

UNIVERSITY OF
CAMBRIDGE

MATHEMATICS TRIPOS

Part III **MTMC ES1**

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Solutions by
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1 Introduction

These are written solutions to Mixing Times of Markov Chains Example Sheet

1. Solutions are based on those handed out by Samuel Thomas and are not endorsed by the lecturer nor necessarily correct.

2 Questions

Question (Question 1). Let P be the transition matrix of a Markov chain with values in E and let μ and ν be two probability distributions on E . Show that

$$\|\mu P - \nu P\|_{\text{TV}} \leq \|\mu - \nu\|_{\text{TV}} .$$

Deduce that $d(t) = \max_x \|P^t(x, \cdot) - \pi\|_{\text{TV}}$ is decreasing as a function of t , where π is the invariant distribution.

Proof. Since P is a stochastic matrix, any eigenvalue λ of P satisfies $|\lambda| \leq 1$ \square

Remark. Equivalently, the number of maximal chains is $n!$ and the number of them containing a given r -set is $r!(n-r)!$, so

$$\sum_{r=0}^n |\mathcal{A}_r| r!(n-r)! \leq n!$$

so this is probability in disguise