UNIVERSITY OF
CAMBRIDGE

MATHEMATICS TRIPOS

Part III

Ramsey Theory

Example Sheet I

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Solutions by
JOSHUA SNYDER

Introduction

These are written solutions to Ramsey TheoryExample Sheet I. Solutions are written based on those seen in examples classes and may contain errors from the author.

Questions

Question (Question 7). Prove that wehenever the collection of finite non-empty sets of $\mathbb{N}$ is finitely coloured there exist disjoint $F_1, F_2, \ldots$ with $\{\bigcup_{i \in I} F_i : \emptyset \neq I \subset \mathbb{N}, I \text{ finite}\}$ monochromatic.

Idea. Use Hindman's Theorem

Solution. By repeatedly applying Ramsey we can find infinite sets $M_1 \supset M_2 \supset \ldots$ with $M_i^{(i)}$ monochromatic. By Hindman's theorem whenever $\mathbb{N}$ is finitely coloured there exists $x_1, x_2, \ldots$ such that $FS(x_1, x_2, \ldots)$ is monochromatic with WLOG $x_1 < x_2 < \ldots$ (If this is not the case take $y_1 = x_1$, $y_2 = x_2 + x_3$, $\ldots$ which has monochromatic finite sums). Then pick an $F_i \in M_{x_i}^{(x_i)}$ for each i and we are done. $\qquad \square$