1. Let $Y = \{M \in \mathbb{N}^{(\omega)} : \text{no two members of } M \text{ are coprime}\}$. Which $M \in \mathbb{N}^{(\omega)}$ accept $\{3\}$? Which M reject $\{3\}$? Which M reject $\{6\}$?

2. Construct a set $Y \subset \mathbb{N}^{(\omega)}$ such that the finite sets rejected by $\mathbb{N}$ are precisely $\emptyset$ and all sets of the form $\{m, m+1, \ldots, n\}$, $m \leq n$.

3. Let E be the set of even numbers and let P be the set of prime numbers. Show that the set $\{M \in \mathbb{N}^{(\omega)} : |M \cap E| = \infty,\ |M \cap P| < \infty\}$ is not $*$-open, but is a countable intersection of $*$-open sets.

4. Let $f_1, f_2, \ldots : \mathbb{C} \to \mathbb{C}$ be bounded complex-valued functions. For any bounded $f : \mathbb{C} \to \mathbb{C}$, write $\|f\|$ for $\sup\{|f(z)| : z \in \mathbb{C}\}$. Show that there is a subsequence $(f_{n_i})_{i=1}^{\infty}$ of $(f_i)_{i=1}^{\infty}$ such that either for every subsequence $(f_{m_i})_{i=1}^{\infty}$ of $(f_{n_i})_{i=1}^{\infty}$ we have $\limsup_{k\to\infty} \|f_{m_1} + \ldots + f_{m_k}\| \geq 1$ or for every subsequence $(f_{m_i})_{i=1}^{\infty}$ of $(f_{n_i})_{i=1}^{\infty}$ we have $\limsup_{k\to\infty} \|f_{m_1} + \ldots + f_{m_k}\| < 1$.

5. Prove carefully that the operation $+$ on $\beta\mathbb{N}$ is not surjective.

6. Prove that the operation $+$ on $\beta\mathbb{N}$ is not commutative.

7. Prove that whenever the collection of finite non-empty subsets of $\mathbb{N}$ is finitely coloured there exist disjoint $F_1, F_2, \ldots$ with $\{\bigcup_{i\in I} F_i : \emptyset \neq I \subset \mathbb{N},\ I \text{ finite}\}$ monochromatic.

8. Do the Ramsey subsets of $\mathbb{N}^{(\omega)}$ form a σ-algebra?

9. Is the $*$-topology on $\mathbb{N}^{(\omega)}$ induced by a metric?

$^+$10. Show that whenever $\mathbb{N}$ is finitely coloured there exist sets $S_1, S_2, \ldots$, with each S_i an arithmetic progression of length i, such that the set

$$\left\{\sum_{i\in I} x_i : \emptyset \neq I \subset \mathbb{N},\ I \text{ finite},\ x_i \in S_i \text{ for all } i \in I\right\}$$

is monochromatic.