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Math 290 - Group Assignment I
8/31/2016

2a.

P	Q	R	$(P \vee R)$	$(P \wedge Q)$	$(P \wedge R)$	$(P \wedge (Q \vee R))$	$(P \wedge Q) \vee (P \wedge R)$
F	F	F	F	F	F	F	F
F	F	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	T	T	T	F	F	F	F
T	F	F	F	F	F	F	F
T	F	T	T	F	T	T	T
T	T	F	T	T	F	T	T
T	T	T	T	T	T	T	T

2b. Not possible for this problem since the problem statement is the definition of the distributive property of and over or

2c. In order to show that $P \wedge (Q \vee R)$ is equivalent to $(P \wedge Q) \vee (P \wedge R)$ we must show two things.

- (1) $P \wedge (Q \vee R)$ then $(P \wedge Q) \vee (P \wedge R)$
- (2) $(P \wedge Q) \vee (P \wedge R)$ then $P \wedge (Q \vee R)$

(1) Suppose $P \wedge (Q \vee R)$ is true. In this case we must also show that $(P \wedge Q) \vee (P \wedge R)$ is true. For $(P \wedge Q) \vee (P \wedge R)$ to be true either $(P \wedge Q)$ must be true OR $(P \wedge R)$ must be true. Since $P \wedge (Q \vee R)$ is true, P must be true and either Q OR R must be true. Since P must be true and either Q OR R must be true, then it follows that at least $(P \wedge Q)$ is true OR $(P \wedge R)$ is true.

(2) assume $(P \wedge R)$ is true. If $(P \wedge R)$ is true then it must be the case that P AND R are both true. Since P AND R are both true, since P AND R must be true, then $(Q \vee R)$ must be true since R is true. and the statement $P \wedge (Q \vee R)$ must also be true since P is also true. Therefore, $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$ must be equivalent.

4a.

P	Q	R	$(Q \wedge R)$	$(P \implies R)$	$(P \implies Q)$	$(P \implies (Q \wedge R))$	$(P \implies Q) \wedge (P \implies R)$
F	F	F	F	F	F	F	F
F	F	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	T	T	T	F	F	F	F
T	F	F	F	F	F	F	F
T	F	T	T	F	T	T	T
T	T	F	T	T	F	T	T
T	T	T	T	T	T	T	T

- 4b $(P \implies (Q \wedge R)) \equiv (\text{Implication removal})$
 $\neg P \vee (Q \wedge R) \equiv (\text{Distribute})$
 $(\neg P \vee Q) \wedge (\neg P \vee R) \equiv (\text{Introduce implications})$
 $(P \implies Q) \wedge (P \implies R)$
- 4c In order to show that $(P \implies (Q \wedge R))$ is equivalent to $(P \implies Q) \wedge (P \implies R)$ we must show two things.

- (1) $(P \implies (Q \wedge R))$ then $(P \implies Q) \wedge (P \implies R)$
(2) $(P \implies Q) \wedge (P \implies R)$ then $(P \implies (Q \wedge R))$

(1) Assume $(P \implies (Q \wedge R))$ is true since the statement is true it follows that Q AND R must both be true. This says nothing about P however, since Q AND R both must be true it is not possible for either $(P \implies Q)$ OR $(P \implies R)$ to ever be false since the only way for the implication to be false is if either Q OR R is also false. Since both $(P \implies Q)$ AND $(P \implies R)$ must be true it must be true when $(P \implies (Q \wedge R))$ is true.

(2) Assume $(P \implies Q) \wedge (P \implies R)$ is true. Which means P, Q AND R can be all false, OR Q AND R can be true. When P, Q, AND R are false. $(Q \wedge R)$ is false, which means since P is false $(P \implies (Q \wedge R))$ must be true. Which is consistent with $(P \implies Q) \wedge (P \implies R)$. When Q AND R are true, it is not possible for $(P \implies Q) \wedge (P \implies R)$ to ever be false since the implication can only evaluate to false when either Q OR R are false. Additionally, since Q AND R are both true, $(Q \wedge R)$ must also be true, forcing $(P \implies (Q \wedge R))$ to also always be true. Therefore, $(P \implies (Q \wedge R))$ must be equivalent to $(P \implies Q) \wedge (P \implies R)$