Dynamic Cash Management Models with Loans

Zimian Zhang

Lancaster University

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Outline

1. Introduction
2. Overall research project
3. Current progress
4. Research plans for next year

Introduction

- What is "cash management problem"?

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- Models from literature

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- Wrap-up from the 1^{st} year's work

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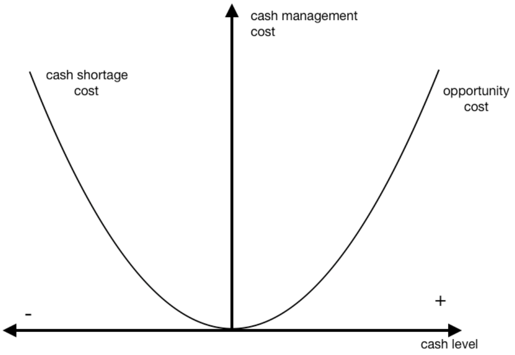
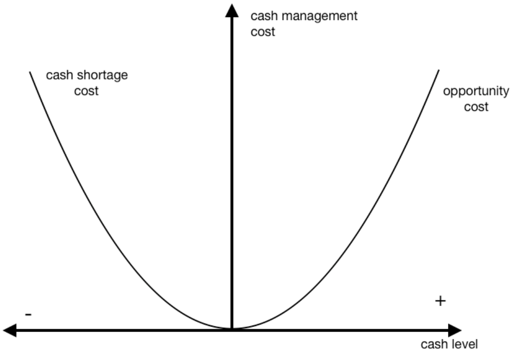


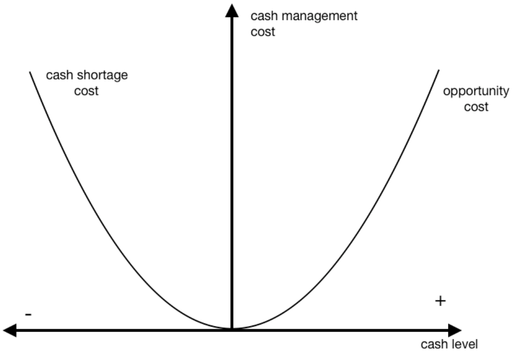
What is cash management problem?

- With insufficient cash holding level, a company might expose to the risk of cash deficit, which might cause a great amount of penalty.



What is cash management problem?

- With insufficient cash holding level, a company might expose to the risk of cash deficit, which might cause a great amount of penalty.
- On the other hand, a high cash-holding level normally means the inefficient use of firm's resource, which would constrain firm's future profitability.



Models from literature

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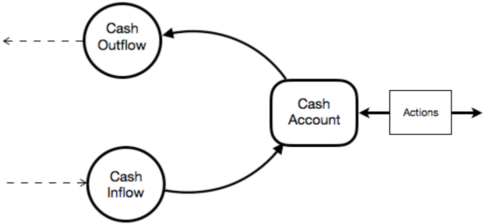

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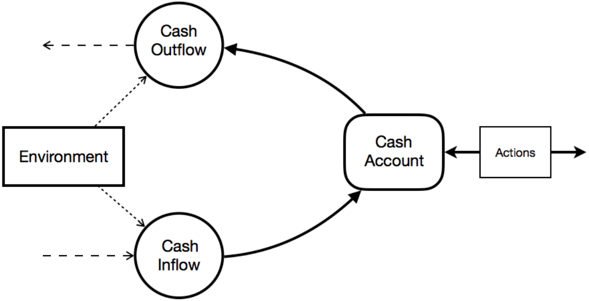

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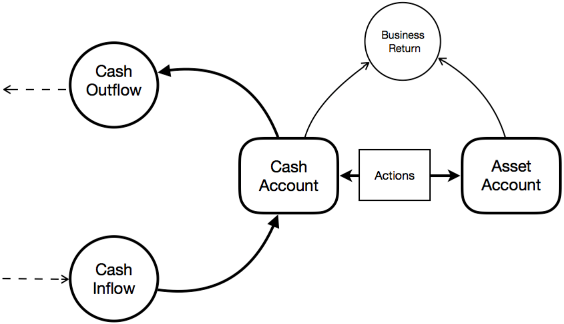

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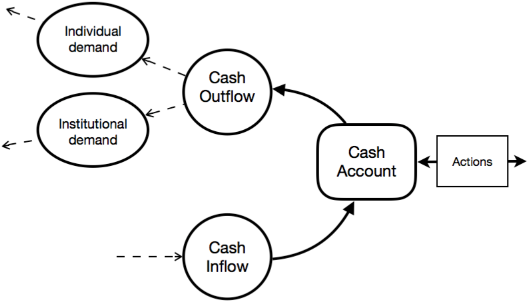

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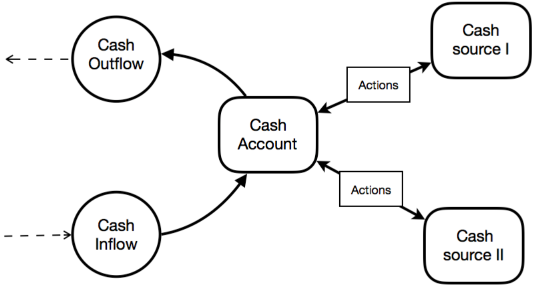

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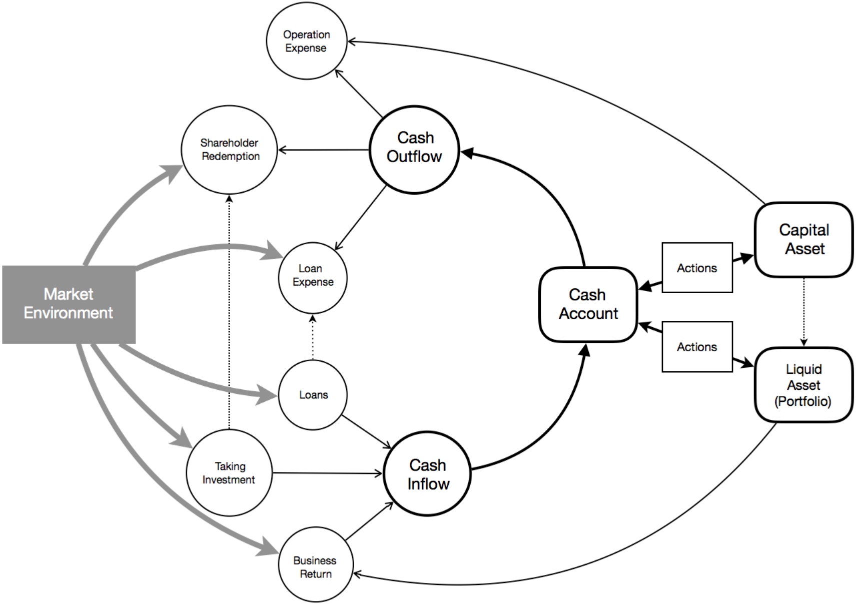
- We study the cash holding strategy with other “cash related activities”.
- We propose a holistic CM model in financial sector (e.g. mutual funds, commercial banks, pension funds or insurance companies)

A holistic model in financial sector



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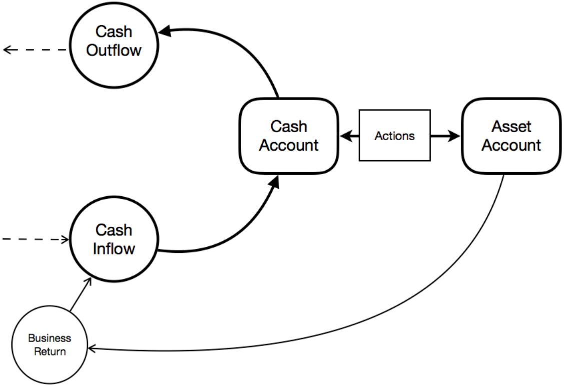

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Objective Function

$$\max_{a \in \prod} \mathbb{E} \left\{ \sum_{t=0}^{T} r y_t \right\}.$$

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  - Recursion Method:

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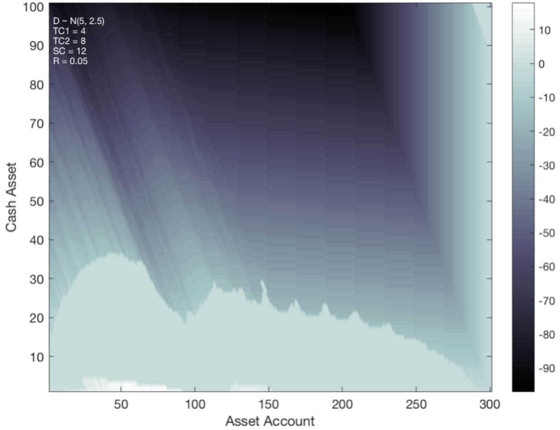
Recursion Method:

1. $$V_{\mathbf{s}}^{T+1} = 0$$
2. $$V_{\mathbf{s}}^{t}(\mathbf{a}) = ry + \int_{-\infty}^{\infty} p_\theta V_{\mathbf{s}'}^{t+1} d\theta.$$
3. $$a^* = \underbrace{\arg\max}_{a \in A_{\mathbf{s}}} \left\{ ry + \int_{-\infty}^{\infty} p_\theta V_{\mathbf{s}'}^{t+1} d\theta \right\}$$
4. $$V_{\mathbf{s}}^{t} = \max_{a \in A_{\mathbf{s}}} \left\{ ry + \int_{-\infty}^{\infty} p_\theta V_{\mathbf{s}'}^{t+1} d\theta \right\}.$$

Wrap-up from 1st year's work

- Result: a solution suggested by DP method



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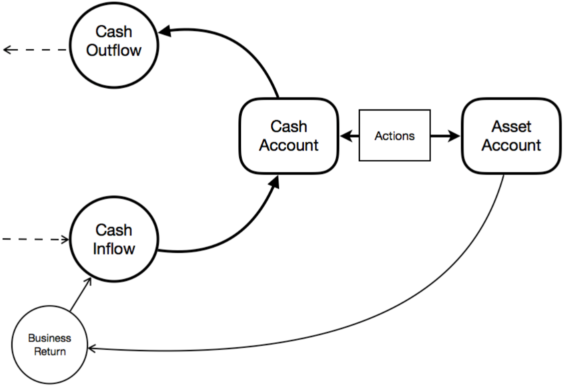

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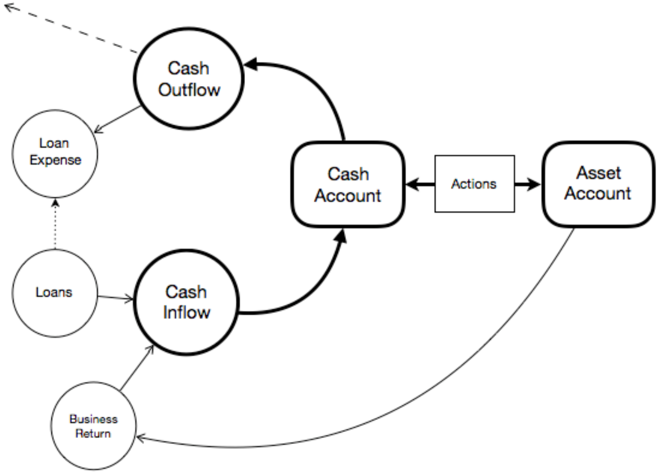
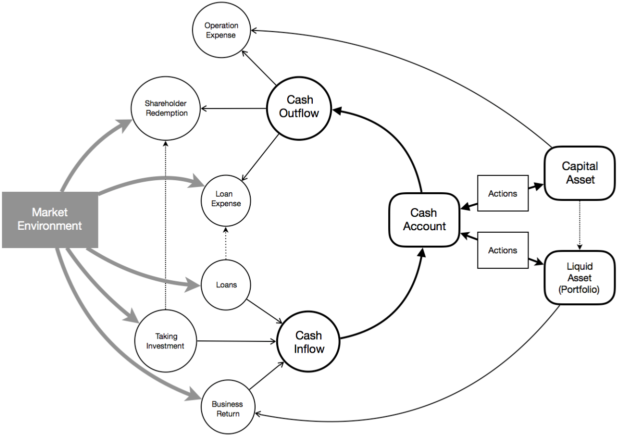


Research Steps

- Step I: Examine the two assets CM model
- Step II: Introduce the loan options into the CM model
- Step III: Complete the holistic CM model and propose an effective way to solve it (approximate dynamic programming).



Thesis structure

- Chapter I. Introduction
- Chapter II. Literature Review
  - 2.1 Literature Review of Cash Management in OR
  - 2.2 Literature Review of Cash Flows in Financial Sector
  - 2.3 A Holistic Model based on Literature
- Chapter III. Methodology
  - 3.1 Introduction to Dynamic Programming
  - 3.2 Introduction to Approximate Dynamic Programming
- Chapter IV. A Two Assets Cash Management Model
  - 4.1 Model Summary
  - 4.2 Dynamic Programming Method
  - 4.3 Numeric Experiment

Thesis structure

- Chapter V. A Cash Management Model with Loans
  - 5.1 Model Summary
  - 5.2 Dynamic Programming Method
  - 5.3 Heuristic Method: One Step Policy Improvement
  - 5.4 Numeric Experiment
- Chapter VI. Approximate Dynamic Programming in Cash Management Models
  - 6.1 Temporal Difference Method
  - 6.2 Eligibility Trace Method
  - 6.3 Function Approximation
- Chapter VII. Conclusion

Current progress

- Modify the two asset model and the performance of the cash holding strategy
- Introduce the loan options into the model and using DP and heuristic method to solve the model
- Approximate Dynamic Programming: Temporal Difference and Eligibility Trace in the CM model

The two assets CM model

- Objective function: Maximising net profits

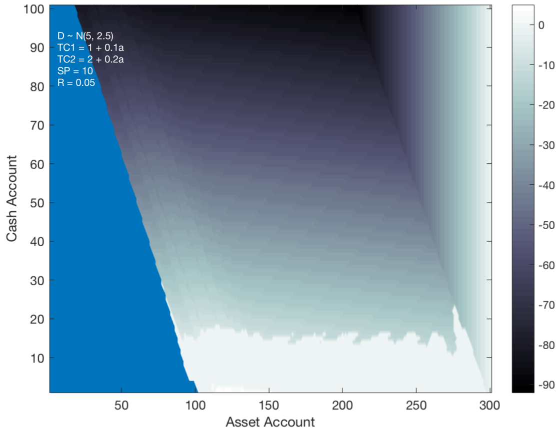
$$\max \sum_{t=0}^{\infty} \gamma^t \{rr \cdot y_t - D_t - \Gamma_t - SC_t\}.$$

- A partially fixed and partially proportional transaction cost function

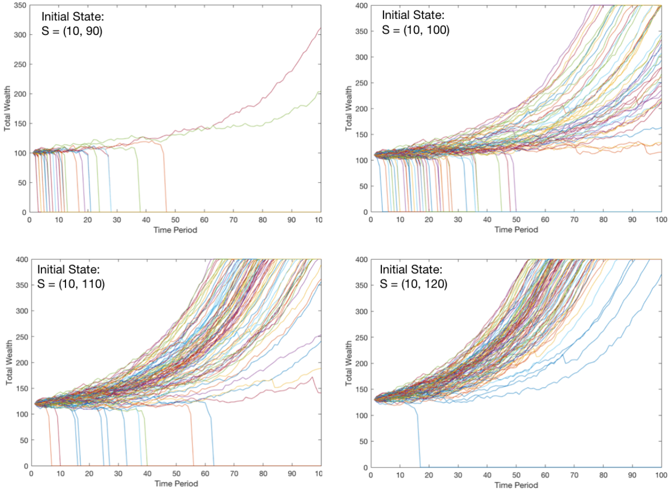
$$\Gamma(a) = (K_c + k_c a) \cdot 1_{\{a<0\}} + (K_a + k_a a) \cdot 1_{\{a>0\}}$$

- Options of declaring bankruptcy

An optimal solution of the modified model



Simulation of the strategy



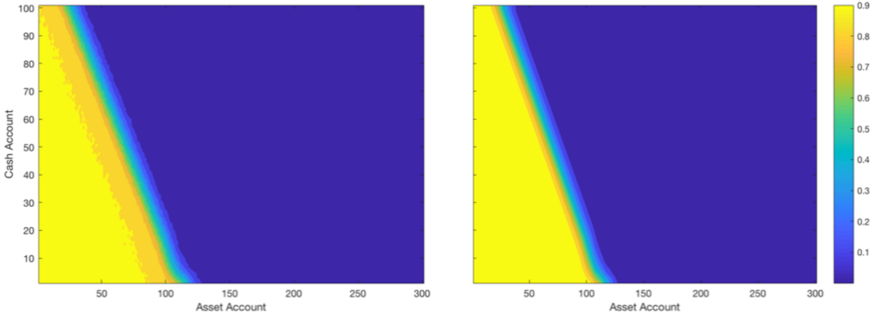
Probability of going bankrupt

- At stage 0 (the last period of time planning horizon), any state $S_{x,y}$ with $y \neq 0$ has value (probability of going bankrupt) equal to 0 and any state $S_{x,y}$ with $y = 0$ has value (probability of going bankrupt) equal to1.
- for any stage $k : k \geq 1$

$$
\begin{aligned}
V^k_{[x,y]} = & \sum P \left\{ S_{(0,0)} : W(S_{x,y}) = S_{(0,0)} \,\middle|\, a = A^*(S_{x,y}) \right\} \\
& + \sum P \left\{ S_{x',y'} : W(S_{x,y}) = S_{x',y'} \,\middle|\, a = A^*(S_{x,y}) \right\} V^{k-1}_{[x',y']}
\end{aligned}
$$

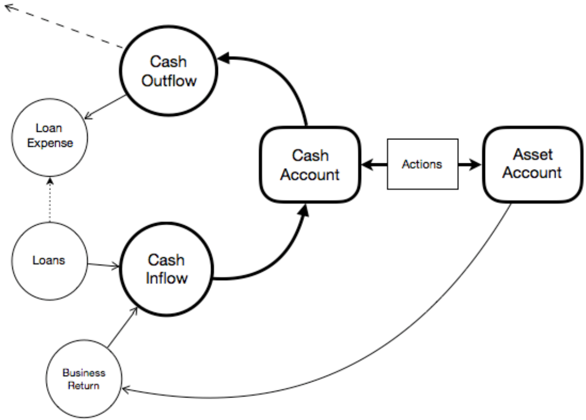
where $V^k_{[x,y]}$ is the probability that the company will eventually going bankrupt if it is in state $S_{x,y}$ at stage k

Probabilities of going bankrupt in each state: Simulation result and Backward calculation result



Cash management with loan options

In reality, once company's income could not cover its cash demand, instead of selling asset and jeopardising future profitabilities, managers tend to take loans from other companies or financial intermediaries.



Cash management with loan options

- State: $S_{x,y,z}$ where x and y represent the current cash and asset level and z represent the remaining times of loan repayment.
- Loan Repayment LP: let L be the loan size, lr be the loan rate and once the manager take the loan, he has to make an equally amount of repayment in following N time periods. Then for each time period, he has to pay

$$LP = L \cdot \frac{lr \cdot (1 + lr)^N}{(1 + lr)^N - 1}$$

- We assume that companies with debt unpaid cannot take more loans.
- At time t, if the manager take the loan, then the cash inflow increases by L amount and its loan state s changes from 0 to N. In the following N time periods, the company's cash demand will increase by LP amount and z value decreases by 1.

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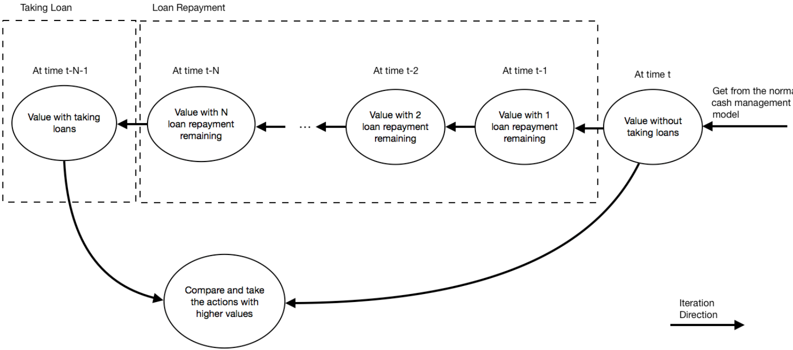
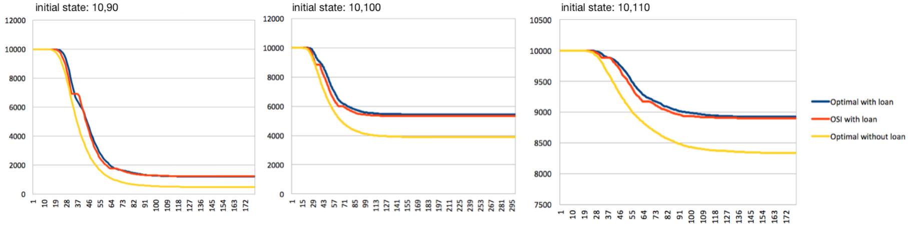


Simulation results of one-step policy improvement

Assume there is only one loan available on the market:
$L = 40, lr = 0.03, N = 40$



The cost of dynamic programming

Updating rule:

$$a(S_{x,y}) = \arg\max \left\{ R_t^a(S_{x,y}) + \gamma \sum_{S_{x',y'} \in S} V_{t-1}^a(S_{x',y'}) \mathbb{P}\left\{ S_{x',y'} : W(S_{x,y}) = S_{x',y'} \right\} \right\}$$

$$V^a(S_{x,y}) = R_t^a(S_{x,y}) + \gamma \sum_{S_{x',y'} \in S} V_{t+1}^a(S_{x',y'}) \mathbb{P}\left\{ S_{x',y'} : W(S_{x,y}) = S_{x',y'} \right\}.$$

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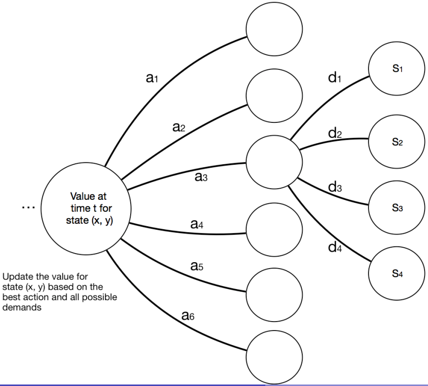

One-step Temporal Difference

Updating rule: $V(S_{x,y}|A = a^{\pi}(S_{x,y})) :=$

$$V(S_{x,y}|A = a^{\pi}(S_{x,y})) + \alpha \left[R + \gamma V(S_{x',y'}|A = a^{\pi}(S_{x',y'}))\right] - V(S_{x,y}|A = a^{\pi}(S_{x,y}))$$

where α is the update step size parameter.

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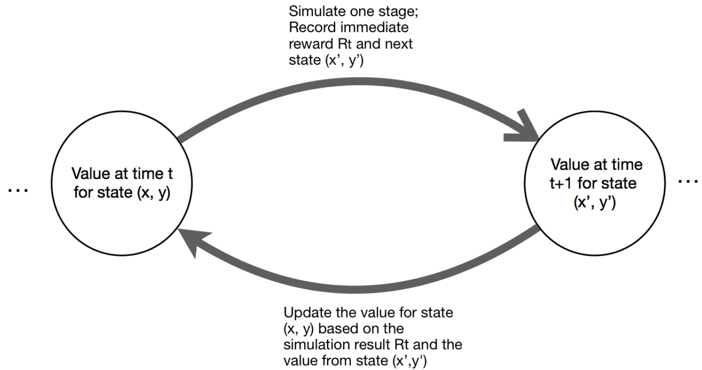
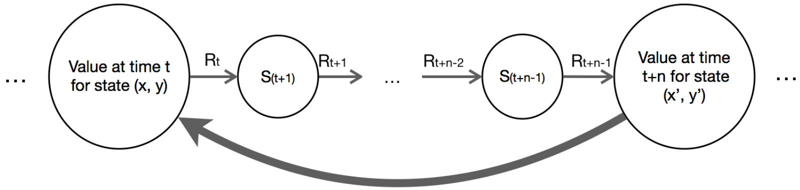


n-step Temporal Difference



Update the value for state (x, y) based on the simulation result R_t, $R_{t+1}, \ldots, R_{t+n-1}$ and the value from state (x',y') at time t+n

Result of 1 step TD

- Limitation of TD method: Speed of convergence.

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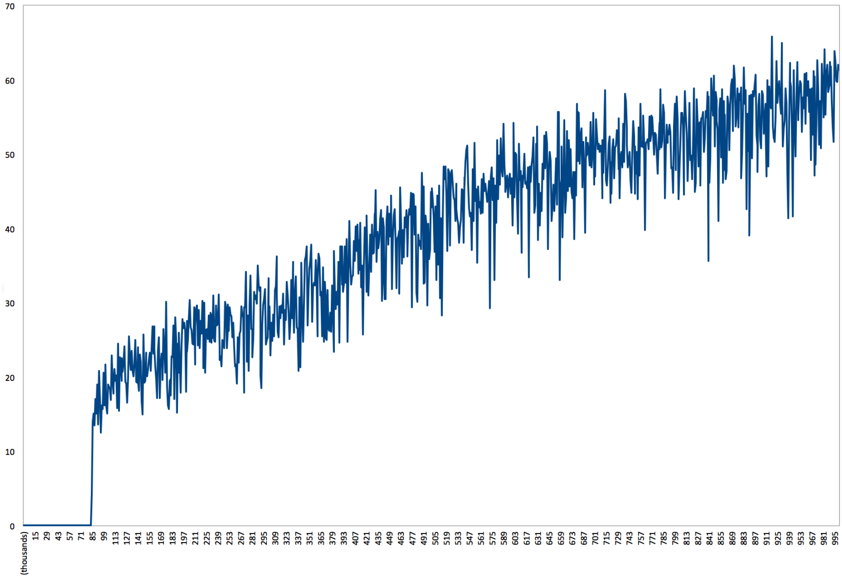

Result of 1 step TD

- Limitation of TD method: Speed of convergence.
- How to improve?
    - change the step size parameter α
    - use eligibility trace method
    - use function approximation

Research Plan for next year

- Study temporal difference and eligibility trace method and apply it into cash management models
- Study approximate functions in ADP; Find a function that can approximate values in DP model.
- Develop the literature review in terms of cash flows in financial companies and modify the holistic model.
- Do numeric experiments for each cash management model and finish the thesis.

Time table for next year

September	Use ET in two assets model;
October	Numeric experiment of two assets model; Study the function approximate in ADP;
November	Continue in the study of the function approximate in ADP; Write the fourth chapter of the thesis;
December	Finish the fourth chapter of the thesis; Continue in the study of the function approximate in ADP;
January	Use TD and ET in the CM model with loans; Write the fifth chapter of the thesis;
Feburary	Use Approximate Functions in the CM model with loans; Write the fifth chapter of the thesis;
March	Numeric experiment of the CM model with loans Finish the fifth chapter of the thesis;

Time table for next year

April	Read more literature about cash flows in financial sector Modify the holistic model
May	Read more literature about cash flows in financial sector; Use TD and ET in the holistic model
June	Write up the sixth chapter of the thesis Use approximate functions in the holistic model
July	Write up the sixth chapter of the thesis Numeric experiment in the holistic model
August	Finish the sixth chapter of the thesis; Write the literature review of the thesis;
September	Write the methodology of the thesis; Finish the first draft of the thesis;

THANK YOU

FOR YOUR ATTENTION

ANY QUESTIONS...?