DISTRIBUTED ROBUST BEAMFORMING FOR MIMO COGNITIVE NETWORKS

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ABSTRACT

Beamforming for multi-input multi-output (MIMO) cognitive networks is considered in the presence of channel uncertainty induced by errors in estimating cognitive-to-primary channels. A robust beamforming problem is formulated to optimize an appropriate cognitive radio network-wide performance metric, while enforcing protection of the primary system. In spite of the non-convexity of the resultant optimization problem, a block coordinate ascent algorithm is developed with provable convergence to a stationary point. Enticingly, the novel scheme also lends itself naturally to a distributed implementation. Numerical results are reported to corroborate the analytical findings.

Index Terms— MIMO cognitive networks, beamforming, channel uncertainty, robust optimization, distributed computation.

1. INTRODUCTION

The key enabler for seamless frequency re-use is the ability of cognitive radios (CRs) to judiciously control the interference inflicted to the incumbent primary user (PU) system [1]. However, as full cooperation between PU and CR nodes is generally infeasible in the advocated hierarchical access model, CR-to-PU channels are difficult to acquire in practice. Consequently, interference is challenging to control. It is thus of paramount importance to take into account the inherent randomness of the CR-to-PU channels, and enforce PU protection throughout the CR network operation [2, 3, 4].

Recently, MIMO CR networks have attracted considerable attention thanks to their ability to mitigate self- and PU-inflicted interference via beamforming, while leveraging spatial multiplexing and diversity to markedly increase transmission rates and reliability. On the other hand, wireless transceiver optimization has been extensively studied in recent years when either perfect or imperfect channel state information (CSI) is available [5, 6]. In the CR context, network utility maximization was investigated under perfect CSI knowledge in e.g., [7, 8] and references therein. CR-to-PU channel uncertainties were considered with single-antenna PUs, identical channel estimation errors for different CR-to-PU links, and only for centralized operation [3, 4, 9].

The present paper considers a MIMO ad-hoc CR network deployed to share the spectrum bands licensed to PUs, who are also equipped with multiple antennas. The inherently stochastic nature of the propagation environment, and the inevitable inaccuracies of the CR-to-PU channel estimates are captured by a Frobenius norm-bounded uncertainty model [10, Ch. 4], which leads to a robust interference constraint ensuring PU protection [2, 4]. Upon recasting the robust constraint in a convenient form, a resource allocation problem is formulated to obtain CR transmit- and receive-beamforming matrices minimizing the overall data symbol estimation error, while ensuring protection of the PU system. To cope with the inherent non-convexity of the novel optimization problem, a block coordinate ascent approach is developed along with a local linear approximation technique to derive an iterative algorithm with provable convergence to a stationary point of the original non-convex problem. The resulting scheme is suitable for distributed operation, where each CR locally solves a convex sub-problem provided that relevant optimization parameters are obtained by measuring the interfering signals [8, 11].

Notation: Boldface lower (upper) case letters represent vectors (matrices). $\mathbb{H}^{n\times n}$, $\mathbb{C}^{n\times n}$ and $\mathbb{R}$ stand for spaces of $n\times n$ Hermitian, $n\times n$ complex matrices, and real numbers, respectively, whereas $\text{Tr}\{\cdot\}$ denotes the trace operator; $(\cdot)^{\mathcal{H}}$ conjugate transpose, and $\text{vec}(\mathbf{A})$ the vector obtained by stacking the columns of a matrix $\mathbf{A}$; $\mathbf{I}_N$ is the $N\times N$ identity matrix. Finally, $\mathbb{E}\{\cdot\}$ denotes the expectation operator.

2. PROBLEM FORMULATION

Consider a wireless MIMO CR network comprising K transmitter-receiver pairs $\{U_k^t, U_k^r\}$, sharing spectrum resources with an incumbent PU system in an underlay setting [1]. Let M_k and N_k, $k \in \mathcal{K} := \{1, 2, \ldots, K\}$, denote the number of antennas of the k-th transmitter and receiver, respectively; and $\mathbf{s}_k$ the $M_k \times 1$ information-bearing symbol vector transmitted by U_k^t per time slot with covariance matrix $\mathbb{E}\{\mathbf{s}_k\mathbf{s}_k^{\mathcal{H}}\} = \mathbf{I}_{M_k}$. In order to alleviate CR mutual interference, U_k^t pre-multiplies $\mathbf{s}_k$ by a transmit-beamforming matrix $\mathbf{F}_k \in \mathbb{C}^{M_k\times M_k}$; that is, U_k^t actually transmits the $M_k \times 1$ symbol vector $\mathbf{x}_k := \mathbf{F}_k\mathbf{s}_k$.

With $\mathbf{H}_{k,j} \in \mathbb{C}^{N_k\times M_j}$ denoting the channel of CR link $U_j^t \to U_k^r$, the $N_k \times 1$ symbol vector received at U_k^r is

$$\mathbf{y}_k = \mathbf{H}_{k,k}\mathbf{x}_k + \sum_{j\in\mathcal{K}\setminus\{k\}} \mathbf{H}_{k,j}\mathbf{x}_j + \mathbf{n}_k \tag{1}$$

where $\mathbf{n}_k \in \mathbb{C}^{N_k}$ denotes the zero-mean complex Gaussian noise, independent of $\mathbf{s}_k$, with covariance matrix $\mathbb{E}\{\mathbf{n}_k\mathbf{n}_k^{\mathcal{H}}\} = \sigma_k^2\mathbf{I}_{N_k}$.

Low-complexity receiver processing motivates the use of a computationally-affordable linear filter $\mathbf{W}_k \in \mathbb{C}^{M_k\times N_k}$ at U_k^r to recover $\mathbf{s}_k$ as $\hat{\mathbf{s}}_k := \mathbf{W}_k\mathbf{y}_k$. Using $\mathbf{W}_k$ at U_k^r, the mean-square error (MSE) matrix $\mathbf{E}_k := \mathbb{E}\{(\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^{\mathcal{H}}\}$, which quantifies the reconstruction error, is given by [cf. (1)]

$$\mathbf{E}_k = \mathbf{W}_k\mathbf{A}_k\mathbf{W}_k^{\mathcal{H}} - \mathbf{W}_k\mathbf{H}_{k,k}\mathbf{F}_k - \mathbf{F}_k^{\mathcal{H}}\mathbf{H}_{k,k}^{\mathcal{H}}\mathbf{W}_k^{\mathcal{H}} + \mathbf{I}_{M_k} \tag{2}$$

where $\mathbf{A}_k := \sum_{j=1}^{K}\mathbf{H}_{k,j}\mathbf{F}_j\mathbf{F}_j^{\mathcal{H}}\mathbf{H}_{k,j}^{\mathcal{H}} + \sigma_k^2\mathbf{I}_{N_k}$. Entry (i,i) of $\mathbf{E}_k$ represents the MSE of the i-th data stream from U_k^t to U_k^r, and $\text{Tr}\{\mathbf{E}_k\}$ corresponds to the MSE of $\hat{\mathbf{s}}_k$. Among candidate network performance metrics, the adopted one in this paper is the sum of MSEs from different data streams. This metric relates to system performance in terms of bit error rate (BER) as explained in [6], and

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facilitates derivation of the optimal filters. To account for different quality-of-service demands, its weighted counterpart can be adopted as in [12].

To complete the formulation, let $\mathbf{G}_k \in \mathbb{C}^{L\times M_k}$ denote the channel between CR U_k^t and a PU receiver, possibly equipped with multiple (L) antennas.[1] Then, the transmit- and receive-beamforming matrices minimizing the overall MSE can be obtained as

$$(P1) \quad \min_{\{\mathbf{F}_k,\mathbf{W}_k\}_{k=1}^K} \sum_{k=1}^{K} \mathrm{Tr}\{\mathbf{E}_k\} \tag{3a}$$
$$\text{s.t.} \quad \mathrm{Tr}\{\mathbf{F}_k\mathbf{F}_k^{\mathcal{H}}\} \le p_k^{\max},\ k\in\mathcal{K} \tag{3b}$$
$$\mathrm{Tr}\{\mathbf{G}_k\mathbf{F}_k\mathbf{F}_k^{\mathcal{H}}\mathbf{G}_k^{\mathcal{H}}\} \le \iota_k^{\max},\ k\in\mathcal{K} \tag{3c}$$

where $p_k^{\max}$ is the maximum transmit-power of U_k^t, and $\iota_k^{\max}$ the maximum interference CR U_k^t can afford to inflict to the PU. As in e.g., [1, 11], partitioning of the interference budget $\iota^{\max} := \sum_k \iota_k^{\max}$ in per-CR transmitter portions $\{\iota_k^{\max}\}$ is assumed carried out beforehand, possibly according to quality-of-service guidelines.

However, due to lack of explicit cooperation between PU and CR nodes, CR-to-PU channels $\{\mathbf{G}_k\}$ are in general difficult to estimate accurately. As PU protection must be enforced strictly, it is important to take into account the inherent channel *uncertainty* in the CR-to-PU links and guarantee that the interference power experienced by the PU receiver stays below a prescribed level for *any* possible (random) channel realization [2, 4]. Before developing a resource allocation approach robust to inaccuracies in channel estimates, problem $(P1)$ is conveniently re-formulated first in order to reduce the number of variables involved.

2.1. Equivalent Optimization Problem

For the sum-MSE cost in (3a), it will turn out that $\mathbf{W}_k$ can be obtained in closed form. To see this, note first that for fixed $\{\mathbf{F}_k\}$, $(P1)$ is convex in $\mathbf{W}_k$, and the optimum $\{\mathbf{W}_k\}$s can be obtained from the first-order optimality conditions. Express the Lagrangian function associated with $(P1)$ as

$$\begin{aligned}\mathcal{L}(\mathcal{P},\mathcal{D}) = &\sum_{k=1}^{K}\mathrm{Tr}\{\mathbf{E}_k\} + \sum_{k=1}^{K}\lambda_k\left(\mathrm{Tr}\{\mathbf{F}_k\mathbf{F}_k^{\mathcal{H}}\} - p_k^{\max}\right)\\ &+\sum_{k=1}^{K}\nu_k\left(\mathrm{Tr}\{\mathbf{G}_k\mathbf{F}_k\mathbf{F}_k^{\mathcal{H}}\mathbf{G}_k^{\mathcal{H}}\} - \iota_k^{\max}\right)\end{aligned} \tag{4}$$

where $\mathcal{P} := \{\{\mathbf{F}_k\},\{\mathbf{W}_k\}\}$ and $\mathcal{D} := \{\{\lambda_k\},\{\nu_k\}\}$ collects the primal and dual variables, respectively. Then, by setting the complex gradient $\partial\mathcal{L}/\partial\mathbf{W}_k^*$ equal to zero, $\mathbf{W}_k^{opt}$ is found as

$$\mathbf{W}_k^{opt} = \mathbf{F}_k^{\mathcal{H}}\mathbf{H}_{k,k}^{\mathcal{H}}\mathbf{A}_k^{-1}, \quad k\in\mathcal{K}. \tag{5}$$

Substituting $\{\mathbf{W}_k^{opt}\}$ back into (3a), and neglecting irrelevant terms, it follows that $(P1)$ can be equivalently re-written as

$$(P2) \quad \max_{\{\mathbf{F}_k\}} \sum_{k=1}^{K} \mathrm{Tr}\left\{\mathbf{H}_{k,k}\mathbf{F}_k\mathbf{F}_k^{\mathcal{H}}\mathbf{H}_{k,k}^{\mathcal{H}}\mathbf{A}_k^{-1}\right\} \tag{6}$$
$$\text{s.t.} \quad (3b), (3c).$$

[1] A single PU receiver is considered throughout the paper. However, extension to multiple receiving PU devices is straightforward.

Using the covariance of transmitted symbols $\mathbf{Q}_k := \mathbb{E}\{\mathbf{x}_k\mathbf{x}_k^{\mathcal{H}}\} = \mathbf{F}_k\mathbf{F}_k^{\mathcal{H}}$ as optimization variable, $(P2)$ can be expressed as

$$(P3) \quad \max_{\{\mathbf{Q}_k\succeq\mathbf{0}\}} \sum_{k=1}^{K} u_k(\{\mathbf{Q}_k\}) \tag{7a}$$
$$\text{s.t.} \quad \mathrm{Tr}\{\mathbf{Q}_k\} \le p_k^{\max},\ k\in\mathcal{K} \tag{7b}$$
$$\mathrm{Tr}\{\mathbf{G}_k\mathbf{Q}_k\mathbf{G}_k^{\mathcal{H}}\} \le \iota_k^{\max},\ k\in\mathcal{K} \tag{7c}$$

where the per-CR link utility $u_k(\{\mathbf{Q}_k\})$ is given by

$$u_k(\{\mathbf{Q}_k\}) := \mathrm{Tr}\left\{\mathbf{H}_{k,k}\mathbf{Q}_k\mathbf{H}_{k,k}^{\mathcal{H}}\left(\mathbf{H}_{k,k}\mathbf{Q}_k\mathbf{H}_{k,k}^{\mathcal{H}} + \mathbf{R}_{k,k}\right)^{-1}\right\} \tag{8}$$

with $\mathbf{R}_{k,k} := \sum_{i\neq k}\mathbf{H}_{k,i}\mathbf{Q}_i\mathbf{H}_{k,i}^{\mathcal{H}} + \sigma_k^2\mathbf{I}_{N_k}$.

Channels $\{\mathbf{G}_k\}$ must be perfectly known in order to solve $(P3)$. A robust version of $(P3)$, which accounts for imperfect channel knowledge, is dealt with in the next section.

3. ROBUST CR BEAMFORMERS

In typical CR scenarios, CR and PU nodes do not generally cooperate. Thus, CR-to-PU channels are challenging to estimate accurately. To capture estimation inaccuracies, consider expressing $\mathbf{G}_k$ as

$$\mathbf{G}_k = \hat{\mathbf{G}}_k + \Delta\mathbf{G}_k,\ k\in\mathcal{K} \tag{9}$$

where $\hat{\mathbf{G}}_k$ is the estimated channel available to U_k^t, and $\{\Delta\mathbf{G}_k\}$ the uncertainty error taking values from the bounded set

$$\mathcal{G}_k := \left\{\Delta\mathbf{G}_k | \mathrm{Tr}\{\Delta\mathbf{G}_k\Delta\mathbf{G}_k^{\mathcal{H}}\} \le \epsilon_k^2\right\},\ k\in\mathcal{K} \tag{10}$$

where $\epsilon_k > 0$ controls the degree of uncertainty associated with $\mathbf{G}_k$. For example, (9) properly models the case where a time division duplex (TDD) strategy is adopted by the PU system, and CRs have prior knowledge of the PUs' pilot symbols [3]. In lieu of pilot symbols, $\{\hat{\mathbf{G}}_k\}$ can be formed using the path loss coefficients, and ϵ_k can be deduced from the fading statistics.

Based on (10), a robust interference constraint can be written as

$$\begin{aligned}\mathrm{Tr}\{(\hat{\mathbf{G}}_k + \Delta\mathbf{G}_k)\mathbf{Q}_k(\hat{\mathbf{G}}_k + \Delta\mathbf{G}_k)^{\mathcal{H}}\} \le \iota_k^{\max},\\ \forall\,\Delta\mathbf{G}_k\in\mathcal{G}_k,\ k\in\mathcal{K}\end{aligned} \tag{11}$$

and thus, a robust counterpart of $(P3)$ is

$$\begin{aligned}(P4) \quad &\max_{\{\mathbf{Q}_k\succeq\mathbf{0}\}} \sum_{k=1}^{K} u_k(\{\mathbf{Q}_k\})\\ &\text{s.t.} \quad (7b), (11).\end{aligned} \tag{12}$$

Clearly, once $\{\mathbf{Q}_k^{opt}\}$ solving $(P4)$ is found, $\{\mathbf{W}_k^{opt}\}$ can be readily computed via (5).

However, $\sum_k u_k(\{\mathbf{Q}_k\})$ is non-convex in $\{\mathbf{Q}_k\}$, and hence $(P4)$ is hard to solve in general. Additionally, constraints (11) are not in a tractable optimization form, and thus further elaboration is needed. These issues are addressed in the next section.

3.1. Distributed algorithm via local approximation

To cope with the non-convexity of the utility function in $(P4)$, a block-coordinate ascent approach is adopted. Define first the sum of all but the k-th utility as $f_k(\mathbf{Q}_k, \mathbf{Q}_{-k}) := \sum_{j \neq k} u_j$, which is convex in $\mathbf{Q}_k$ [cf. Lemma 2]. By keeping only the linear term of the Taylor's expansion of $f_k(\cdot)$ around a feasible point $\{\bar{\mathbf{Q}}_k\}$, the objective function in (12) can be approximated as (see also [8])

$$\sum_{k=1}^{K} u_k(\{\mathbf{Q}_k\}) = u_k(\{\mathbf{Q}_k\}) + f_k(\mathbf{Q}_k, \mathbf{Q}_{-k})$$
$$\approx u_k(\{\mathbf{Q}_k\}) + f_k(\bar{\mathbf{Q}}_k, \mathbf{Q}_{-k}) + \mathrm{Tr}\left\{\mathbf{D}_k^{\mathcal{H}}(\mathbf{Q}_k - \bar{\mathbf{Q}}_k)\right\} \quad (13)$$

where $\mathbf{D}_k := \nabla_{\mathbf{Q}_k} f_k(\bar{\mathbf{Q}}_k, \mathbf{Q}_{-k}) := \left.\frac{\partial f_k}{\partial \mathbf{Q}_k^*}\right|_{\mathbf{Q}_k = \bar{\mathbf{Q}}_k}$. Matrix $\mathbf{Q}_k$ can be obtained by solving the following sub-problem

$$(P5) \quad \max_{\mathbf{Q}_k \succeq \mathbf{0}} \; u_k(\mathbf{Q}_k, \mathbf{Q}_{-k}) + \mathrm{Tr}\left\{\mathbf{D}_k^{\mathcal{H}} \mathbf{Q}_k\right\} \quad (14a)$$
$$\text{s.t.} \quad \mathrm{Tr}\{\mathbf{Q}_k\} \leq p_k^{\max} \quad (14b)$$
$$\mathrm{Tr}\{\mathbf{G}_k \mathbf{Q}_k \mathbf{G}_k^{\mathcal{H}}\} \leq \iota_k^{\max}, \; \forall\, \Delta\mathbf{G}_k \in \mathcal{G}_k \quad (14c)$$

where

$$\mathbf{D}_k := -\sum_{j \neq k} \left.\mathbf{H}_{j,k}^{\mathcal{H}} \mathbf{B}_j^{-1} \mathbf{V}_j \mathbf{B}_j^{-1} \mathbf{H}_{j,k}\right|_{\mathbf{Q}_k = \bar{\mathbf{Q}}_k} \quad (15)$$

$$\mathbf{B}_j := \sum_{i=1}^{K} \mathbf{H}_{j,i} \mathbf{Q}_i \mathbf{H}_{j,i}^{\mathcal{H}} + \sigma_j^2 \mathbf{I}_{N_j}, \text{ and } \mathbf{V}_j := \mathbf{H}_{j,j} \mathbf{Q}_j \mathbf{H}_{j,j}^{\mathcal{H}}. \quad (16)$$

Problem $(P4)$ can clearly be solved centrally at a CR fusion center, but the coordinate ascent approach suggests also a distributed optimization procedure; in fact, each CR U_k^t can update locally $\mathbf{Q}_k$ based on: *i)* its covariance matrix $\bar{\mathbf{Q}}_k$ obtained at the previous iteration of the algorithm, that is to be used in (13); *ii)* a measurement of the interference perceived $\mathbf{R}_{k,k}$ (see also [8]); and *iii)* matrices $\{\mathbf{B}_j\}$, $\{\mathbf{V}_j\}$, and $\{\mathbf{H}_{j,k}\}$ obtained from the neighboring CR links via local message passing.

3.2. Equivalent robust interference constraint

Constraint (14c) renders $(P5)$ a semi-infinite program. An equivalent constraint in linear matrix inequality (LMI) form will be derived in this section. This will turn $(P5)$ into an equivalent semidefinite program (SDP), which can be efficiently solved in polynomial time by standard interior point methods [10]. To this end, the following lemma is useful.

Lemma 1 *(S-Procedure [10, p. 655]) Let* $\mathbf{A}, \mathbf{D} \in \mathbb{H}^{n \times n}$, $\mathbf{b} \in \mathbb{C}^n$, *and* $c, e \in \mathbb{R}$, *and assume that there exists an* $\bar{\mathbf{x}}$ *satisfying* $\bar{\mathbf{x}}^{\mathcal{H}} \mathbf{D} \bar{\mathbf{x}} < e$. *Then, the inequality*

$$\mathbf{x}^{\mathcal{H}} \mathbf{A} \mathbf{x} + 2\Re(\mathbf{b}^{\mathcal{H}} \mathbf{x}) + c \geq 0, \; \forall\, \mathbf{x}^{\mathcal{H}} \mathbf{D} \mathbf{x} \leq e \quad (17)$$

holds if and only if there exists a scalar $s \geq 0$ *such that*

$$\begin{bmatrix} s\mathbf{D} + \mathbf{A} & \mathbf{b} \\ \mathbf{b}^{\mathcal{H}} & c - es \end{bmatrix} \succeq \mathbf{0}. \quad (18)$$

Using Lemma 1, (14c) can be equivalently reformulated as follows.[2]

[2]Proofs can be found in [13].

Algorithm 1 Distributed robust sum-MSE minimization

1: Initialize $\mathbf{Q}_k = \mathbf{0}, \forall\, k \in \mathcal{K}$.
2: **repeat**
3:     **for** $k = 1, 2, \ldots, K$ **do**
4:         Measure $\mathbf{R}_{k,k}$.
5:         Exchange $\{\mathbf{B}_j, \mathbf{V}_j, \mathbf{H}_{j,k}\}$ with interfering CR links.
6:         Update $\mathbf{Q}_k$ by solving $(P6)$.
7:     **end for**
8: **until** convergence
9: Update $\mathbf{W}_k$ via (5).

Proposition 1 *There exists* $s_k \geq 0$ *for which (14c) is equivalent to*

$$\begin{bmatrix} s_k \mathbf{I}_{L \times M_k} - (\mathbf{I}_L \otimes \mathbf{Q}_k) & -\mathrm{vec}(\mathbf{Q}_k^{\mathcal{H}} \hat{\mathbf{G}}_k^{\mathcal{H}}) \\ -\mathrm{vec}(\mathbf{Q}_k^{\mathcal{H}} \hat{\mathbf{G}}_k^{\mathcal{H}})^{\mathcal{H}} & \iota_k^{\max} - \mathrm{Tr}\{\hat{\mathbf{G}}_k \mathbf{Q}_k \hat{\mathbf{G}}_k^{\mathcal{H}}\} - \epsilon_k^2 s_k \end{bmatrix} \succeq \mathbf{0}. \quad (19)$$

Using Schur complement, $(P5)$ can be *equivalently* reformulated as

$$(P6) \quad \min_{\substack{\mathbf{Q}_k \succeq \mathbf{0} \\ \mathbf{T}, s_k \geq 0}} \mathrm{Tr}\{\mathbf{T}\} - \mathrm{Tr}\left\{\mathbf{D}_k^{\mathcal{H}} \mathbf{Q}_k\right\} \quad (20a)$$
$$\text{s.t.} \quad \mathrm{Tr}\{\mathbf{Q}_k\} \leq p_k^{\max} \quad (20b)$$
$$\begin{bmatrix} \mathbf{H}_{k,k} \mathbf{Q}_k \mathbf{H}_{k,k}^{\mathcal{H}} + \mathbf{R}_{k,k} & \mathbf{R}_{k,k}^{1/2} \\ \mathbf{R}_{k,k}^{1/2} & \mathbf{T} \end{bmatrix} \succeq \mathbf{0} \quad (20c)$$
$$\begin{bmatrix} s_k \mathbf{I}_{L \times M_k} - (\mathbf{I}_L \otimes \mathbf{Q}_k) & -\mathrm{vec}(\mathbf{Q}_k^{\mathcal{H}} \hat{\mathbf{G}}_k^{\mathcal{H}}) \\ -\mathrm{vec}(\mathbf{Q}_k^{\mathcal{H}} \hat{\mathbf{G}}_k^{\mathcal{H}})^{\mathcal{H}} & \iota_k^{\max} - \mathrm{Tr}\{\hat{\mathbf{G}}_k \mathbf{Q}_k \hat{\mathbf{G}}_k^{\mathcal{H}}\} - \epsilon_k^2 s_k \end{bmatrix} \succeq \mathbf{0}. \quad (20d)$$

The overall distributed scheme implemented via nonlinear Gauss-Seidel iterations is tabulated as Algorithm 1. Notice that covariances can be alternatively updated using the Jacobi iteration [14, Ch. 2].

4. CONVERGENCE

Since the original optimization problem $(P4)$ is non-convex, convergence of the coordinate ascent solver has to be established. To this end, recall that $(P5)$ and $(P6)$ are equivalent; thus, convergence can be asserted by supposing that $(P5)$ is solved per Gauss-Seidel iteration instead of $(P6)$. The following lemma is first needed.

Lemma 2 *For each* $k \in \mathcal{K}$, *the feasible set of problem* $(P5)$, *namely* $\mathcal{Q}_k := \{\mathbf{Q}_k | \mathbf{Q}_k \in (14b), (14c)\}$ *is convex. And the real-valued function* $f_k(\mathbf{Q}_k, \mathbf{Q}_{-k})$ *is convex in* $\mathbf{Q}_k$ *over the feasible set* $\mathcal{Q}_k$, *whenever* $\{\mathbf{Q}_j, j \neq k\}$ *are fixed.*

Based on Lemma 2, convergence is established next.

Proposition 2 *The sequence of objective function values obtained by the coordinate ascent Algorithm* 1 *converges.*

Interestingly, by inspecting the structure of the channel matrices $\{\mathbf{H}_{k,k}, k \in \mathcal{K}\}$ of links $\{U_k^t \to U_k^r\}$, it is possible to show that the coordinate ascent algorithm not only converges, but also that the limit point satisfies the first-order optimality conditions as summarized in the following theorem.

Theorem 1 *If matrices* $\mathbf{H}_{k,k}$, $k \in \mathcal{K}$, *have full column rank, then every limit point of Algorithm* 1 *is a stationary point of* $(P4)$.

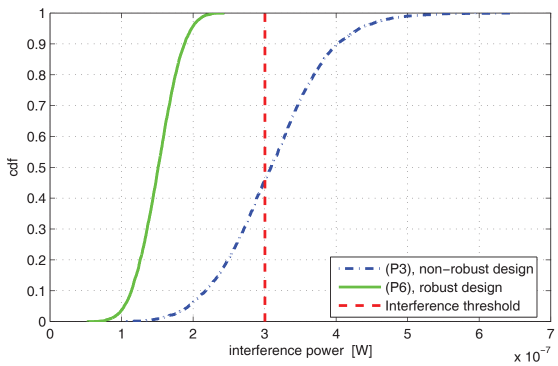

Fig. 1: Interference cumulative distribution function.

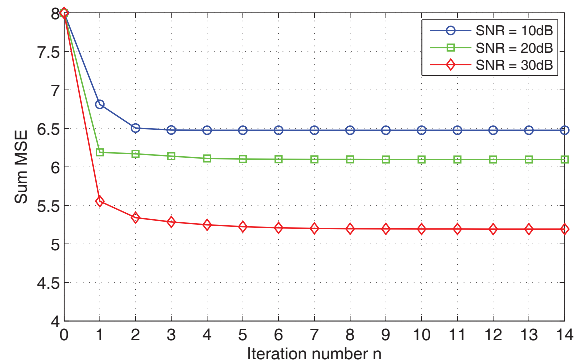

Fig. 2: Algorithm convergence.

The proof of Theorem 1 is in the spirit of the convergence claim of the block coordinate descent method in [14, Ch. 2], [5]. What is basically needed to show is that the limit point of the algorithm satisfies the first-order optimality conditions over the Cartesian product of closed convex sets; see [13] for detailed proof.

5. NUMERICAL RESULTS

In this section, numerical results are presented to verify the performance merits of the novel design. Four CR pairs and one PU receiver are considered, all are equipped with 2 antennas. The path loss obeys the model $d^{-\eta}$, with d the distance between nodes, and $\eta = 3.5$. A flat Rayleigh fading model is employed. For simplicity, the distances of links $U_k^t \to U_k^r$ are all set to $d_{k,k} = 30$ m; for the interfering links $\{U_k^t \to U_j^r, j \neq k\}$ distances are uniformly distributed in the interval $30 - 100$ m. Finally, CR-to-PU distances are uniformly distributed in $70 - 100$ m. The maximum transmit-power and the noise power are identical for all CRs.

To validate the effect of the robust interference constraint, the cumulative distribution functions (CDF) of the interference power perceived by the PU are depicted in Fig. 1. Transmit-powers and noise powers are set so that the signal-to-noise ratio defined as SNR $:= p_k^{\max}(d_{k,k}^{-\eta})/\sigma_k^2$ equals 10 dB. The total interference threshold $\iota^{\max} = 3 \cdot 10^{-7}$ W is equally split among the CR transmitters. The channel uncertainty is set to $\epsilon_k^2 = 0.08 \cdot \|\hat{\mathbf{G}}_k\|_F^2$. CDF curves are obtained over $4,000$ independent channel realizations using Monte Carlo simulations. As expected, the proposed robust scheme enforces the interference constraint strictly. In fact, the interference never exceeds the tolerable limit (shown as the vertical red dashed line). On the contrary, its non-robust counterpart frequently violates the interference limit (more than half of the times).

Fig. 2 illustrates the convergence of the proposed iterative algorithm for a given channel realization with different SNRs. It is clearly seen that the total MSE decreases monotonically across fast-converging iterations.

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