EECS E6720: Bayesian Models for Machine Learning
Homework 4

Josh Rutta

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Problem 1

1(a)

Josh Rit[illegible]

Bayes ML

HW 4

$\binom{20}{x_i}\theta_{c_i}^{x_i}(1-\theta_{c_i})^{20-x_i}$

1. $x_i|c_i \sim Bin(20, \theta_{c_i})$, $c_i \overset{iid}{\sim} Discrete(\pi)$

Likelihood: $p(x_1, \dots, x_n | \pi, \theta)$

a)

E-step:

$$p(x|\pi,\theta) = \prod_{i=1}^{n} p(x_i|\pi,\theta)$$

$$\prod_{i=1}^{n} p(x_i, c_i|\pi,\theta) = \prod_{i=1}^{n} p(x_i|c_i,\pi,\theta)\,p(c_i|\pi) = \prod_{i=1}^{n}\prod_{j=1}^{k} (\pi_j Bin(20,\theta_{c_i}))^{\mathbb{1}(c_i=j)} = \prod_{i=1}^{n} p(x_i, c_i=j|\pi,\theta)$$

$$p(x|\pi,\theta) = \sum_{c_1=1}^{k}\cdots\sum_{c_n=1}^{k}\prod_{i=1}^{n} p(x_i, c_i|\pi,\theta) = \prod_{i=1}^{n}\sum_{j=1}^{k} p(x_i, c_i=j|\pi,\theta) = \prod_{i=1}^{n}\sum_{j=1}^{k}\pi_j Bin(20,\theta_j)$$

$$\ln p(x|\pi,\theta) = \sum_c q(c)\ln\frac{p(x,c|\pi,\theta)}{q(c)} + \sum_c q(c)\ln\frac{q(c)}{p(c|x,\pi,\theta)}$$

$$q(c) = p(c|x,\pi,\theta) \propto p(x|c,\pi,\theta)\,p(c|\pi) = \prod_{i=1}^{n}\frac{p(x_i|c_i,\pi,\theta)\,p(c_i|\pi)}{\sum_{j=1}^{k} p(x_i|c_i=j,\pi,\theta)\,p(c_i=j|\pi)}$$

$$= \prod_{i=1}^{n} p(c_i|x_i,\pi,\theta) = \prod_{i=1}^{n} q(c_i)$$

$$\phi_i(j) = p(c_i=j|x_i,\pi,\theta) = \frac{\pi_j Bin(20,\theta_j)}{\sum_{k=1}^{K}\pi_k Bin(20,\theta_k)}$$

$$\mathcal{L}(\pi,\theta) = \sum_{i=1}^{n}\mathbb{E}_{q(c_i)}[\ln p(x_i,c_i|\pi,\theta)] + const.$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{k}\underbrace{\phi_i(j)}_{\text{using } \pi \text{ and } \theta \text{ from previous iteration}}\left[\ln\pi_j + \ln\binom{20}{x_i} + x_i\ln\theta_j + (20-x_i)\ln(1-\theta_j)\right] + const.$$

M-step: $\nabla_{\pi_j}\mathcal{L} = 0$ s.t. $\sum_j \pi_j = 1$, $0 \le \pi_j \le 1$: $\pi_j = \sum_{i=1}^{n}\frac{\phi_i(j)}{n}$

$\nabla_{\theta_j}\mathcal{L} = 0$; $0 \le \theta_j \le 1$: $\sum_{i=1}^{n}\phi_i(j)\left[\frac{x_i}{\theta_j} - \frac{(20-x_i)}{1-\theta_j}\right] = 0$

$$\frac{\sum_{i=1}^{n}\phi_i(j)x_i}{\theta_j} = \frac{\sum_{i=1}^{n}\phi_i(j)(20-x_i)}{(1-\theta_j)}$$

$$\frac{1}{\theta_j} - \cancel{1} = \frac{\sum_{i=1}^{n}\phi_i(j)(20-\cancel{x_i})}{\sum_{i=1}^{n}\phi_i(j)x_i} \Rightarrow \theta_j = \frac{\sum_{i=1}^{n}\phi_i(j)x_i}{20\sum_{i=1}^{n}\phi_i(j)}$$

Pseudo-Code:

Input: Data $x_1, \dots, x_n$, $x_i \in \{0, \dots, 20\}$, Number of clusters K

Output: Binomial mixture model parameters π, θ, and cluster assignment distributions ϕ_i

1. Initialize $\pi^{(0)}$ and $\theta^{(0)}$ in some way

2. At iteration t,

   (a) E-step: for $i = 1, \dots, n$ and $j = 1, \dots, K$ set

   $$\phi_i^{(t)}(j) = \frac{\pi_j^{(t-1)} Bin(20, \theta_j^{(t-1)})}{\sum_{k=1}^{K}\pi_k Bin(20, \theta_k^{(t-1)})}$$

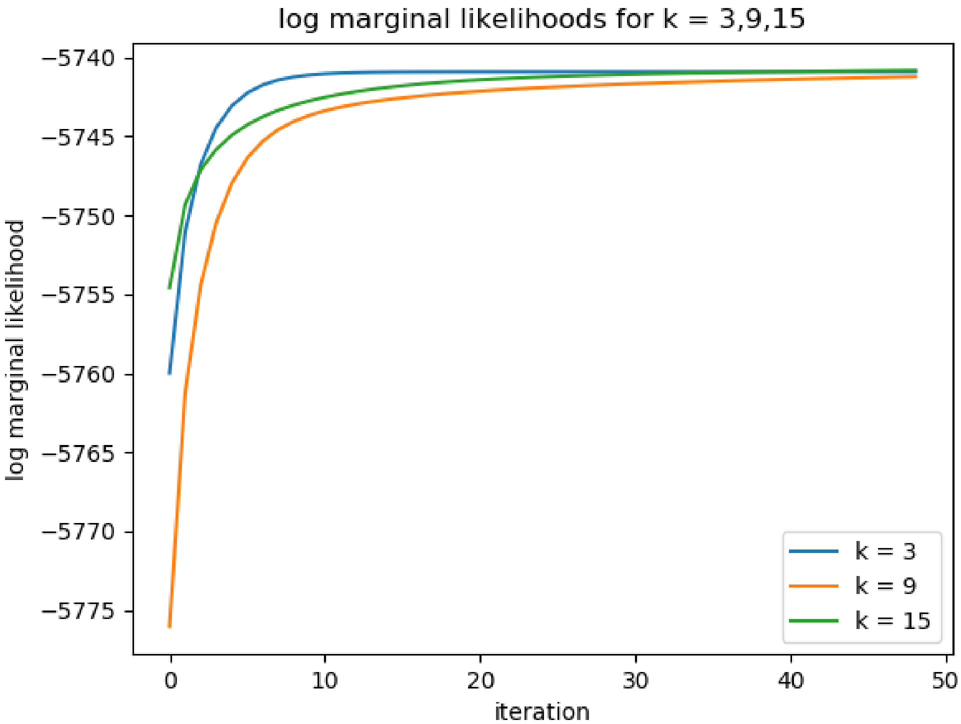
   (b) M-step: Set

   $$\pi_j^{(t)} = \frac{\sum_{i=1}^{n}\phi_i^{(t)}(j)}{n}$$

   $$\theta_j^{(t)} = \frac{\sum_{i=1}^{n}\phi_i(j)x_i}{20\sum_{i=1}^{n}\phi_i(j)}$$

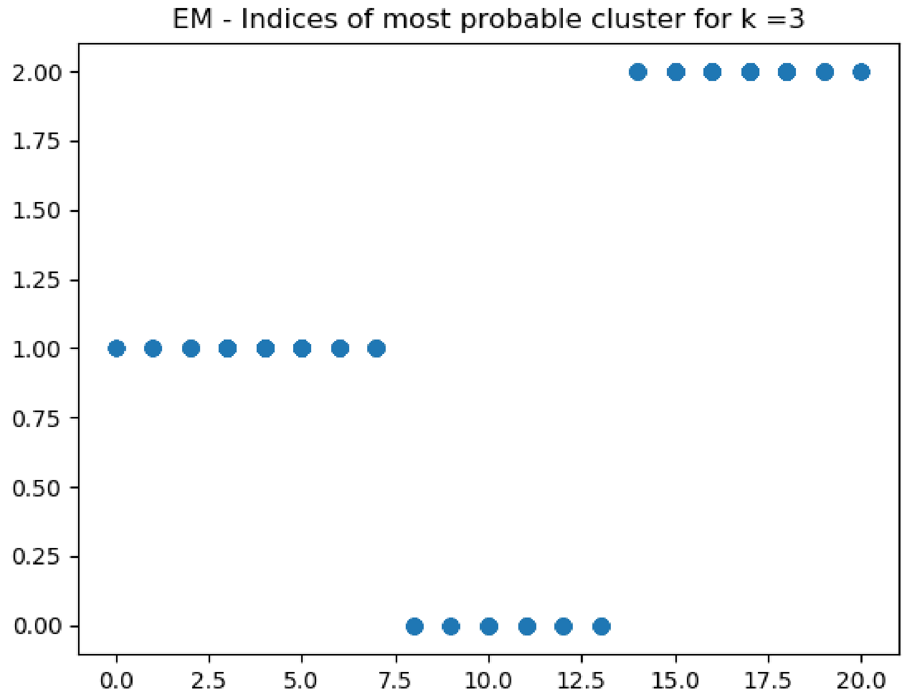
3. Calculate $\ln p(c|X,\pi,\theta) = \sum_{i=1}^{n}\ln\left(\sum_{k=1}^{K}\pi_k Bin(20,\theta_k)\right)$

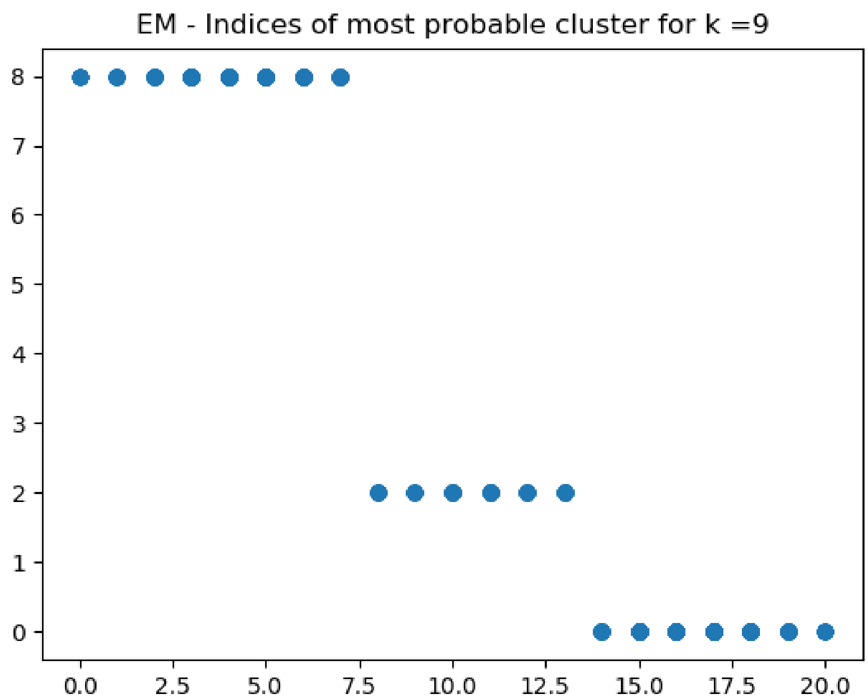
1 (b)

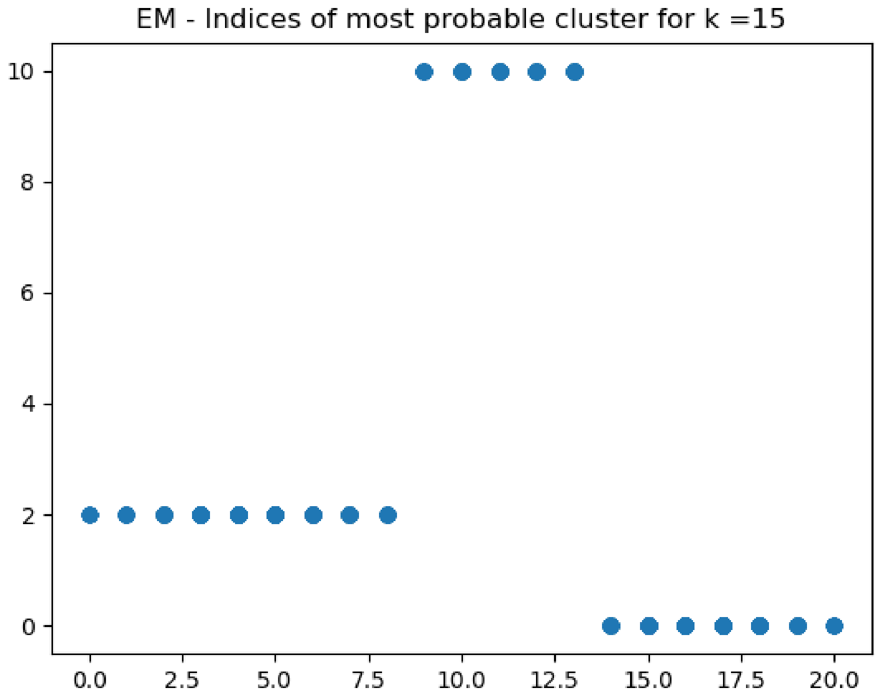


log marginal likelihoods for k = 3,9,15
log marginal likelihood
iteration
−5740
−5745
−5750
−5755
−5760
−5765
−5770
−5775
0
10
20
30
40
50
k = 3
k = 9
k = 15

1(c)


EM - Indices of most probable cluster for k =3
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


EM - Indices of most probable cluster for k =9
8
7
6
5
4
3
2
1
0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


EM - Indices of most probable cluster for k =15
10
8
6
4
2
0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

Problem 2

2(a)

$$\mathrm{Bin}(20,\theta_{c_i}) = \binom{20}{x_i}\theta_{c_i}^{x_i}(1-\theta_{c_i})^{20-x_i}$$

$$\mathrm{Beta}(a,b) = \underbrace{\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}}_{\frac{1}{B(a,b)}}\theta_k^{a-1}(1-\theta_k)^{b-1}$$

$$\mathrm{Dir}(\alpha) = \frac{\Gamma\left(\sum_{i=1}^K \alpha_i\right)}{\prod_{i=1}^K \Gamma(\alpha_i)}\prod_{i=1}^K \pi_i^{\alpha_i - 1} = \frac{1}{B(\alpha)}\prod_{i=1}^K \pi_i^{\alpha_i-1}$$

2. $\pi \sim \mathrm{Dir}(\alpha)$, $\theta_k \overset{iid}{\sim} \mathrm{Beta}(a,b)$

$\alpha = \frac{1}{10}$ $\quad a = 0.5,\ b = 0.5$

a) $q(\pi,\theta,c) = q(\pi)\left[\prod_{k=1}^K q(\theta_k)\right]\left[\prod_{i=1}^n q(c_i)\right]$

$$p(x,c,\pi,\theta) = p(x|c,\theta)\,p(c|\pi)\,p(\pi)\,p(\theta)$$
$$= \left[\prod_{i=1}^n \mathrm{Bin}(20,\theta_{c_i})\right]\left[\prod_{i=1}^n\prod_{j=1}^K \pi_j^{\mathbb{1}(c_i=j)}\right]\left[\mathrm{Dir}(\alpha)\right]\left[\prod_{k=1}^K \mathrm{Beta}(a,b)\right]$$

$$\ln p(x,c,\pi,\theta) = \sum_{i=1}^n\sum_{j=1}^k \mathbb{1}(c_i=j)\left(\ln\binom{20}{x_i} + x_i\ln\theta_{c_i} + (20-x_i)\ln(1-\theta_{c_i})\right) - \ln(B(\alpha)) + k\ln\left(\frac{1}{B(a,b)}\right) + \sum_{k=1}^K\left[(\alpha_i-1)\ln\pi_k + (a-1)\ln\theta_k + (b-1)\ln(1-\theta_k)\right]$$

$$q(\pi) \propto \exp\left\{\sum_{i=1}^n \mathbb{E}_{-q}\left[\ln p(c_i|\pi)\right] + \ln p(\pi)\right\} \propto \exp\left\{\sum_{i=1}^n \mathbb{E}_{-q}\Big[\underbrace{\sum_{j=1}^K \mathbb{1}(c_i=j)}\ln\pi_j\Big] + \sum_{j=1}^K(\alpha-1)\ln\pi_j\right\}$$

$$\mathbb{E}\left[\mathbb{1}(c_i=j)\right] = q(c_i=j) = \phi_i(j)$$

$$q(\pi) \propto \exp\left\{\sum_{i=1}^n\left(\sum_{j=1}^K \phi_i(j)\ln\pi_j\right) + \sum_{j=1}^K(\alpha-1)\ln\pi_j\right\} \propto \prod_{j=1}^K \pi_j^{\alpha-1+\sum_{i=1}^n\phi_i(j)} = \mathrm{Dir}(\alpha'),\ \alpha_j' = \alpha_j + \sum_{i=1}^n \phi_i(j)$$

$$q(\theta_k) \propto \exp\left\{\sum_{i=1}^n\left[\mathbb{E}_{-q}\left[\ln(\mathrm{Bin}(20,\theta_{c_i=k}))\right]\right] + (a-1)\ln\theta_k + (b-1)\ln(1-\theta_k)\right\}$$

$$q(\theta_k) \propto \exp\left\{\sum_{i=1}^n \mathbb{E}_{-q}\left[\mathbb{1}[c_i=k]\left[x_i\ln\theta_k + (20-x_i)\ln(1-\theta_k)\right]\right] + (a-1)\ln\theta_k + (b-1)\ln(1-\theta_k)\right\}$$

$$\propto \exp\left\{\sum_{i=1}^n\left[\phi_i(k)\left(x_i\ln\theta_k + (20-x_i)\ln(1-\theta_k)\right)\right] + (a-1)\ln\theta_k + (b-1)\ln(1-\theta_k)\right\} \propto \theta_k^{\sum_{i=1}^n x_i\phi_i(k)+a-1}(1-\theta_k)^{\sum_{i=1}^n\phi_i(k)(20-x_i)+b-1}$$

$$q(\theta_k) = \mathrm{Beta}(a_k', b_k'),\quad a_k' = \sum_{i=1}^n x_i\phi_i(k) + a,\quad b_k' = \sum_{i=1}^n \phi_i(k)(20-x_i) + b$$

$$q(c_i=j) \propto \exp\left\{\mathbb{E}_{-q}\left[\left[\left(x_i\ln\theta_j + (20-x_i)\ln(1-\theta_j)\right) + \ln\pi_j\right]\right]\right\} \propto \exp\left\{\left[x_i(\psi(a_j') - \psi(a_j'+b_j')) + (20-x_i)(\psi(b_j') - \psi(a_j'+b_j')) + \psi(\alpha_j') - \cancel{\psi\left(\sum_{k=1}^K \alpha_k'\right)}\right]\right\}$$

(some constant – can ignore

Input: Data $x_1,\dots,x_n$, $x_i \in \{0,\dots,20\}$, Number of clusters K

Output: Pameters for $q(\pi)$, $q(\theta_k)$, and $q(c_i)$

1. Initialize $(\alpha_1^{(0)},\dots,\alpha_K^{(0)})$, $(a_1^{(0)},\dots,a_K^{(0)})$, $(b_1^{(0)},\dots,b_K^{(0)})$ in some way

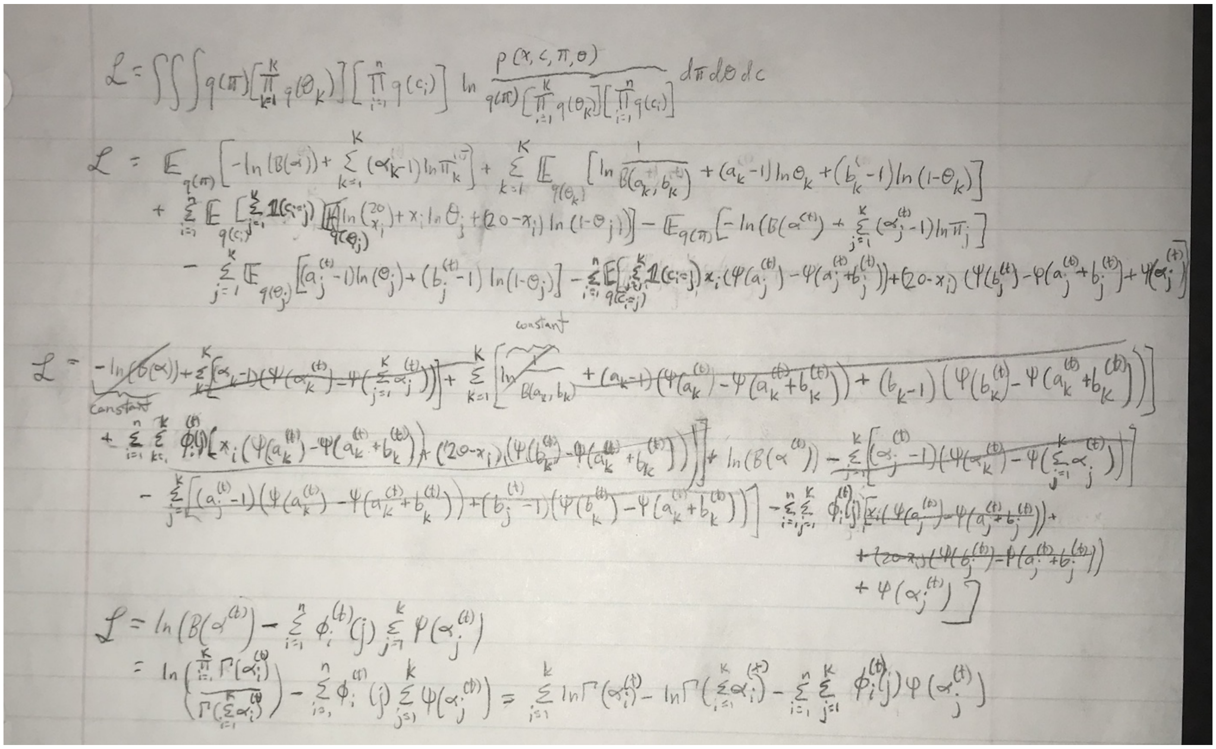
2. At iteration t: Update $q(c_i)$ for $i=1,\dots,n$

(a) $$\phi_i^{(t)}(j) = q(c_i=j) = \frac{\exp\{x_i(\psi(a_j^{(t-1)}) - \psi(a_j^{(t-1)}+b_j^{(t-1)})) + (20-x_i)(\psi(b_j^{(t-1)}) - \psi(a_j^{(t-1)}+b_j^{(t-1)})) + \psi(\alpha_j^{(t-1)}) - \}}{\sum_{\ell=1}^K \exp\{x_i(\psi(a_\ell^{(t-1)}) - \psi(a_\ell^{(t-1)}+b_\ell^{(t-1)})) + (20-x_i)(\psi(b_\ell^{(t-1)}) - \psi(a_\ell^{(t-1)}+b_\ell^{(t-1)})) + \psi(\alpha_\ell^{(t-1)}) - \}}$$

(b) Update $q(\theta_k)$ for $k=1,\dots,K$: $a_k^{(t)} = \sum_{i=1}^n x_i\phi_i^{(t)}(k) + a$, $b_k^{(t)} = \sum_{i=1}^n \phi_i^{(t)}(k)(20-x_i) + b$

(c) Update $q(\pi)$: $\alpha_j^{(t)} = \alpha_j + \sum_{i=1}^n \phi_i^{(t)}(j)$

(d) Calculate Variational Objective $f^{\underline{n}}$: $\mathcal{L} = \iiint q(\pi)\left[\prod_{i=1}^K q(\theta_k)\right]\left[\prod_{i=1}^n q(c_i)\right]\ln\frac{p(x,c,\pi,\theta)}{q(\pi)\left[\prod_{i=1}^K q(\theta_k)\right]\left[\prod_{i=1}^n q(c_i)\right]}\,d\pi\,d\theta\,dc$



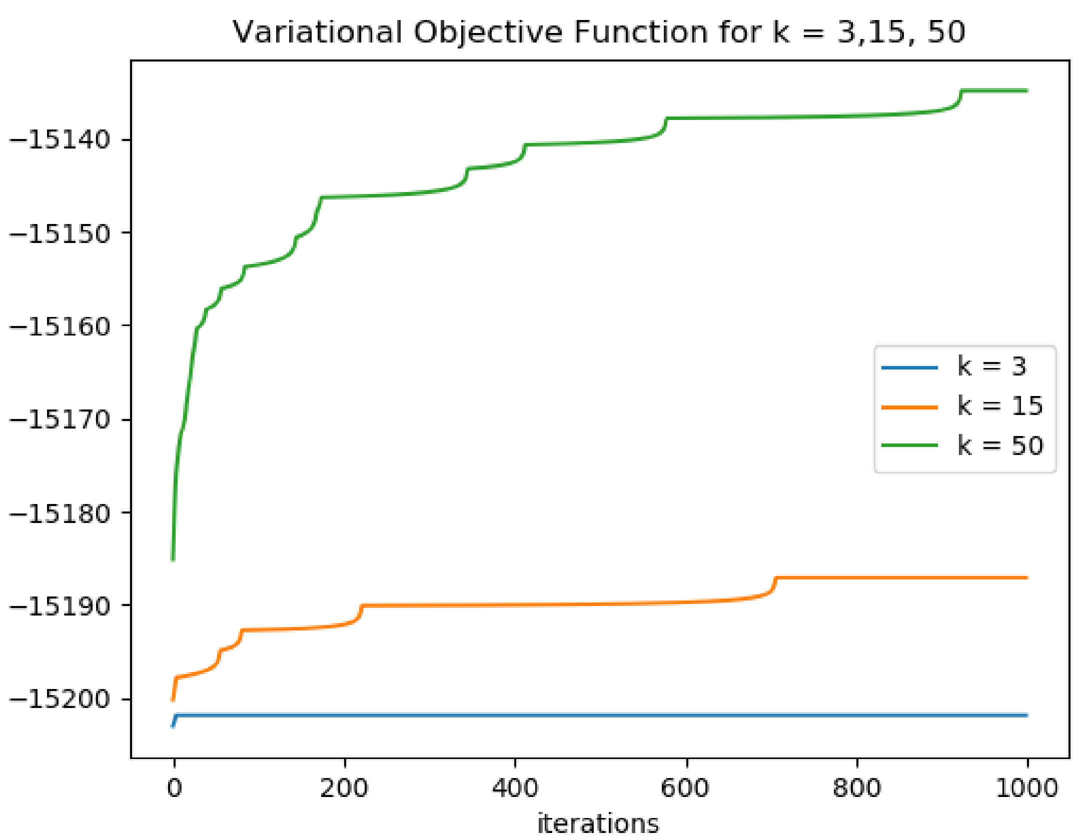
$$\mathcal{L} = \iiint q(\pi)\left[\prod_{k=1}^{K} q(\theta_k)\right]\left[\prod_{i=1}^{n} q(c_i)\right] \ln \frac{p(x, c, \pi, \theta)}{q(\pi)\left[\prod_{i=1}^{K} q(\theta_k)\right]\left[\prod_{i=1}^{n} q(c_i)\right]}\, d\pi\, d\theta\, dc$$

$$\mathcal{L} = \mathbb{E}_{q(\pi)}\left[-\ln(B(\alpha)) + \sum_{k=1}^{K} (\alpha_k - 1)\ln \pi_k\right] + \sum_{k=1}^{K} \mathbb{E}_{q(\theta_k)}\left[\ln \frac{1}{B(a_k, b_k)} + (a_k - 1)\ln\theta_k + (b_k - 1)\ln(1-\theta_k)\right]$$
$$+ \sum_{i=1}^{n} \mathbb{E}_{q(c_i)}\left[\sum_{j=1}^{K} \mathbb{1}(c_i = j)\, \mathbb{E}_{q(\theta_j)}\left[\ln\binom{20}{x_i} + x_i \ln\theta_j + (20 - x_i)\ln(1-\theta_j)\right]\right] - \mathbb{E}_{q(\pi)}\left[-\ln(B(\alpha^{(t)})) + \sum_{j=1}^{K}(\alpha_j^{(t)} - 1)\ln\pi_j\right]$$
$$- \sum_{j=1}^{K} \mathbb{E}_{q(\theta_j)}\left[(a_j^{(t)} - 1)\ln(\theta_j) + (b_j^{(t)} - 1)\ln(1-\theta_j)\right] - \sum_{i=1}^{n} \mathbb{E}_{q(c_i = j)}\left[\sum_{j=1}^{K} \mathbb{1}(c_i = j)\, x_i(\psi(a_j^{(t)}) - \psi(a_j^{(t)} + b_j^{(t)})) + (20 - x_i)(\psi(b_j^{(t)}) - \psi(a_j^{(t)} + b_j^{(t)})) + \psi(\alpha_j^{(t)})\right]$$

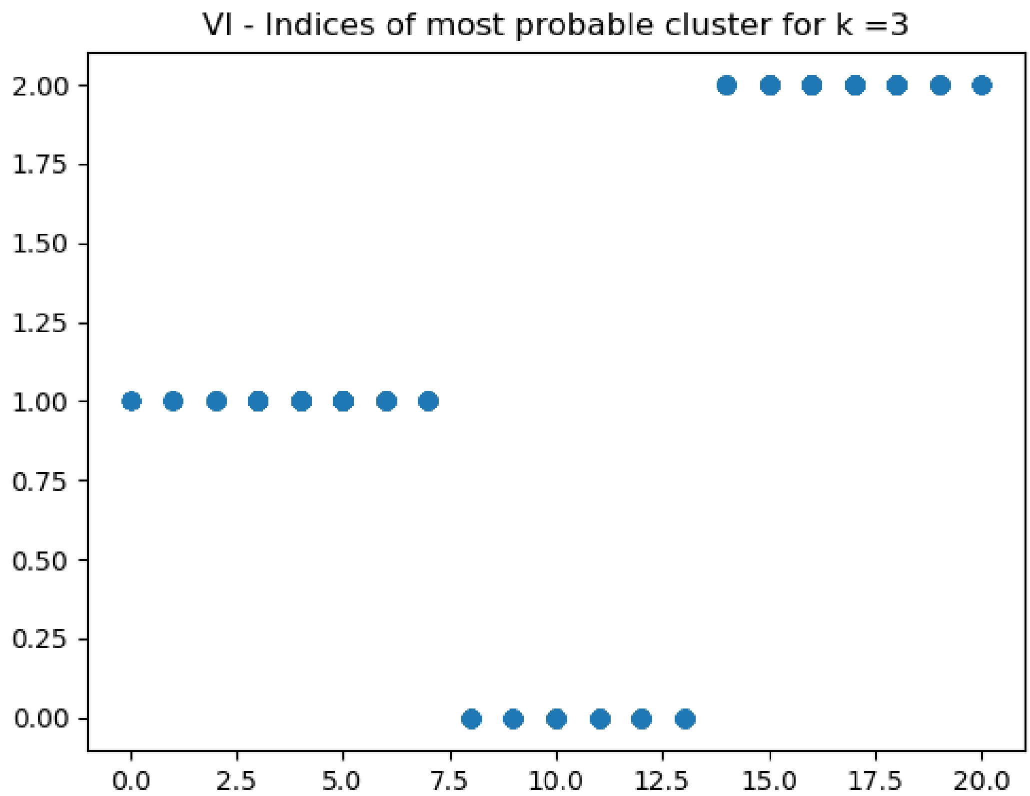
$$\mathcal{L} = \underbrace{\cancel{-\ln(B(\alpha))}}_{\text{constant}} + \cancel{\sum_{k=1}^{K}\left[(\alpha_k - 1)(\psi(\alpha_k^{(t)}) - \psi(\sum_{j=1}^{K}\alpha_j^{(t)}))\right]} + \sum_{k=1}^{K}\left[\overbrace{\cancel{\ln\frac{1}{B(a_k, b_k)}}}^{\text{constant}} + \cancel{(a_k - 1)(\psi(a_k^{(t)}) - \psi(a_k^{(t)} + b_k^{(t)})) + (b_k - 1)(\psi(b_k^{(t)}) - \psi(a_k^{(t)} + b_k^{(t)}))}\right]$$
$$+ \sum_{i=1}^{n}\sum_{k=1}^{K} \phi_i^{(t)}(j)\left[\cancel{x_i(\psi(a_k^{(t)}) - \psi(a_k^{(t)} + b_k^{(t)})) + (20 - x_i)(\psi(b_k^{(t)}) - \psi(a_k^{(t)} + b_k^{(t)}))}\right] + \ln(B(\alpha^{(t)})) - \cancel{\sum_{j=1}^{K}\left[(\alpha_j^{(t)} - 1)(\psi(\alpha_k^{(t)}) - \psi(\sum_{j=1}^{K}\alpha_j^{(t)}))\right]}$$
$$- \cancel{\sum_{j=1}^{K}\left[(a_j^{(t)} - 1)(\psi(a_k^{(t)}) - \psi(a_k^{(t)} + b_k^{(t)})) + (b_j^{(t)} - 1)(\psi(b_k^{(t)}) - \psi(a_k^{(t)} + b_k^{(t)}))\right]} - \sum_{i=1}^{n}\sum_{j=1}^{K} \phi_i^{(t)}(j)\left[\cancel{x_i(\psi(a_j^{(t)}) - \psi(a_j^{(t)} + b_j^{(t)})) + (20 - x_i)(\psi(b_j^{(t)}) - \psi(a_j^{(t)} + b_j^{(t)}))} + \psi(\alpha_j^{(t)})\right]$$

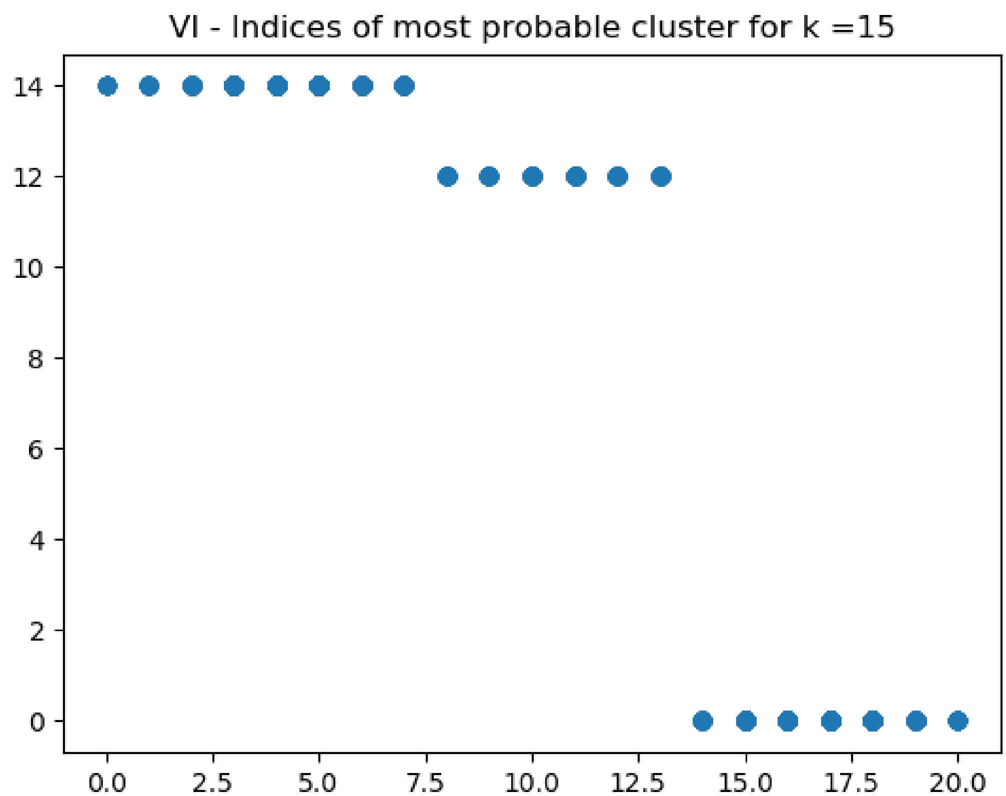
$$\mathcal{L} = \ln(B(\alpha^{(t)})) - \sum_{i=1}^{n} \phi_i^{(t)}(j) \sum_{j=1}^{K} \psi(\alpha_j^{(t)})$$
$$= \ln\left(\frac{\prod_{i=1}^{K}\Gamma(\alpha_i^{(t)})}{\Gamma(\sum_{i=1}^{K}\alpha_i^{(t)})}\right) - \sum_{i=1}^{n}\phi_i^{(t)}(j)\sum_{j=1}^{K}\psi(\alpha_j^{(t)}) = \sum_{i=1}^{K}\ln\Gamma(\alpha_i^{(t)}) - \ln\Gamma\left(\sum_{i=1}^{K}\alpha_i^{(t)}\right) - \sum_{i=1}^{n}\sum_{j=1}^{K}\phi_i^{(t)}(j)\,\psi(\alpha_j^{(t)})$$

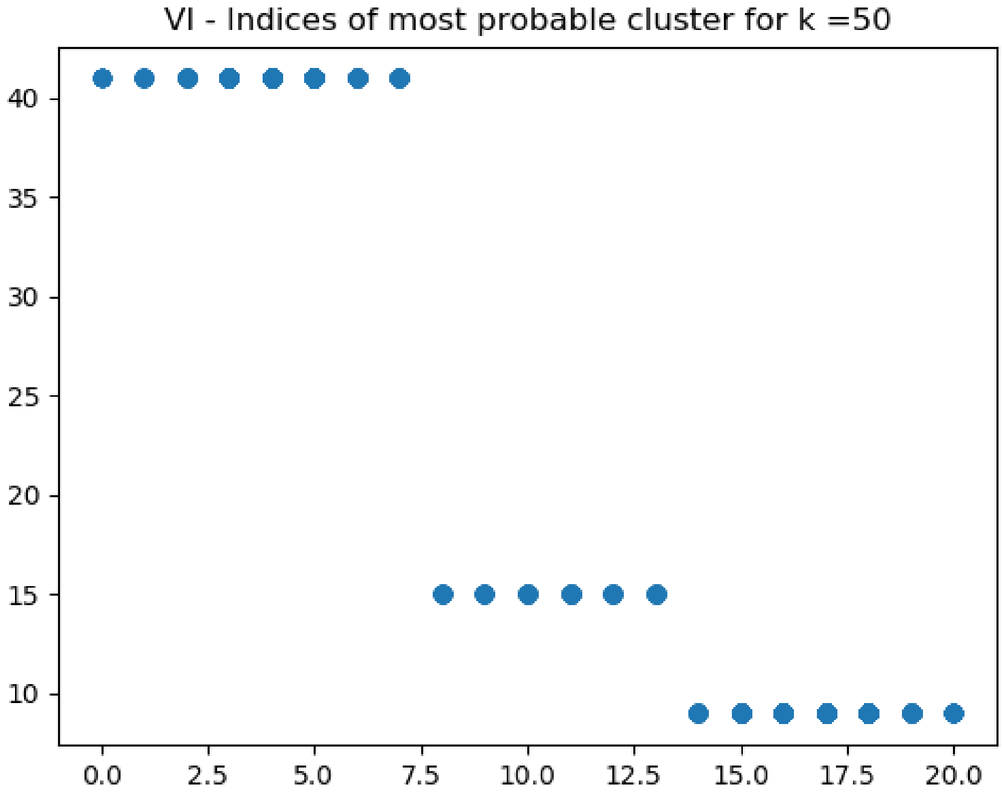
2(b)



2(c)


VI - Indices of most probable cluster for k =3
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


VI - Indices of most probable cluster for k =15
14
12
10
8
6
4
2
0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


VI - Indices of most probable cluster for k =50
40
35
30
25
20
15
10
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

Problem 3

3(a)

3. Sampling θ_j: $p(\theta_j | \{x_i : c_i = j\}) \propto p(x | \theta_j, c) p(\theta_j) \propto [\text{Bin}(20, \theta_j)][\text{Beta}(a,b)]$

$$p(\theta_j | \{x_i : c_i = j\}) \propto \theta_j^{\sum_{i:c_i=j} x_i} (1-\theta_j)^{20n_j - \sum_{i:c_i=j} x_i} \theta_j^{a-1} (1-\theta_j)^{b-1} \; ; \; n_j = |\{x_i : c_i = j\}|$$

$$= \text{Beta}\left(a + \sum_{i:c_i=j} x_i ,\; b + 20n_j - \sum_{i:c_i=j} x_i\right)$$

Sampling c_i: $p(c_i | x_i, \theta, c_{-i}) \propto p(x | c_i = j, \theta) p(c_i = j | c_{-i})$

$$\propto p(x_i | \theta_j) p(c_i = j | c_{-i})$$

$$p(c_i = j | c_{-i}) = \int p(c_i = j | \pi) p(\pi | c_{-i}) d\pi$$

$$= \int \pi_j \cdot \text{Dir}\left(\pi \,\middle|\, \frac{\alpha}{k} + n_1^{(-i)}, \ldots, \frac{\alpha}{k} + n_k^{(-i)}\right) d\pi \quad \left(n_j^{(-i)} = \sum_{s \neq i} \mathbb{1}(c_s = j)\right)$$

$$= \int \pi_j \frac{\Gamma(\alpha + n - 1)}{\prod_\ell \Gamma(\frac{\alpha}{k} + n_\ell^{(-i)})} \prod_{\ell=1}^{K} \pi_\ell^{\frac{\alpha}{k} + n_\ell^{(-i)} - 1} d\pi = \frac{\Gamma(\alpha+n-1)}{\prod_\ell \Gamma(\frac{\alpha}{k} + n_\ell^{(-i)})} \underbrace{\int \pi_j^{\frac{\alpha}{k} + n_j^{(-i)}} \prod_{\ell \neq j} \pi_\ell^{\frac{\alpha}{k} + n_\ell^{(-i)} - 1} d\pi}_{\frac{\Gamma(\frac{\alpha}{k} + n_j^{(-i)} + 1) \prod_{\ell \neq j} \Gamma(\frac{\alpha}{k} + n_\ell^{(-i)})}{\Gamma(\alpha + n)}}$$

$$p(c_i = j | c_{-i}) = \frac{\Gamma(\alpha+n-1)}{\Gamma(\alpha+n)} \frac{\Gamma(\frac{\alpha}{k} + n_j^{(-i)} + 1)}{\Gamma(\frac{\alpha}{k} + n_j^{(-i)})} = \frac{\frac{\alpha}{k} + n_j^{(-i)}}{\alpha + n - 1}$$

$$p(c_i | x_i, \theta, c_{-i}) \propto p(x_i | \theta_j) \left(\frac{\frac{\alpha}{k} + n_j^{(-i)}}{\alpha + n - 1}\right)$$

Letting $K \to \infty$: $p(c_i = j | x_i, \theta, c_{-i}) \propto p(x_i | \theta_j) \frac{n_j^{(-i)}}{\alpha + n - 1} = \text{Bin}(20, \theta_j) \frac{n_j^{(-i)}}{\alpha + n - 1}$ for $n_j^{(-i)} > 0$

$$p(c_i = j | x_i, \theta, c_{-i}) \propto 0 \quad \text{for } n_j^{(-i)} = 0$$

$$p(c_i = \text{new} | x_i, \theta, c_{-i}) = \sum_{j : n_j^{(-i)} = 0} p(c_i = j | X, \theta, c_{-i}) \propto \lim_{k \to \infty} \sum_{j : n_j^{(-i)} = 0} p(x_i | \theta_j) \frac{\alpha / k}{\alpha + n - 1}$$

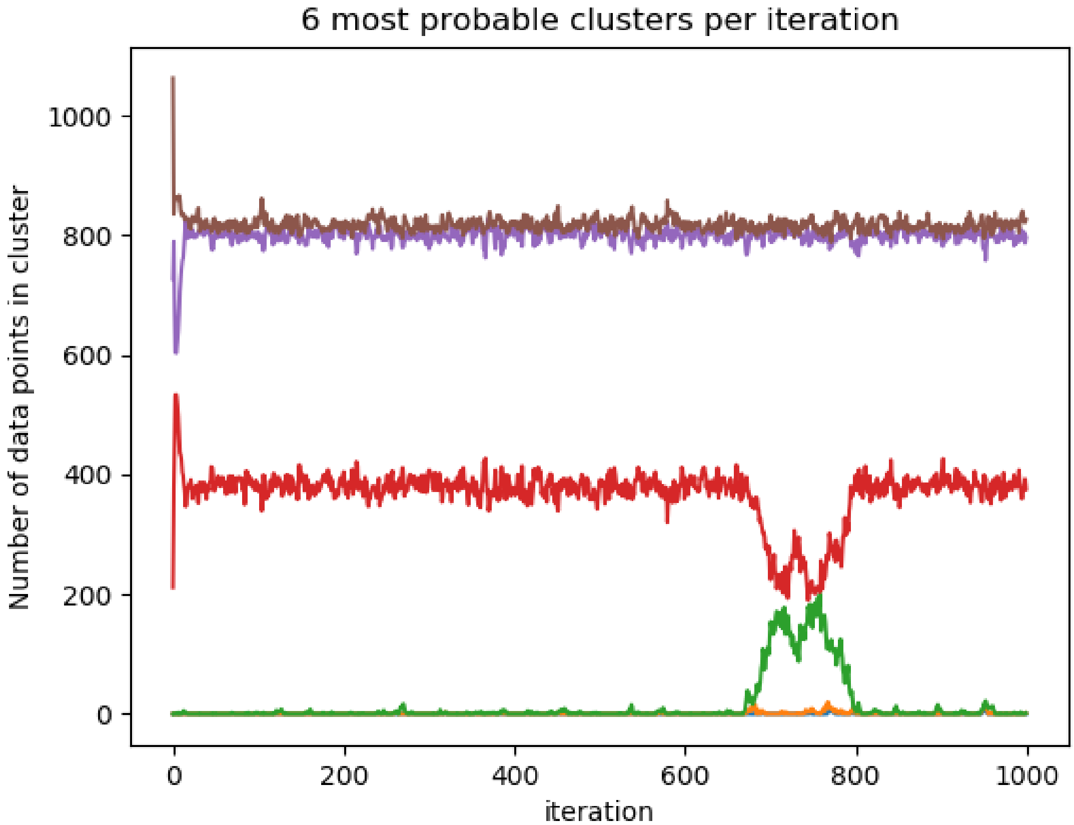
$$p(c_i = \text{new} | x_i, \theta, c_{-i}) \propto \frac{\alpha}{\alpha + n - 1} \int p(x_i | \theta) p(\theta) d\theta$$

$$\propto \frac{\alpha}{\alpha + n - 1} \int \binom{20}{x_i} \theta^{x_i} (1-\theta)^{20 - x_i} \theta^{a-1} (1-\theta)^{b-1} d\theta$$

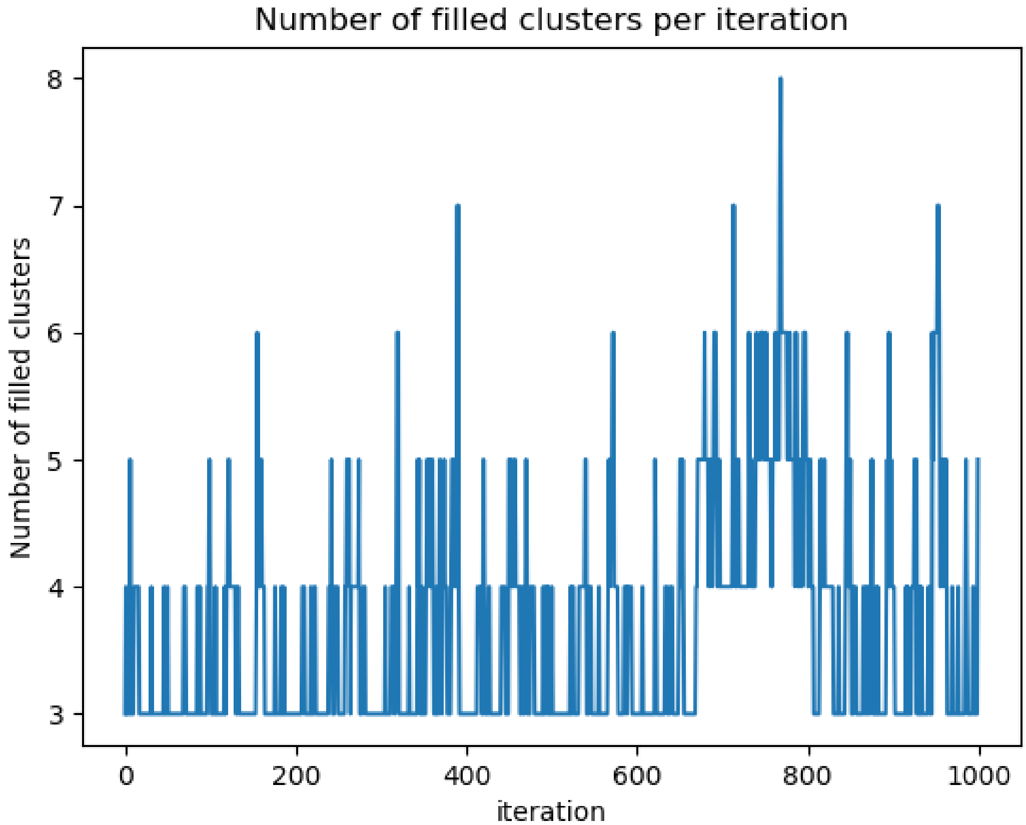
$$\propto \underbrace{\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}}_{\frac{1}{B(a,b)}} \frac{\alpha}{\alpha + n - 1} \binom{20}{x_i} \underbrace{\int \theta^{x_i + a - 1} (1-\theta)^{20 - x_i + b - 1} d\theta}_{= \frac{\Gamma(x_i + a)\Gamma(20 - x_i + b)}{\Gamma(20 + a + b)} = B(x_i + a, 20 - x_i + b)}$$

$$p(c_i = \text{new} | x_i, \theta, c_{-i}) \propto \frac{\alpha}{\alpha + n - 1} \binom{20}{x_i} \frac{B(x_i + a, 20 - x_i + b)}{B(a,b)}$$

3(b)


6 most probable clusters per iteration
Number of data points in cluster
iteration
0
200
400
600
800
1000

3(c)


Number of filled clusters per iteration
Number of filled clusters
iteration
8
7
6
5
4
3
0
200
400
600
800
1000