Lecture 13

Thermal Design

Convective Heat Transfer

March 6, 2025

Reminders and Announcements

- Homework #3 due Friday, March 7th, by 11:59pm
- Office Hours: Friday 2:00pm-3:00pm
- Please put the course number in the subject line in emails to the instructor and TA (this helps us filter our inboxes so we don’t miss your messages)
- Midterm exam: Thursday, March 20th, 5:00pm-6:15pm
- Midterm exam review session during the lecture on Tuesday, March 18th

Midterm: Thursday, March 20th, 5:00pm-6:15pm

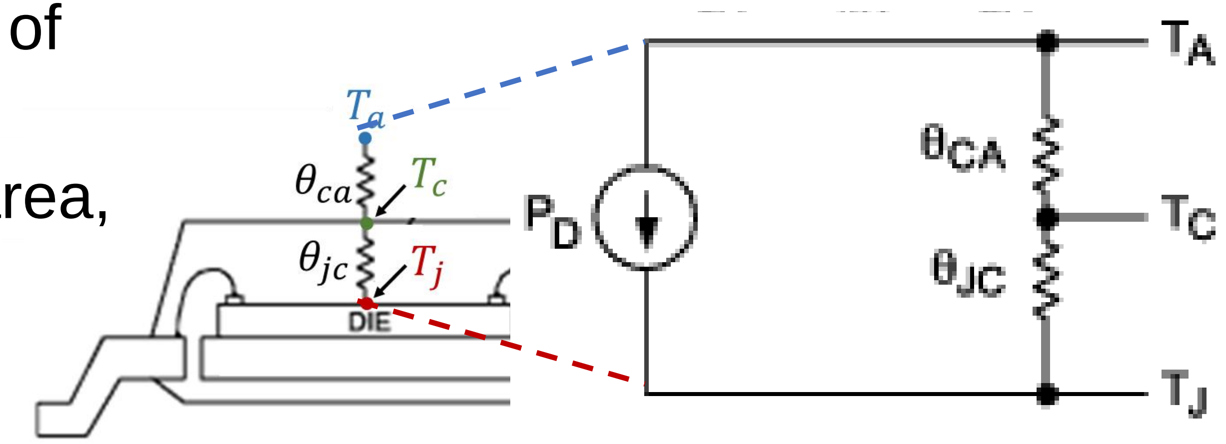
- If you are located in Blacksburg, then you should take the exam in-person in TORG 1050, even if you are enrolled in the virtual section of the course
- If you are enrolled in the *virtual campus*, please email me by March 5th indicating if you will take the exam virtually or in-person in Arlington or Blacksburg
- Will cover the topics in lectures 1-6 (packaging overview and electrical design) and 9-13 (transmission lines and thermal design)
- The lecture on March 18th will be a review session → come ready with questions or topics you would like to cover
- You will <u>not</u> be asked to do any simulations for the midterm
- The problems will be a mix of conceptual short response and calculation problems
- Things to bring to the exam: writing utensils & *non-programmable* calculator
- An equation/reference sheet and extra paper will be provided by the proctor
- The reference sheet will be uploaded to Canvas next week; you do <u>not</u> need to print out the reference sheet

Package Thermal Resistance

- θ_{ja} can be separated into two parts:
  - Junction-to-case, θ_{jc}
  - Case-to-ambient, θ_{ca}

$$\boxed{\theta_{ja} = \theta_{jc} + \theta_{ca}}$$

- Junction-to-case, θ_{jc}
  - Depends on the internal construction of the package
  - Depends on length, cross-sectional area, and k
- Case-to-ambient, θ_{ca}
  - Depends on the mounting and cooling techniques
  - Depends on wetted surface area and h



Convection

- Transfer of heat between the surface of a body and a fluid in motion
- Newton's Law of Cooling:

$$q = hA_s(T_s - T_f)$$

- q = heat (W)
- h = convective heat transfer coefficient (W/(m^{2}K))
- A_s = wetted surface area (m^2)
- T_s = surface temperature (°C)
- T_f = bulk temperature of fluid (°C)
- Rearranging the above equation:

$$\frac{1}{hA_s} = \frac{(T_s - T_f)}{q} \quad \rightarrow \quad R_{th,conv} = \frac{1}{hA_s}$$

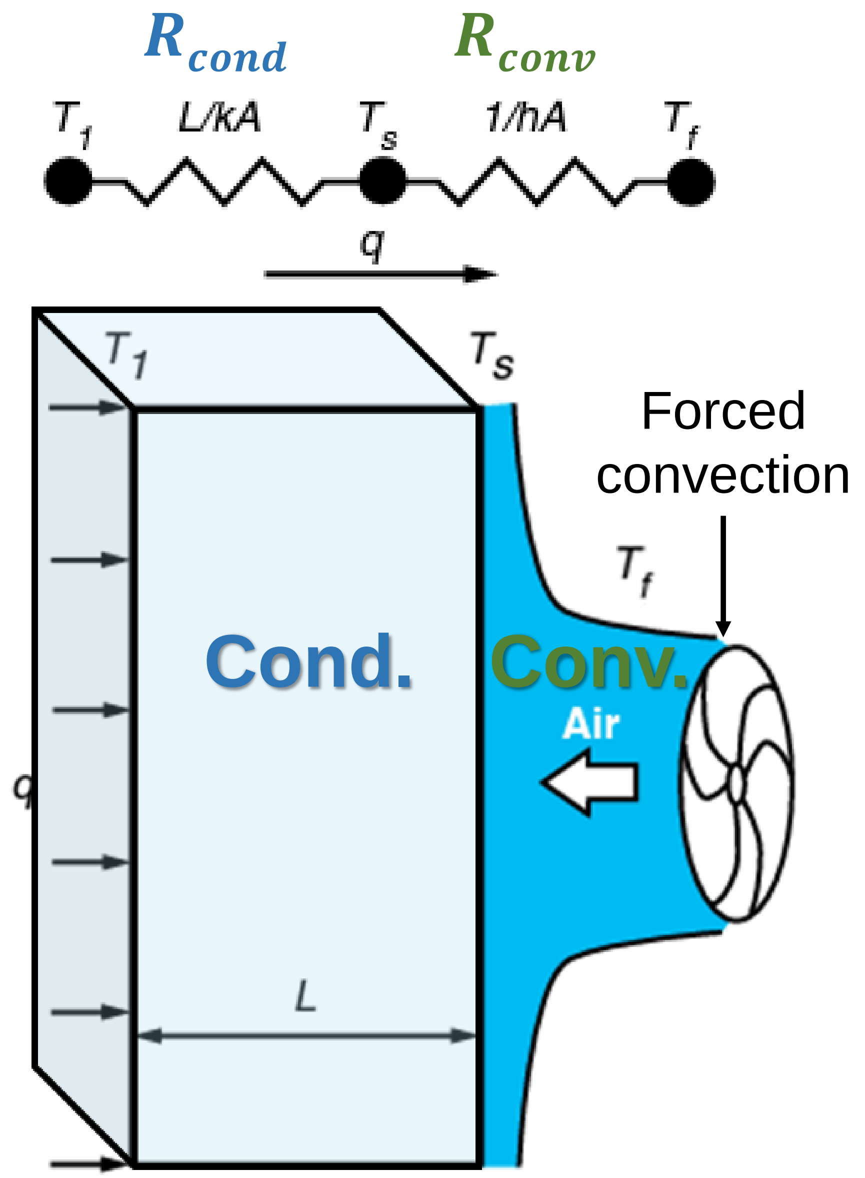
Conduction & Convection Thermal Resistances

$$q = \frac{kA_c(T_h - T_c)}{L} \qquad R_{th,cond} = \frac{L}{kA_c}$$

q = heat (W)
k = thermal conductivity (W/(m·K))
A_c = cross-sectional area (m²)
L = length q needs to travel (m)
T_h = hot temperature (°C)
T_c = cold temperature (°C)

$$q = hA_s(T_s - T_f) \qquad R_{th,conv} = \frac{1}{hA_s}$$

h = heat transfer coefficient (W/(m²K))
A_s = wetted surface area (m²)
T_s = surface temperature (°C)
T_f = bulk temperature of fluid (°C)



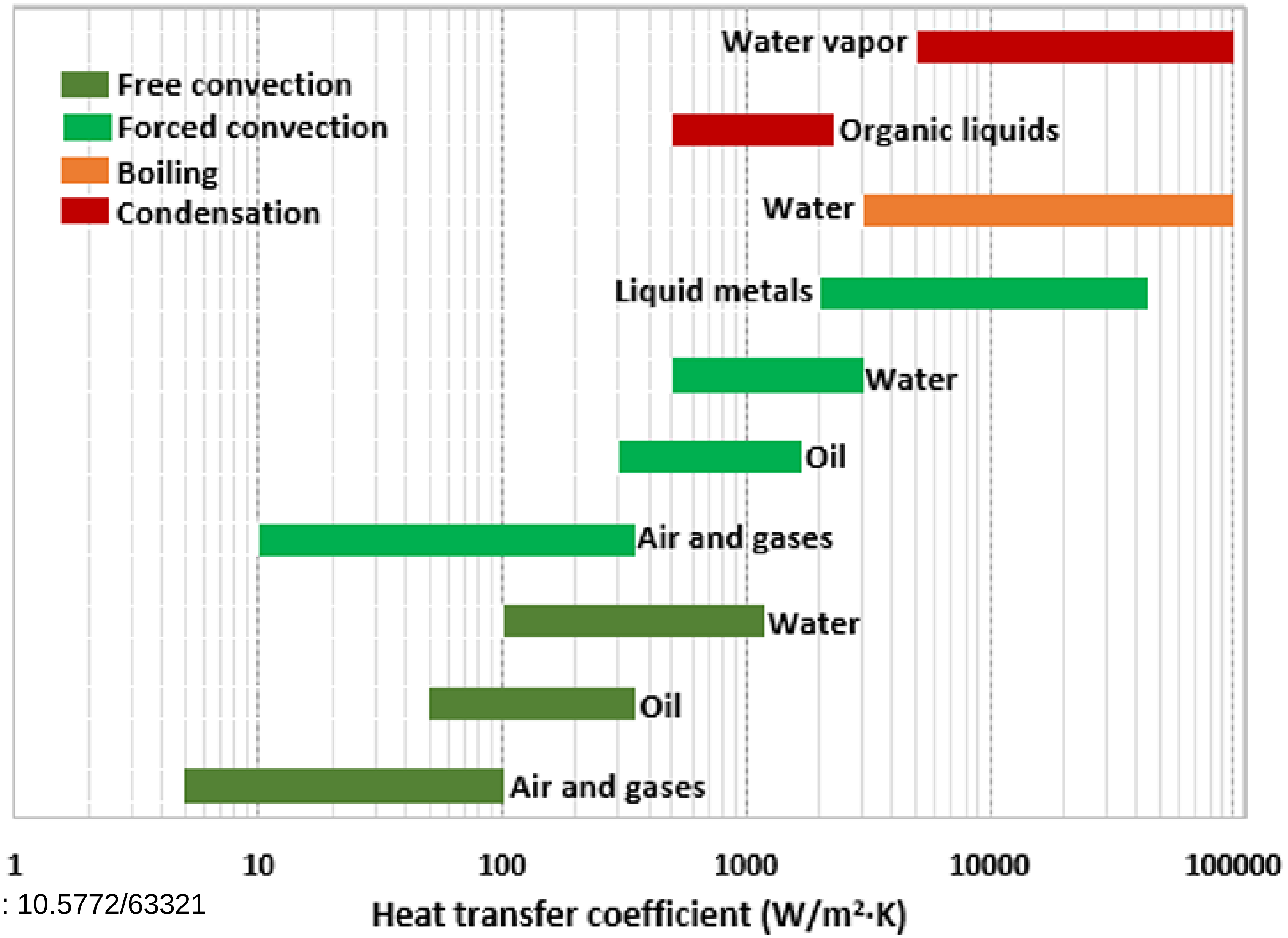
Convection Heat Transfer Coefficient h

$$q = hA_s(T_s - T_f)$$

- h depends on the properties of the fluid, the velocity of the fluid, and the surface geometry
- h can be determined empirically or analytically

Cooling Method	h (W/(m²K))
Free (natural) convection	5 – 25
Forced convection, air	25 – 250
Forced convection, water	100 – 10,000
Boiling water	1,000 – 50,000
Condensing steam	5,000 – 100,000

Heat Transfer Coefficients

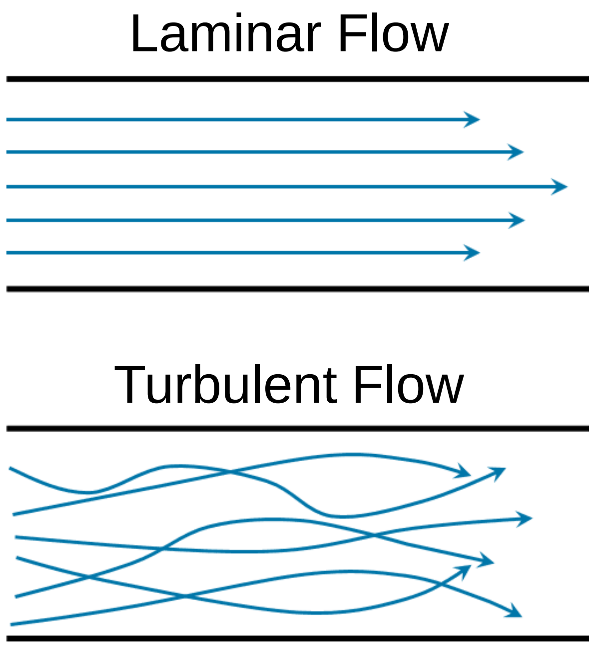


S M Sohel Murshed, DOI: 10.5772/63321

Types of Convection

- **Free** (or natural)
  - Occurs due to buoyancy effects: hotter fluid adjacent to a hot surface rises, leading to the transfer of heat from the hot surface
- **Forced**
  - Occurs when heat is transported from a hot surface by a fluid stream moved by an external stimulant (e.g., fan, pump)
- **Mixed** (combination of free and forced)
  - Occurs when the forced fluid velocity is low such that heat transfer due to free and forced convection are of similar magnitudes

Flow Regimes



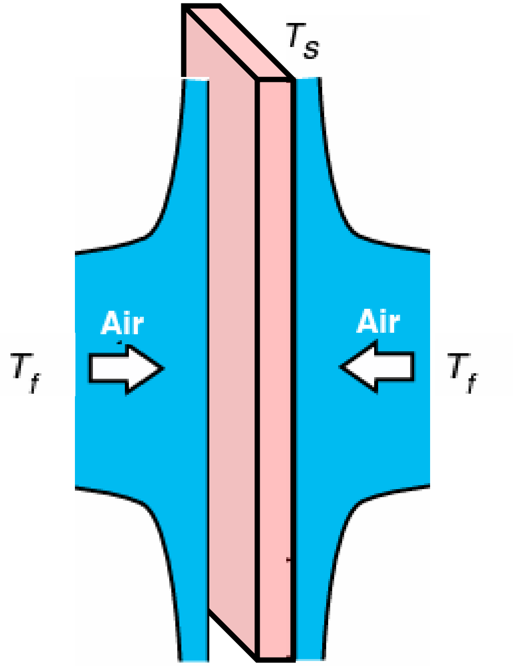
- **Laminar**
  - Orderly flow
  - Streamlines can be clearly traced
- **Turbulent**
  - Spatially and time-varying features in the flow
  - Preferred for cooling

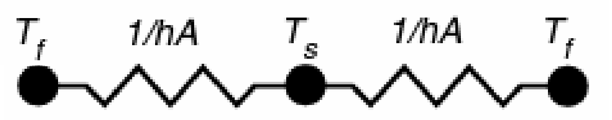
- **Reynolds number**: used to characterize the laminar-turbulent transition for *forced* convection
- **Rayleigh number**: used to characterize laminar-turbulent transition for *free* convection

Example: Convection

Calculate the average temperature of a 20 cm x 20 cm PWB dissipating 10 W cooled by natural convection in air at 35 °C from both sides.

- $q = 10$ W
- $h = 5$ W/m^{2}K
- $A_s = 20$ cm x 20 cm= 0.04 m^2 (per side)
- $R_{th,conv,total} = \left(\frac{1}{hA_s}\right) || \left(\frac{1}{hA_s}\right) = \frac{1}{2hA_s}$
- $T_f = 35$ °C
- $T_s = \left(\frac{q}{2hA_s}\right) + T_f = \frac{(10\ \mathrm{W})}{2\left(5\frac{\mathrm{W}}{\mathrm{m^2K}}\right)(0.04\ \mathrm{m^2})} + 35°\mathrm{C} = \mathbf{60°C}$



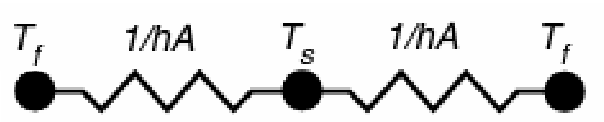


$q = hA_s(T_s - T_f)$

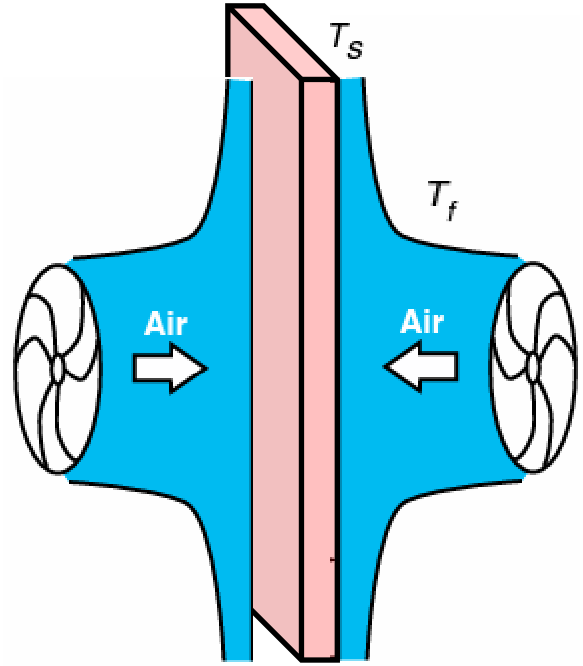
Example: Convection

Estimate the power dissipation from the same PWB to maintain the same average temperature if it were cooled using air in two-sided forced convection, flowing at a sufficiently high velocity (4–5 m/s) across the surface of the PWB to yield an h of 25 W/m²K.

- T_s = 60 °C
- T_f = 35 °C
- h = *25 W/m²K*
- A_s = (20 cm x 20 cm)= 0.04 m² (per side)
- q = ?



$$q = hA_s\left(T_s - T_f\right)$$



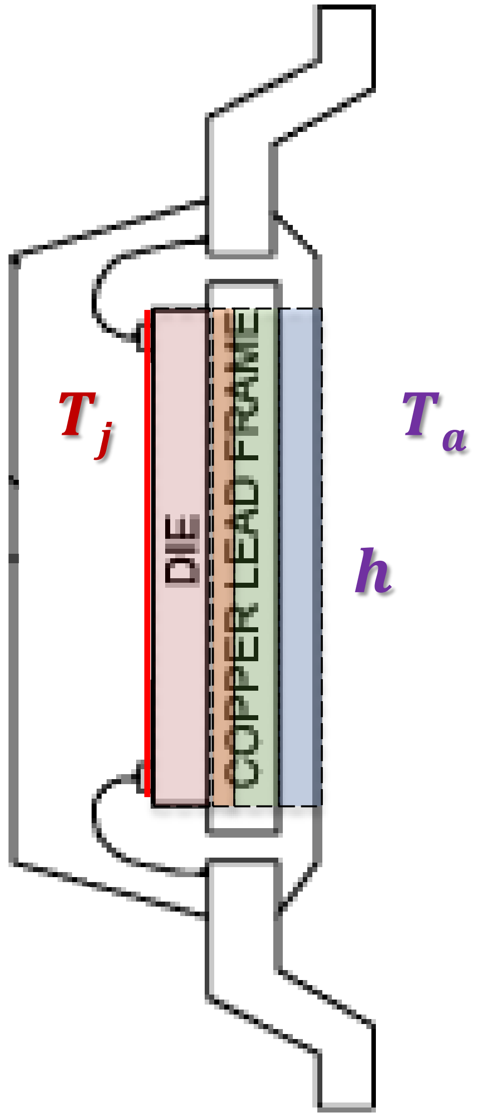
- $q = 2h(A_s)\left(T_s - T_f\right) = 2\left(25\,\frac{\mathrm{W}}{\mathrm{m^2K}}\right)(0.04\ \mathrm{m}^2)(60°\mathrm{C} - 35°\mathrm{C}) = \mathbf{50\ W}$

➢ 5x higher power dissipation with forced convection

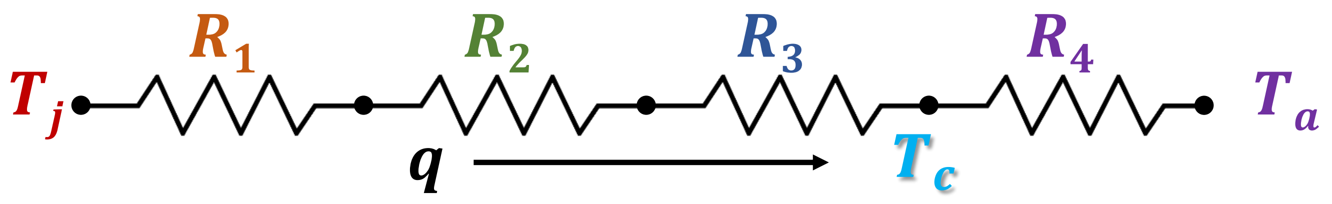
Example: Package Thermal Resistance

Find the junction temperature, T_j, of the die if it dissipates 1 W of heat and has the below specifications.

- $T_a = 25$ °C
- $A_c = 10$ x 10 mm^2 (for all components)
- Solder die attach: $k_1 = 50$ W/(m·K), $L_1 = 0.1$ mm
- Cu lead frame: $k_2 = 390$ W/(m·K), $L_2 = 1$ mm
- EMC: $k_3 = 0.23$ W/(m·K), $L_3 = 1$ mm
- Forced convection (bottom): $h = 200$ W/m^{2}K
- Assume other sides are thermally insulated, that the die is at a uniform temperature, and that the heat flow is uniformly distributed in each layer.



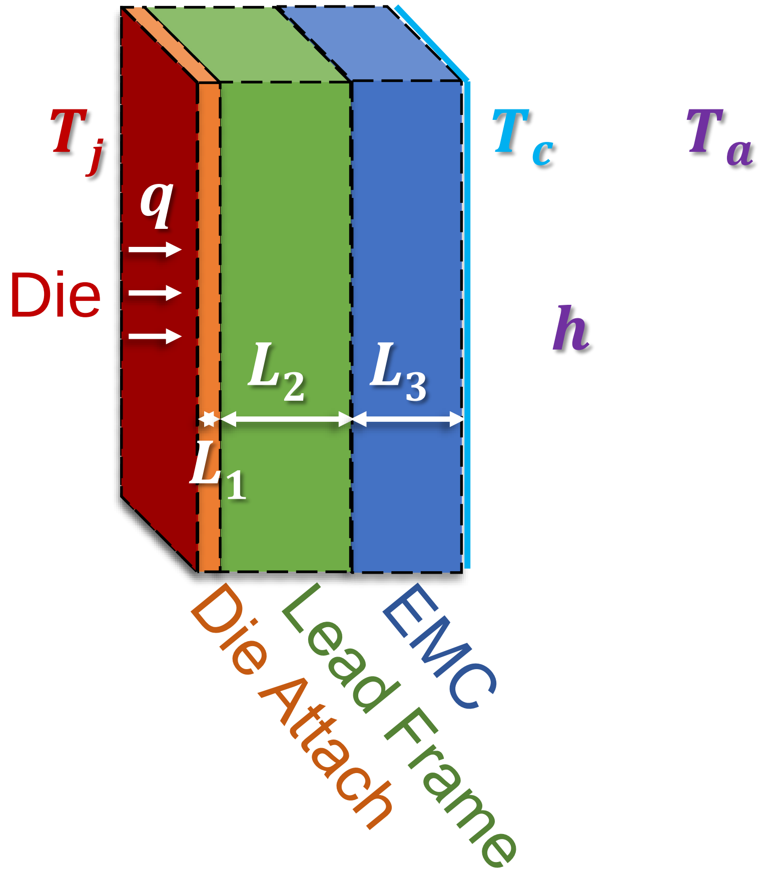
Example: Package Thermal Resistance



$$\boldsymbol{R_{th,cond} = \frac{L}{kA_c}}$$

$$\boldsymbol{R_{th,conv} = \frac{1}{hA_s}}$$

- $T_j - T_a = qR_{th,j-a}$
- $R_{th,j-a} = R_1 + R_2 + R_3 + R_4$
- $R_1 = \frac{L_1}{k_1 A} = \frac{100\times10^{-6}\text{m}}{\left(50\frac{\text{W}}{\text{mK}}\right)(1\times10^{-4}\text{m}^2)} = 0.02\ \text{K/W}$
- $R_2 = \frac{L_2}{k_2 A} = \frac{1\times10^{-3}\text{m}}{\left(390\frac{\text{W}}{\text{mK}}\right)(1\times10^{-4}\text{m}^2)} = 0.03\ \text{K/W}$
- $R_3 = \frac{L_3}{k_3 A} = \frac{1\times10^{-3}\text{m}}{\left(0.23\frac{\text{W}}{\text{mK}}\right)(1\times10^{-4}\text{m}^2)} = \mathbf{43\ K/W}$
- $R_4 = \frac{1}{hA} = \frac{1}{\left(200\frac{\text{W}}{\text{m}^2\text{K}}\right)(1\times10^{-4}\text{m}^2)} = \mathbf{50\ K/W}$



Example: Package Thermal Resistance

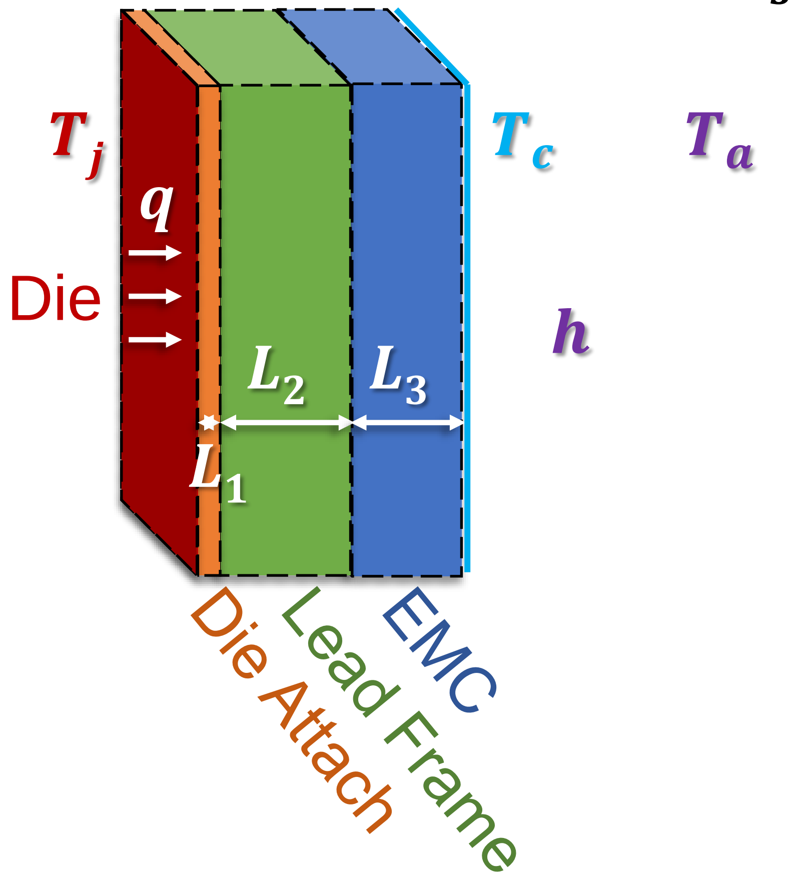
- Find T_j
- $R_{th,j-a} = R_1 + R_2 + R_3 + R_4$
- $R_{th,j-a} = 0.02\ \mathrm{K/W} + 0.03\ \mathrm{K/W}$

  $+ 43\ \mathrm{K/W} + 50\ \mathrm{K/W} = \mathbf{93\ K/W}$
- $T_j = qR_{th,j-a} + T_a$
- $T_j = (1\mathrm{W})\left(93\,\frac{°\mathrm{C}}{\mathrm{W}}\right) + 25°\mathrm{C} = \mathbf{118°C}$

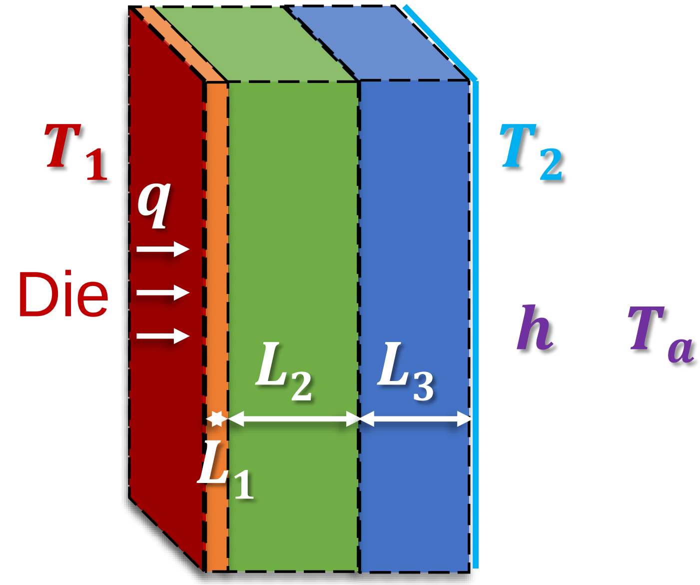
➢ Conduction through the EMC and the convection to ambient are the greatest contributors to the $R_{th,j-a}$

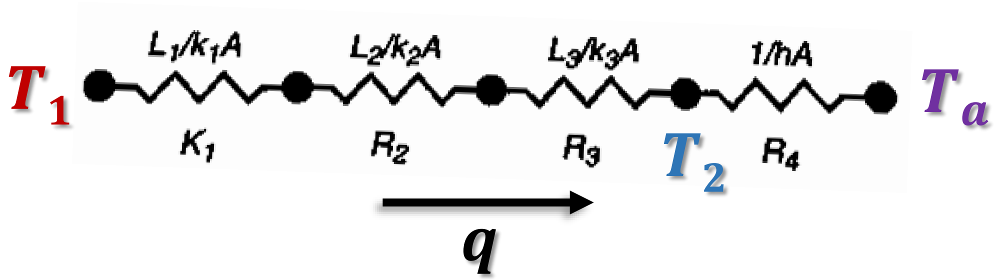
$$\boldsymbol{R_{th,cond} = \frac{L}{kA_c}}$$

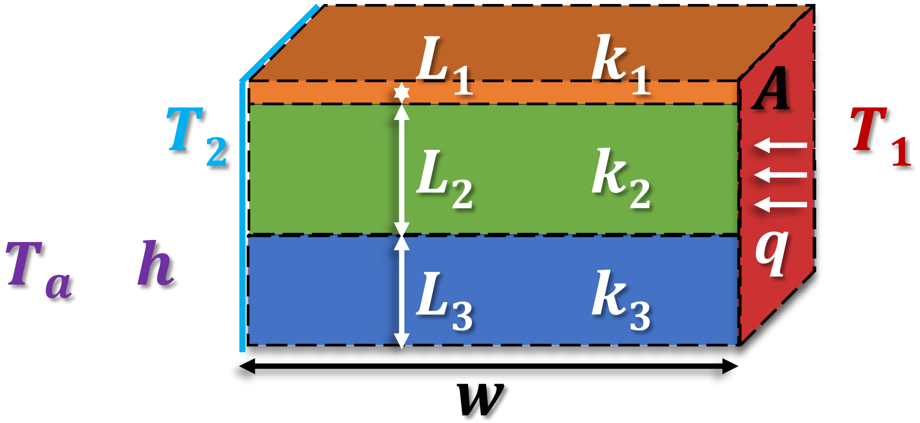
$$\boldsymbol{R_{th,conv} = \frac{1}{hA_s}}$$

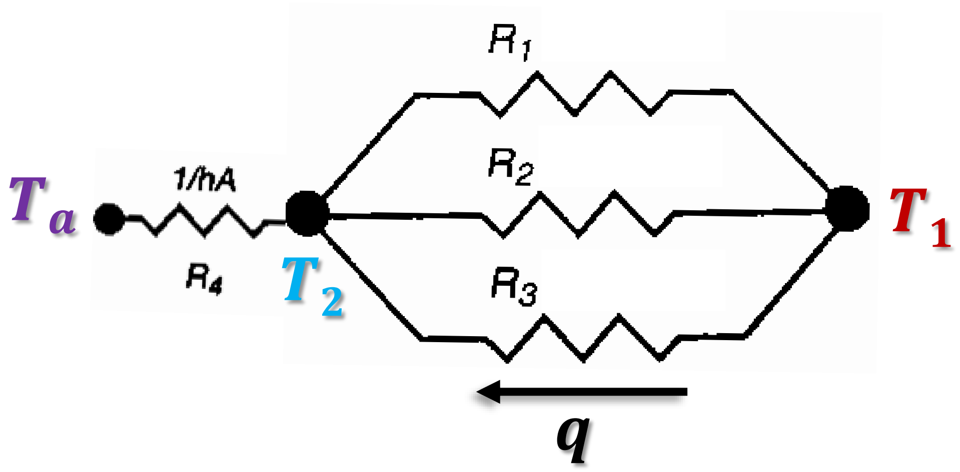


Thermal Resistances in Series and Parallel





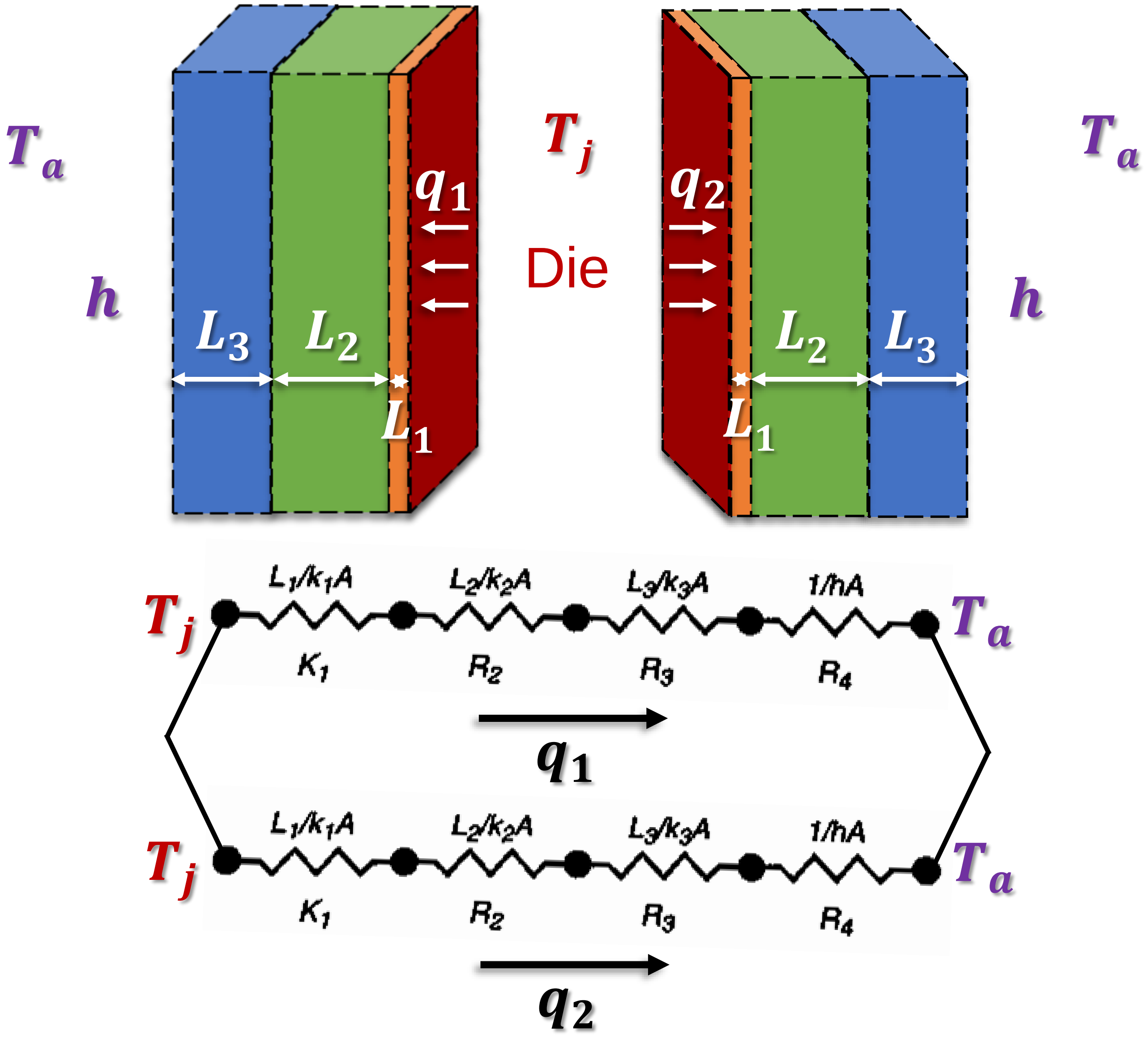




$$R_{th,1-a} = R_1 + R_2 + R_3 + R_4$$

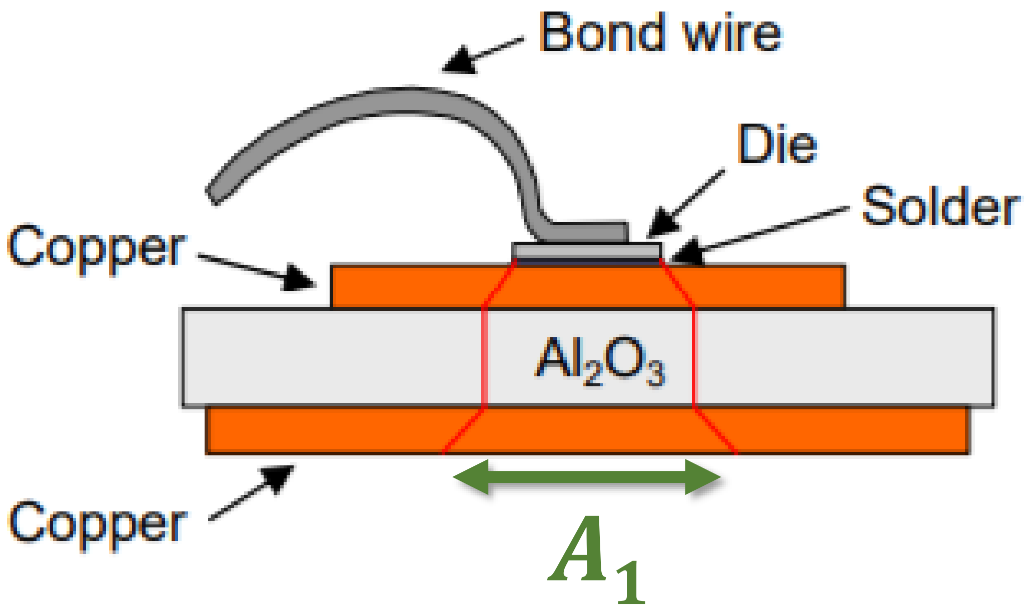
$$R_{th,1-a} = R_{th,1-2} + R_{th,2-a} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} + R_4$$

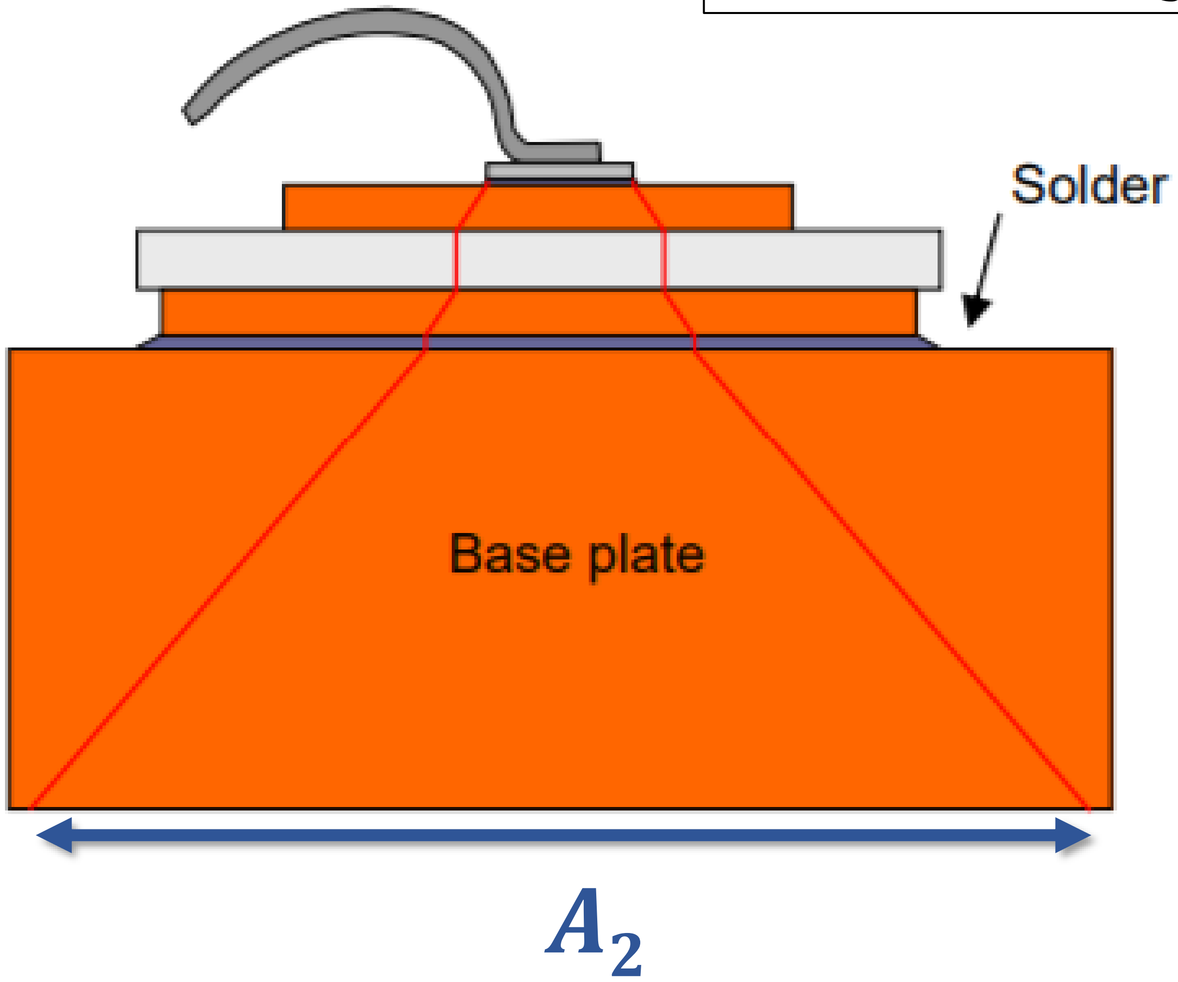
Thermal Resistances in Series and Parallel



Base Plate/Heat Spreader

$$R_{th,conv} = \frac{1}{hA_s}$$





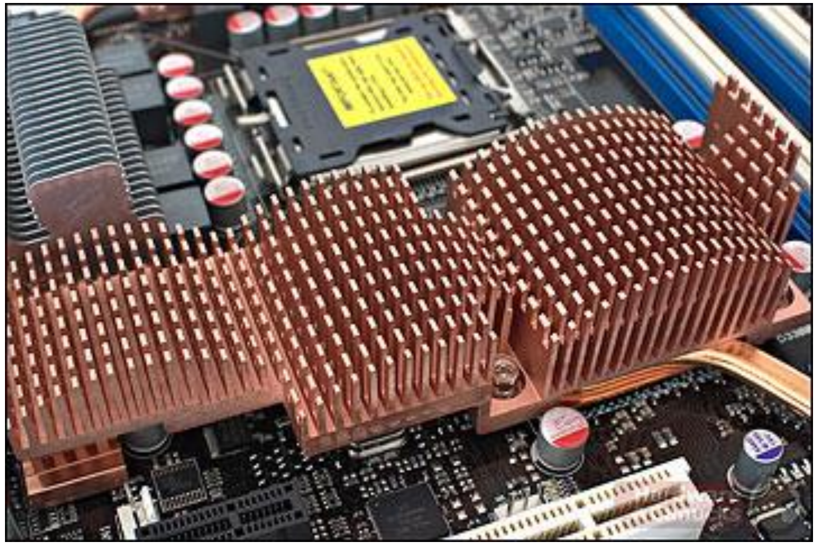
For the same h,

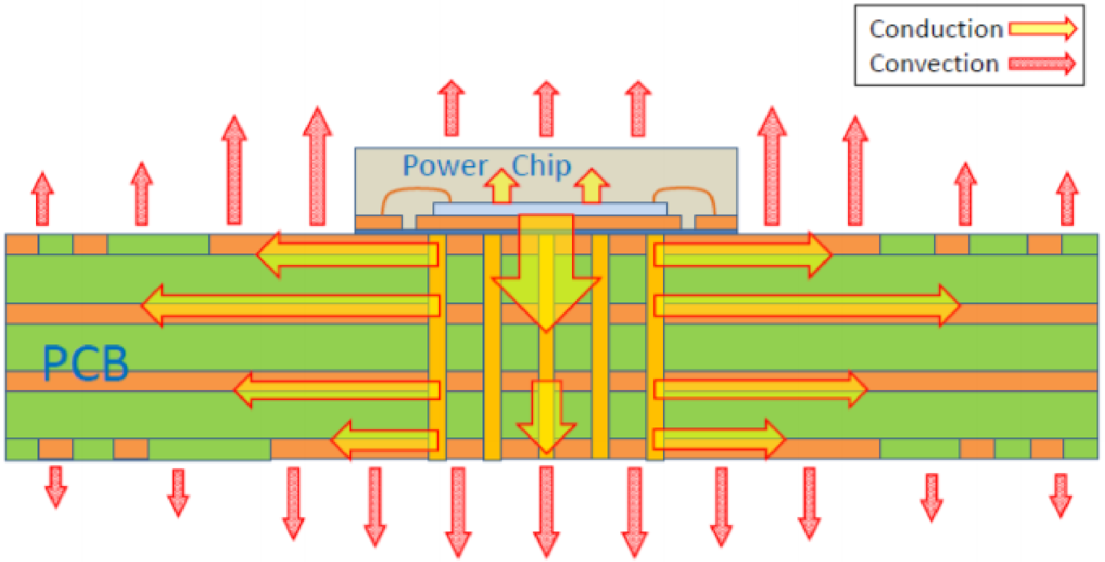
$$R_{th,convA_1} > R_{th,convA_2}$$

*note: if h is high, then Z heat flow > X,Y heat flow, so heat spreading is low and the baseplate becomes less effective.

Cooling Methods

- Heat sink
- Thermal vias
- Cold plate
- Heat pipe
- Vapor chamber
- Jet impingement
- Immersion cooling

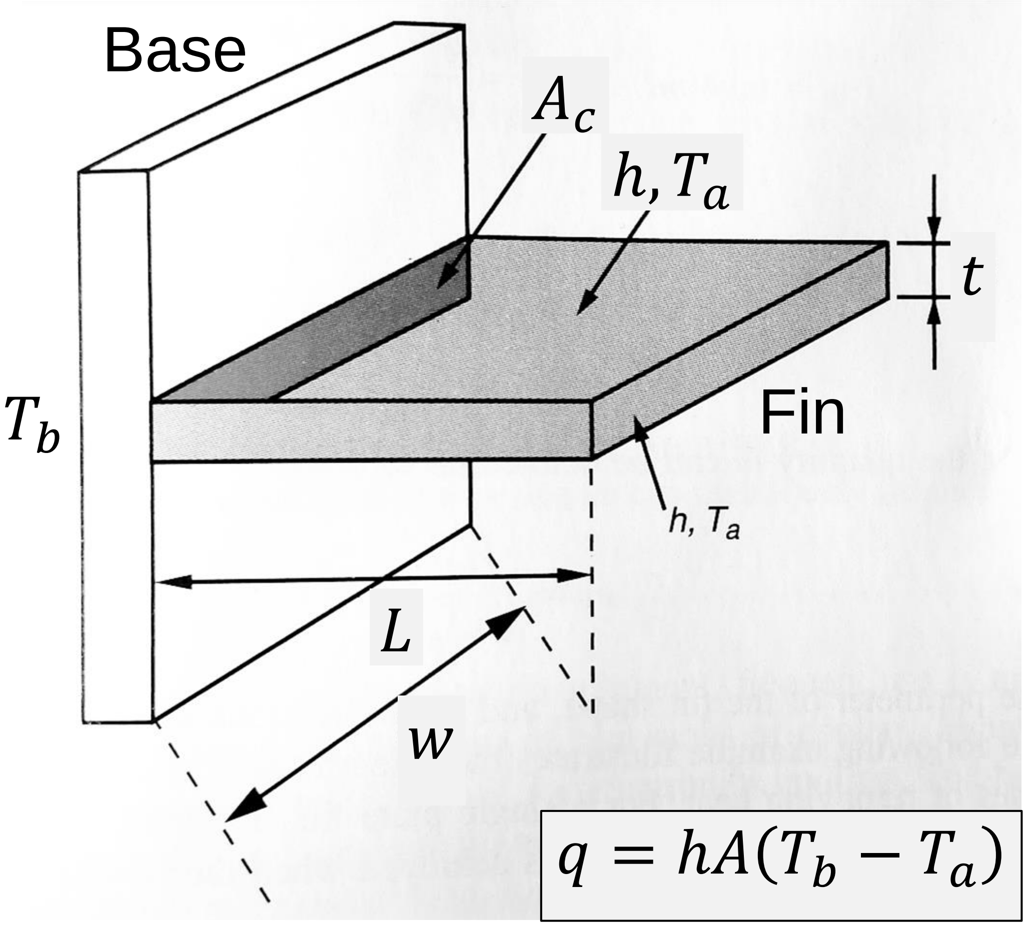




Heat Sinks

$$R_{th} = \frac{T_b - T_a}{q}$$

- Maximum heat transfer would occur if the entire heat sink was at the base temperature T_b
- However, due to $R_{th,cond}$, the temperature will decrease from the base to the end
- Convective heat transfer depends on the temperature difference between the fin and the ambient
- The actual heat transfer will be lower compared to assuming the entire heat sink is at T_b



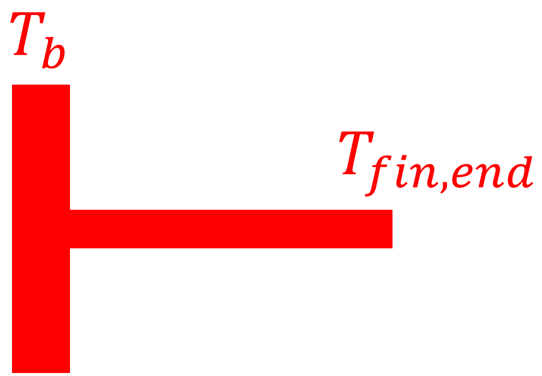
$$q = hA(T_b - T_a)$$

Tummala, 3.4

Fin Efficiency η_{fin}

- The actual heat transferred by the fin, divided by the heat transfer were the fin to be isothermal (i.e., have an infinite thermal conductivity and thus no conductive thermal resistance)
- Going from the base to the end of the fin, the temperature reduces (due to conductive thermal resistance of the fin and heat lost due to convection) and therefore the heat transfer to the fluid also decreases
- η_{fin} is a fraction between 0 and 1. A value of 1 would occur if the fin had infinite thermal conductivity and thus the entire fin would be at the same temperature as the base

$T_b > T_{fin,end} \rightarrow \eta_{fin} < 1$ $\qquad$ $T_b = T_{fin,end} \rightarrow \eta_{fin} = 1$



Heat Transfer from Fin

η = fin efficiency
P = perimeter of fin
A_c = cross-sectional area of the fin

$$q_{fin} = \eta_{fin} h A_{fin} (T_b - T_a)$$

where $A_{fin} \approx 2(wL)$,

and η_{fin} is the fin efficiency:

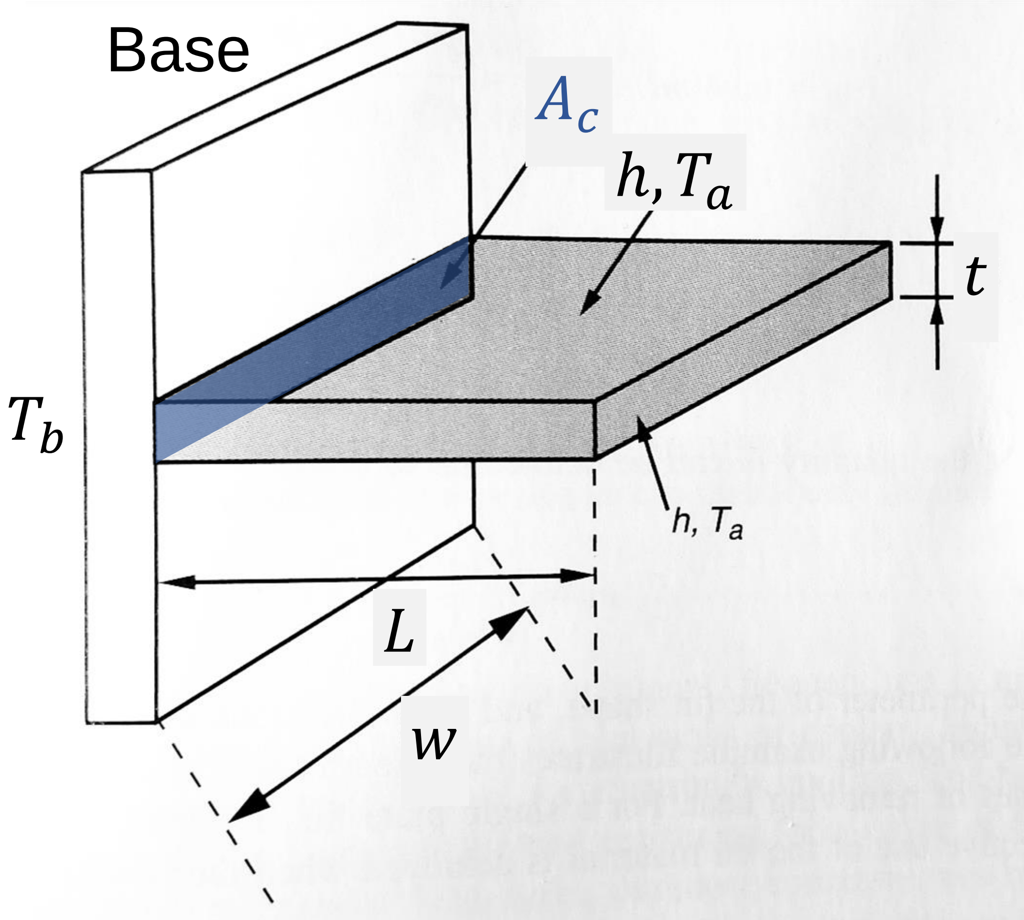
$$\eta_{fin} = \tanh(mL)/mL$$

where m is:

$$m = \sqrt{hP/kA_c}$$

where $P = 2(w + t)$, $A_c = wt$, h is the convective heat transfer coefficient, and k is the thermal conductivity of the fin

For a fin tip with convection, a corrected length is used in place of L: $L_c = L + t/2$



Multiple-Fin Heat Transfer

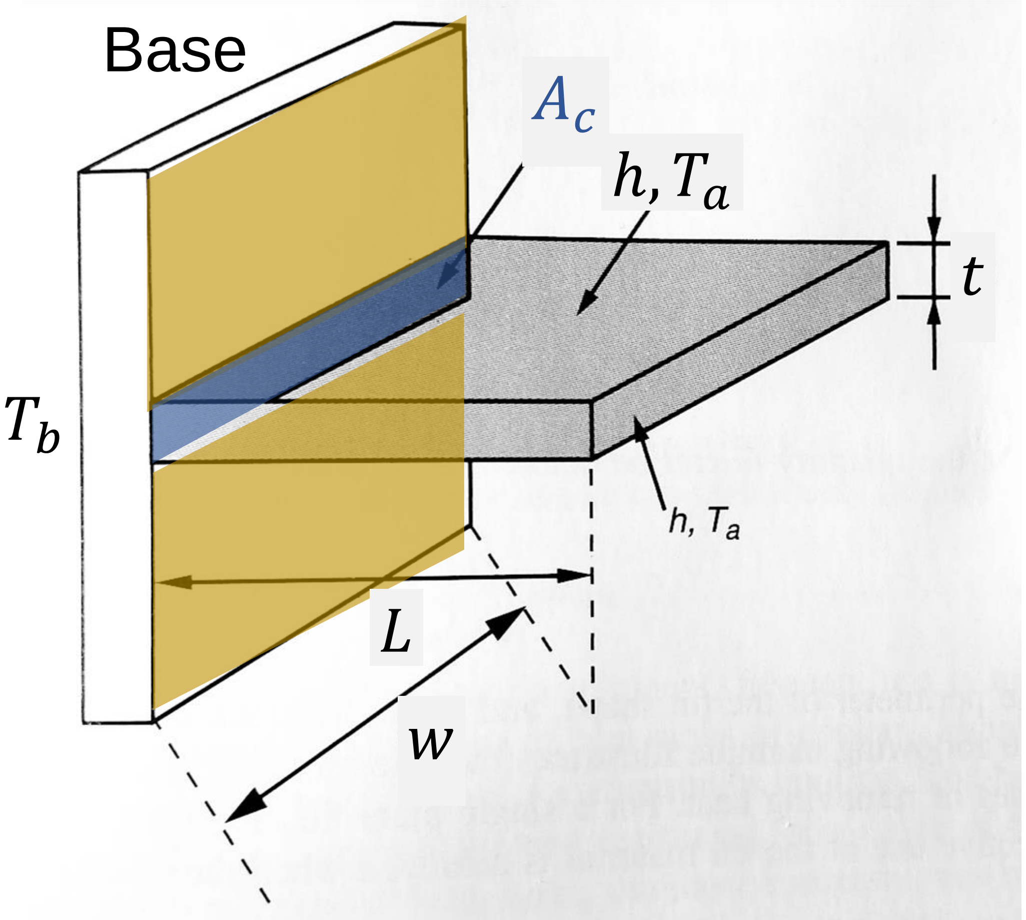
For N fins, the heat transfer is found by adding the contributions of each fin, as well as that from the inter-fin spaces:

$$q_{multi-fin} = Nq_{fin} + hA_{interfin}(T_b - T_a)$$

where $A_{interfin}$ is the unfinned area, or the base surface area between the fins.

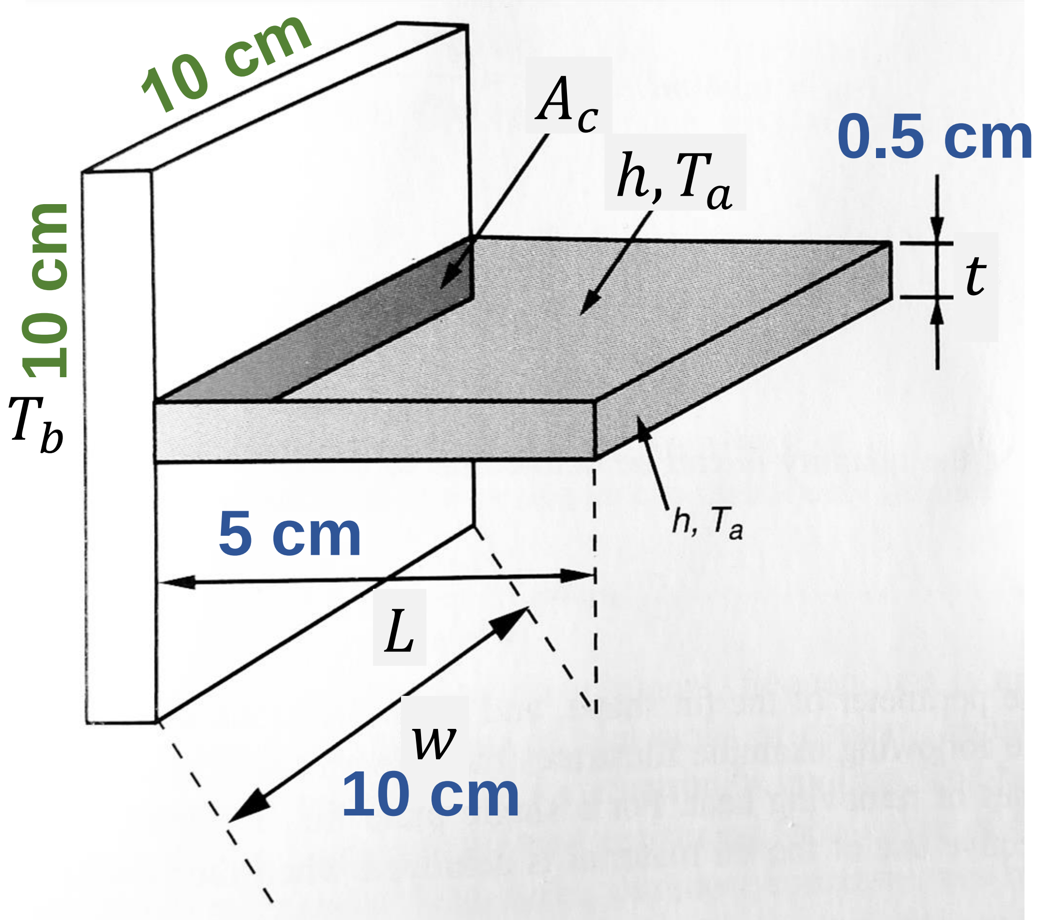
$$A_{interfin} = A_{base} - NA_{c,fin}$$

The inter-fin area is assumed to have a convective heat transfer coefficient of h



Example: Heat Sink Heat Transfer

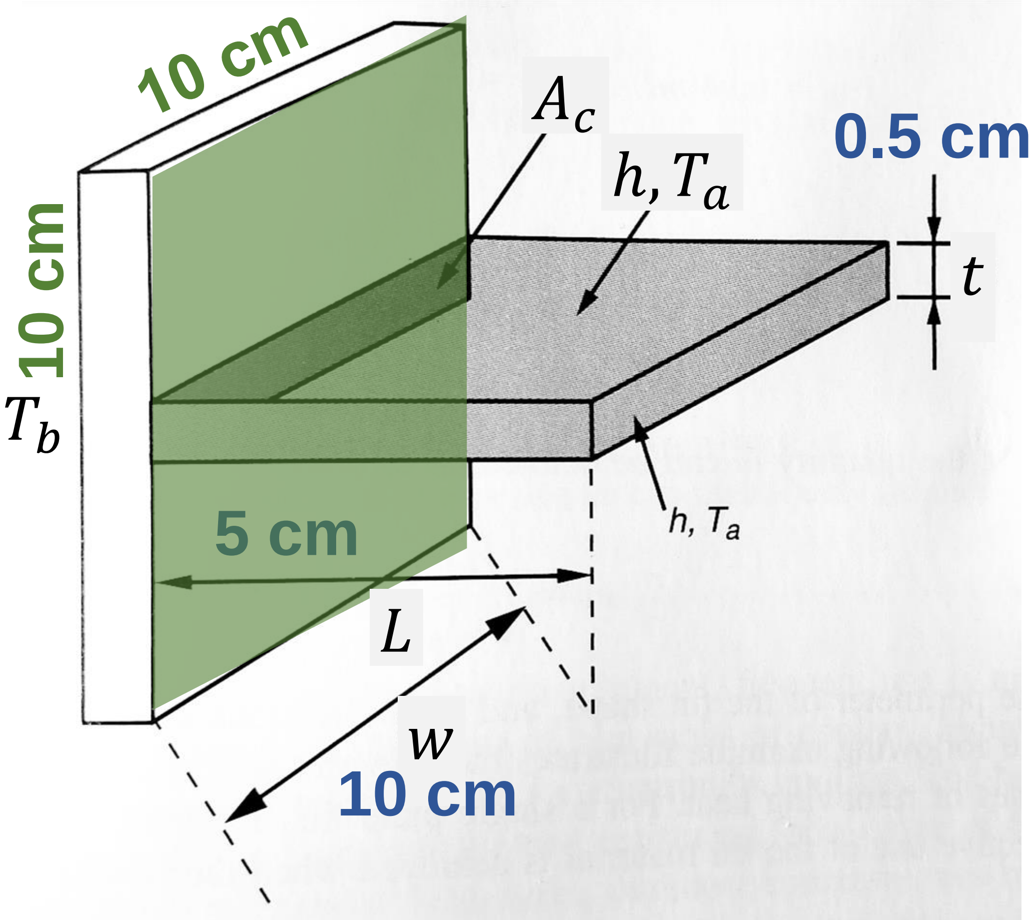
- A 10 x 10 cm^2 Al plate is maintained at 100°C.
- Heat is removed from the other face of this plate by forced convection to an ambient temperature of 50°C.
- A rectangular fin that is 5 cm x 10 cm x 0.5 cm is attached to the exposed face of the Al plate.
- Calculate the increase in heat transfer from the plate resulting from the presence of the fin.
- Assume $h = 100$ W/m^{2}K



Example: Heat Sink Heat Transfer

- **First, find heat transfer from the Al plate *without* the fin:**
- $h = 100$ W/m²K
- $A_{Al} = 100$ cm² = 0.01 m²
- $T_b = 100$°C
- $T_a = 50$°C
- The heat that can be removed from the Al plate *without* the fin is:

$$q_{Al} = hA_{Al}(T_{Al} - T_a)$$
$$q_{Al} = (100\text{W/m}^2\text{K})(0.01\text{m}^2)(50°\text{C})$$
$$= \mathbf{50W}$$



Example: Heat Sink Heat Transfer

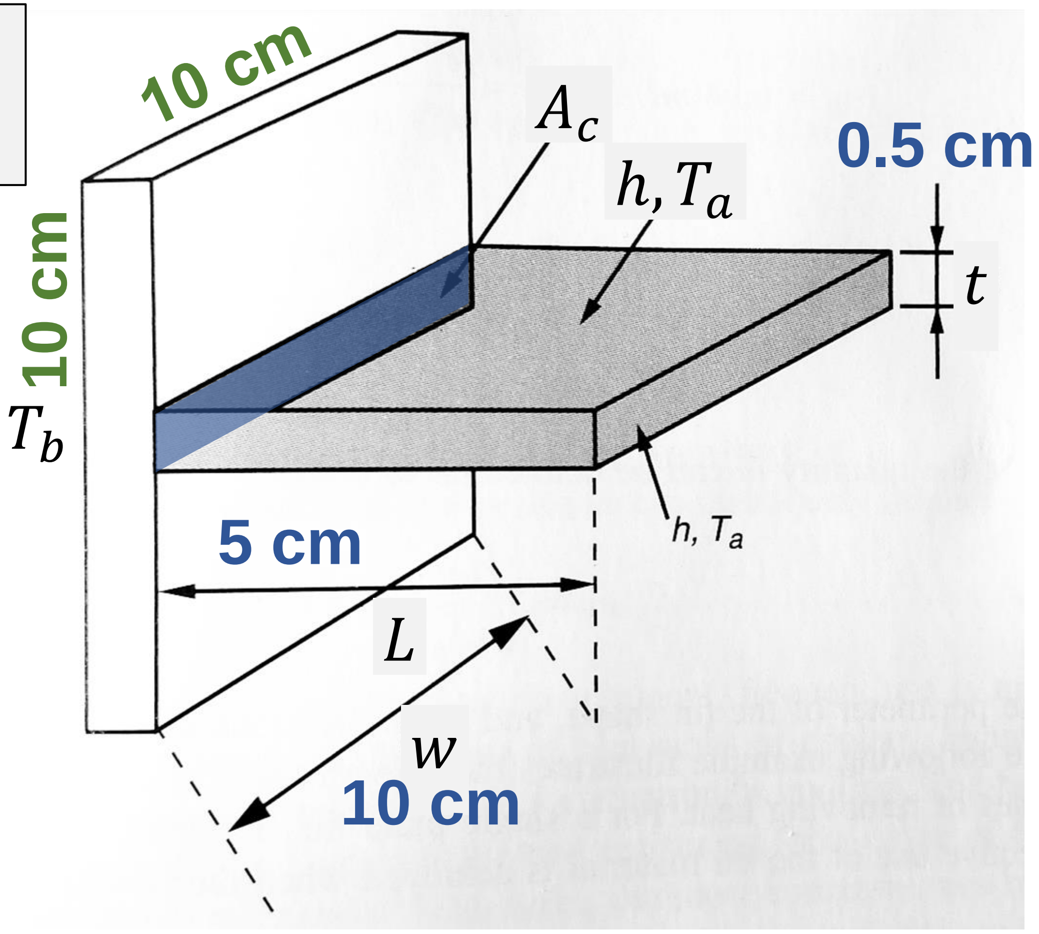
- **Now, find heat transfer from the fin:**
- $k_{Al} = 180$ W/mK
- $L = 5$ cm = 0.05 m
- $A_c = 10$ cm x 0.5 cm = 5 cm^2

  = 0.0005 m^2
- $P = 2$ x (10 cm + 0.5 cm) = 21 cm

  = 0.21 m
- $mL = \sqrt{\frac{hP}{kA_c}}L =$

$$(0.05\text{m})\sqrt{\frac{(100\text{W/m}^2\text{K})(0.21\text{m})}{(180\text{W/mK})(0.0005\text{m}^2)}} = 0.764$$

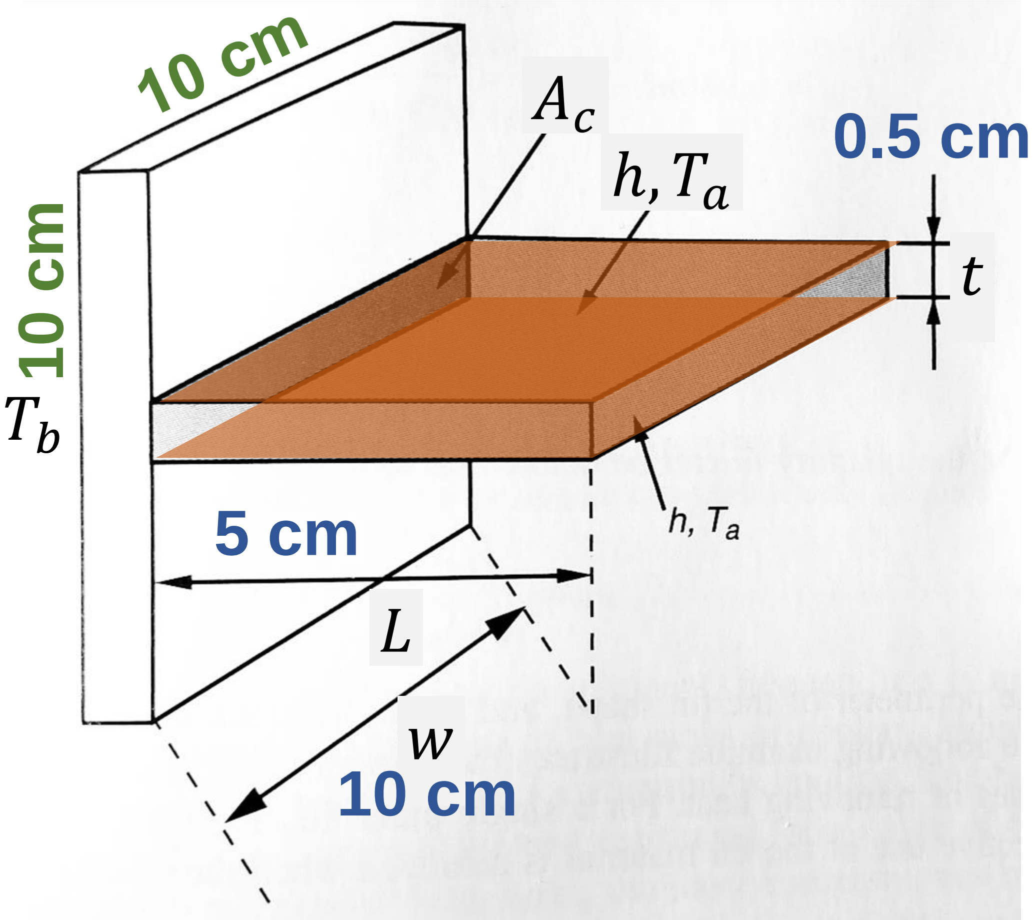
$$q_{fin} = \eta h A_{fin}(T_b - T_a)$$
$$\eta = \tanh(mL)/mL$$



Example: Heat Sink Heat Transfer

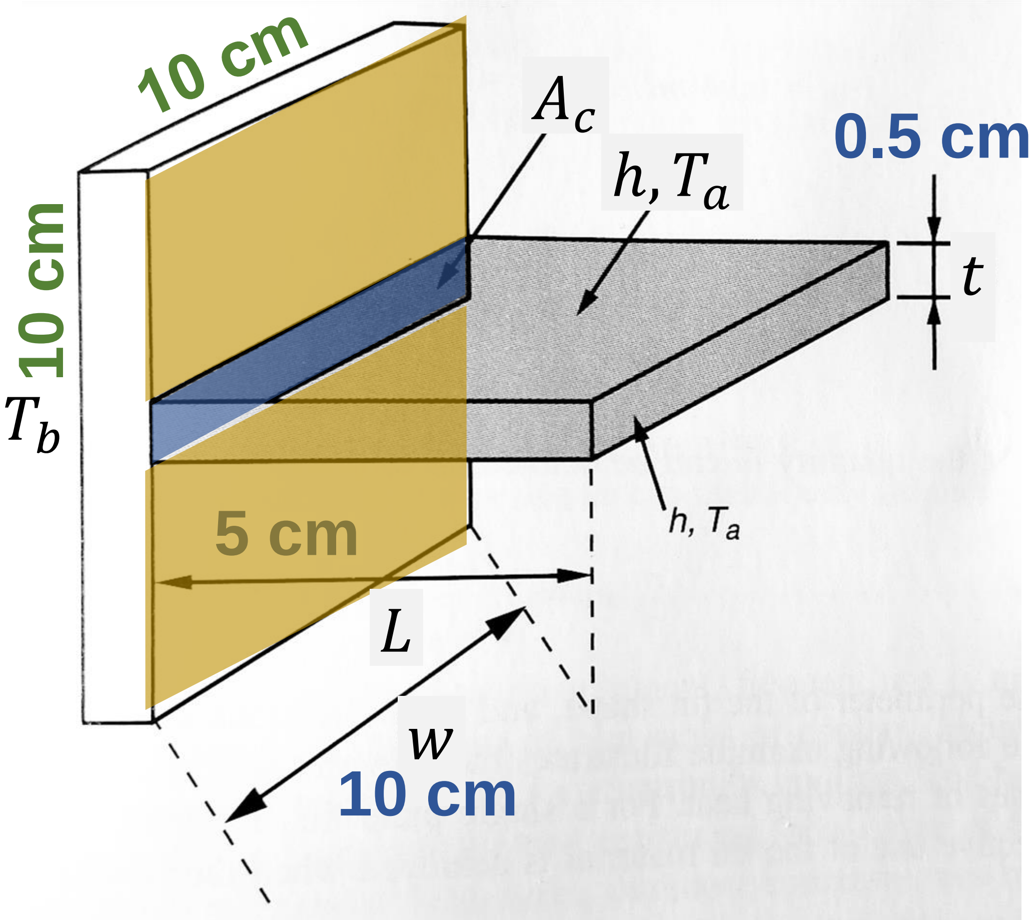
- $\eta_{fin} = \tanh(mL)/mL = \frac{\tanh(0.764)}{0.764} = 0.842$
- A_{fin} = 2 x (10 cm x 5 cm)
  = 0.01 m^2
- $q_{fin} = \eta h A_{fin}(T_b - T_a)$
  $= (0.842)(100\mathrm{W/m^2K})(0.01\mathrm{m^2}) \cdot (100°\mathrm{C} - 50°\mathrm{C})$
  $= \mathbf{42.1W}$

- **42.1 W** is removed from the fin alone



Example: Heat Sink Heat Transfer

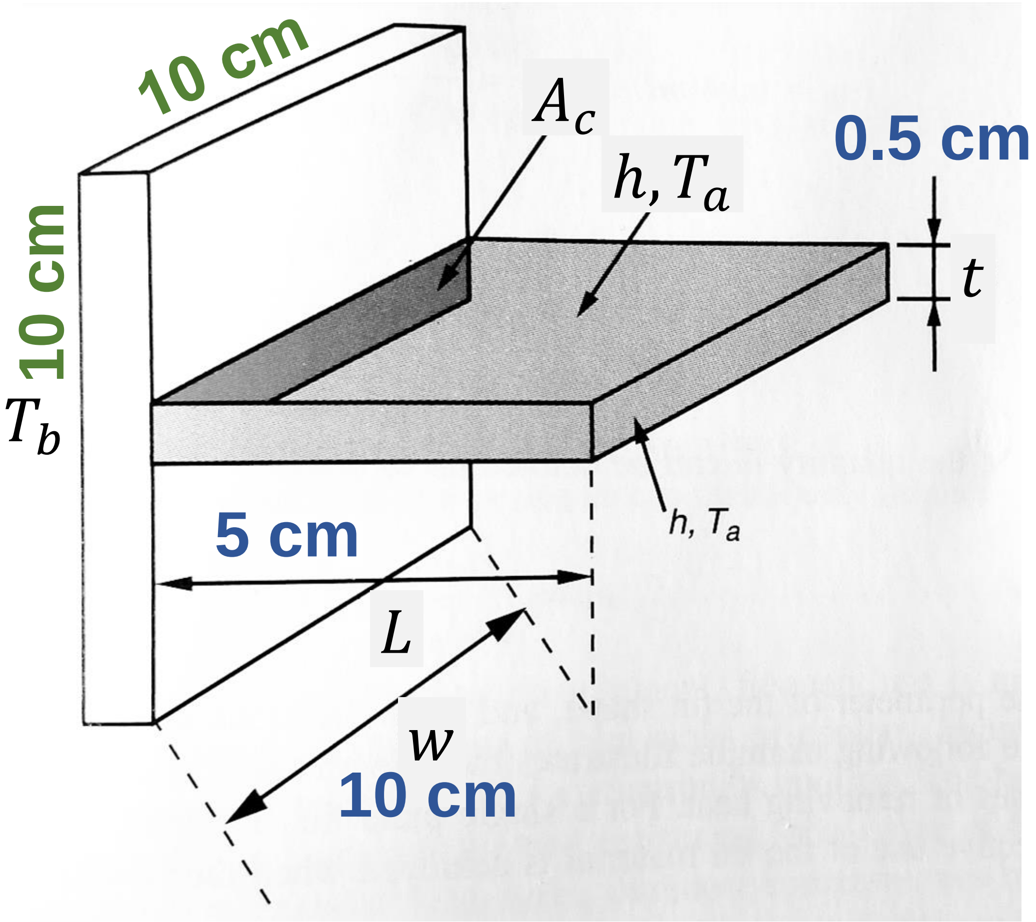
- **Now, find the heat transfer from regions of the Al plate *not* occupied by the fin:**
- $q_{unfinned} = h(A_{Al} - A_c)(T_b - T_a)$
  $= (100\mathrm{W/m^2K})(0.01\mathrm{m}^2 - 0.0005\mathrm{m}^2)(100°\mathrm{C} - 50°\mathrm{C})$
  $= \mathbf{47.5W}$
- **47.5 W** is removed from the areas of the Al plate *not* occupied by the fin



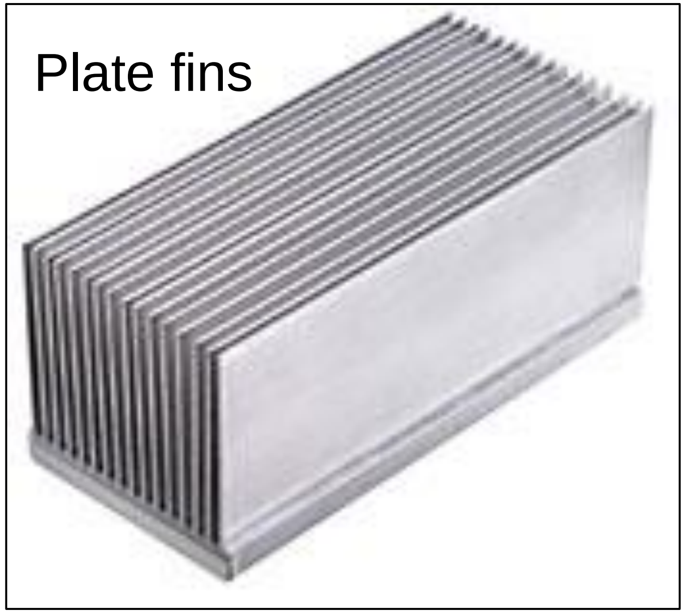
Example: Heat Sink Heat Transfer

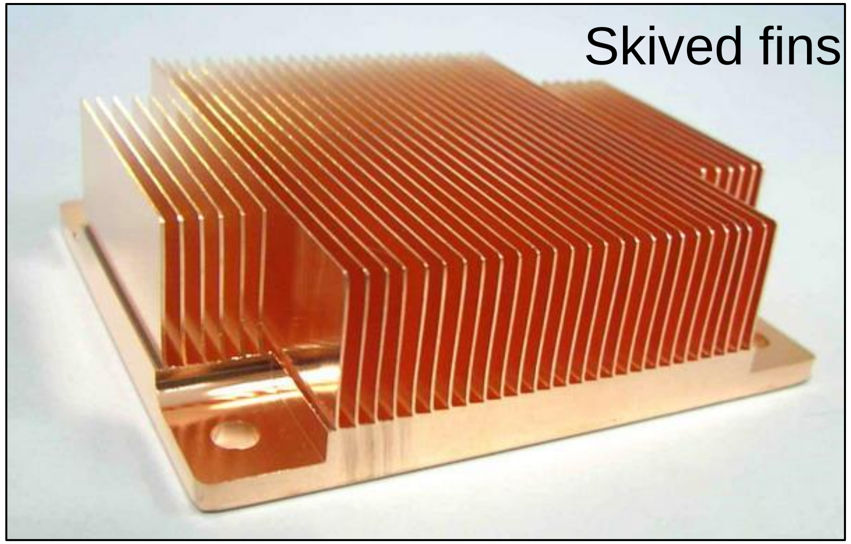
- **The total heat transfer from the Al plate with one fin is:**
- $q_{total} = q_{unfinned} + q_{fin}$
  $= \mathbf{47.5W} + \mathbf{42.1W} = \mathbf{89.6W}$
- **The additional heat transfer due to the fin is:**
- $q = q_{finned} - q_{Al}$
  $= \mathbf{89.6W} - \mathbf{50W} = \mathbf{39.6W}$

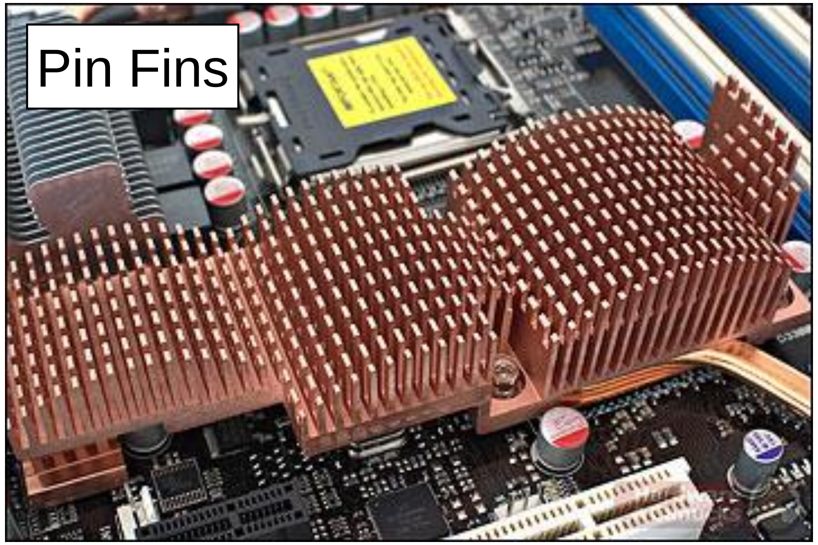
➢ The addition of *one* fin increased the heat transfer from the surface by 80%



Finned Heat Sinks







Radial fins