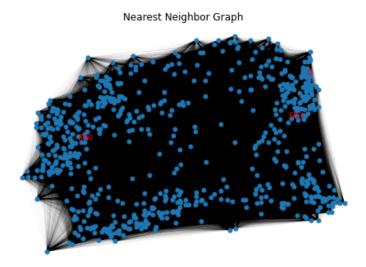
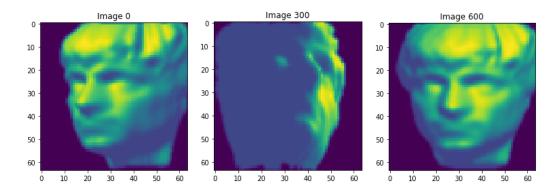
Order of faces using ISOMAP

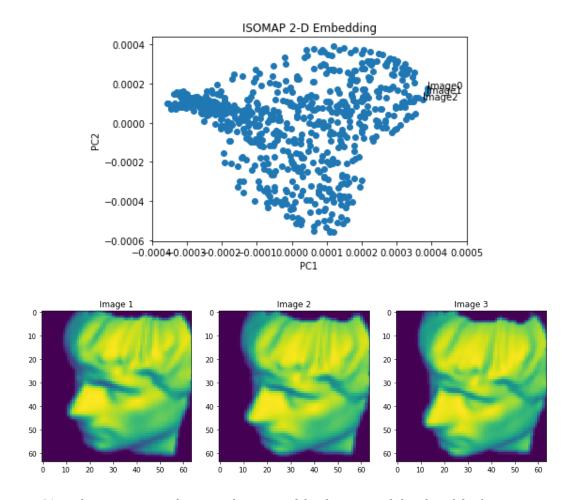
To begin, a weighted adjacency matrix was created for the data. No threshold limits nor neighbor minimums were set for this process, and the Euclidean distance was used to weight the adjacency within each neighborhood. Afterwards a *network* graph was created using the weighted adjacency matrix to create the nearest neighbor graph below:



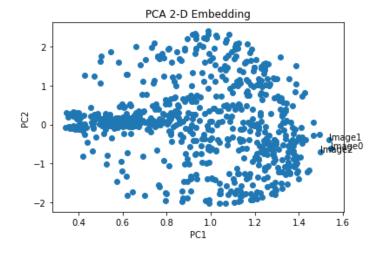
The three points chosen came from varying parts of the graph. Image 0 and Image 600 were somewhat close to each other, but much further from Image 300. Notably, the direction the face is pointing appears to be different at different parts of the graph, with Image 0 and Image 600 both showing faces that are pointing up, though at different angle, while Image 300 shows a face pointing down.



To implement the ISOMAP algorithm, the weighted adjacency matrix computed previously was used to create a pairwise distance matrix using Dijkstra's algorithm to find the shortest length. Next, the centering matrix was computed to then construct the covariance matrix. Afterward, eigen decomposition was performed on the covariance matrix. The top two principal components were then plotted, and the three images corresponding to the three largest eigenvalues of the first principal component were found. Notably, the faces in all three of these images are pointing the same direction, which is similar to the results found in the paper.



The same PCA implementation used previously was used for the step, and the plot of the first two principal components can be seen below:



Notably, the grouping of data points is less even than the ISOMAP implementation and the overall plot appears to be more circular. This may suggest that the PCA Implementation contains some different data in its first two principal components compared to ISOMAP.

When looking at the photos below corresponding to the three largest eigenvalues of the first principal component, it can be seen that the photos selected are more "two-dimensional" than those chosen of ISOMAP. The images chosen by ISOMAP are more of a top-down view and capture more geometry to the faces, while those chosen by PCA are more of a side view and appear flatter. This is to be expected, as PCA is less suitable when non-linear structures are present since the Euclidean distance cannot measure distance globally. Given that the images could appear at different angles, the projection provided by ISOMAP is more meaningful as it better captures this geometry of face shape in its approach.

