1

1.a

1.a.1 Find the following probabilities

- $P(A) = \frac{5}{20}$
- $P(B) = \frac{10}{20}$
- $P(A \bigcap B) = \frac{2}{20} = \frac{1}{10}$

1.a.2 Are A and B independent

$P(A \bigcap B) = \frac{1}{10} \neq \frac{5}{20} * \frac{10}{20} \therefore$ No, they are not independent

1.b Roll 3 times

$4(DDD), 16(DDN), 16(DNN), 16(NNN)$

$$P(X) = \frac{4}{20}$$
$$p(0) = \frac{3}{5} * \frac{1}{8} = \frac{3}{40}$$
$$p(1) = \frac{3}{5} * \frac{3}{8} = \frac{9}{40}$$
$$p(2) = \frac{3}{5} * \frac{3}{8} = \frac{9}{40}$$
$$p(3) = \frac{3}{5} * \frac{1}{8} = \frac{3}{40}$$

2

2.a

$$\sum_{k=1}^{n} x_{kp(x_k)}$$

$$p(Y = -2 \bigcup -1) = \frac{1}{2}$$

$$\frac{1}{10} + a = \frac{1}{2}$$

$$a = \frac{4}{10}$$

$$-2(\frac{1}{10}) - \frac{4}{10} + 0(b) + 1(\frac{3}{10}) + 2(c) = 0$$

$$-\frac{2}{10} - \frac{4}{10} + 0(b) + \frac{3}{10} + 2(c) = 0$$

$$-\frac{6}{10} + \frac{3}{10} + 2(c) = 0$$

$$-\frac{3}{10} + 2(c) = 0$$

$$c = \frac{3}{20}$$

$$b = 1 - \frac{4}{10} - \frac{3}{10} - \frac{1}{10} - \frac{3}{20}$$

$$b = 1 - \frac{8}{10} - \frac{3}{20}$$

$$b = 1 - \frac{16}{20} - \frac{3}{20}$$

$$b = 1 - \frac{19}{20} = \frac{1}{20}$$

2.b Find $E[Y^2]$

$$-2(\frac{1}{10}) - \frac{4}{10} + 0(b) + 1(\frac{3}{10}) + 2(c) = 0$$

3

3.a

3.a.1

$p(1) = \frac{c}{1}$

4

4.a a_1

$$\frac{1}{2(1)+4} = \frac{1}{6}$$

4.b $\sum_{k=1}^{\infty} a_k$

$$\lim_{a\to\infty} \int_1^a 1 - \frac{n}{2n+4} dn$$
$$\lim_{a\to\infty} n\Big|_1^a - \int_1^a \frac{n}{4(\frac{n}{2}+1)}$$

4.c $\lim_{k\to\infty}$

$$\lim_{n\to\infty} \frac{n}{2n+4} = \frac{1}{2}$$
$$1 - \frac{1}{2} = \frac{1}{2}$$

4.d $\sum_{k=101}^{200} a_k$

4.e a_{100}

$$1 - \frac{100}{200+4} = 1 - \frac{100}{204}$$

5

5.a

$$\sum_{k=1}^{\infty}(9^{\frac{1}{k}} - 9^{\frac{1}{k+2}})$$

$$\begin{aligned}
S_1 &= 9^1 - 9^{\frac{1}{3}} \\
S_2 &= (9^1 - 9^{\frac{1}{3}}) + (9^{\frac{1}{2}} - 9^{\frac{1}{4}}) \\
S_3 &= (9^1 - \cancel{9^{\frac{1}{3}}}) + (9^{\frac{1}{2}} - 9^{\frac{1}{4}}) + (\cancel{9^{\frac{1}{3}}} - 9^{\frac{1}{5}}) \\
S_3 &= (9^1) + (3 - 9^{\frac{1}{4}}) + (-9^{\frac{1}{5}}) \\
S_n &= 9 + 3 - 9^{\frac{1}{n+1}} - 9^{\frac{1}{n+2}} \\
S_n &= 12 - 9^{\frac{1}{n+1}} - 9^{\frac{1}{n+2}} \\
\lim_{n\to\infty} S_n &= 12 - 9^0 - 9^0 = 10
\end{aligned}$$

5.b $\sum_{k=1}^{\infty} e^{\sin(\frac{1}{k})}$

$$\lim_{k\to\infty} e^{\sin(0)}$$

$$\lim_{k\to\infty} e^0 = 1 \neq 0 \therefore \text{ Divergent because of limit test}$$

5.c $\sum_{k=1}^{\infty} \frac{1+e^{-k}}{4e^{-k}+3}$

$$\lim_{k\to\infty} \frac{1+e^{-k}}{4e^{-k}+3}$$

$$\lim_{k\to\infty} \frac{1}{4+3} = \frac{1}{7} \neq 0 \therefore \text{ Divergent because of limit test}$$

5.d $\sum_{k=1}^{\infty}(\cos(\frac{\pi}{k}) - \cos(\frac{\pi}{k+2}))$

$$S_1 = \cos(\frac{\pi}{1}) - \cos(\frac{\pi}{3})$$
$$S_2 = (\cos(\frac{\pi}{1}) - \cos(\frac{\pi}{3})) + (\cos(\frac{\pi}{2}) - \cos(\frac{\pi}{4}))$$
$$S_3 = (\cos(\frac{\pi}{1}) - \cancel{\cos(\frac{\pi}{3})}) + (\cos(\frac{\pi}{2}) - \cos(\frac{\pi}{4})) + (\cancel{\cos(\frac{\pi}{3})} - \cos(\frac{\pi}{5}))$$
$$S_3 = (\cos(\frac{\pi}{1})) + (\cos(\frac{\pi}{2}) - \cos(\frac{\pi}{4})) - \cos(\frac{\pi}{5}))$$
$$S_n = -1 + 0 - \cos(\frac{\pi}{n+1}) - \cos(\frac{\pi}{n+2})$$
$$S_n = -1 - \cos(\frac{\pi}{n+1}) - \cos(\frac{\pi}{n+2})$$
$$\lim_{n\to\infty} = -1 - \cos(0) - \cos(0) = -1 - 1 - 1 = -3$$