University of Cambridge

Mathematics Tripos

Part III

Mixing Times of Markov Chains

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Introduction

These are written solutions to Mixing Times of Markov Chains Example Sheet I. Solutions are written based on those seen in examples classes and may contain errors, likely due to the author. Solutions may be incomplete and do not usually include starred questions. These are to be used as a reference for revision **after** examples classes and should never be used beforehand. Doing so will severely impair your ability to learn and study mathematics.

Questions

Question (Question 1).

Question (Question 5). Let X be an irreducible, lazy and reversible Markov Chain on a finite state space with transition matrix P and stationary distribution π.

(i) Show that
$$\mathbb{E}_\pi[\tau_\pi] := \sum_{x,y} \pi(x)\pi(y)\mathbb{E}_x[\tau_y] = \sum_{i \geq 2} \frac{1}{1 - \lambda_i}$$
where (λ_i) are the eigenvalues. (*Hint:* Use question 12(b) from example sheet 1)

(ii) Show that
$$\sum_{t=k}^{\infty} (P^t(x,x) - \pi(x)) \leq e^{-\frac{k}{t_{\text{rel}}}} \mathbb{E}_\pi[\tau_x]$$