



Lecture 11

Thermal Design

*(Conductive) Thermal Resistance, Contact Resistance,
and Heat Spreading*

February 27, 2025

Reminders and Announcements

- Homework #1 grades posted
- Homework #2 solutions will be posted to Canvas tomorrow morning
- Homework #3 will be assigned today and due Thursday, March 6th, by 11:59pm
- Office Hours: Monday & Wednesday 3:30pm-4:30pm
- Please put the course number in the subject line in emails to the instructor and TA (this helps us filter our inboxes so we don't miss your messages)

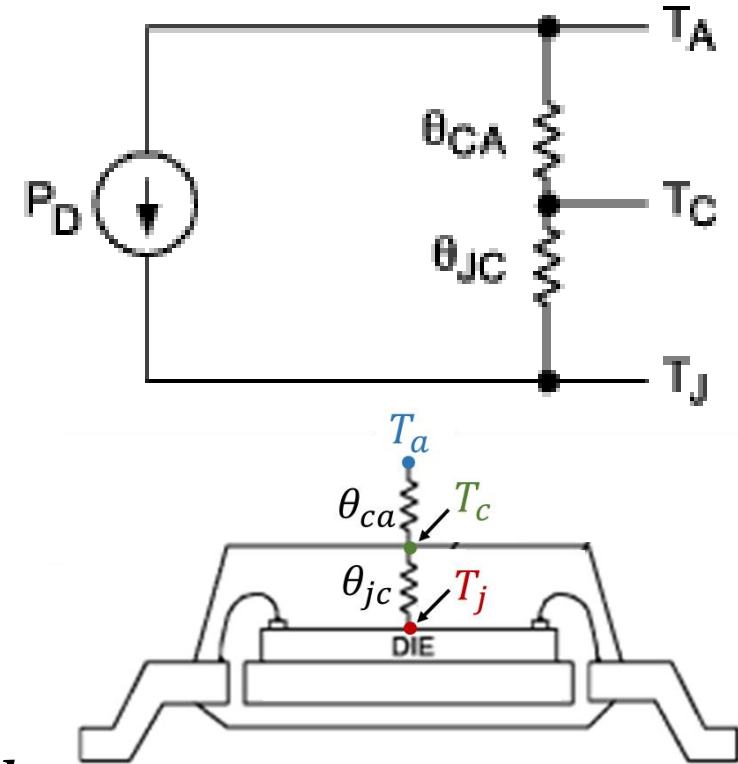
Midterm: Thursday, March 20th, 5:00pm-6:15pm

- If you are located in Blacksburg, then you should take the exam in-person in TORG 1050, even if you are enrolled in the virtual section of the course
- If you are enrolled in the *virtual campus*, please email me by March 5th indicating if you will take the exam virtually or in-person in Arlington or Blacksburg
- Will cover the topics in lectures 1-6 (packaging overview and electrical design) and 9-13 (transmission lines and thermal design)
- The lecture on March 18th will be a review session → come ready with questions or topics you would like to cover
- You will not be asked to do any simulations for the midterm
- The problems will be a mix of conceptual short response and calculation problems
- Things to bring to the exam: writing utensils & *non-programmable* calculator
- An equation/reference sheet and extra paper will be provided by the proctor
- The reference sheet will be uploaded to Canvas next week; you do not need to print out the reference sheet

Package Thermal Resistance

- θ_{ja} can be separated into two parts:
 - Junction-to-case, θ_{jc}
 - Case-to-ambient, θ_{ca}
- Junction-to-case, θ_{jc}
 - Depends on the internal construction of the package
 - Depends on length, cross-sectional area, and k
- Case-to-ambient, θ_{ca}
 - Depends on the mounting and cooling techniques
 - Depends on wetted surface area and h

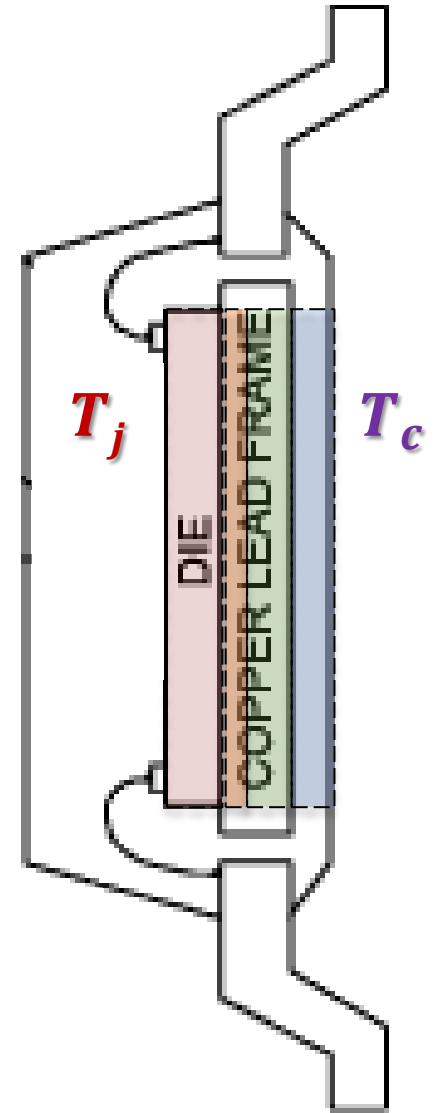
$$\theta_{ja} = \theta_{jc} + \theta_{ca}$$



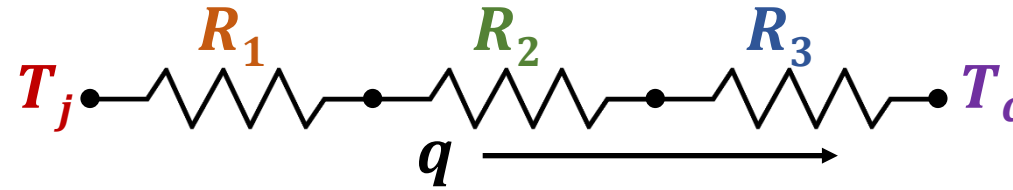
Example: Package Thermal Resistance

Find the junction temperature, T_j , of the die if it dissipates 1 W of heat and has the below specifications.

- $T_c = 50\text{ }^{\circ}\text{C}$
- $A_c = 10 \times 10\text{ mm}^2$ (for all components)
- **Solder die attach:** $k_1 = 50\text{ W/(m}\cdot\text{K)}$, $L_1 = 0.1\text{ mm}$
- **Cu lead frame:** $k_2 = 390\text{ W/(m}\cdot\text{K)}$, $L_2 = 1\text{ mm}$
- **EMC:** $k_3 = 0.23\text{ W/(m}\cdot\text{K)}$, $L_3 = 1\text{ mm}$
- Assume other sides are thermally insulated, that the die is at a uniform temperature, and neglect heat spreading.

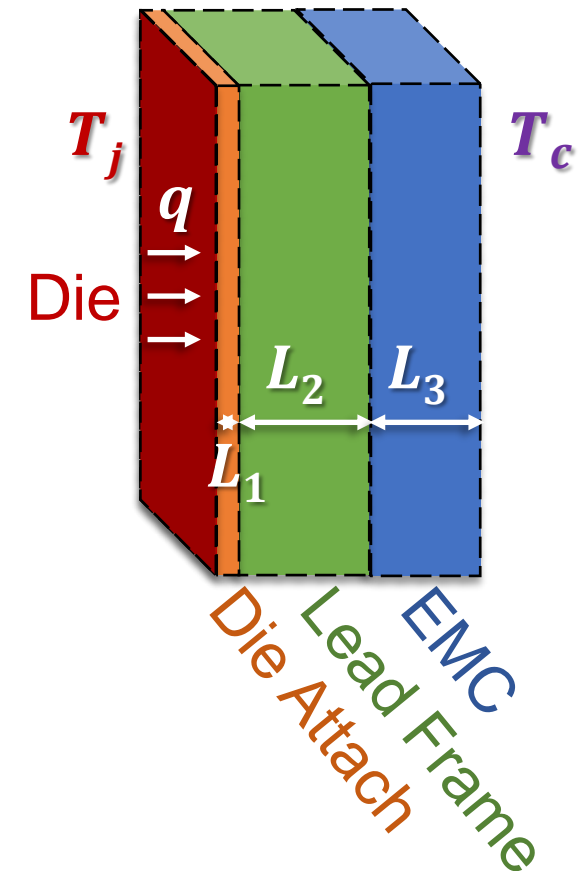


Example: Package Thermal Resistance



$$R_{th,cond} = \frac{L}{kA}$$

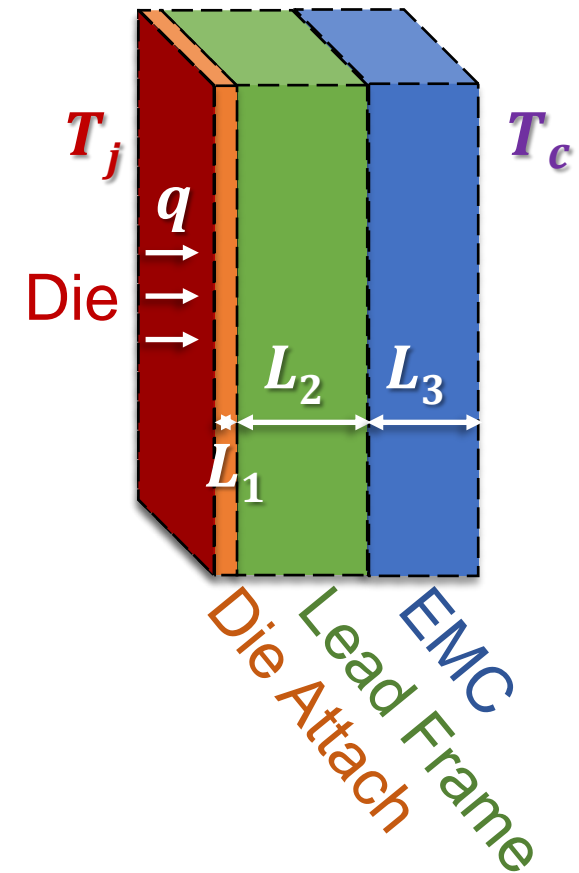
- $T_j - T_c = q R_{th,j-c}$
- $R_{th,j-c} = R_1 + R_2 + R_3$
- $R_1 = \frac{L_1}{k_1 A} = \frac{100 \times 10^{-6} \text{ m}}{\left(50 \frac{\text{W}}{\text{mK}}\right)(1 \times 10^{-4} \text{ m}^2)} = 0.02 \text{ K/W}$
- $R_2 = \frac{L_2}{k_2 A} = \frac{1 \times 10^{-3} \text{ m}}{\left(390 \frac{\text{W}}{\text{mK}}\right)(1 \times 10^{-4} \text{ m}^2)} = 0.03 \text{ K/W}$
- $R_3 = \frac{L_3}{k_3 A} = \frac{1 \times 10^{-3} \text{ m}}{\left(0.23 \frac{\text{W}}{\text{mK}}\right)(1 \times 10^{-4} \text{ m}^2)} = \mathbf{43 \text{ K/W}}$



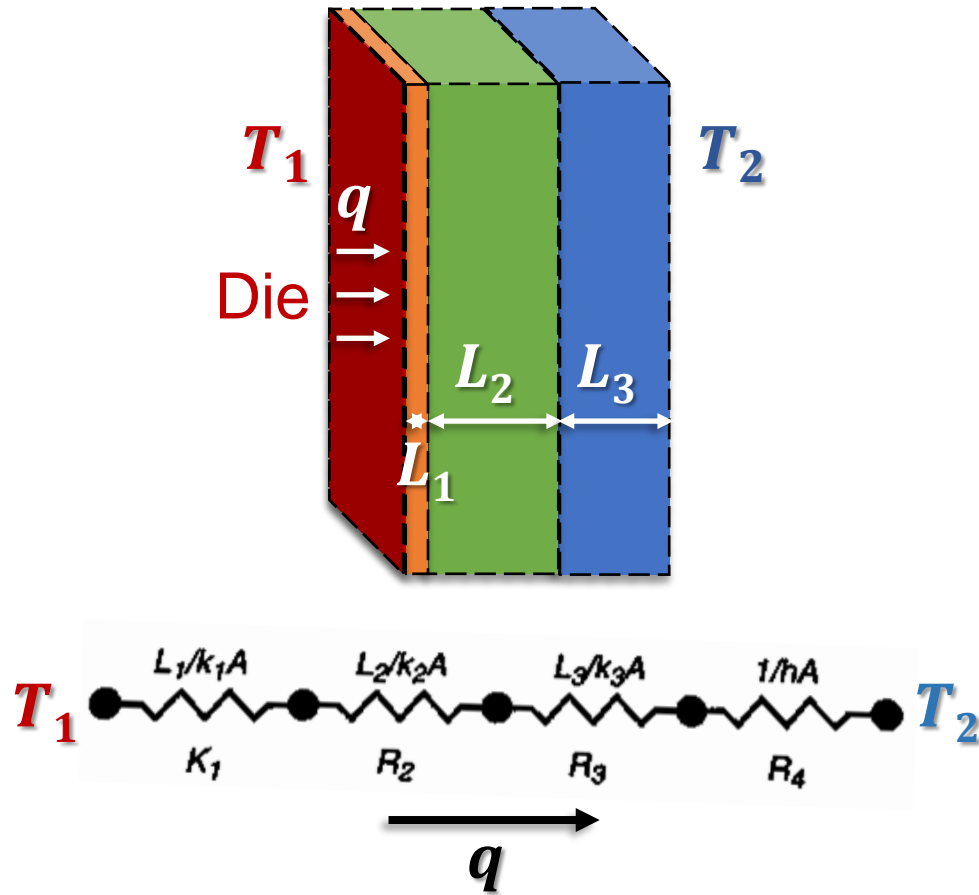
Example: Package Thermal Resistance

$$R_{th,cond} = \frac{L}{kA}$$

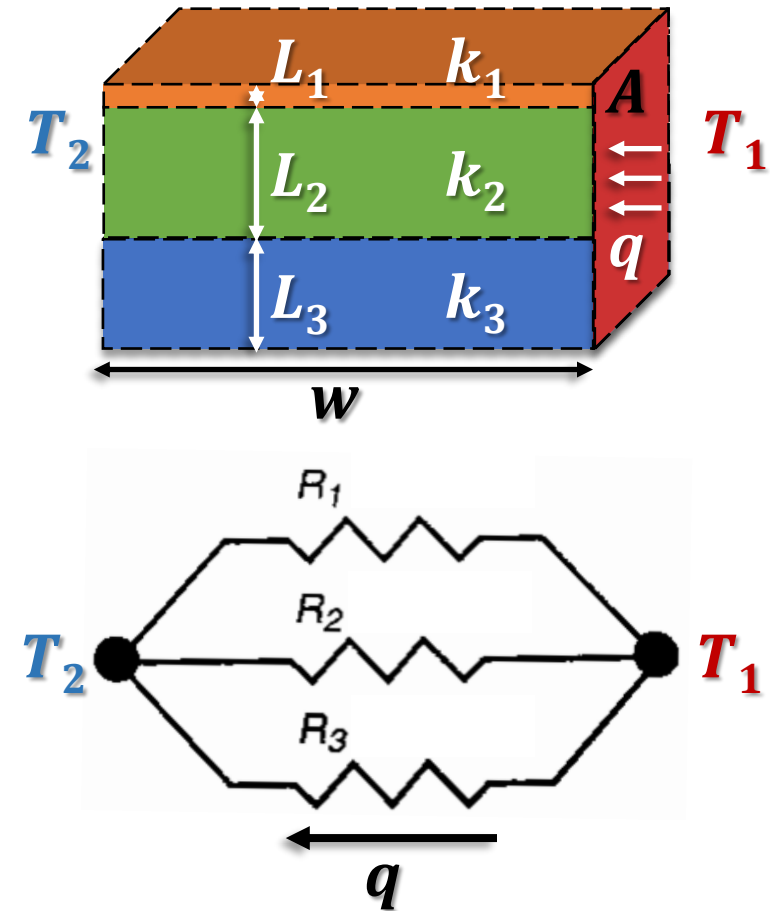
- Find T_j
- $R_{th,j-c} = R_1 + R_2 + R_3$
- $R_{th,j-c} = 0.02 \text{ K/W} + 0.03 \text{ K/W}$
 $+ 43 \frac{\text{K}}{\text{W}} = 43.05 \text{ K/W}$
- $T_j = qR_{th,j-c} + T_c$
- $T_j = (1\text{W}) \left(43 \frac{^\circ\text{C}}{\text{W}} \right) + 50^\circ\text{C} = 93^\circ\text{C}$
- Conduction through the EMC is the greatest contributor to the $R_{th,j-c}$



Thermal Resistances in Series and Parallel

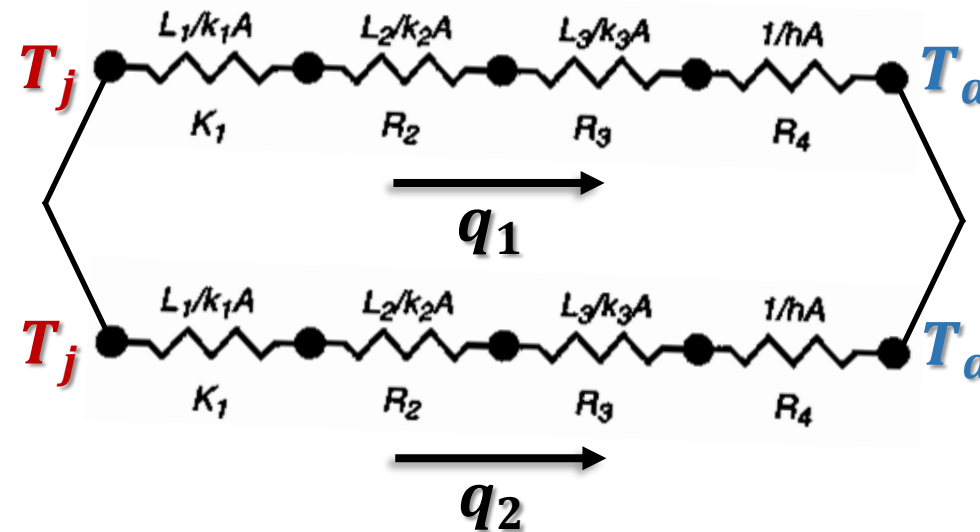
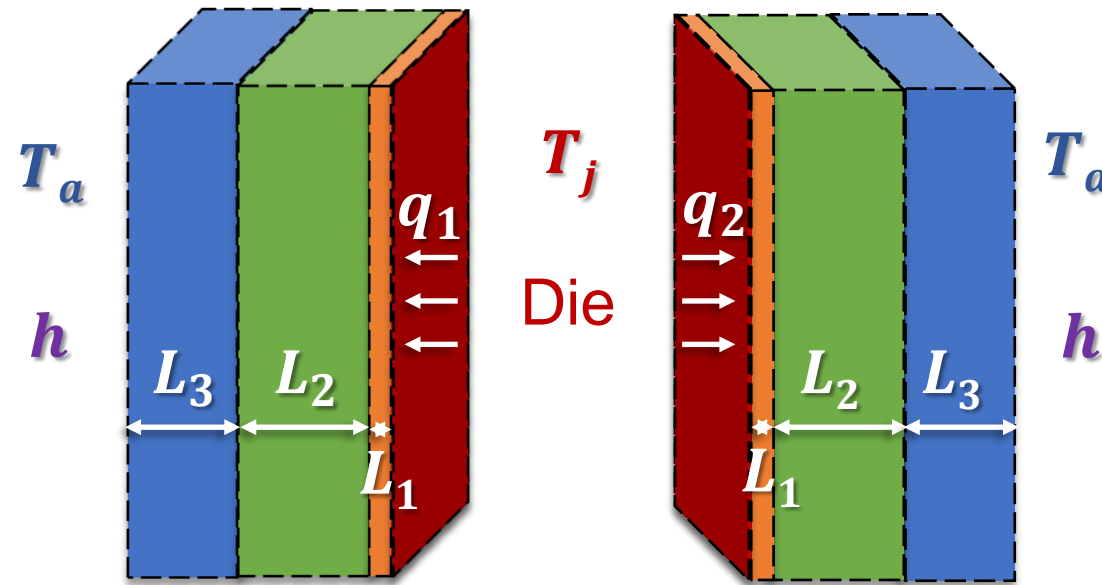


$$R_{th,1-2} = R_1 + R_2 + R_3 + R_4$$

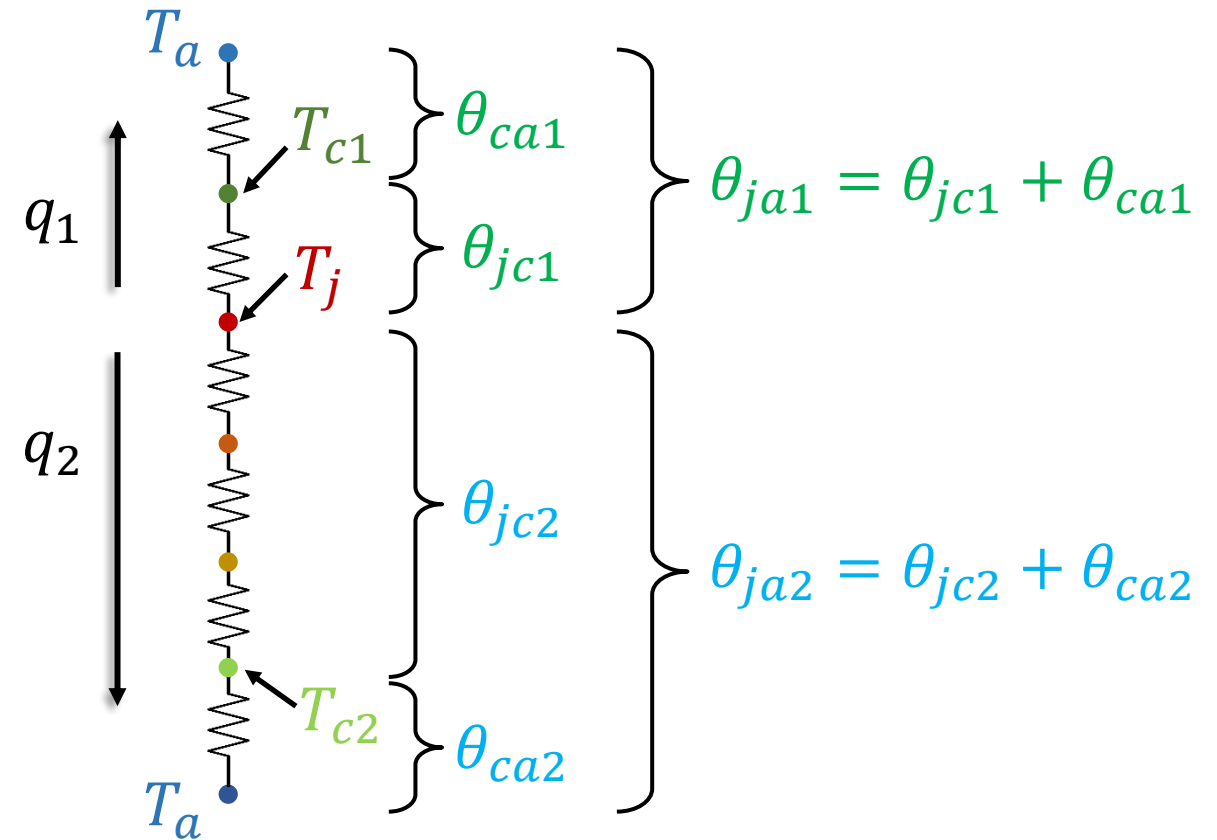
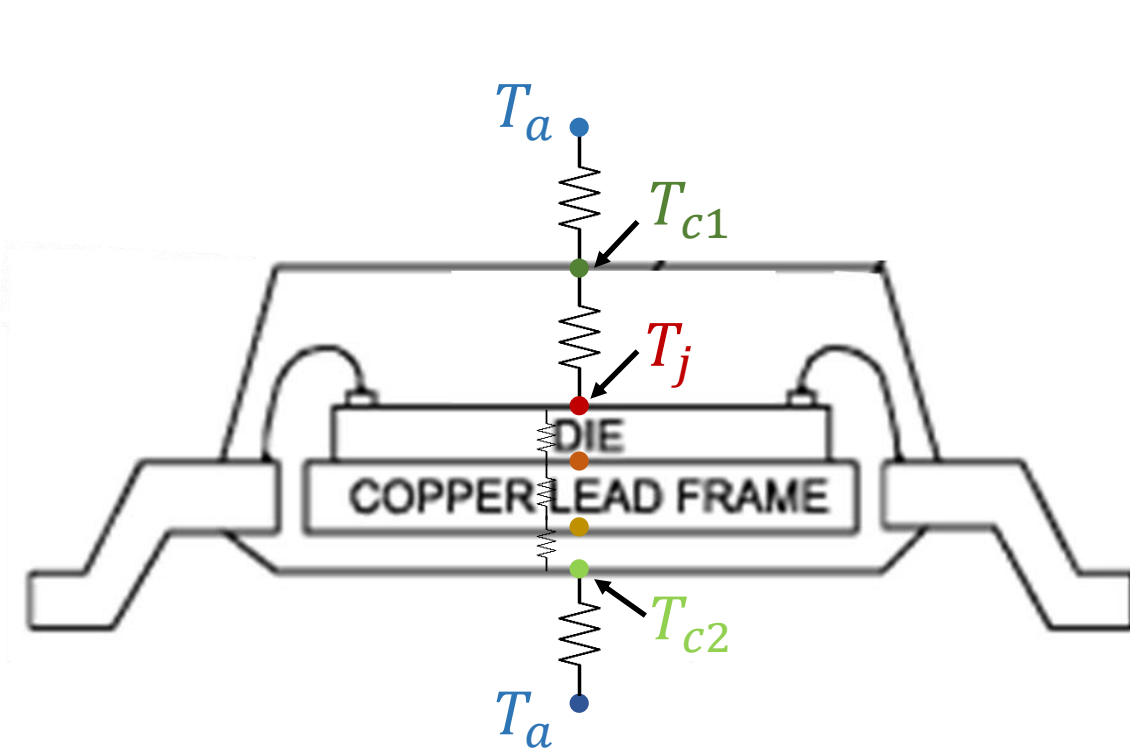


$$R_{th,1-2} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

Thermal Resistances in Series and Parallel



Thermal Resistances in Series and Parallel

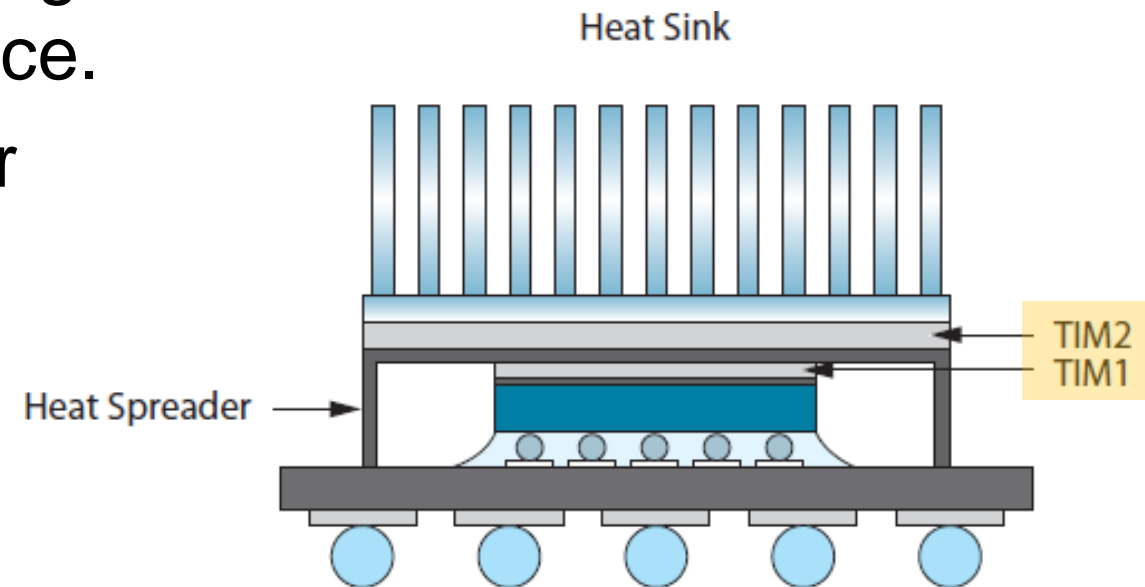


Total junction-to-ambient thermal resistance:

$$\theta_{ja} = \frac{\theta_{ja1} \theta_{ja2}}{\theta_{ja1} + \theta_{ja2}} = \frac{(\theta_{jc1} + \theta_{ca1})(\theta_{jc2} + \theta_{ca2})}{(\theta_{jc1} + \theta_{ca1}) + (\theta_{jc2} + \theta_{ca2})}$$

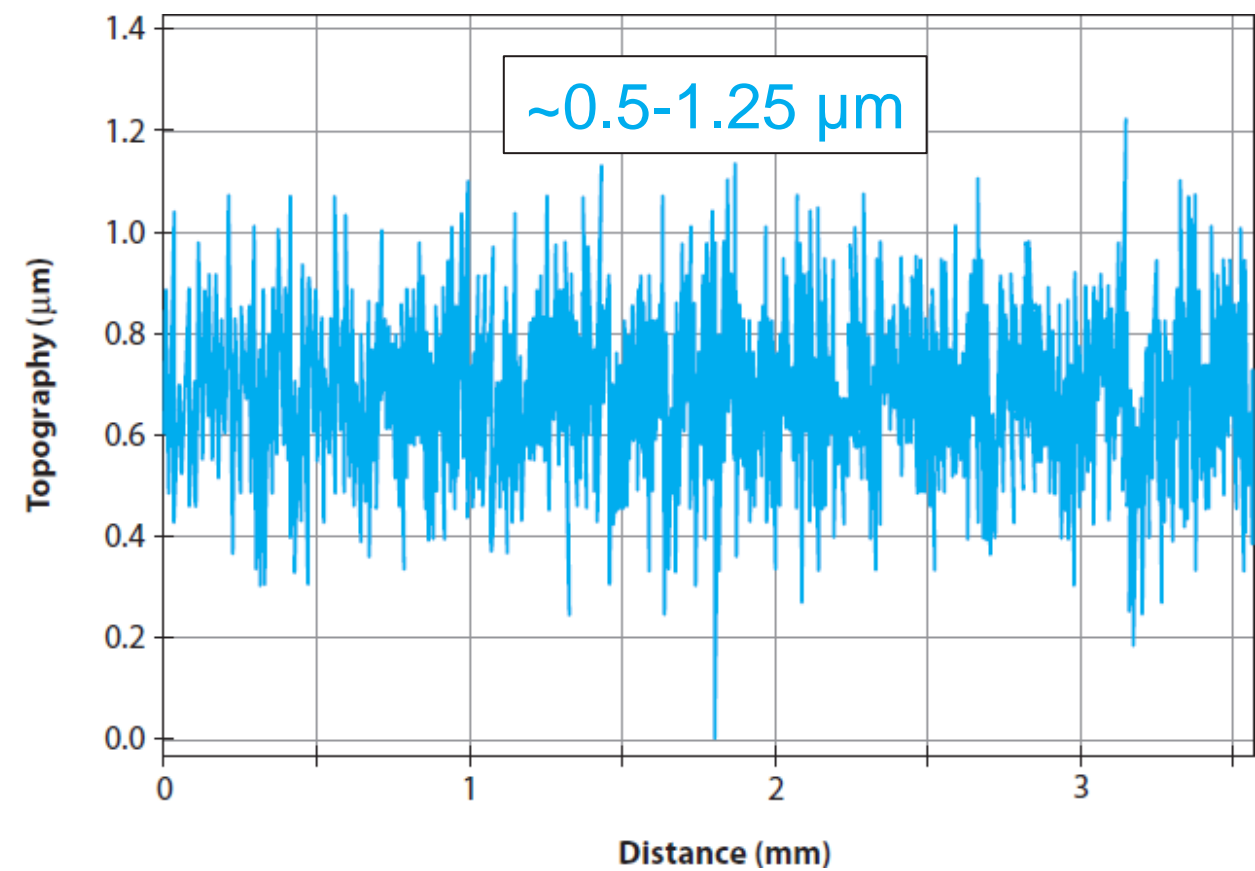
Contact Resistance

- **Contact resistance** is the thermal resistance that occurs between two adjacent solid bodies as a result of surface imperfections and non-uniformities.
- **Thermal interface materials (TIM)** are used to fill the interface between mating surfaces to decrease contact resistance.
 - Higher thermal conductivity than air
 - Examples of interfaces include:
 - between the chip and package lid/heat spreader, or between the heat spreader and the heat sink

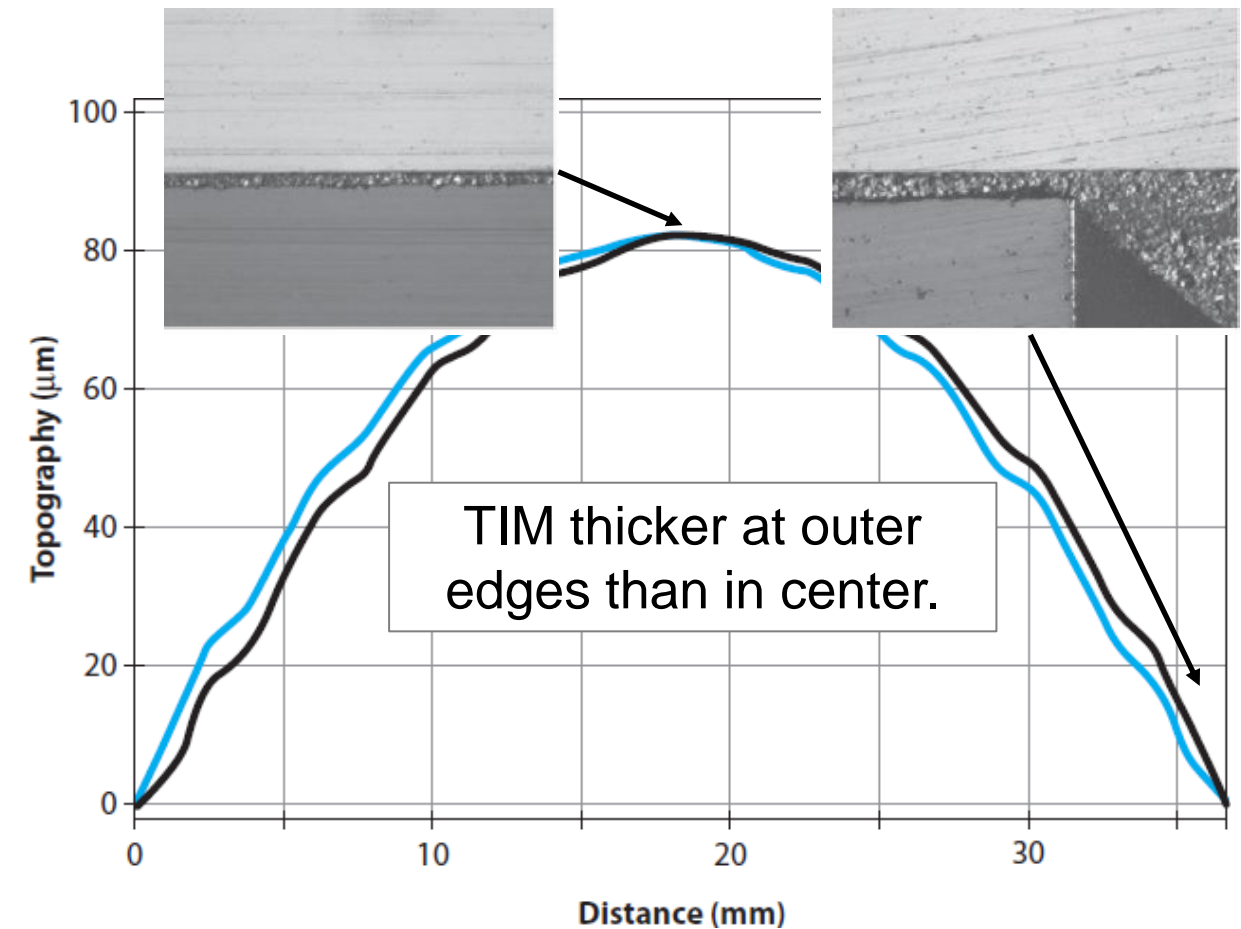


Surface Roughness and Flatness

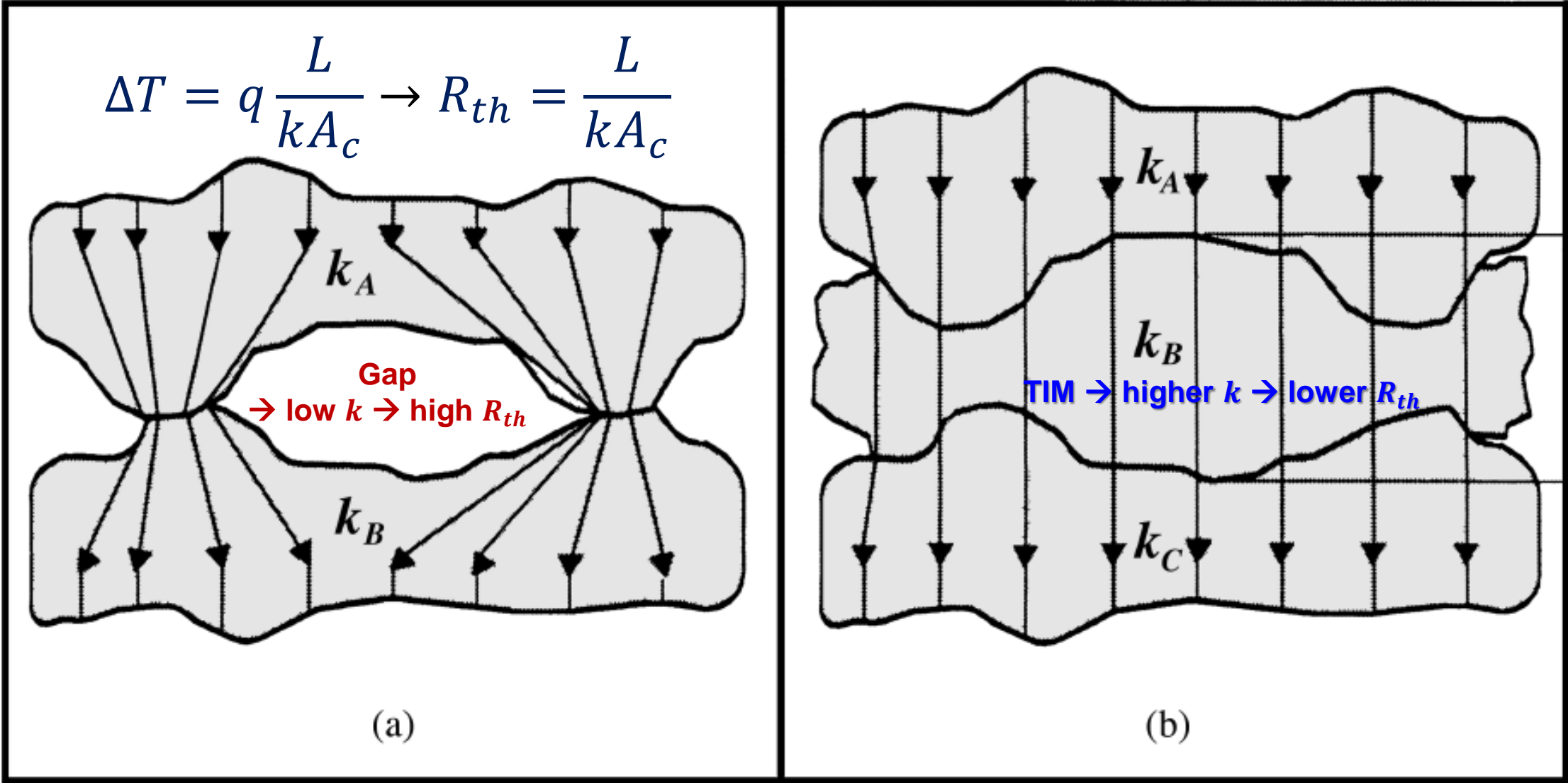
Copper Lid Surface Roughness



Chip Flatness Along Diagonal Axes

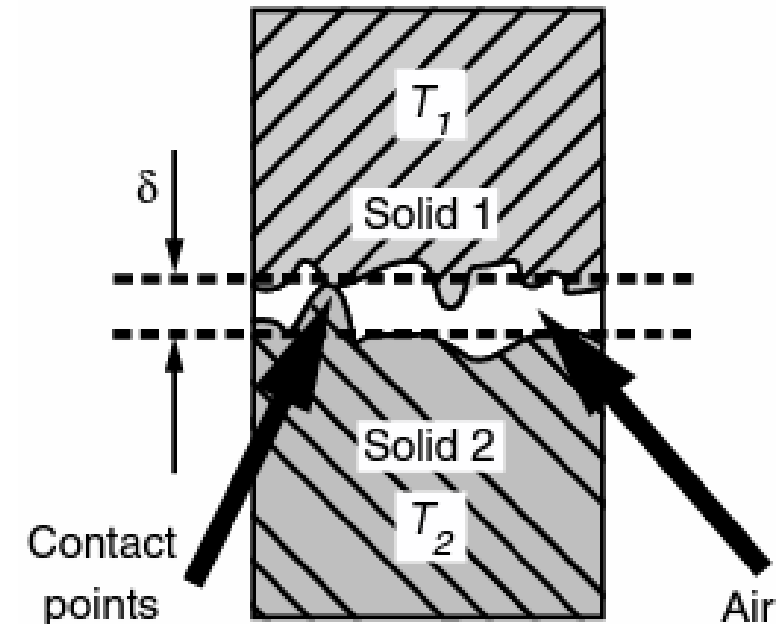


Interface Thermal Resistance



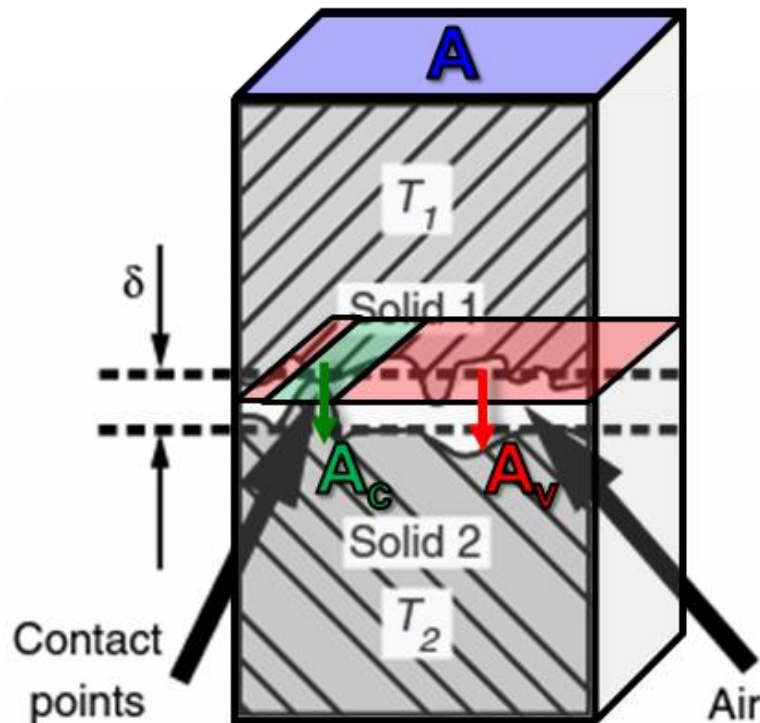
Example: Interface Thermal Resistance

Consider a lightly loaded, 10 cm^2 alumina-to-silicon interface with an actual contact percentage of 0.5% and $\delta = 25 \text{ }\mu\text{m}$.



Example: Interface Thermal Resistance

Consider a lightly loaded, 10 cm² alumina-to-silicon interface with an actual contact percentage of 0.5% and $\delta = 25\text{ }\mu\text{m}$.



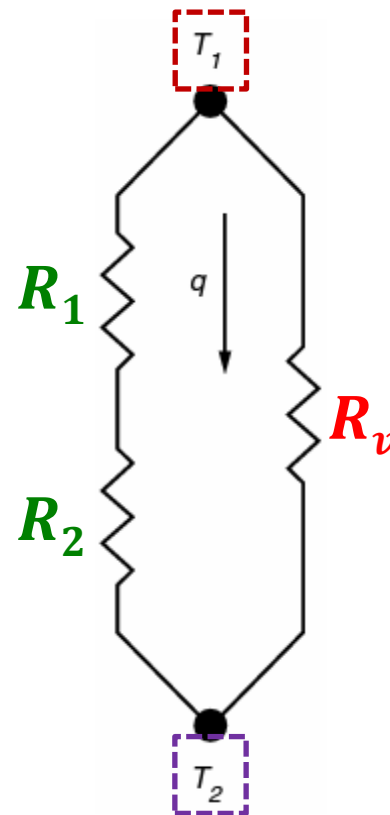
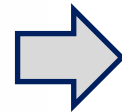
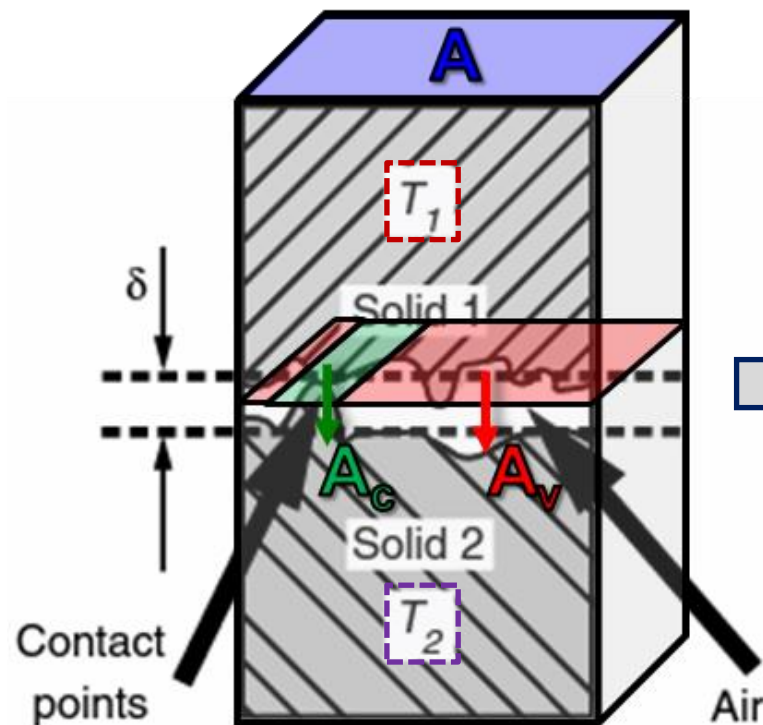
A = Total area of interface = 10 cm²

A_c = Contact area between solid 1 and solid 2
 $= 0.005 \cdot A = 0.05\text{ cm}^2$

A_v = Void area between solid 1 and solid 2
 $= A - 0.005 \cdot A = 10\text{ cm}^2 - 0.05\text{ cm}^2 = 9.95\text{ cm}^2$

Example: Interface Thermal Resistance

Consider a lightly loaded, 10 cm² alumina-to-silicon interface with an actual contact percentage of 0.5% and $\delta = 25 \mu\text{m}$.



$$\begin{aligned}
 R_1 &= \frac{L}{k_1 A_c} = \frac{\delta}{2k_1 A_c} \\
 R_2 &= \frac{L}{k_2 A_c} = \frac{\delta}{2k_2 A_c} \\
 R_v &= \frac{L}{k_f A_v} = \frac{\delta}{k_f A_v}
 \end{aligned}
 \left. \vphantom{\begin{aligned} R_1 \\ R_2 \\ R_v \end{aligned}} \right\} R_{int} = (R_1 + R_2) \parallel R_v = \left(\frac{1}{R_1 + R_2} + \frac{1}{R_v} \right)^{-1}$$

k_1 = thermal conductivity of solid 1

k_2 = thermal conductivity of solid 2

k_f = thermal conductivity of fluid in void

A_c = contact area between solid 1 and 2

A_v = area of void

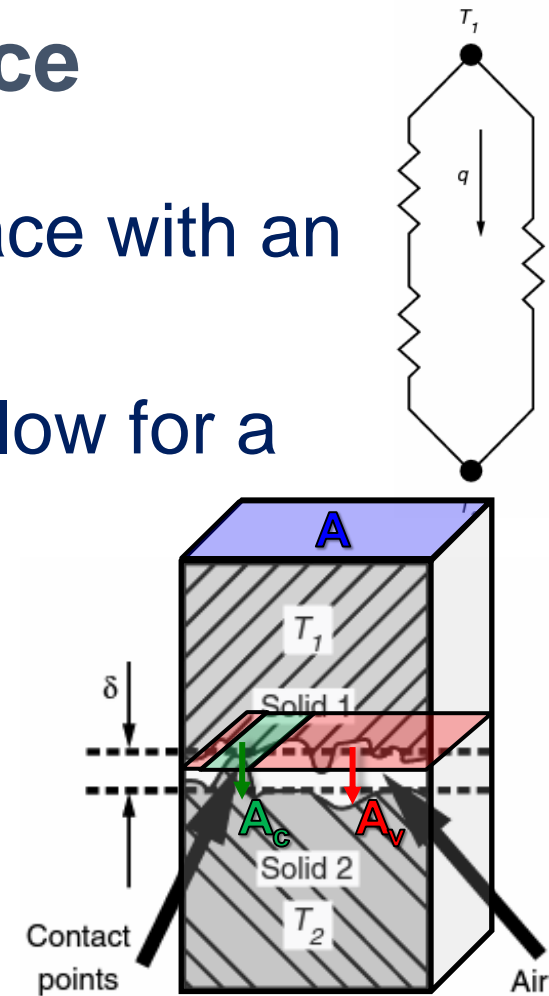
δ = length of interface

Example: Interface Thermal Resistance

Consider a lightly loaded, 10 cm² alumina-to-silicon interface with an actual contact percentage of 0.5% and $\delta = 25 \mu\text{m}$.

If the interfacial gaps are filled with air, calculate the heat flow for a 5°C temperature difference across the interface.

- $R = \frac{L}{kA}$
- $q = \left(\frac{kA}{L}\right) \Delta T = \left(\frac{1}{R}\right) \Delta T$
- $q = \left(\frac{1}{R_{int}}\right) \Delta T = \Delta T \left(\frac{1}{R_1 + R_2} + \frac{1}{R_v} \right) = \Delta T \left(\frac{1}{\frac{\delta}{2k_1 A_c} + \frac{\delta}{2k_2 A_c}} + \frac{1}{\frac{\delta}{k_f A_v}} \right)$



Example: Interface Thermal Resistance

Consider a lightly loaded, 10 cm² alumina-to-silicon interface with an actual contact percentage of 0.5% and $\delta = 25 \mu\text{m}$.

If the interfacial gaps are filled with air, calculate the heat flow for a 5°C temperature difference across the interface.

$$q = \frac{T_1 - T_2}{\frac{\delta}{2k_1A_c} + \frac{\delta}{2k_2A_c}} + \frac{T_1 - T_2}{\delta/(k_fA_v)}$$

$$q = \frac{5}{[25 \times 10^{-6}/(2 \times 22 \times 0.05 \times 10^{-4})] + [25 \times 10^{-6}/(2 \times 120 \times 0.05 \times 10^{-4})]}$$

$$+ \frac{5}{[25 \times 10^{-6}/(0.0261 \times 9.95 \times 10^{-4})]}$$

$$= \frac{5}{0.116 + 0.021} + \frac{5}{0.963}$$

$$= 42.4 \text{ W}$$

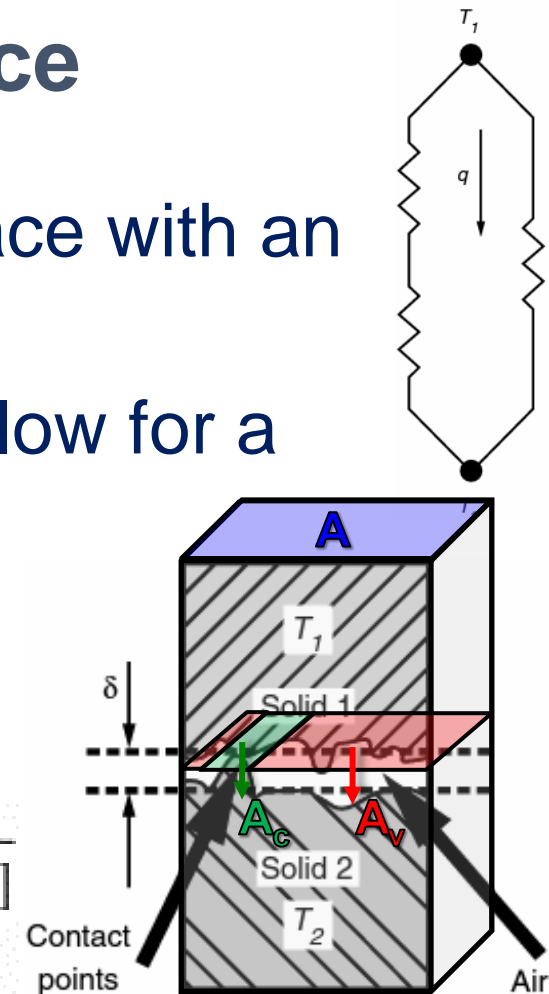
$$A_v = A - A_c$$

$$A_c = (0.005)A$$

$$k_1 = k_{Al_2O_3} = 22 \text{ W/mK}$$

$$k_2 = k_{Si} = 120 \text{ W/mK}$$

$$k_f = k_{air} = 0.026 \text{ W/mK}$$



Thermal Interface Materials

Material	Thermal Conductivity (W/mK)	Ratio
Air	0.024	1
Epoxy (dielectric)	0.23	9.6
Epoxy (conductive)	0.35	14.6
Polyimide	0.33	13.8
FR4	0.30	12.5
Water	0.59	24.6
Thermal grease	1.10	46
Alumina	22.0	916
Aluminum	150	6250
Silicon	120	5000
Copper	390	16,250
Gold	300	12,500
Diamond	2000	83,330

Electrically NON-Isolating

Thermal Greases

Sil-Free™

Ther-O-Link

Ultrastick

Thermalcote™



Double Sided Tapes

Ther-A-Grip 1070/T405R

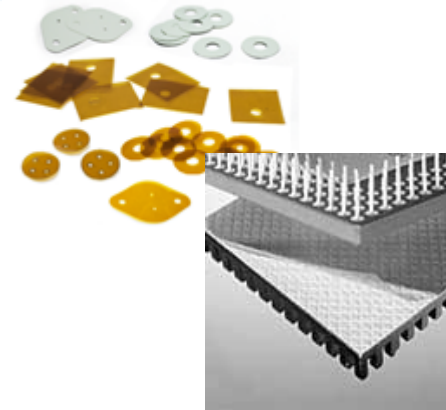
Ther-A-Grip 1090/T412

T410R/T411

Thin Films

Kon Dux

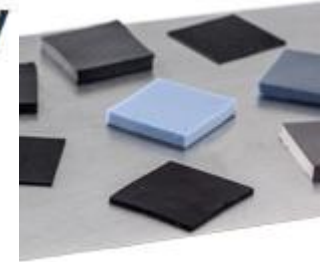
Grafoil™



Electrically Isolating

Gap Fillers

A-Pli™



Double Sided Tapes

Ther-A-Grip 1050 T404

Pads

In-Sil-8

T-gard™500

Thermalsil™ III

Insulating Films

Thermalfilm™

Thermalfilm™MT

Thermal Adhesives

Ther-O-Bond 1500

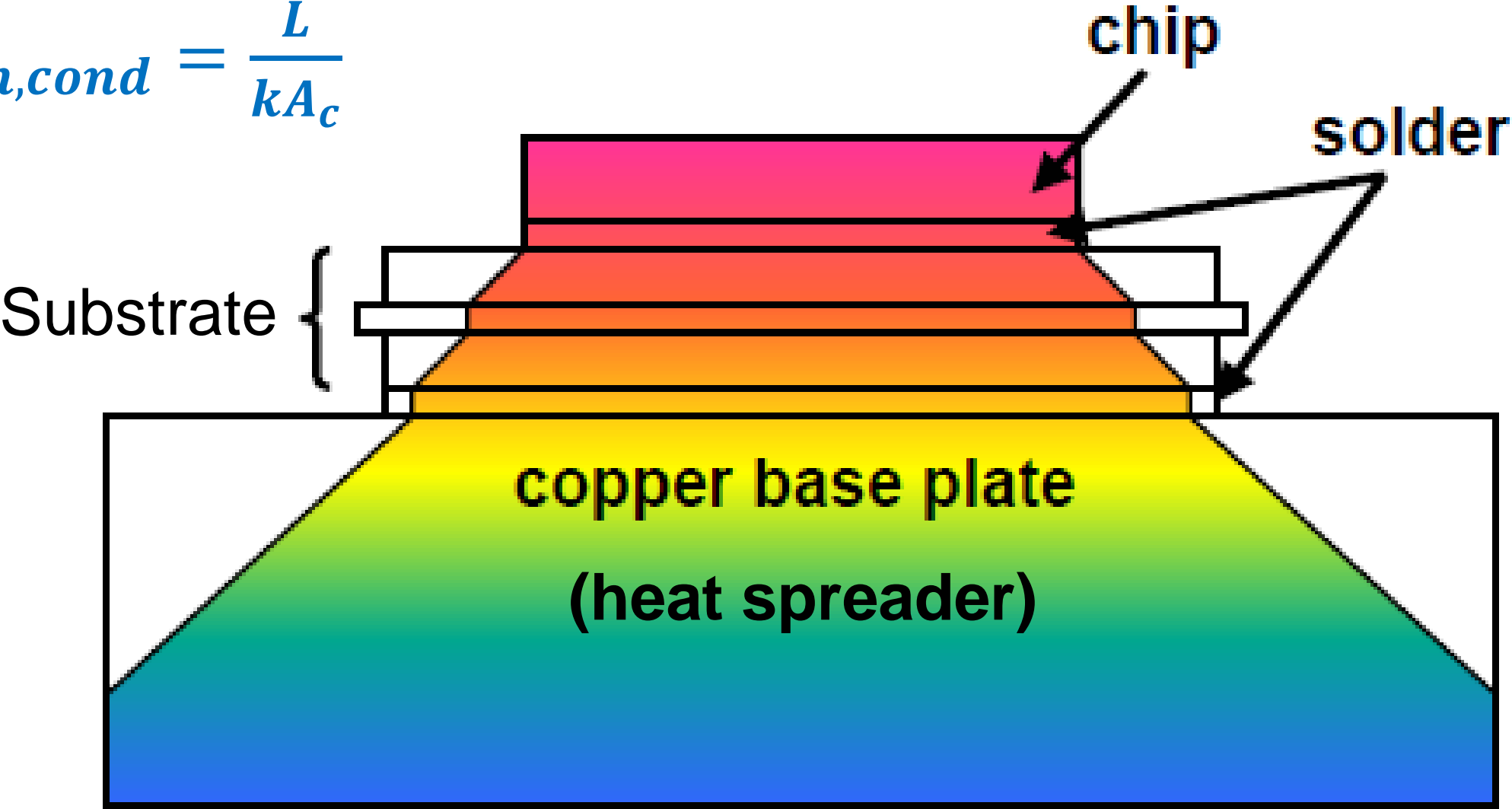
Ther-O-Bond 1600

Ther-O-Bond 2000

Thermalbond™

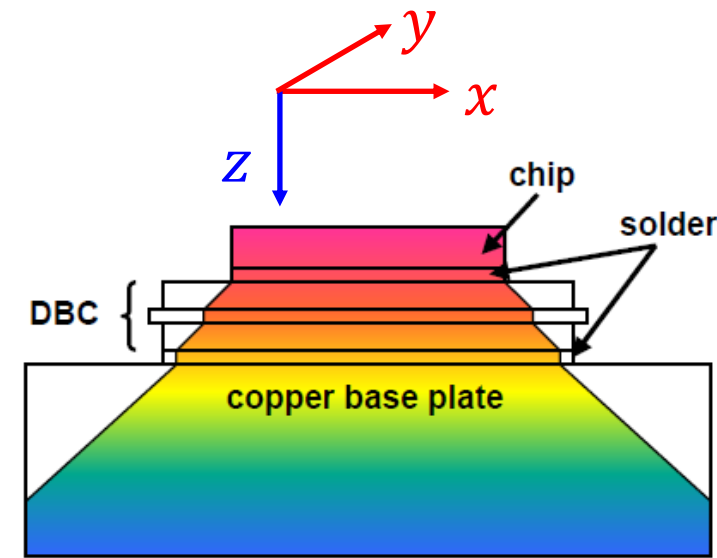
Lateral Heat Spreading

$$R_{th,cond} = \frac{L}{kA_c}$$



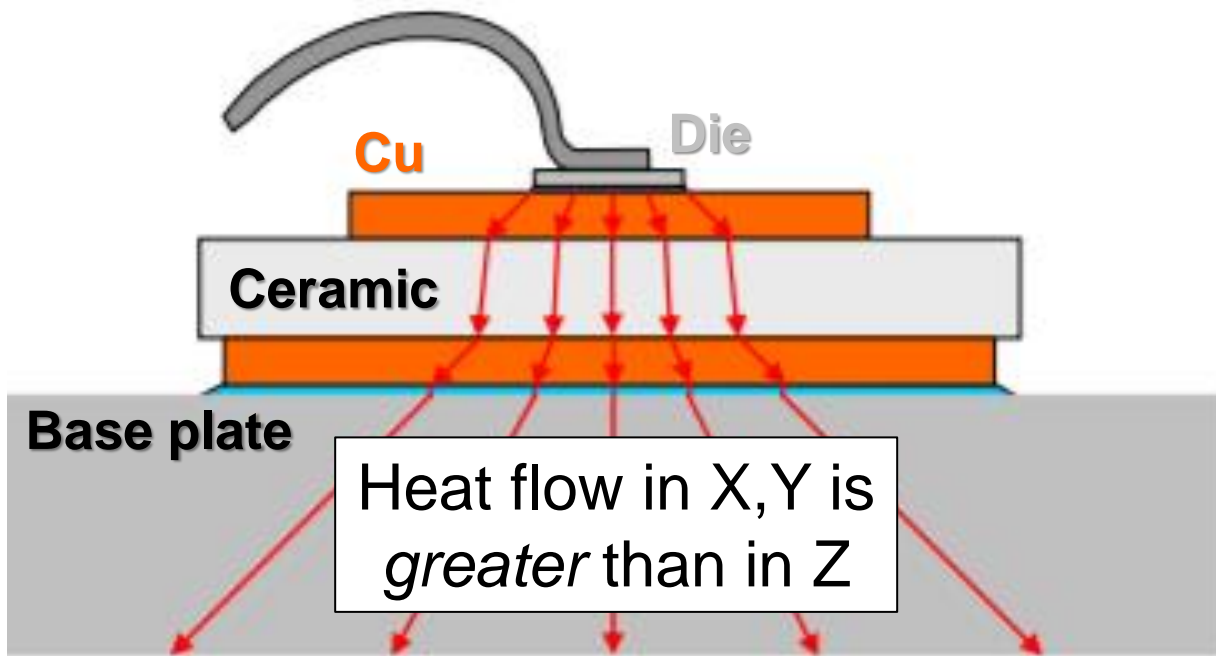
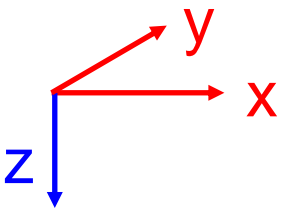
Heat Spreading

- Heat spreading occurs when the heat flow in the x and y (horizontal) directions is *greater* than the heat flow in the z (vertical direction)
- This can occur when:
 - A material with *high thermal conductivity* k is placed near the heat path. This is called a *heat spreader*
 - The *heat transfer coefficient* is low (meaning the heat flow in the z direction is low)
- The heat spreading determines the effective area used in the thermal resistance calculations

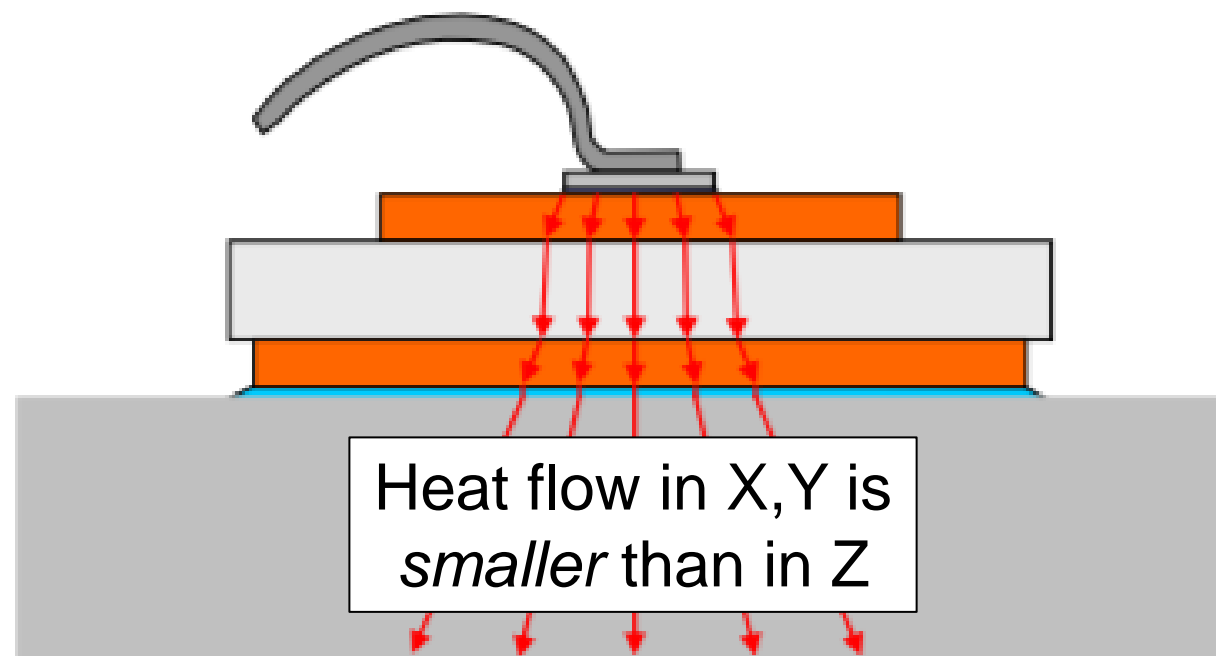


$$R_{th,cond} = \frac{L}{kA_c}$$

Heat Spreading

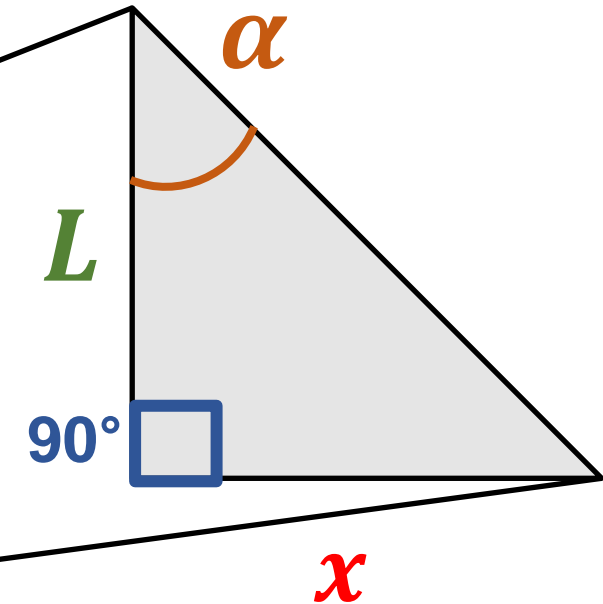
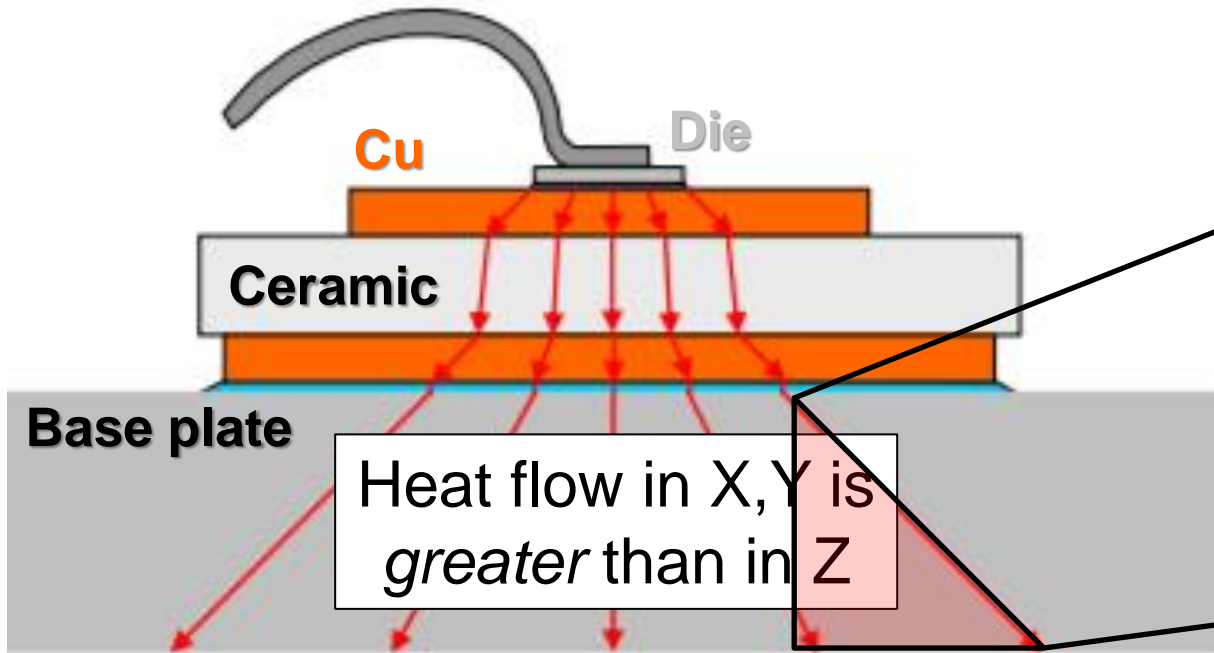
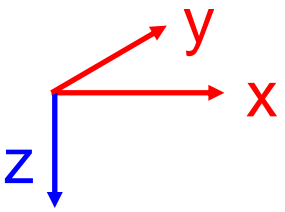


- Heatsink thermal resistance is *high*
- *Low* h (e.g., natural convection)
- Heatsink has *low* k



- Heatsink thermal resistance is *low*
- *High* h (e.g., forced liquid cooling)
- Heatsink has *high* k

Heat Spreading Angle α



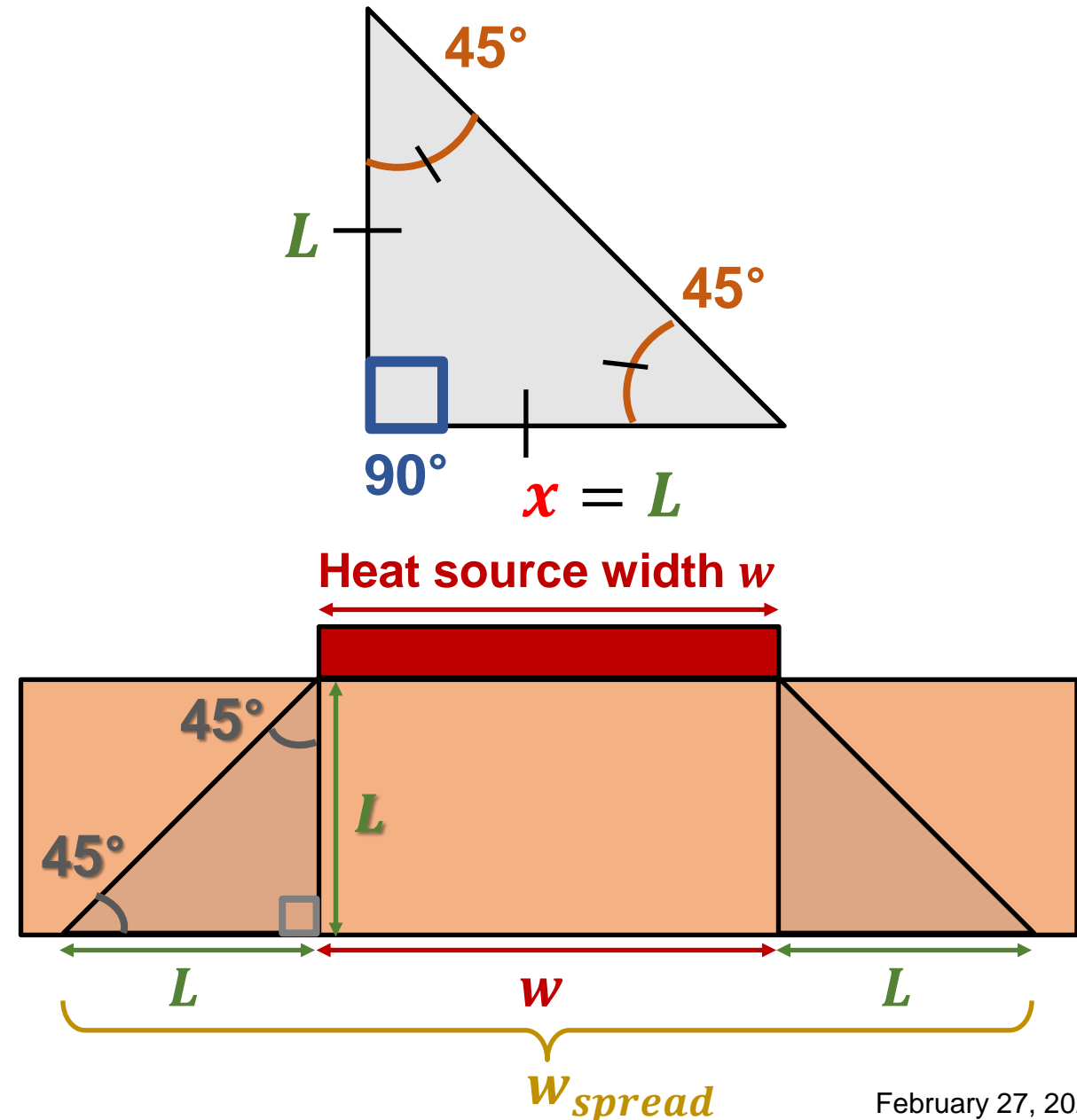
- Heatsink thermal resistance is *high*
- *Low* h (e.g., natural convection)
- Heatsink has *low* k

$$R_{th,cond} = \frac{L}{kA_c}$$

Heat Spreading Approximation

- A 45° spreading angle is a common approximation/simplification for heat spreading in materials with high thermal conductivity
- For a 45° spreading angle, the width at the base of the heat spreading is

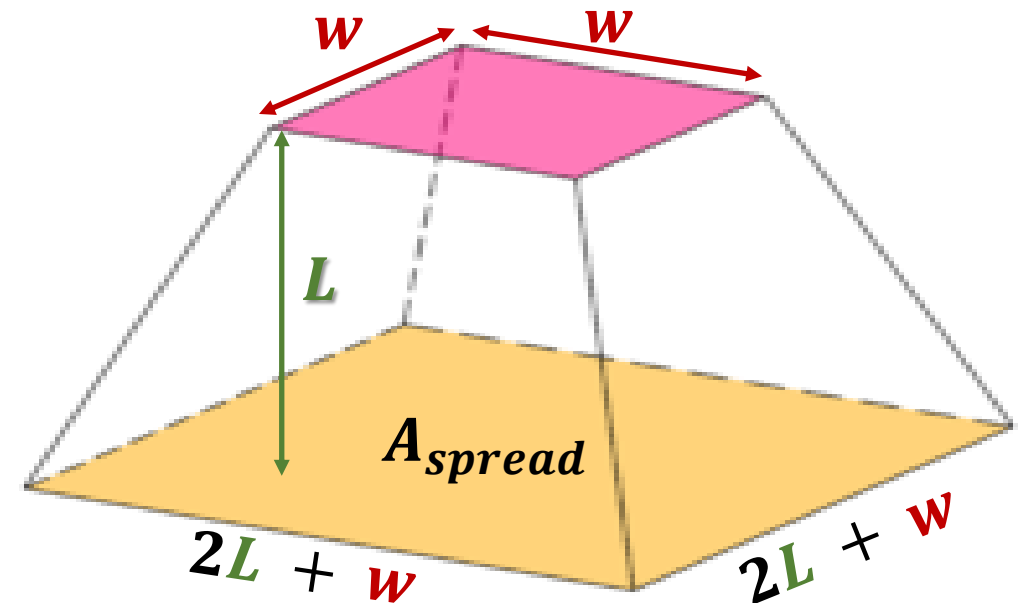
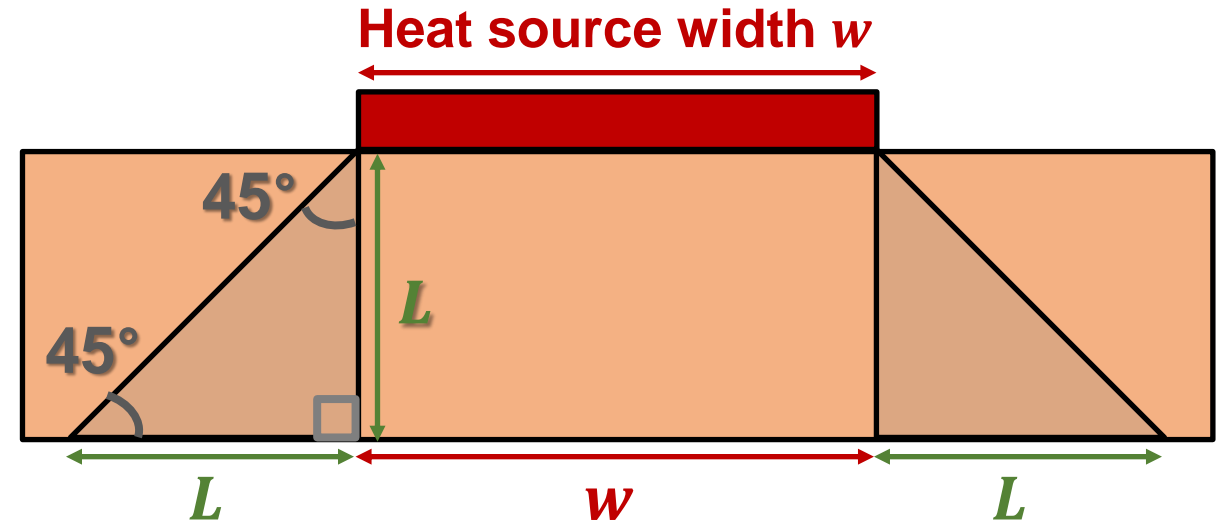
$$w_{spread} = 2L + w$$



Heat Spreading Approximation

- If the heat source is square, then the base area of the heat spreading is

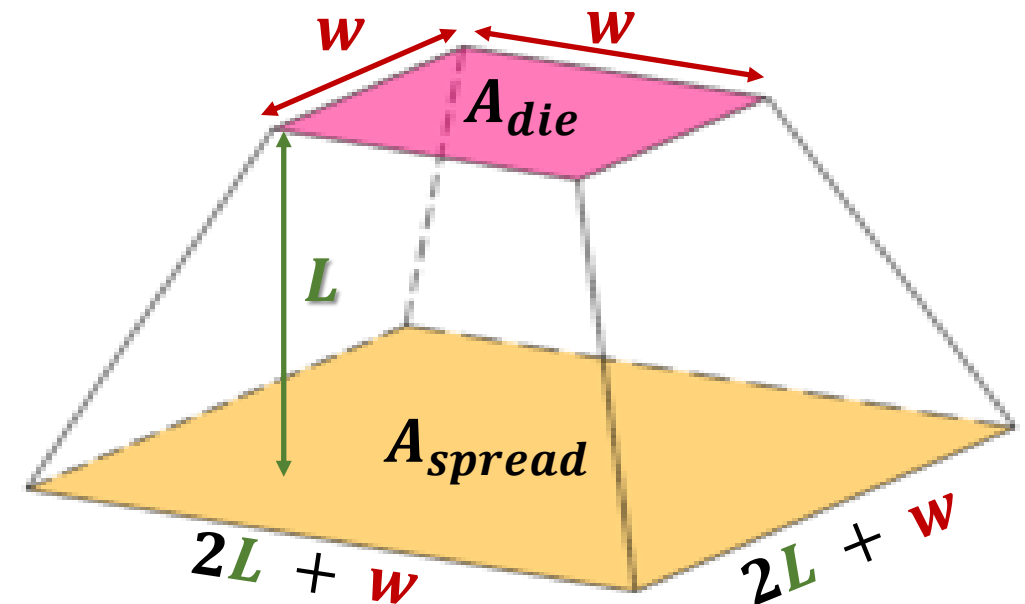
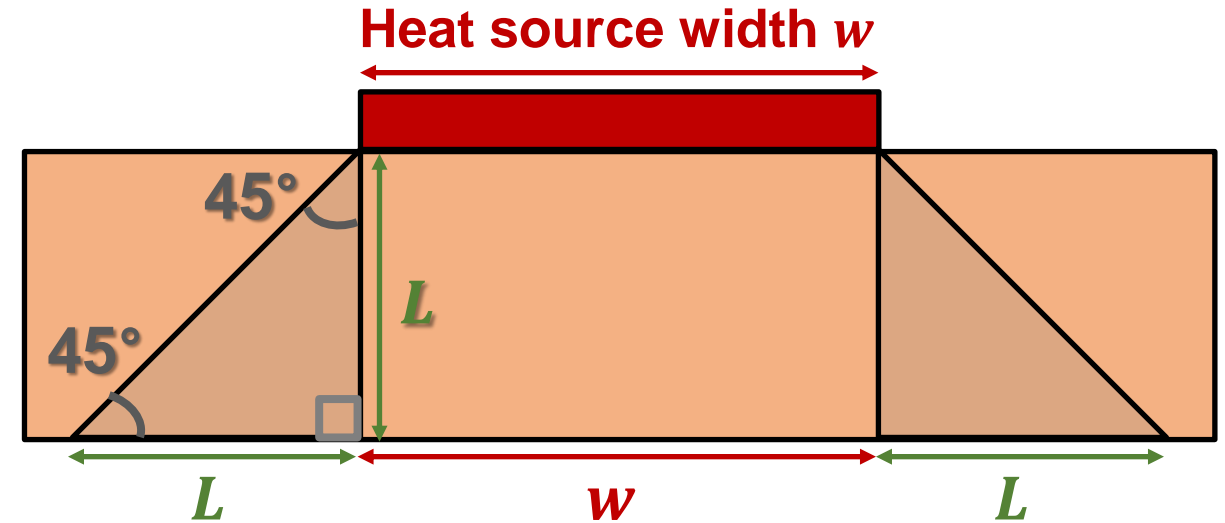
$$\begin{aligned} A_{spread} &= w_{spread} \times w_{spread} \\ &= (2L + w)(2L + w) \end{aligned}$$



Heat Spreading Approximation

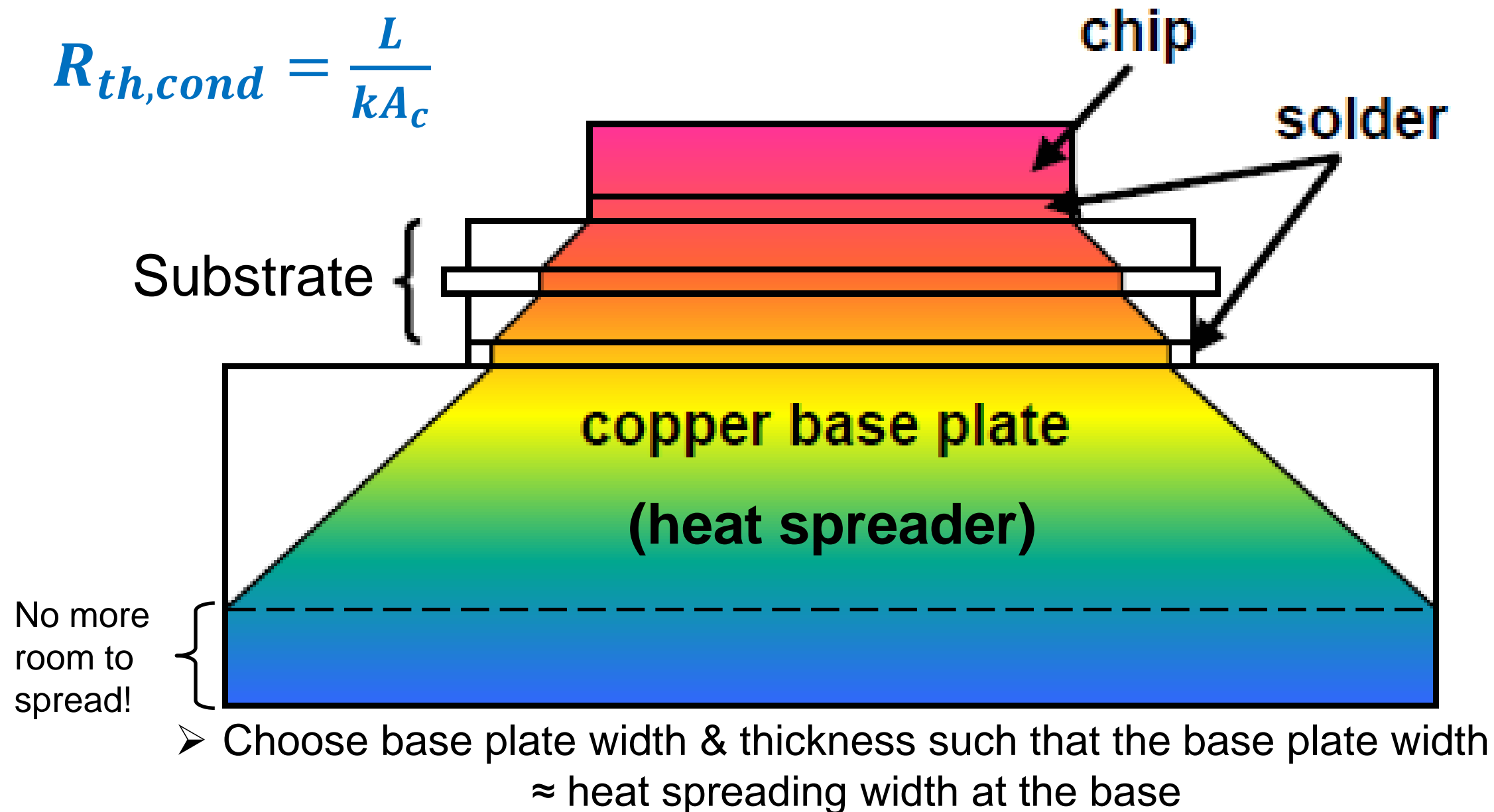
- The effective area A_{eff} for the heat flow through this layer can be approximated by averaging the heat source area A_{die} and the base area A_{spread} :

$$\begin{aligned} A_{eff} &= (A_{spread} + A_{die}) / 2 \\ &= [(2L + w)(2L + w) + (w \times w)] / 2 \end{aligned}$$



Lateral Heat Spreading

$$R_{th,cond} = \frac{L}{kA_c}$$



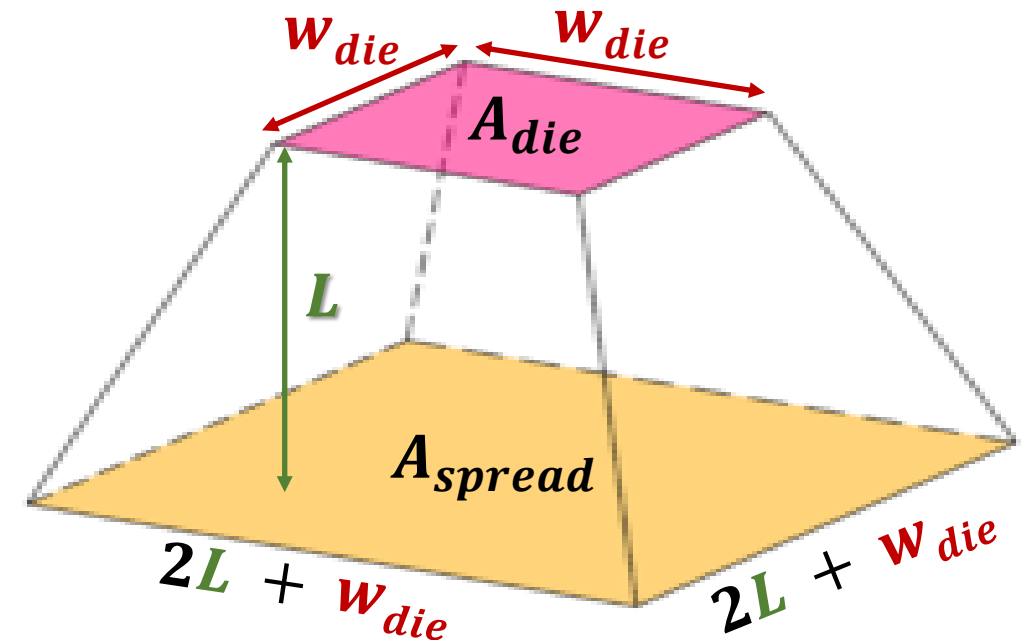
Example: Heat Spreading

Find the thermal resistance of a copper base plate with dimensions of 15 x 15 x 4 mm³. The dimensions of the heat-generating component (die) on top of the base plate are 5 x 5 x 1 mm³. Assume a heat spreading angle of 45°.

- $w_{die} = 5 \text{ mm}$
- $A_{die} = 5 \text{ mm} \times 5 \text{ mm} = 25 \text{ mm}^2$
- $L_{BP} = 4 \text{ mm}$
- $w_{BP} = 15 \text{ mm}$

Check that $w_{spread} \leq w_{BP}$:

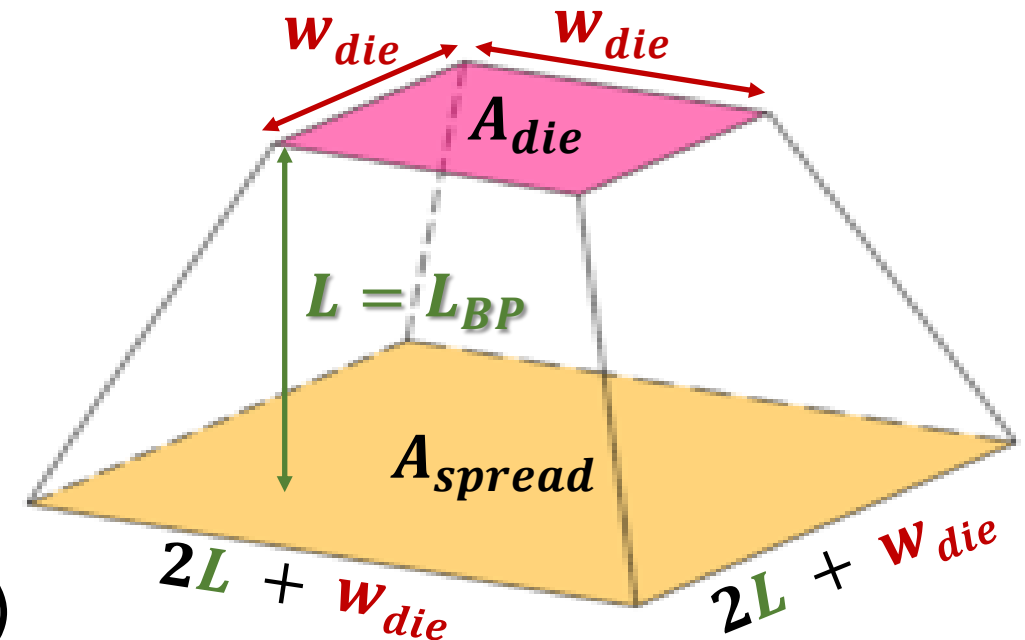
- $w_{spread} = (2L + w_{die}) = 2(4\text{mm}) + 5\text{mm} = \mathbf{11 \text{ mm}} < 15 \text{ mm}$



Example: Heat Spreading

Find the thermal resistance of a copper base plate with dimensions of 15 x 15 x 4 mm³. The dimensions of the heat-generating component (die) on top of the base plate are 5 x 5 x 1 mm³. Assume a heat spreading angle of 45°.

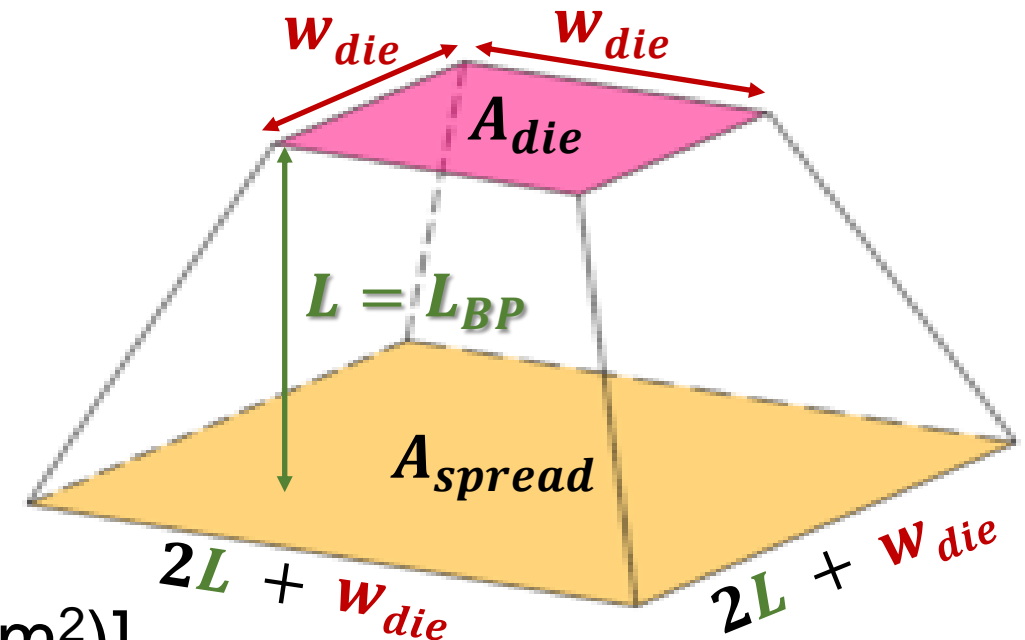
- $w_{die} = 5 \text{ mm}$
- $A_{die} = 5 \text{ mm} \times 5 \text{ mm} = 25 \text{ mm}^2$
- $L = L_{BP} = 4 \text{ mm}$
- $A_{spread} = (2L_{BP} + w_{die})(2L_{BP} + w_{die})$
 $= (2(4\text{mm}) + 5\text{mm})(2(4\text{mm}) + 5\text{mm})$
 $= \mathbf{169 \text{ mm}^2}$



Example: Heat Spreading

Find the thermal resistance of a copper base plate with dimensions of 15 x 15 x 4 mm³. The dimensions of the heat-generating component (die) on top of the base plate are 5 x 5 x 1 mm³. Assume a heat spreading angle of 45°.

- $A_{eff} = (A_{spread} + A_{die}) / 2$
 $= (169 \text{ mm}^2 + 25 \text{ mm}^2) / 2$
 $= 97 \text{ mm}^2 = 0.000097 \text{ m}^2$
- $R_{th,BP} = L_{BP} / (k_{BP} A_{eff})$
 $= 0.004 \text{ m} / [(390 \text{ W/(mK)})(9.7\text{e-}5 \text{ m}^2)]$
 $= \mathbf{0.106 \text{ K/W}}$



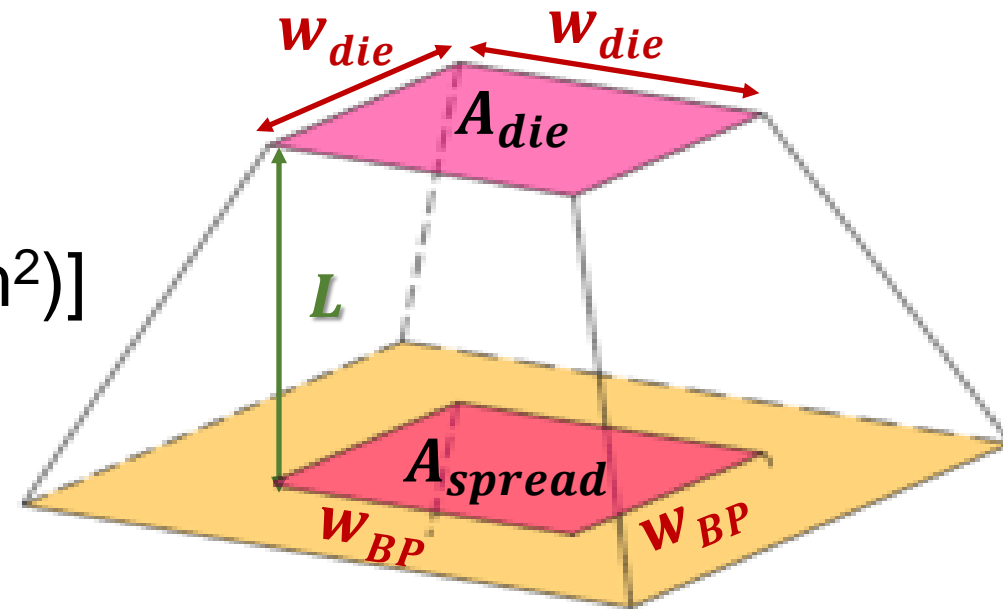
Example: Smaller Base Plate Area

- Silicon die: 5 x 5 x 1 mm
- Copper baseplate: 5 x 5 x 4 mm
- Find the thermal resistance of the base plate.

- $A_{spread} = A_{BP} = A_{die}$

- $$R_{th,BP} = L_{BP} / (k_{BP} A_{BP})$$
$$= 0.004 \text{ m} / [(390 \text{ W/(mK)})(2.5\text{e-}5 \text{ m}^2)]$$
$$= \mathbf{0.410 \text{ K/W}}$$

➤ $R_{th,BP}$ increases by **4x** because there is no room for heat spreading (A_{BP} is smaller)

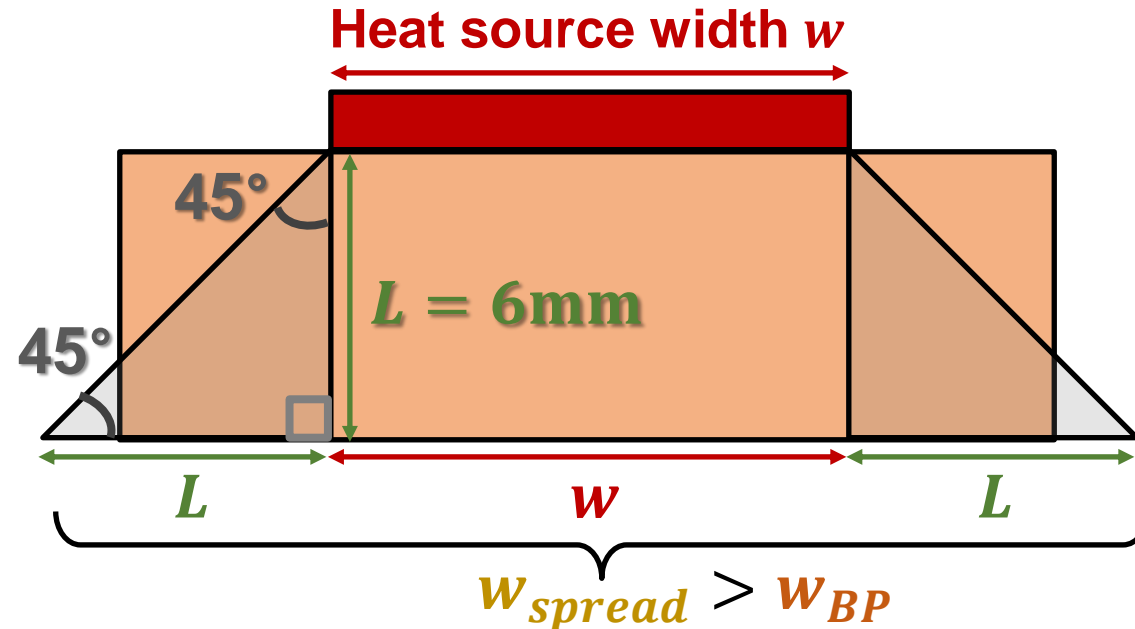


Example: Thicker Base Plate Area

- Silicon die: 5 x 5 x 1 mm
- Copper baseplate: 15 x 15 x 6 mm

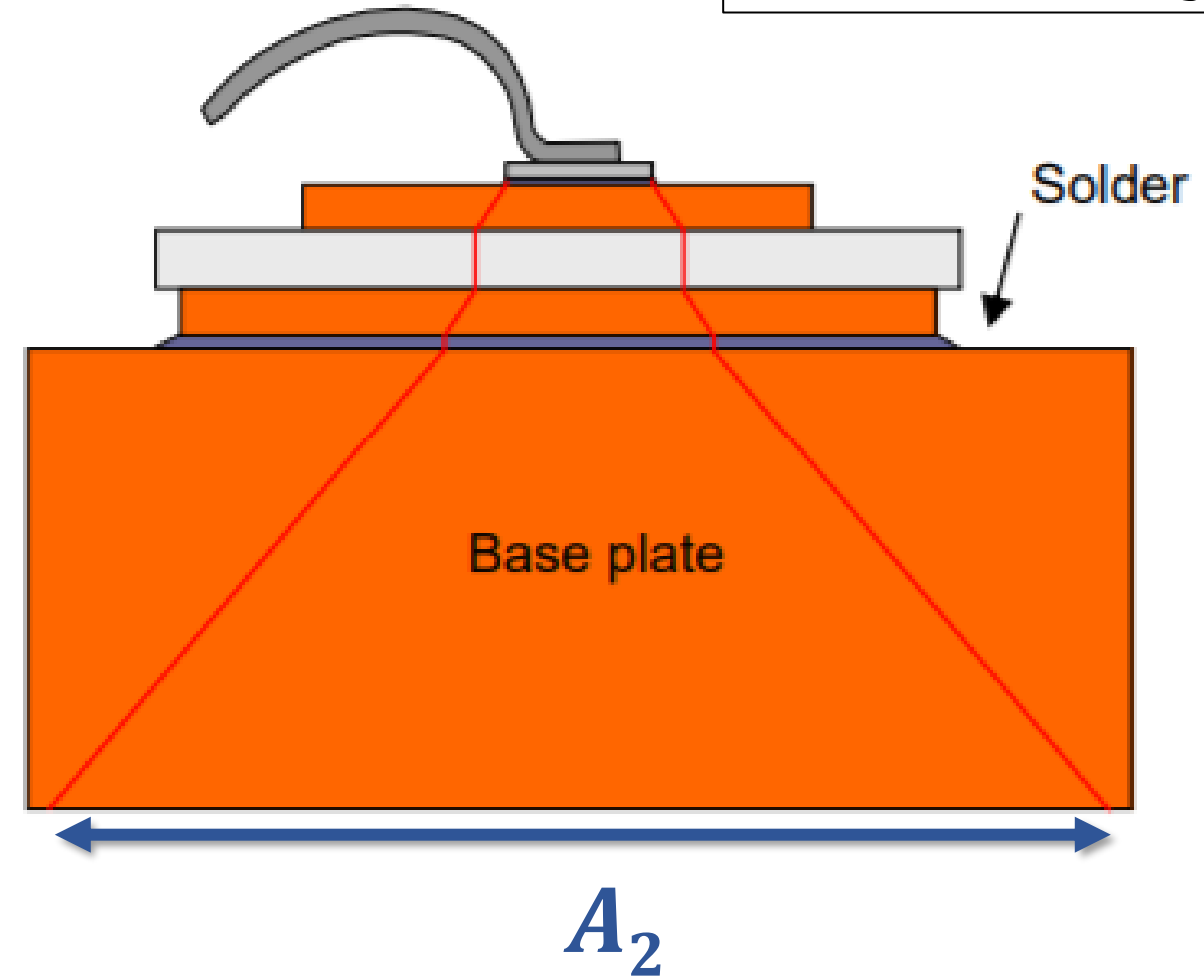
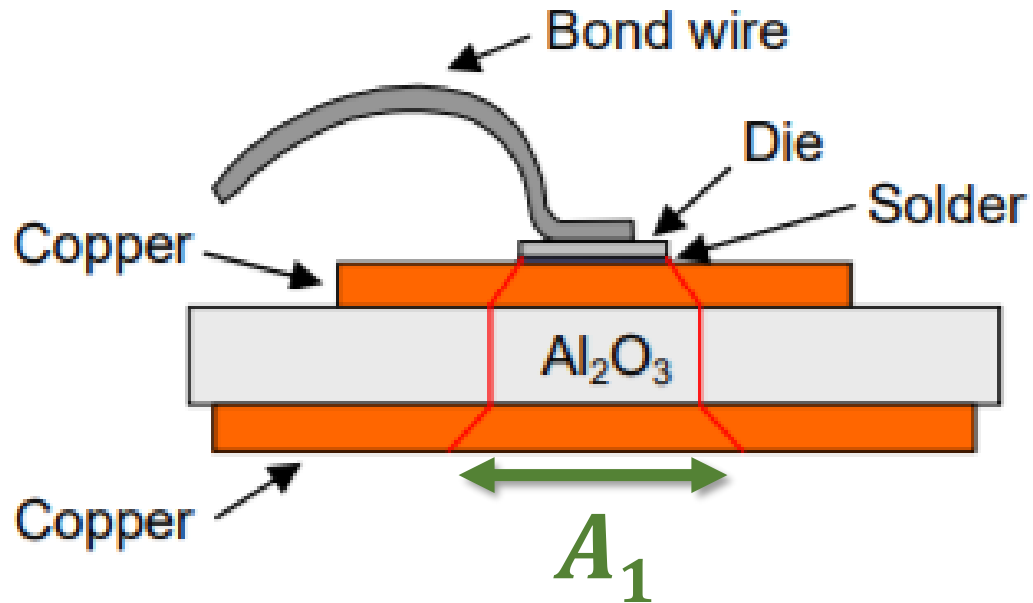
Check that $w_{spread} \leq w_{BP}$:

- $w_{spread} = (2L + w_{die}) = 2(6\text{mm}) + 5\text{mm} = \mathbf{17\text{ mm}} > 15\text{ mm}$!
- The bottom of the base plate is not helping with the heat spreading



Base Plate/Heat Spreader

$$R_{th,conv} = \frac{1}{hA_s}$$



For the same h ,

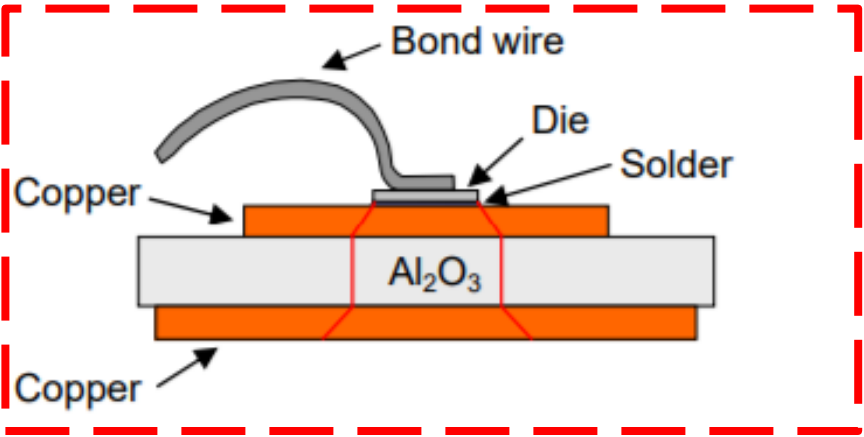
$$R_{th,conv}A_1 > R_{th,conv}A_2$$

*note: if h is high, then Z heat flow > X,Y heat flow, so heat spreading is low and the baseplate becomes less effective.

Impact of Base Plate/Heat Spreader

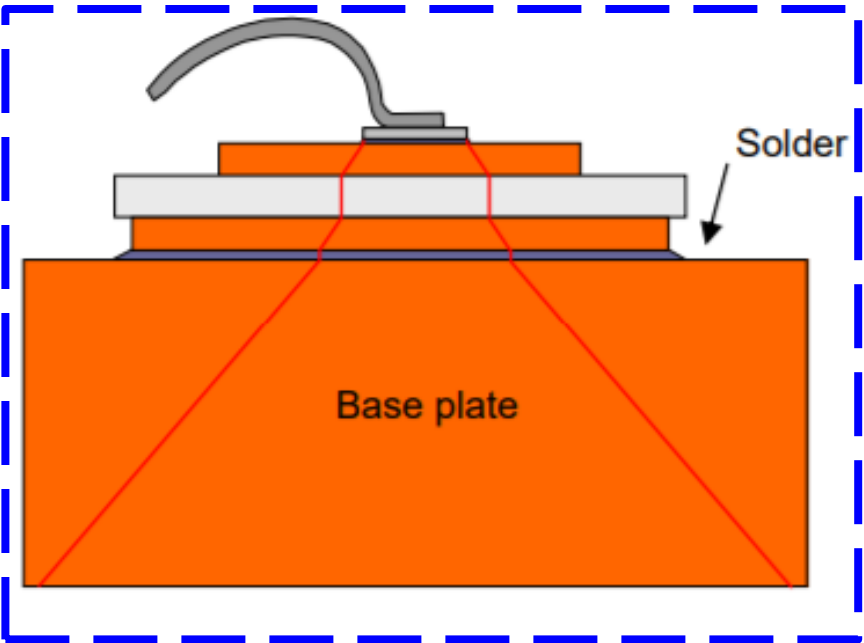
Thermal Conductivities

Part	DBC module		Baseplate modules	
Die [W/mK]	Silicon [148]		Silicon [148]	
Solder [W/mK]	SnAg [62]		SnAg [62]	
DBC [W/mK]	Al ₂ O ₃ [25]	AlN [155]	Al ₂ O ₃ [25]	AlN [155]
Solder [W/mK]			SnAg [62]	
Baseplate [W/mK]			Cu [401]	AlSiC [180]



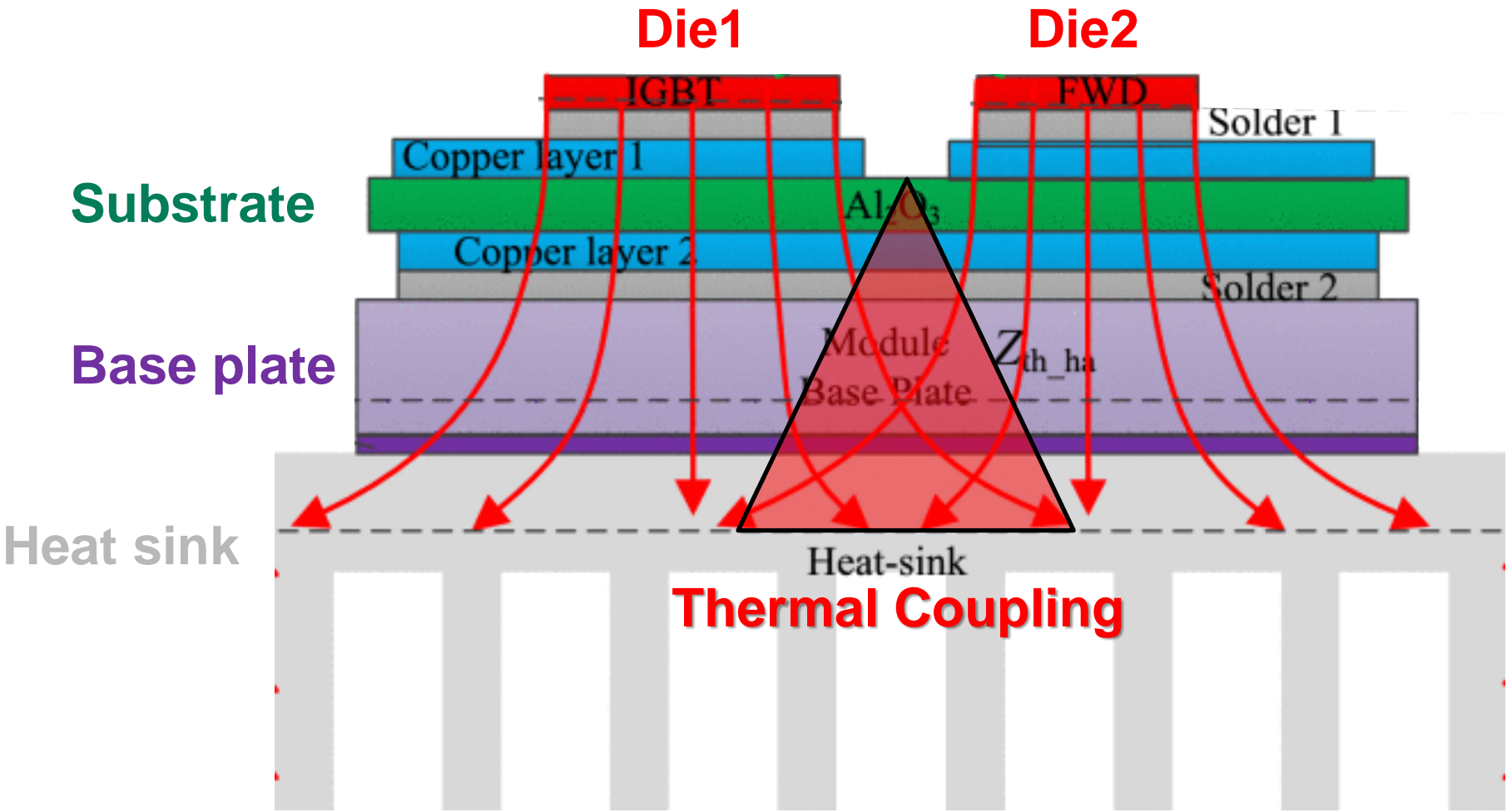
Coefficients of Thermal Expansion (CTE)

Part	DBC module		Baseplate modules	
Die [10 ⁻⁶ /K]	Silicon [2.8]		Silicon [2.8]	
Solder [10 ⁻⁶ /K]	SnAg [22.1]		SnAg [22.1]	
DBC [10 ⁻⁶ /K]	Al ₂ O ₃ [8.2]	AlN [4.5]	Al ₂ O ₃ [8.2]	AlN [4.5]
Solder [10 ⁻⁶ /K]			SnAg [22.1]	
Baseplate [10 ⁻⁶ /K]			Cu [16.5]	AlSiC [8.4]

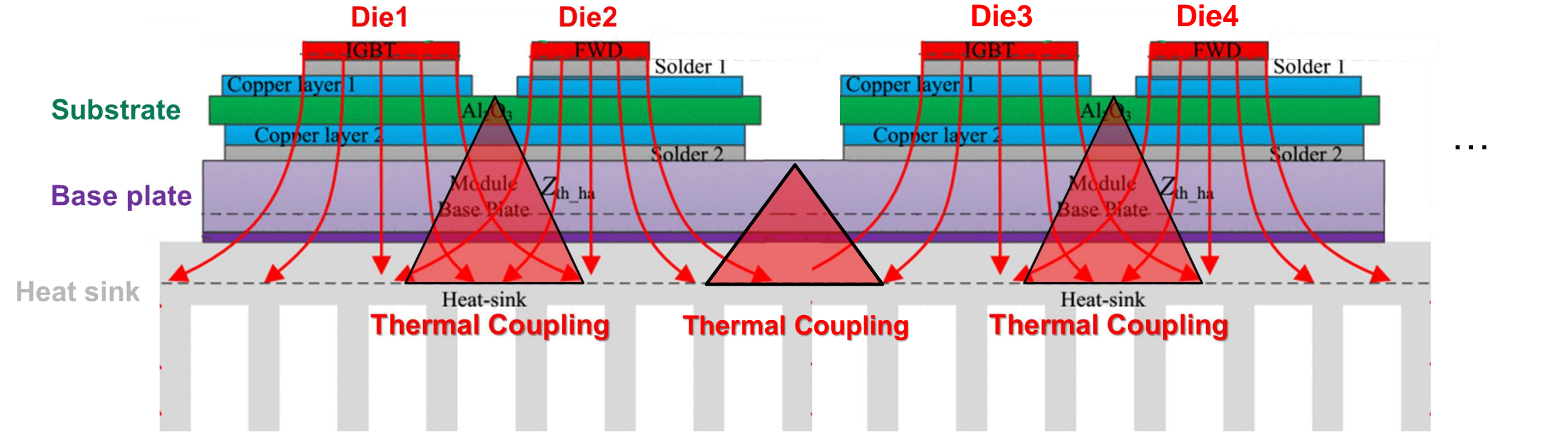


Reliability trade-off: baseplate reduces T_j , but increases CTE mismatch

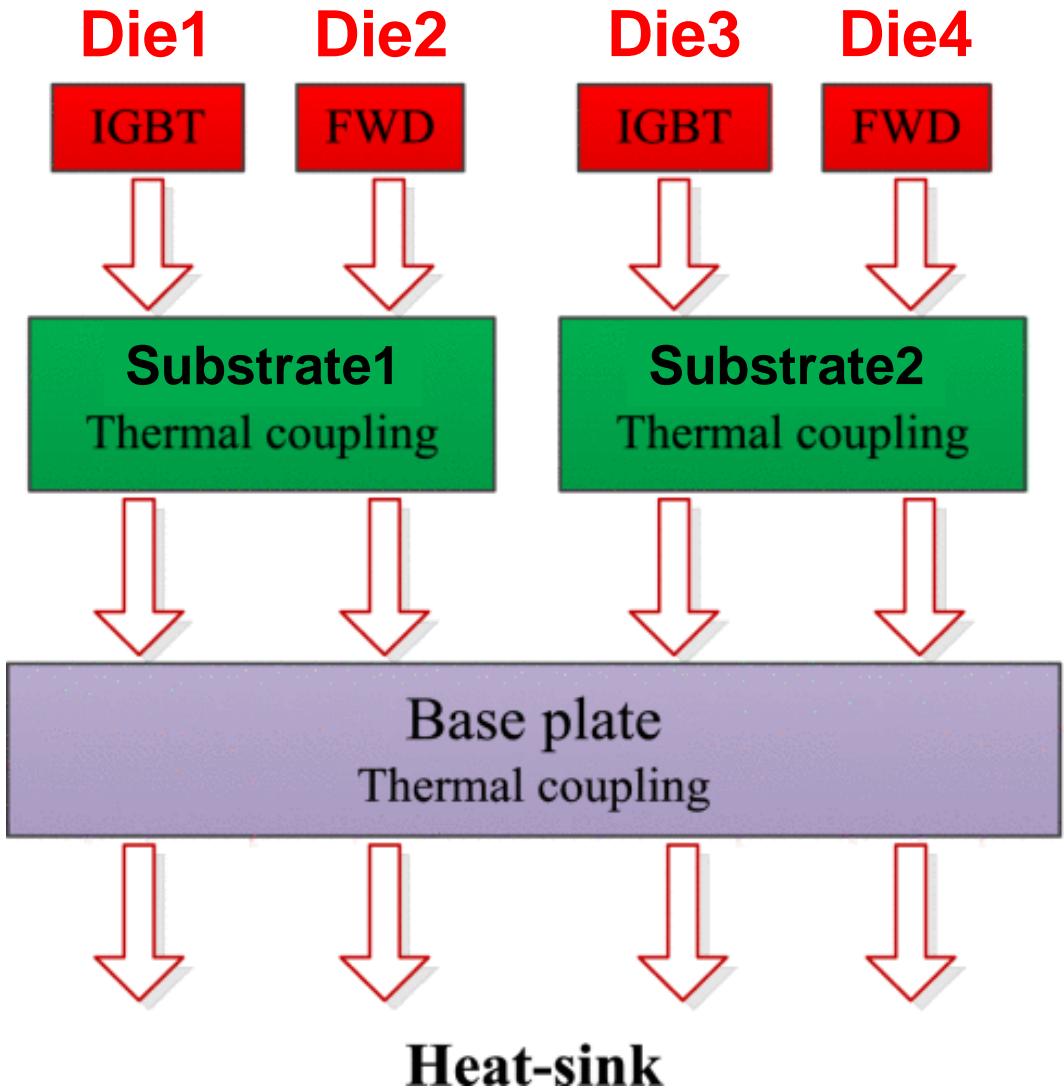
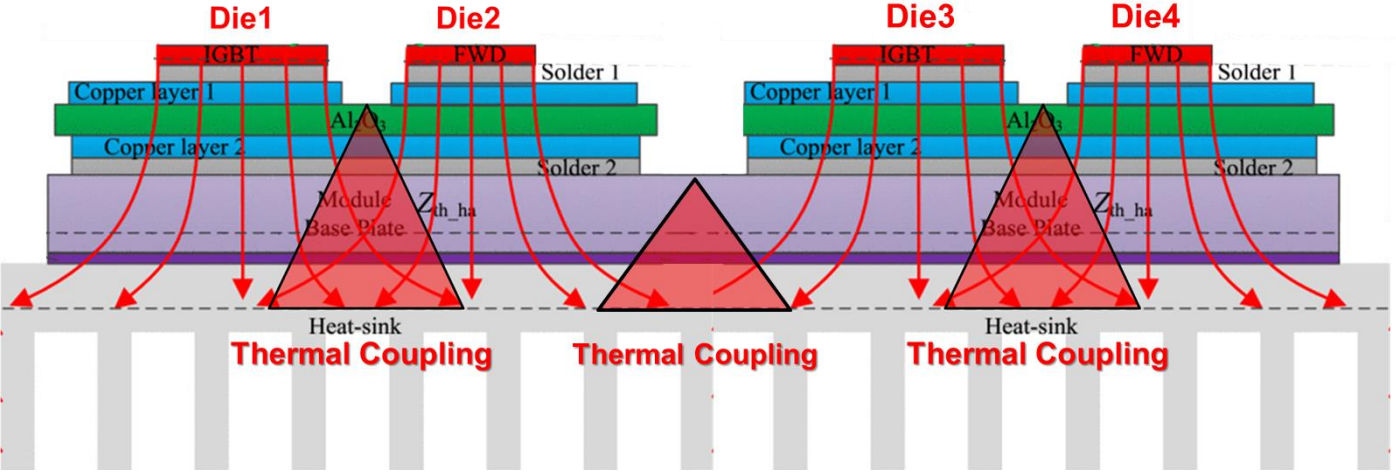
Heat Spreading in MCM = Thermal Coupling: Common Substrate



Heat Spreading in MCM = Thermal Coupling: Common Base Plate



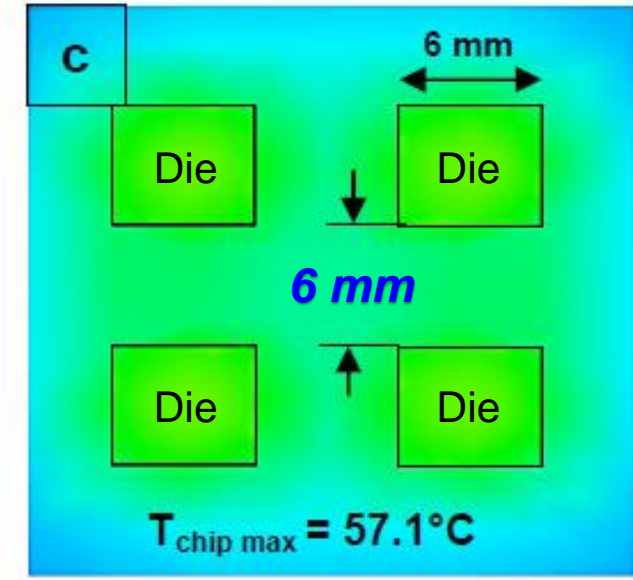
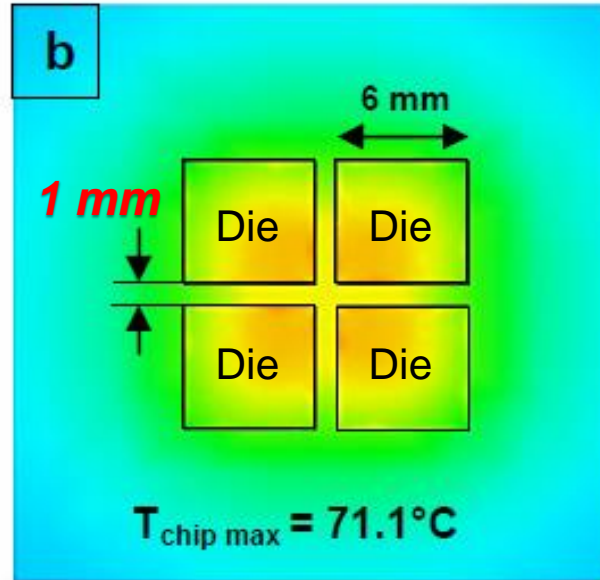
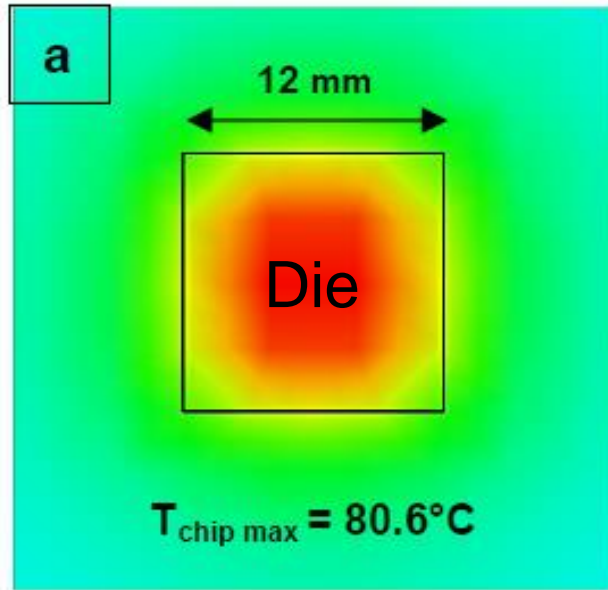
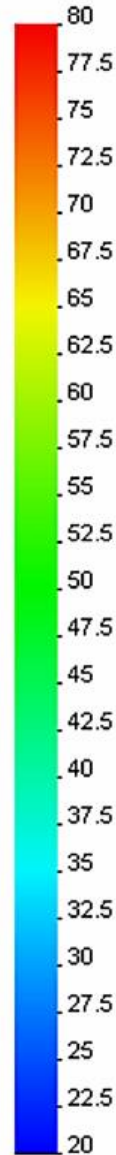
Simplified Heat Flow Path



- Thermal coupling at **substrate** level due to multiple **dies**
- Thermal coupling at **base plate** level due to multiple **substrates**

Impact of Thermal Coupling

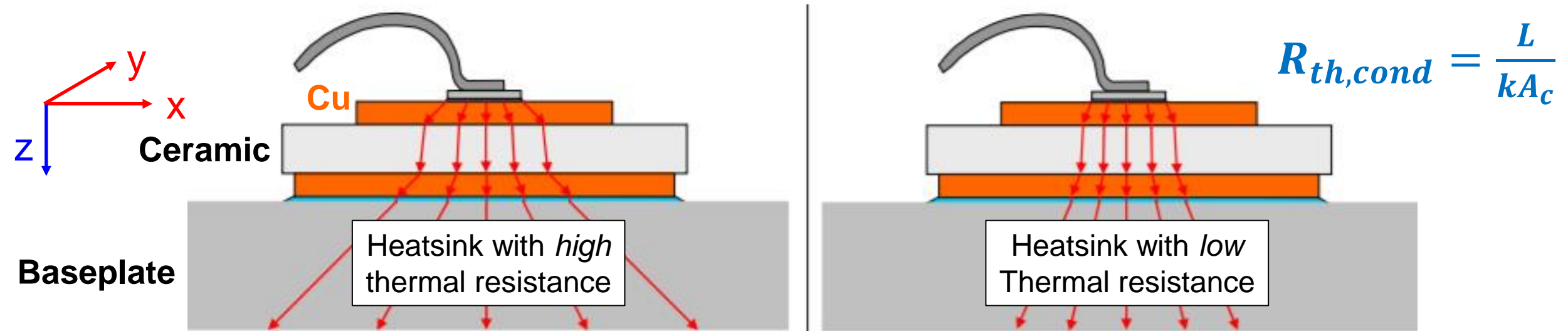
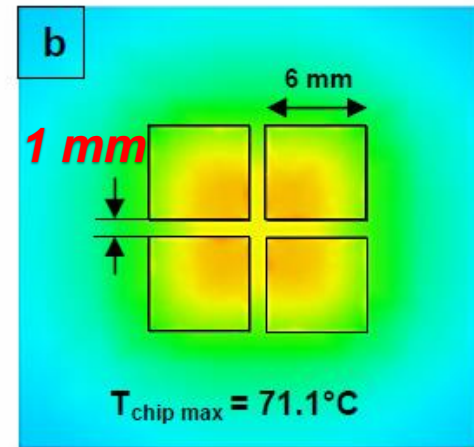
Temp (Celsius)



- **Large dies** may have **greater ΔT** across the area, and therefore **worse thermal spreading** than smaller dies
- **Several smaller dies** with the same overall area have a **lower R_{th}**
- If the **spacing between chips is small**, the **chips heat up one another** (thermal coupling)
- **Greater spacing** between chips further **lowers R_{th}**

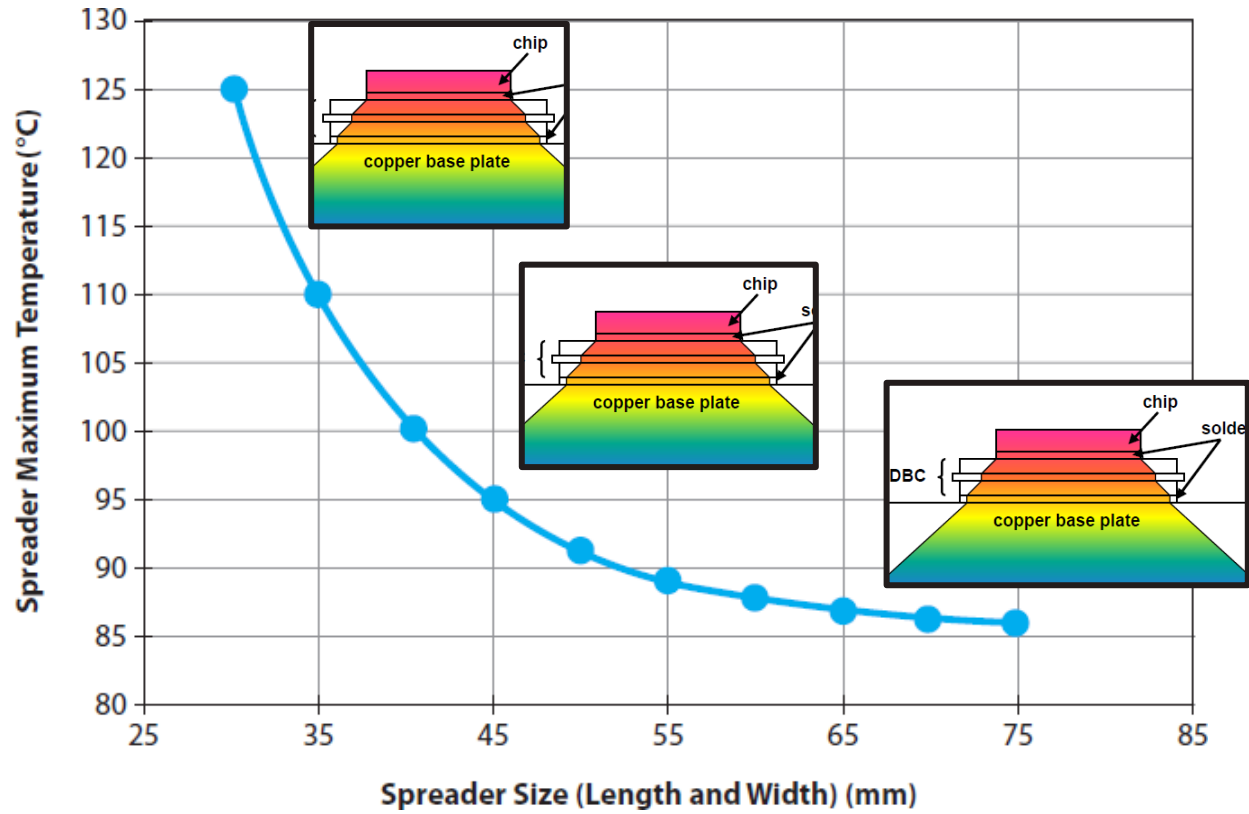
Heat Spreading Summary

- Heat spreading occurs when:
 - Heat flow in X, Y > heat flow in Z
 - k and/or h of downward layer is low (high R_{th} , low q)
- 45° heat spreading angle is a good approximation for high- k materials
 - Use to find effective heat transfer area through the spreading layer
- Close spacing of chips can increase T_j due to thermal coupling



Impact of Heat Spreader Area and Thickness

Heat Spreader Temp. vs. Size



Heat Spreader Temp. vs. Thickness

