In exam, a lot of proofs become obvious with a picture, if so how much detail to give?

Q1 Grand

Q2 Y rejects Intervals & only intervals

We want to add stuff to Y to not reject sets but not add clumsily so that we accidentally accept more stuff.

Easy: split $\mathbb{N}$ into countably many disjoint ∞ sets. M_i then enumerate all the sets that we don't want to reject $B \subset \mathbb{N}^{(<\omega)}$ countable, then add
(A_i, M_i) $\forall i$
and then done.

Q3 To show not open, need a elt not contained in an open nbhd. M = all evens won't work
So take M = all composites.

"You've defined the set so it MUST be borel"

So $\mathcal{A} = \bigcap X_n$

$$X_n = \{ M : M \cap E \geq n,\ M \cap P < \infty \}$$

X_n trivially open

Just need to show $\forall\ x \in X_n$, $\exists$ open nbhd U st $x \in U \subset X_n$

but take ISA of x that contains n, evens then (A, x) is an open nbhd so done.

Q4 Want to show the given property is a Ramsey property. $\leftarrow$ Borel?

- Yes because we've defined it

$\bigcup_N$ of limsup $\leq 1 - \frac{1}{N}$

?

$$= \bigcup_t \underbrace{\| f_{i_1} + \dots + f_{i_n} \| \leq 1 - \tfrac{1}{N}\ \ \forall n \geq t}_{\text{show this Borel}}$$

obvious becuse is a condition on the first n terms

"Build backwards from the answer"

5. Never have $\mathcal{U}+\mathcal{V}=\tilde{1}$

$\forall_{\mathcal{U}} x \forall_{\mathcal{V}} y \quad x+y=1$

$\forall x \nexists y$ s.t $x+y=1$

Done

6. Something similar to example in Lectures.

$A=\{2^a+3^b : a<b\} \qquad \{2^a+3^b : b<a\}$

$\mathcal{U}$ has $\{3^n, 3^{n+1}, \dots\}$ $\forall n$

$\mathcal{V}$ has $\{2^n, 2^{n+1}, \dots\}$ $\forall n$

Exist because they form a filter.

"When need to get assymetry between $\forall_{\mathcal{U}}\forall_{\mathcal{V}}$ & $\forall_{\mathcal{V}}\forall_{\mathcal{U}}$ do something with $x<y$ as in Lectures"

7. "Hindman for sets"

"Can biject sets & numbers via powers of 2"

8. Y non-Ramsey

Write $Y = \bigcap X_i$ X_i all Ramsey then done

$$X_i = Y \cup \{M : [i] \notin M\}$$

9 Thing that makes this space V. weird is

Meagre = nowhere dense

i.e

$\bigcup_n$ nowhere dense = nowhere dense.

→ This is the only purely topological fact we proved about the top

Real were Ramsey facts

Take, in X, a maximal 1-seperation family

($\exists$ by Zorn)

i.e $d(x,y) \geq 1$ $\forall x,y \in$ set

Note $\forall$ $x \in$ set, $\exists$ point in family y s.t.

$d(x,y) < 1$ (else not maximal)

cleerly nowhere dense

now take $\frac{1}{2}$-sep family

$\vdots$

$\frac{1}{n}$-sep family

Then the union is dense $\divideontimes$

[key point is nowhere dense stuff is only topological fact we learn]

JJds2