Canonical Sphere

$$x^2 + y^2 + z^2 = 1$$ where radius $= 1$
center $= (0,0,0)$

Ray: $p(t) = \underset{\text{start}}{e} + t\underset{\text{direction}}{d}$

$$\begin{cases} x: x_e + t d_x \\ y: y_e + t d_y \\ z: z_e + t d_z \end{cases}$$

Sphere Equation

$$(x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 = r^2$$

$x \to x_e + t d_x$, $y \to y_e + t d_y$, $z \to z_e + t d_z$

(c_x, c_y, c_z) = center / pos of sphere

r = radius

$$\underbrace{(x_e + t d_x - c_x)^2}_{} + (y_e + t d_y - c_y)^2 + (z_e + t d_z - c_z)^2 = r^2$$

$\boxed{\text{solve for } t}$

$$(x_e + t d_x - c_x)(x_e + t d_x - c_x)$$

$$\underbrace{(x_e + t\,dx - cx)^2}_{} + (y_e + t\,dy - cy)^2 + (z_e + t\,dz - cz)^2 = r^2$$

$\boxed{\text{solve for } t}$

$$(x_e + t\,dx - cx)(x_e + t\,dx - cx)$$

$$x_e^2 + x_e t\,dx - x_e cx + x_e t\,dx + t^2 dx^2 - t\,dx\,cx - x_e cx - t\,dx\,cx + cx^2$$

$$x_e^2 + 2x_e t\,dx - 2x_e cx + t^2 dx^2 - 2t\,dx\,cx + cx^2$$

$$t^2 \underbrace{dx^2}_{1} + t\underbrace{(2x_e dx - 2dx\,cx)}_{2} + \underbrace{x_e^2 + cx^2 - 2x_e cx}_{3}$$

$$t^2 (1) + t(2) + (3)$$

similar/same derivation for

$(y_e + t\,dy - cy)^2$

and

$(z_e + t\,dz - cz)^2$

(1) $a = dx^2 + dy^2 + dz^2$

(2) $b = (2x_e dx - 2dx\,cx) + (2y_e dy - 2dy\,cy) + (2z_e dz - 2dz\,cz)$

(3) $c = (x_e^2 + cx^2 - 2x_e cx) + (y_e^2 + cy^2 - 2y_e cy) + (z_e^2 + cz^2 - 2z_e cz)$

Don't forget → was set equal to r^2

↳ we want $= 0$

(3) $C = (x_e^2 + cx^2 - 2x_e cx) + (y_e^2 + cy^2 - 2y_e cy) + (z_e^2 + cz^2 - 2z_e cz) - r^2$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

you can get 2 t values → take smallest nonnegative t