquad u = x^2 + 2x - 3 \\ \lim_{a \rightarrow 1} \frac{1}{2} \int \frac{du}{u} + \frac{1}{2} \lim_{b \rightarrow 5} \int \frac{du}{u} & \qquad \frac{1}{2} du = x + 1 \\ \frac{1}{2} \ln(|u|) + \frac{1}{2} \ln(|u|) & \\ \lim_{a \rightarrow 1} \ln(\sqrt{x^2 + 2x - 3}) \Big|_0^a + \dots & \\ \ln(\sqrt{1^2 + 2 - 3}) & \\ \ln(0) \rightarrow DNE \therefore \text{Diverges} & \end{aligned}$$

2 Find using given values

2.a $\int_0^2 3(f(x) - 2)dx$

$$\begin{aligned} 3 \int_0^2 f(x)dx - 3 \int_0^2 2dx & \\ 3\left(\frac{1}{2} * 6\right) - 3(2x \Big|_0^2) & \qquad \int_0^2 f(x) = \frac{1}{2} \int_{-2}^2 f(x)dx \\ 9 - 3(2(2) - 0) & \\ 9 - 3(4) = -3 & \end{aligned}$$

2.b $\int_{-2}^{-1} f(x)dx$

$$\begin{aligned}\int_{-2}^0 f(x) &= 3 \\ \int_0^1 f(x) &= \int_{-1}^0 f(x) = 1 \\ \int_{-2}^0 f(x) - \int_{-1}^0 f(x) &= \int_{-2}^{-1} f(x) \\ 3 - 1 &= \int_{-2}^{-1} f(x) \\ \int_{-2}^{-1} f(x) &= 2\end{aligned}$$

2.c $\int_{-1}^2 xf(x)$

$$\begin{aligned}\int_0^1 xf(x)dx &= \int_{-1}^0 xf(x)dx = -\frac{1}{2} \\ \int_{-1}^2 xf(x) &= \int_{-1}^0 xf(x) + \int_0^2 xf(x)dx \\ &= -\frac{1}{2} + 4 \\ &= \frac{7}{2}\end{aligned}$$

3 $f(x)$ and $f'(x)$ are continuous. Use the table

3.a $\int_0^1 f'(x)$

$$\begin{aligned}&f(x)\big|_0^1 \\ &f(1) - f(0) \\ &5 - 3 = 2\end{aligned}$$

$$\mathbf{3.b} \quad \int_0^2 (f'(x) \sin(x) + f(x) \cos(x))$$

$$\int_0^2 f'(x) \sin(x) dx + \int_0^2 f(x) \cos(x)$$

$$\int_0^2 f'(x) \sin(x) dx$$

$$u = \sin(x)$$

$$dv = f'(x)$$

$$du = \cos(x)$$

$$v = f(x)$$

$$f(x) \sin(x) \Big|_0^2 - \int_0^2 f(x) \cos(x) dx$$

$$f(2) \sin(2) - f(0) \sin(0) - \int_0^2 f(x) \cos(x) dx$$

$$8 \sin(2) - \int_0^2 f(x) \cos(x) + \int_0^2 f(x) \cos(x)$$

$$8 \sin(2)$$

$$\mathbf{3.c} \quad \int_0^3 \frac{f'(x)e^x - f(x)e^x}{(e^x)^2} dx$$

$$\int_0^3 \frac{f'(x) - f(x)}{e^x} dx$$

$$\int_0^3 \frac{f'(x)}{e^x} - \int_0^3 \frac{f(x)}{e^x}$$

$$u = e^{-x}$$

$$dv = f'(x)$$

$$du = -e^{-x}$$

$$v = f(x)$$

$$\frac{f(x)}{e^x} \Big|_0^3 + \int_0^3 \frac{f(x)}{e^x}$$

$$\frac{f(3)}{e^3} - \frac{f(0)}{1} + \int_0^3 \frac{f(x)}{e^x} - \int_0^3 \frac{f(x)}{e^x}$$

$$\frac{9}{e^3} - 3$$

$$9e^{-3} - 3$$

$$\mathbf{3.d} \quad \int_0^1 f(x)f'(x)$$

$$\int u du \qquad u = f(x)$$

$$\int_3^5 u du \qquad du = f'(x)$$

$$\frac{1}{2}u^2 \Big|_3^5$$

$$\frac{1}{2}(5)^2 - \frac{1}{2}3^2$$

$$\frac{25}{2} - \frac{9}{2} = 8$$

$$\mathbf{4} \quad \text{Average value} = \frac{1}{b-a} \int_a^b f(x) dx$$

4.a Find number(s) **b**, so a.v of $f(x) = 2 + 6x - 3x^2$ on interval $[0, b]$ equal to **3**

$$\frac{1}{b-0} \int_0^b 2 + 6x - 3x^2 dx = 3$$

$$\frac{1}{b} (2x + 3x^2 - x^3) \Big|_0^b = 3$$

$$\frac{1}{b} (2b + 3b^2 - b^3 - 0) = 3$$

$$\frac{1}{b} (2b + 3b^2 - b^3) = 3$$

$$2 + 3b - b^2 = 3$$

$$-1 + 3b - b^2 = 0$$

$$b = \frac{-3 \pm \sqrt{9 - 4(-1)(-1)}}{-2}$$

$$b = \frac{-3 \pm \sqrt{9 - 4}}{-2}$$

$$b = \frac{-3 \pm \sqrt{5}}{-2}$$

4.b

$$\frac{1}{8-1} \int_1^8 f(x) dx$$

$$\int_1^6 f(x) = 5 * A.V(1 < x < 6) \quad \text{Equals inverse of } \frac{1}{b-a} * value$$
$$= 5 * 4 = 20$$

$$\int_6^8 f(x) = 2 * A.V(6 < x < 8)$$
$$= 2(5) = 10$$

$$\frac{1}{7} \int_1^8 f(x) = \frac{1}{7} \left(\int_1^6 f(x) + \int_6^8 f(x) \right)$$
$$= \frac{1}{7} * (20 + 10)$$
$$= \frac{30}{7}$$

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5.a $\int_1^\infty \frac{1}{(x+1)(2x+3)} dx$

$$\lim_{a \rightarrow \infty} \int_1^a \frac{1}{(x+1)(2x+3)} dx$$

$$\frac{A}{x+1} + \frac{B}{2x+3}$$

$$A(2x+3) + B(x+1) = 1$$

$$A(1) = 1$$

$$x = -1$$

$$B(-\frac{3}{2} + 1) = 1$$

$$x = \frac{-3}{2}$$

$$-\frac{1}{2}B = 1$$

$$B = -2$$

$$\lim_{a \rightarrow \infty} \int_1^a \frac{1}{x+1} - \lim_{a \rightarrow \infty} 2 \int_1^a \frac{1}{2x+3}$$

$$\lim_{a \rightarrow \infty} \ln(|x+1|) \Big|_1^a - 2 \lim_{a \rightarrow \infty} \ln(|2x+3|) * \frac{1}{2}$$

$$\lim_{a \rightarrow \infty} \ln(|x+1|) \Big|_1^a - \lim_{a \rightarrow \infty} \ln(|2x+3|)$$

$$\lim_{a \rightarrow \infty} \ln \frac{|x+1|}{|2x+3|} \Big|_1^a$$

$$LH \rightarrow \lim_{a \rightarrow \infty} \ln \frac{a+1}{2a+3} - \ln \frac{2}{5}$$

$$\lim_{a \rightarrow \infty} \ln(\frac{1}{2}) + \ln(\frac{5}{2})$$

$$\ln(\frac{5}{4})$$

$$\mathbf{5.b} \quad \int_0^e \ln(x) dx$$

$$\lim_{a \rightarrow 0} \int_a^e \ln(x)$$

$$u = \ln(x)$$

$$dv = 1$$

$$du = \frac{1}{x}$$

$$v = x$$

$$\lim_{a \rightarrow 0} x \ln(x) \Big|_a^e - \lim_{a \rightarrow 0} \int_a^e 1 dx$$

$$\lim_{a \rightarrow 0} \frac{\ln(x)}{\frac{1}{x}} \Big|_a^e - \lim_{a \rightarrow 0} x \Big|_a^e$$

$$LH \frac{1}{1/e} - \frac{\ln(a)}{1/a} - e$$

$$e - \frac{1/a}{-1/a^2} - e$$

$$\lim_{a \rightarrow 0} \frac{a^{-1}}{-a^{-2}}$$

$$\lim_{a \rightarrow 0} -a^1 = 0$$

$$\mathbf{5.c} \quad \int_{-\pi/2}^{\pi/2} \frac{x \cos(x^2)}{(\sin(x^2))^2}$$

$$\lim_{a \rightarrow 0} \int_{-\pi/2}^a \frac{x \cos(x^2)}{(\sin(x^2))^2} + \lim_{b \rightarrow 0} \int_b^{\pi/2} \frac{x \cos(x^2)}{(\sin(x^2))^2}$$

$$\lim_{b \rightarrow 0} \frac{1}{2} \int_b^{\pi/2} \frac{du}{u^2}$$

$$u = \sin(x^2)$$

$$\lim_{b \rightarrow 0} \frac{1}{2} \int_b^{\pi/2} u^{-2}$$

$$du = 2x \cos(x^2)$$

$$\frac{1}{2} \lim_{b \rightarrow 0} (-u^{-1}) \Big|_b^{\pi/2}$$

$$\frac{1}{2} du = x \cos(x^2)$$

$$\frac{1}{2} \lim_{b \rightarrow 0} \left(-\frac{1}{\sin(x^2)} \right) \Big|_b^{\pi/2}$$

$$\frac{1}{2} \left(-\frac{1}{\sin(\pi/4)} + \frac{1}{\sin(0)} \right)$$

$$\frac{1}{\sin(0)} = \frac{1}{0} \rightarrow \infty \therefore \text{Divergent}$$