VIRGINIA TECH

Lecture 4

Electrical Design

AC Resistance & Inductance

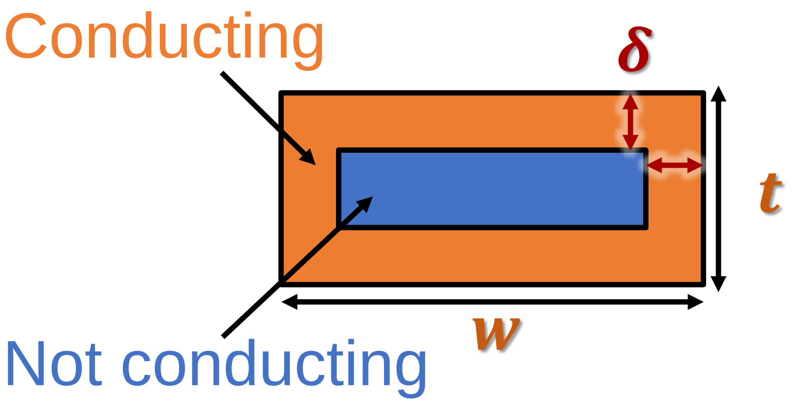
January 31, 2025

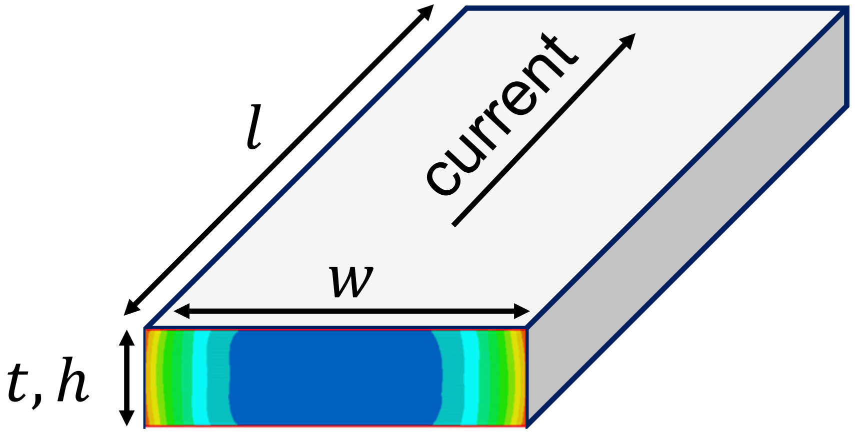
Reminders and Announcements

- Office hours: Monday, 3:30pm-4:30pm
- Quiz #1 answers and grades on Canvas
- Homework #1 will be assigned tomorrow (tentative)
- Download and install ANSYS Electronics Desktop and LTspice and check that you can open them before Feb. 3rd

AC Resistance of Rectangular Conductors
(First-Order Approximation)

- $R_{AC} = \rho l / A_{eff}$
- When $2\delta \ll\ w$ and t
  - $A_{eff} = wt - (w - 2\delta)(t - 2\delta)$
  - $A_{eff} = 2w\delta + 2t\delta - 4\delta^2$
  - $\boldsymbol{A_{eff} = 2\delta(w + t - 2\delta)}$
- $\delta = \sqrt{(\rho\ /\ (\pi\ f\ \mu))}$
- When $2\delta \geq w$ or t
  - $A_{eff} = wt$





Keller, R.B. (2023). Skin Effect. In: Design for Electromagnetic Compatibility--In a Nutshell. Springer, Cham. https://doi.org/10.1007/978-3-031-14186-7_10

Example: AC Resistance of PCB Trace

- 200-mm-long, 0.1-mm-wide PCB trace with 1 oz* copper
- ρ_{copper} = 1.7 x 10^{-8} Ω·m
- 1 oz copper = 1.37 mils = 0.00137 in = 0.034798 mm
- What is the AC resistance of the copper trace at 150 MHz?

$$\boldsymbol{R_{AC} = \rho l / A_{eff}}$$

$$\boldsymbol{\delta = \sqrt{(\rho \,/\, (\pi\, f\, \mu))}}$$

$$= \sqrt{(1.7 \cdot 10^{-8}\,\Omega \cdot \mathrm{m} / \,(\pi\,(150\mathrm{MHz})(4\pi * 10^{-7}\mathrm{H/m})))} = \mathbf{5.36 \cdot 10^{-6} m}$$

Check 2$\delta \ll$ *w* and *t* : 10.7 µm << 100 µm and 35 µm

$$\boldsymbol{A_{eff} = 2\delta(w + t - 2\delta)}$$

$$= 2(5.36 \cdot 10^{-6}\mathrm{m})(1 \cdot 10^{-4}\mathrm{m} + 3.5 \cdot 10^{-5}\mathrm{m} - 2(5.36 \cdot 10^{-6}\mathrm{m}))$$

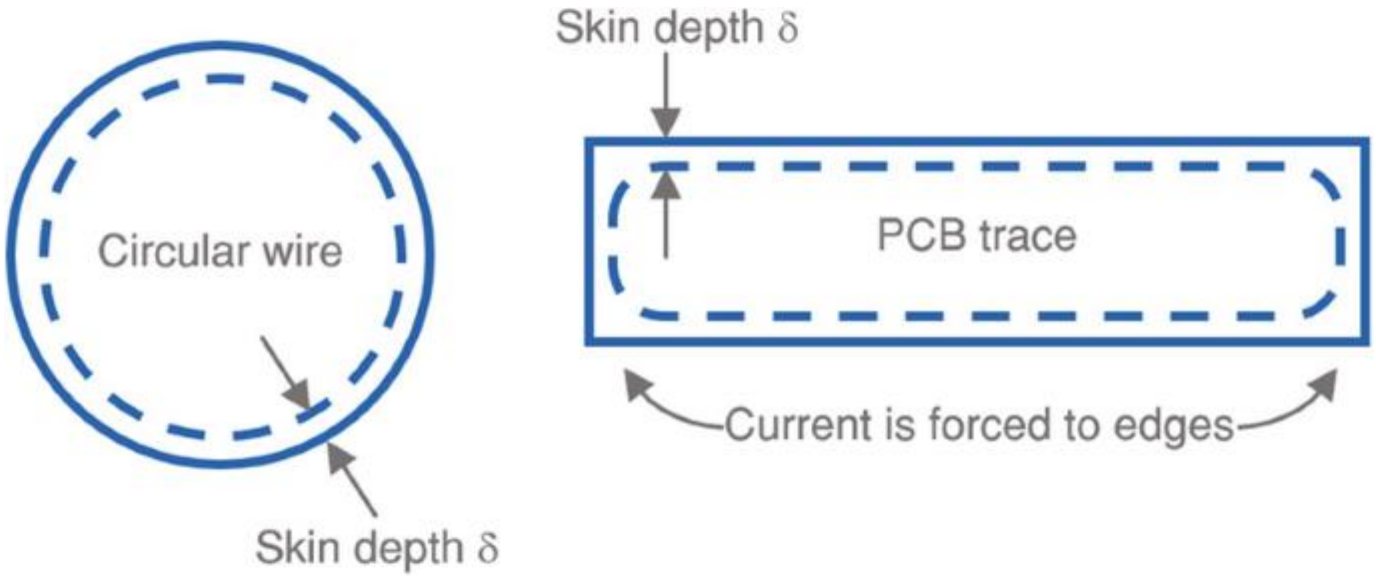
$$= \mathbf{1.33 \cdot 10^{-9} m^2}$$

R_{AC} = (1.7 x 10^{-8} Ω·m) · (0.2 m) / (1.33 x 10^{-9} m^2) = **2.56 Ω**

→ 2.6x higher than the DC resistance

AC Resistance: Circular vs Rectangular Cross-Sections

- Circles have less perimeter for a given cross-sectional area, so the skin effect is more pronounced
- Flat, wide conductors with rectangular cross-sections are preferred for high-frequency applications



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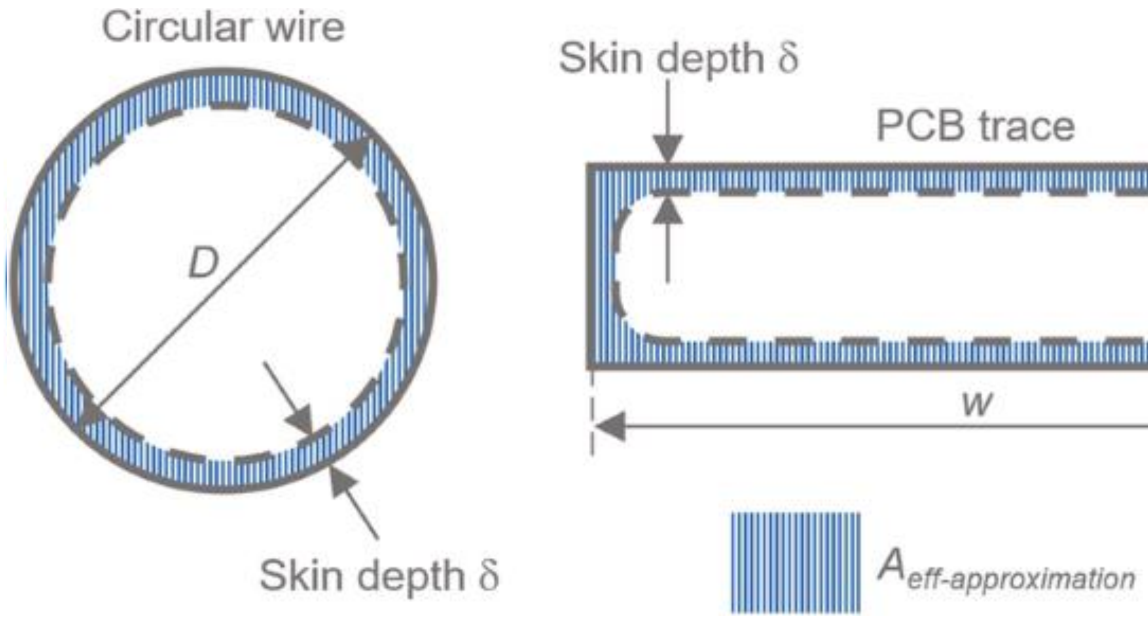

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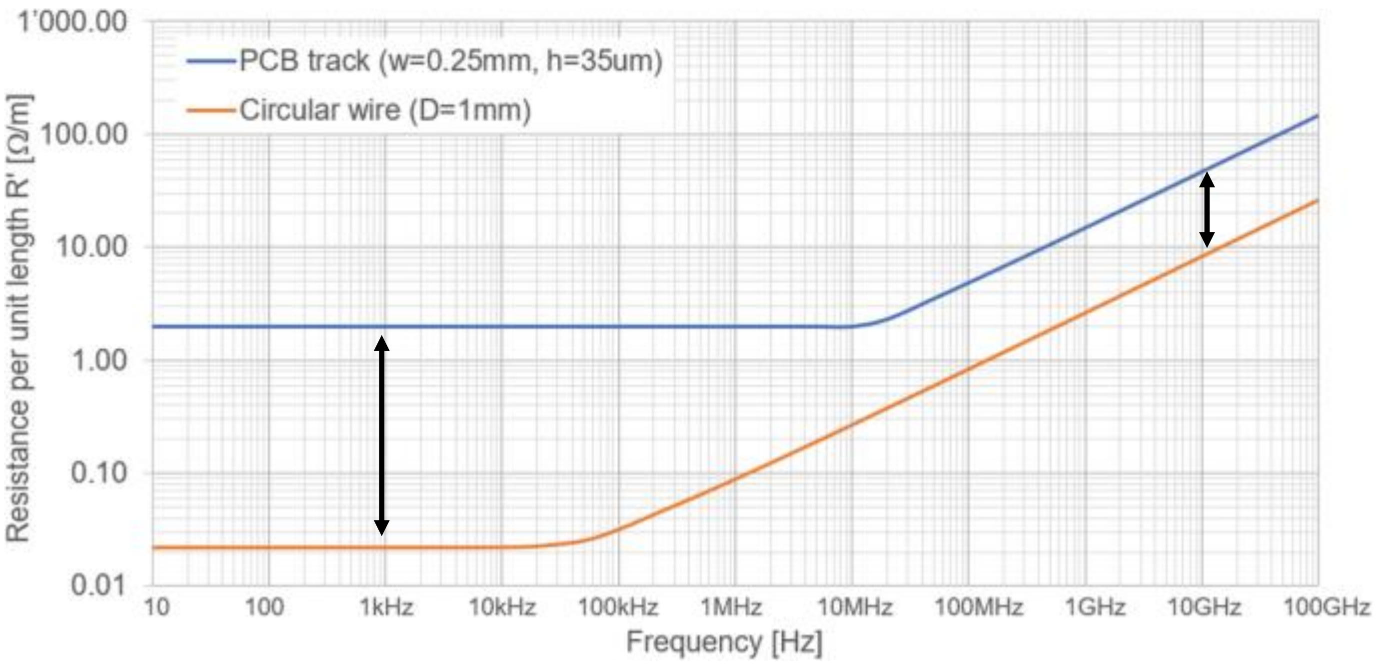

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Resistance Reduction Methods

DC Resistance

$$R_1 = \rho_0 l/A \cdot [1 + \alpha(T_1 - T_0)]$$

- Material
  - High electrical conductivity (low electrical resistivity)
- Geometry
  - Increase cross-sectional area
  - Decrease length
  - Parallel conductors
- Application
  - Decrease temperature
  - Decrease current

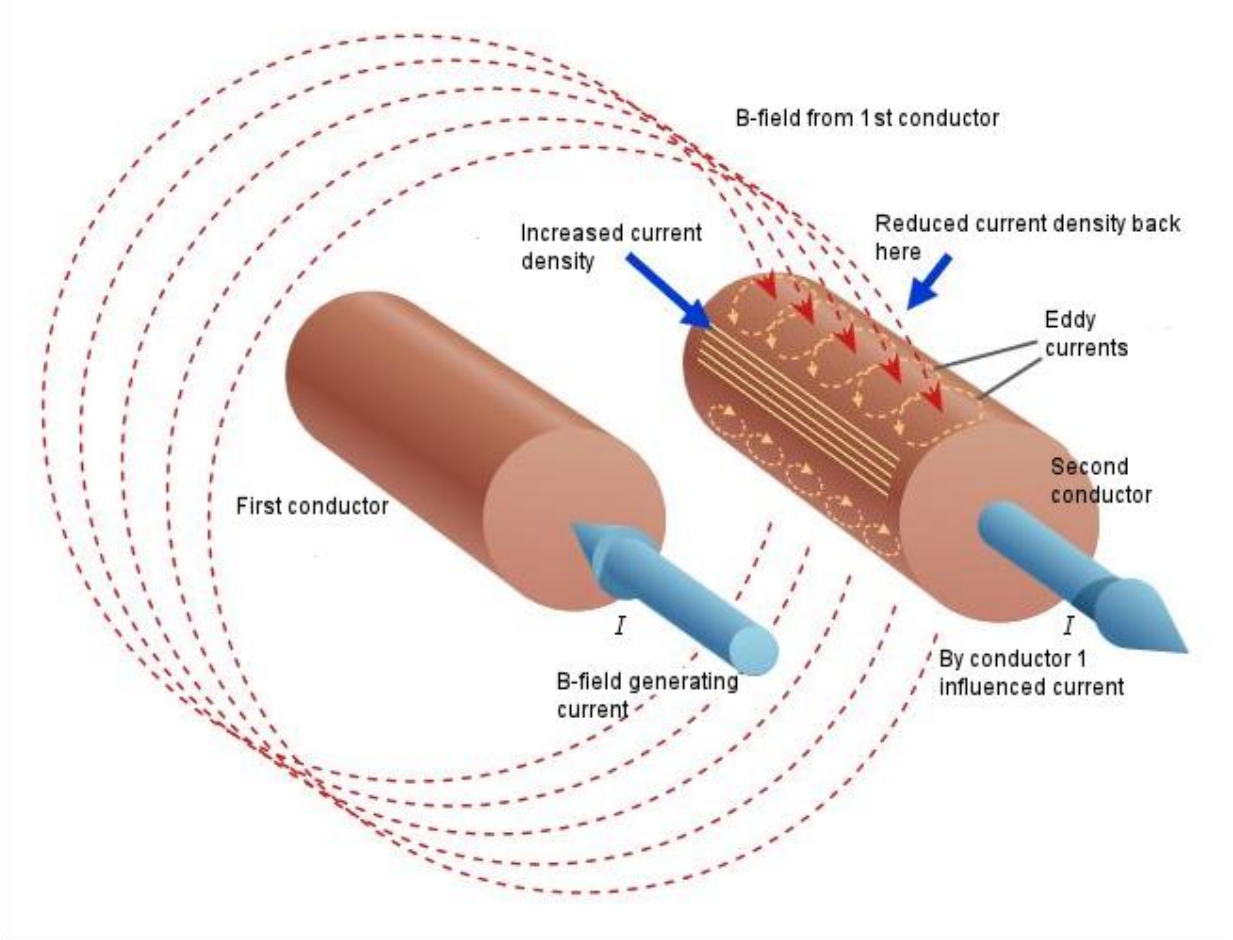
AC Resistance

$$\delta = \sqrt{(\rho \,/\, (\pi\, f\, \mu))}$$

- Material
  - Conductors with low permeability and low resistivity
- Geometry
  - Increase circumference/perimeter
  - Use wide flat conductor
  - Parallel smaller conductors
- Application
  - Decrease frequency

Skin and Proximity Effects

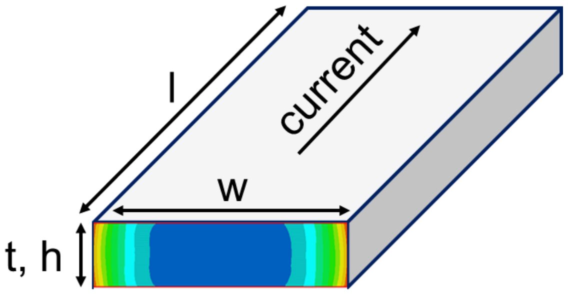
- Skin and proximity effects result from eddy currents
- When the magnetic field is generated by the conductor itself, this phenomena is called “skin effect”
- If the magnetic field is generated by an adjacent conductor, the phenomena is called “proximity effect”

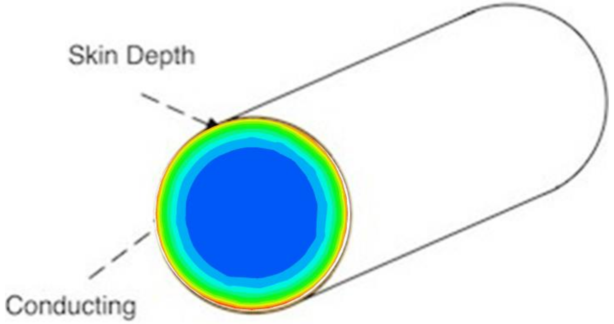


Proximity Effect

- Alternating current (AC) creates an alternating magnetic field around the conductor
- The alternating magnetic field induces eddy currents in adjacent conductors, changing the distribution of the current
- The result is current becoming concentrated in areas of the conductor closest to the first conductor
- Proximity effect significantly increases AC resistance of adjacent conductors
- The effect increases with frequency

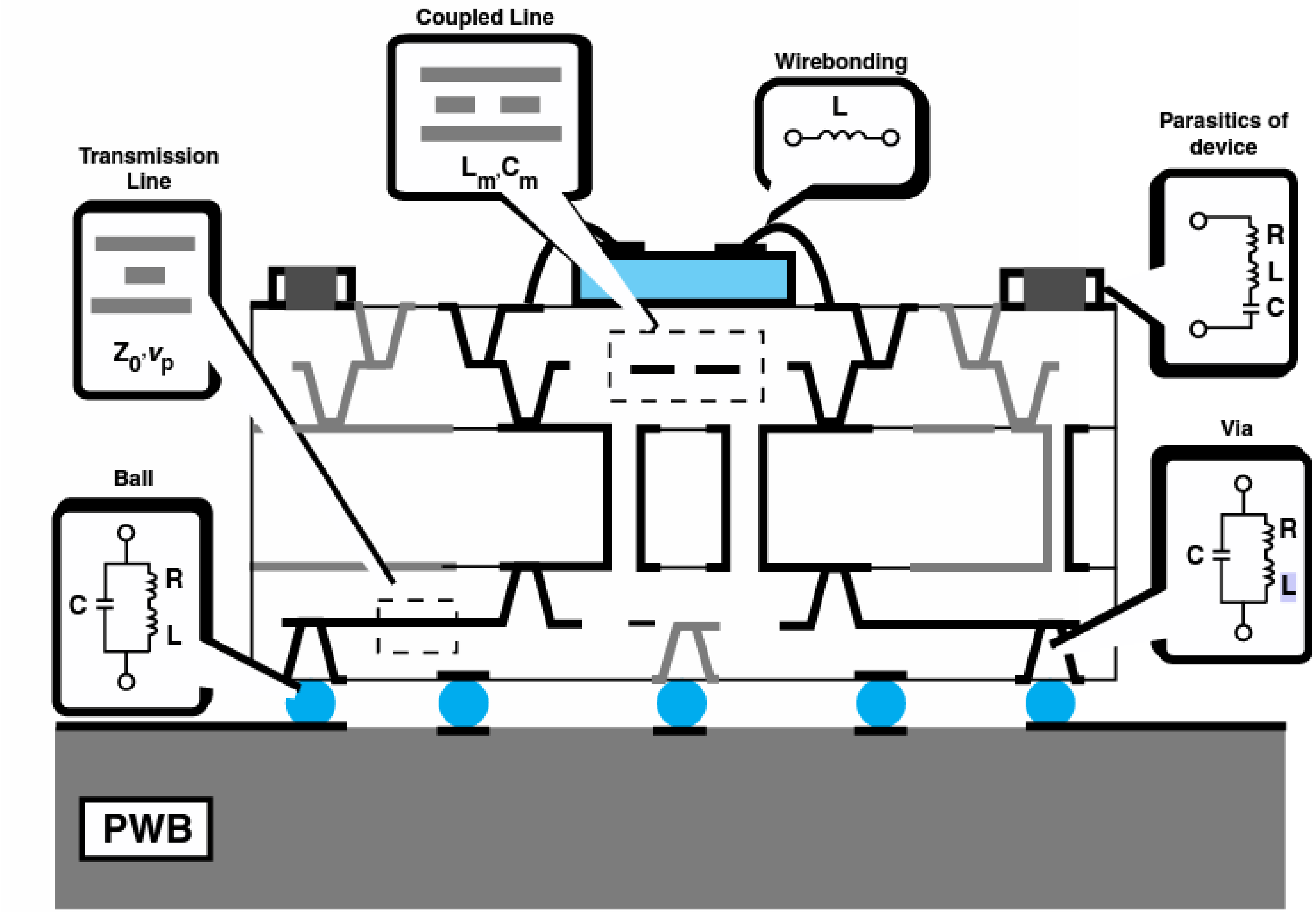
Summary: Resistance





- DC resistance at room temperature: $R = \rho l / A$
- Temperature correction: $R = R_0 \cdot [1 + \alpha\,(T - T_0)]$
- AC resistance
  - Skin effect: $\delta = \sqrt{(\rho\,/\,(\pi\, f\, \mu))}$
    - AC current → alternating magnetic field → opposing EMF → opposes current flow in center of wire → majority of current flows through outer surface
  - Related to circumference, ρ, and μ of conductor, and frequency
  - Effective area for circular conductors: $A_{eff} = \pi(d\delta - \delta^2)$, when $\delta \ll r$
  - Effective area for rectangular conductors: $A_{eff} = 2\delta(w + t - 2\delta)$, when $2\delta \ll\ w$ and t

Package Parasitics



Inductance

- Any current-carrying conductor can have parasitic inductance:
  - Wire / wire bond
  - Lead / terminals
  - PCB trace
- $V = L\, di/dt$, oscillations, coupling
- Types:
  - Self/partial inductance
  - Mutual inductance
  - Loop/effective/total inductance

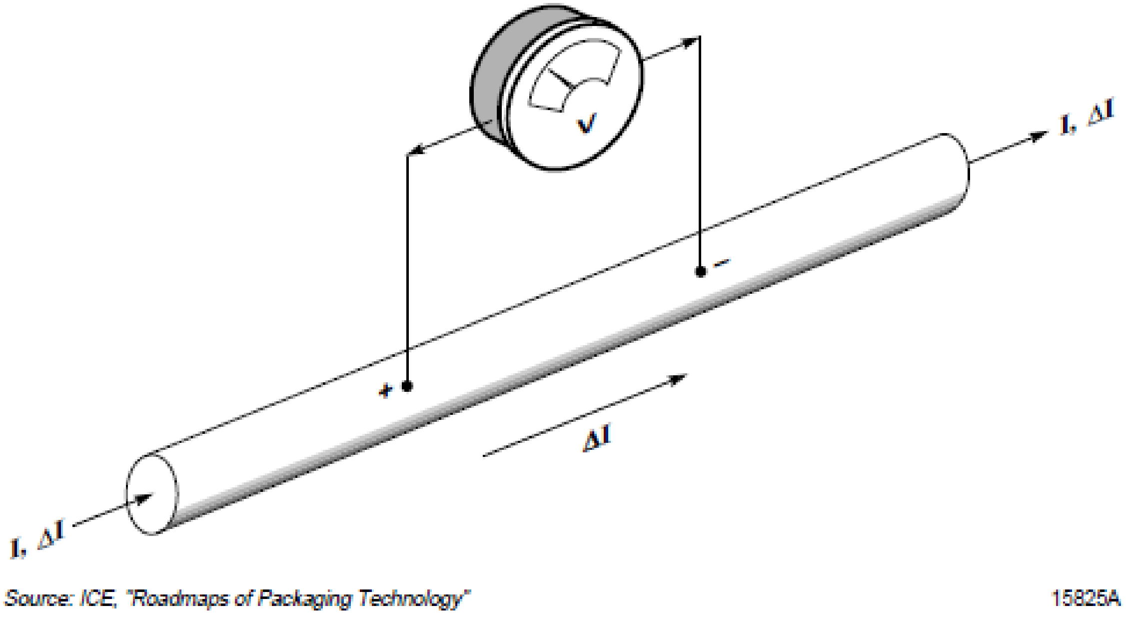


Figure 7-21. Voltage Induced in a Circuit Element Due to a Change in Current

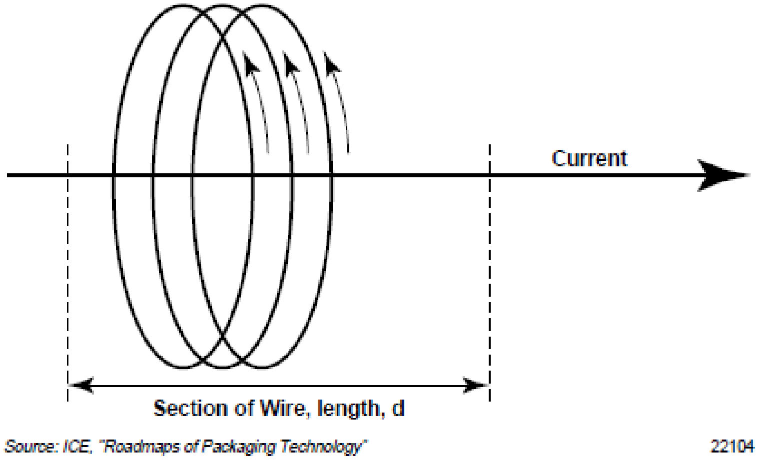
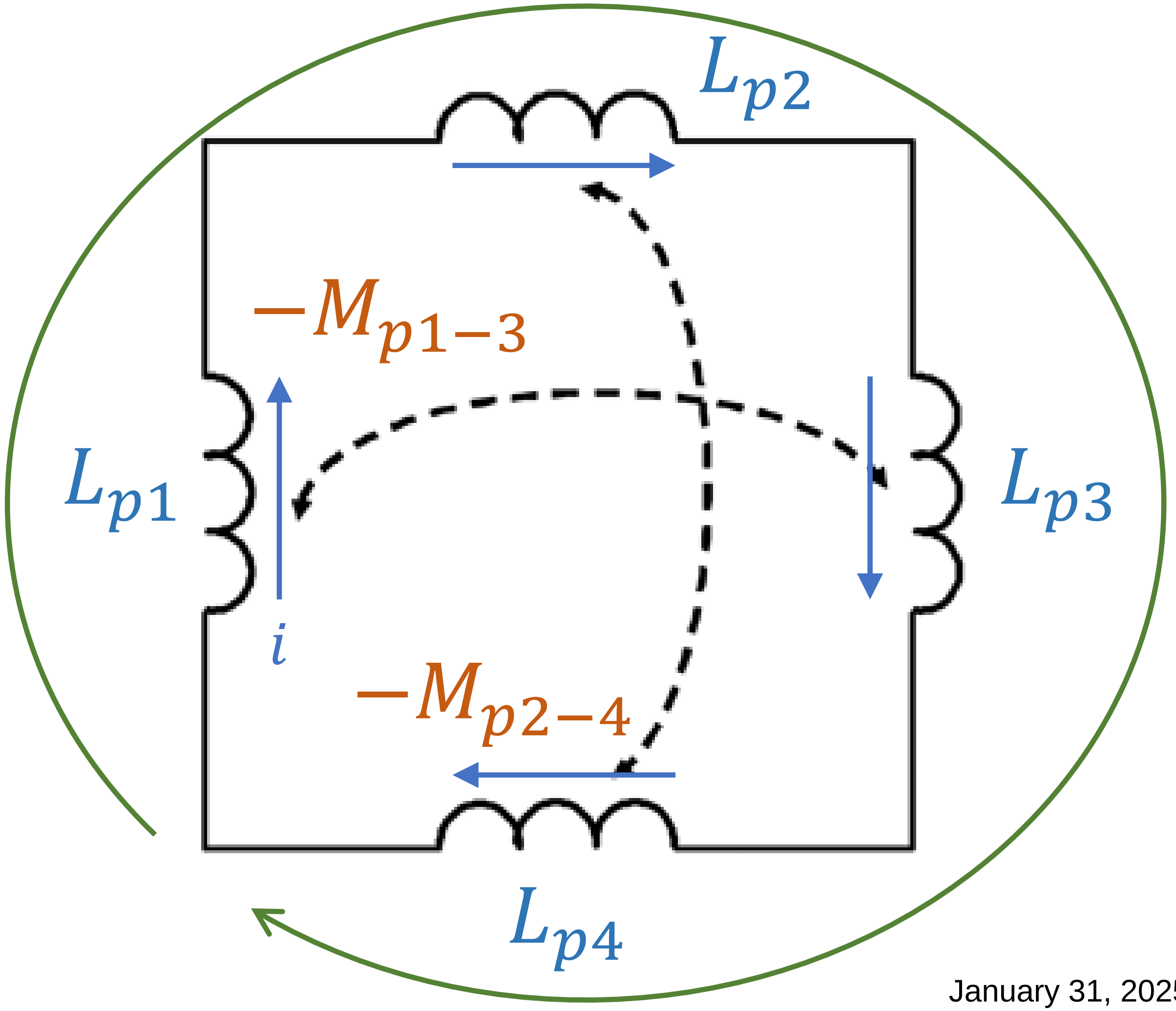


Figure 7-22. Magnetic Field Lines About a Current Carrying Wire

Types of Inductance

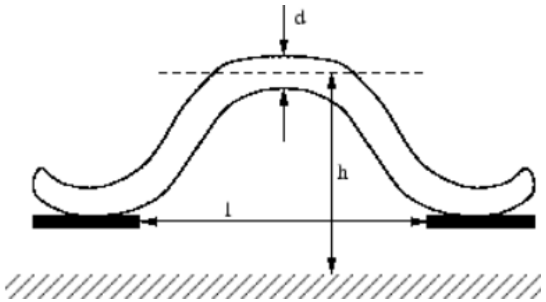
- Self/partial inductance, L_p
  - Its own magnetic field opposes any change in current
- Mutual inductance, M_p
  - Magnetic field lines from one conductor pass through another
- Loop/total/effective inductance, L_{total}
  - Inductance of the overall loop

$$L_{total} = L_{p1} + L_{p2} + L_{p3} + L_{p4} - 2M_{p1-3} - 2M_{p2-4}$$



Self Inductance: Wire (Round Conductor)

- $L_{self} = \frac{\mu}{2\pi} l \left[\ln\left(\frac{l}{r} + \sqrt{1 + \frac{l^2}{r^2}} \right) - \sqrt{1 + \frac{r^2}{l^2}} + \frac{r}{l} + \frac{1}{4} \right]$
    - l = length in meters
    - r = radius in meters
    - $\mu = \mu_0 \mu_{\mathrm{r}}$ ≈ (4π x 10^{-7} H/m)(1) ≈ (4π x 10^{-7} H/m) (for non-ferromagnetic conductors)
    - For straight wires (does not consider curvature)



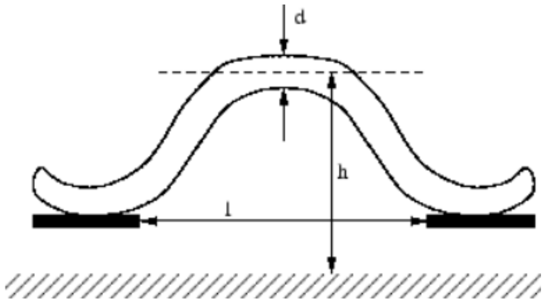
Self Inductance: Wire (Round Conductor)

When $l \gg r$:

- $L_{self} = 0.002l\left[\ln\left(\frac{2l}{r}\right) - \frac{3}{4}\right]$

  (μH) (cm)

  - l = length in centimeters
  - r = radius in centimeters
  - For straight wires (does not consider curvature)



- Rule of thumb for inductance of a wire per unit length:
  - Self inductance ~25 nH/in (~1 nH/mm = 10 nH/cm = 0.01 μH/cm)

Example: Self Inductance of a Wire Bond

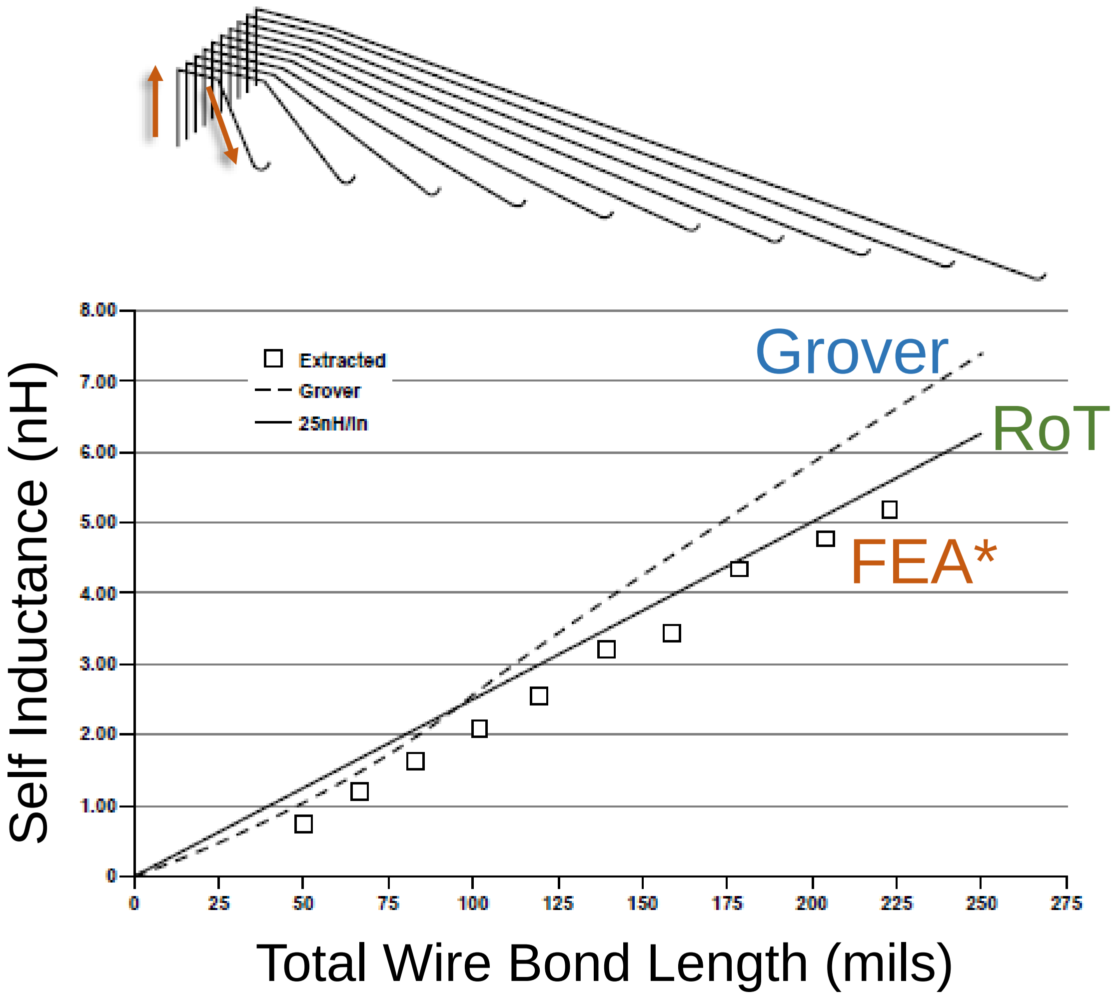
(μH) (cm)

- $L_{self} = 0.002l\left[\ln\left(\frac{2l}{r}\right) - \frac{3}{4}\right]$
  - l = 10 mm = 1 cm
  - r = 5 mils = 0.0127 cm

- $L_{self} = 0.002(1\text{cm})\left[\ln\left(\frac{2(1\text{cm})}{(0.0127\text{cm})}\right) - \frac{3}{4}\right]$

  $=$ **8.6 nH**

- Rule of thumb:
  - 1 nH/mm x 10 mm = **10 nH**



Source: Ansoft Corp./ICE, "Roadmaps of Packaging Technology"

*FEA = finite element analysis

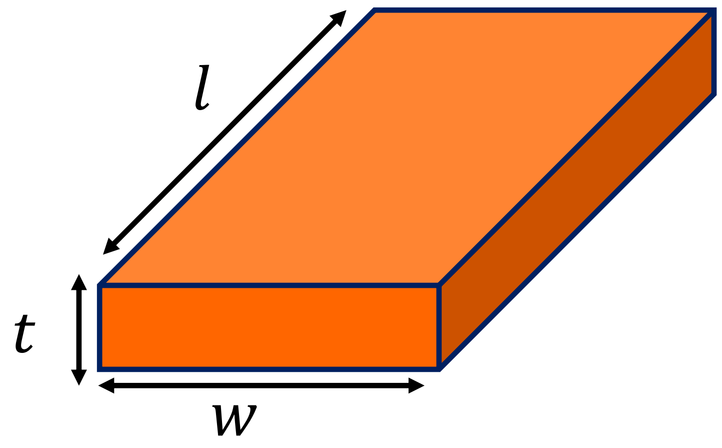
Self Inductance: Rectangular Conductor

- In case of DC, low frequency, or a thin rectangular conductor, Grover gives the following self inductance formula:

- $$L_{self} = 0.002l\left(\ln\left(\frac{2l}{w+t}\right) + 0.50049 + \frac{w+t}{3l}\right)$$

  (μH) (cm)

  - $w =$ width in centimeters
  - $l =$ length in centimeters
  - $t =$ thickness in centimeters



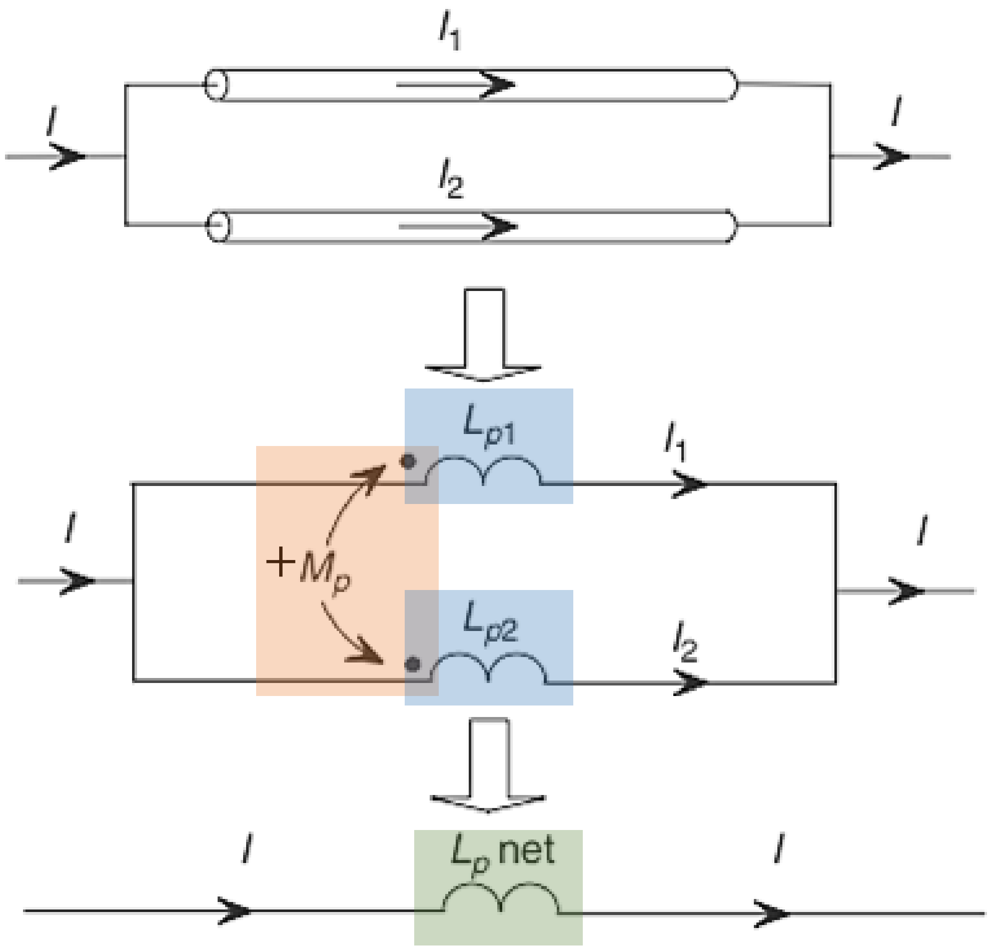
Example: Inductance of Parallel Wire Bonds

- If we have two identical wire bonds in parallel (electrically and physically), would $L_{eq} = L_p/2$?
  - Only if the wires are far apart!
  - If the wires are close together, there will be mutual inductance, M

$$L_{parallel} = \frac{L_{p1}L_{p2} - M^2}{L_{p1} + L_{p2} - 2M}$$

If $L_{p1} = L_{p2} = L_p$,

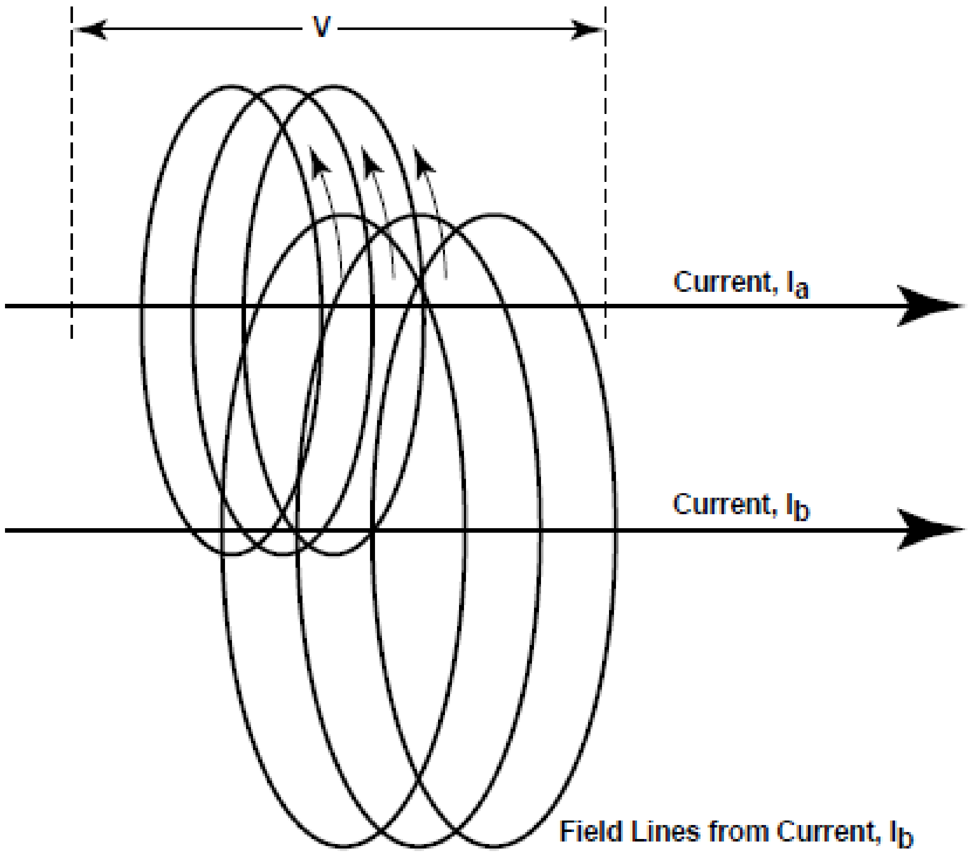
$$L_{parallel} = \frac{L_p + M}{2}$$



Mutual Inductance

- M = mutual inductance
  - Measure of shared field lines per amp of current in one conductor
- Crosstalk
  - Δi in one conductor induces V in the other
- Can increase total L for conductors carrying current in the same direction
- Return path cancellation could reduce total L

$$V_{induced} = M\frac{dI}{dt}$$



Source: ICE, "Roadmaps of Packaging Technology"

Figure 7-24. Origin of Mutual Inductance

Mutual Inductance

(μH) (cm)

- $M = 0.002l\left[\ln\left(\frac{l}{s} + \sqrt{1 + \left(\frac{l}{s}\right)^2}\right) - \sqrt{1 + \left(\frac{s}{l}\right)^2} + \frac{s}{l}\right]$
  - $l =$ length in centimeters
  - $s =$ conductor spacing in centimeters

- When $s \ll l$: $M = \frac{\mu l}{2\pi}\left[\ln\left(\frac{2l}{s}\right) - 1\right]$

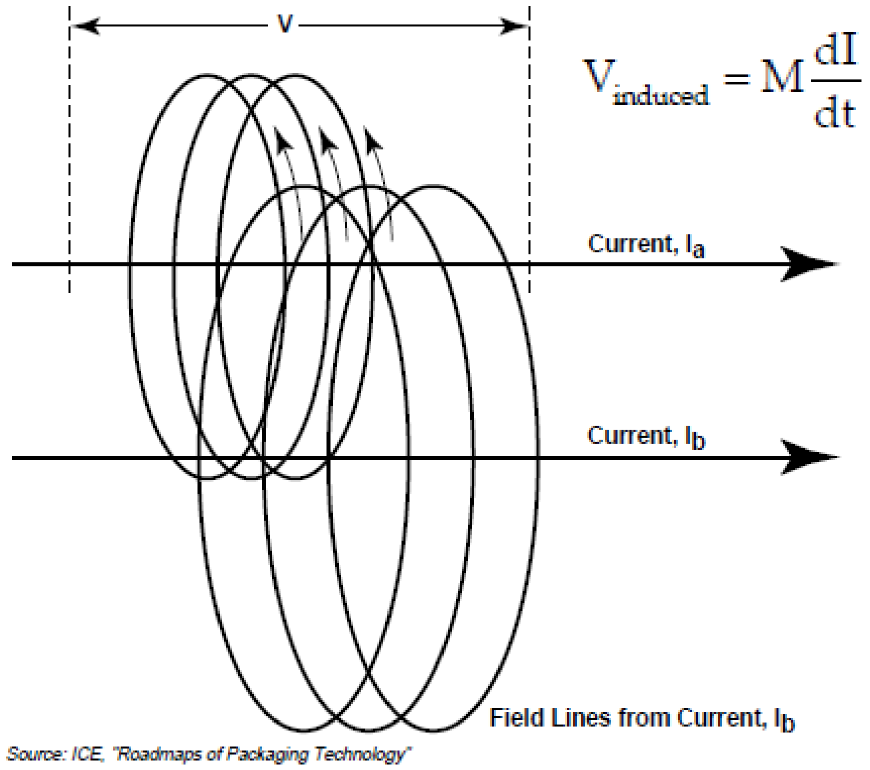


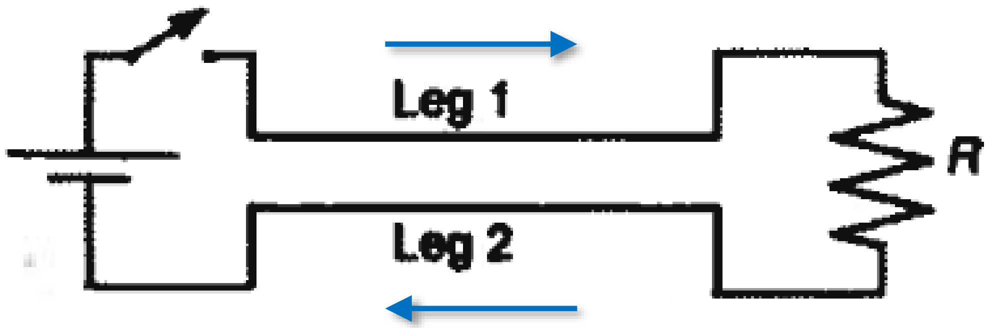
Figure 7-24. Origin of Mutual Inductance

Mutual Inductance

(μH) (cm)

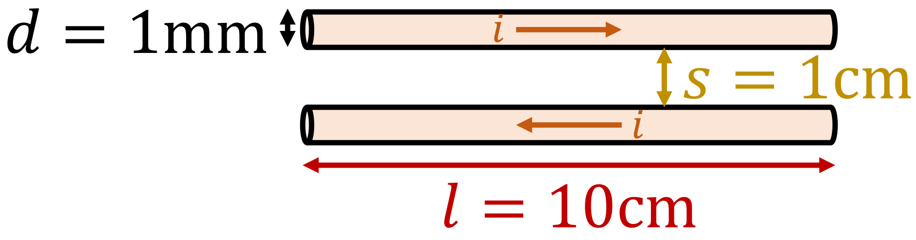
- $M = 0.002l\left[\ln\left(\frac{l}{s} + \sqrt{1 + \left(\frac{l}{s}\right)^2}\right) - \sqrt{1 + \left(\frac{s}{l}\right)^2} + \frac{s}{l}\right]$
  - $l =$ length in centimeters
  - $s =$ conductor spacing in centimeters

- $V = L_1 \frac{dI}{dt} - M_{12} \frac{dI}{dt} + IR + L_2 \frac{dI}{dt} - M_{12} \frac{dI}{dt}$
- $V = (L_1 + L_2 - 2M_{12}) \frac{dI}{dt} + IR$



Example: Inductance of Adjacent Wires

- Find the equivalent inductance of two adjacent wires carrying current in opposite directions with 1-mm diameter, 10-cm length, and spaced 10 mm apart.



- For $r \ll l$ ($0.05\text{cm} \ll 10\text{cm}$), the self inductance of each wire is:

$$L(\mu H) = 0.002(10\text{cm})\left[\ln\frac{(2)(10\text{cm})}{0.05\text{cm}} - \frac{3}{4}\right] = 0.02[5.99 - 0.75] = \mathbf{0.105\ \mu H}$$

- For $s \ll l$ ($1\text{cm} \ll 10\text{cm}$), the mutual inductance is:

$$M = 0.002(10\text{cm})\left[\ln\left(\frac{2(10\text{cm})}{(1\text{cm})}\right) - 1\right] = \mathbf{0.040\ \mu H}$$

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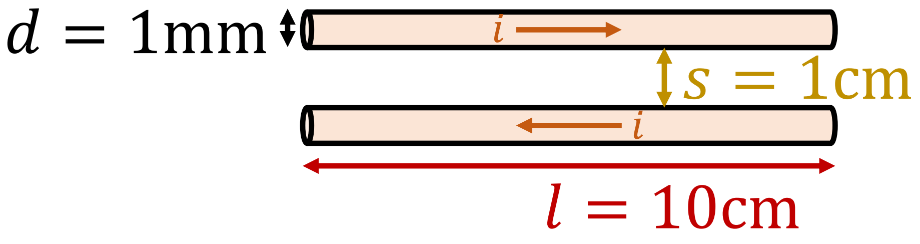


- Total inductance for the return circuit:

$$L_{eq} = L_1 + L_2 - 2M_{12}$$

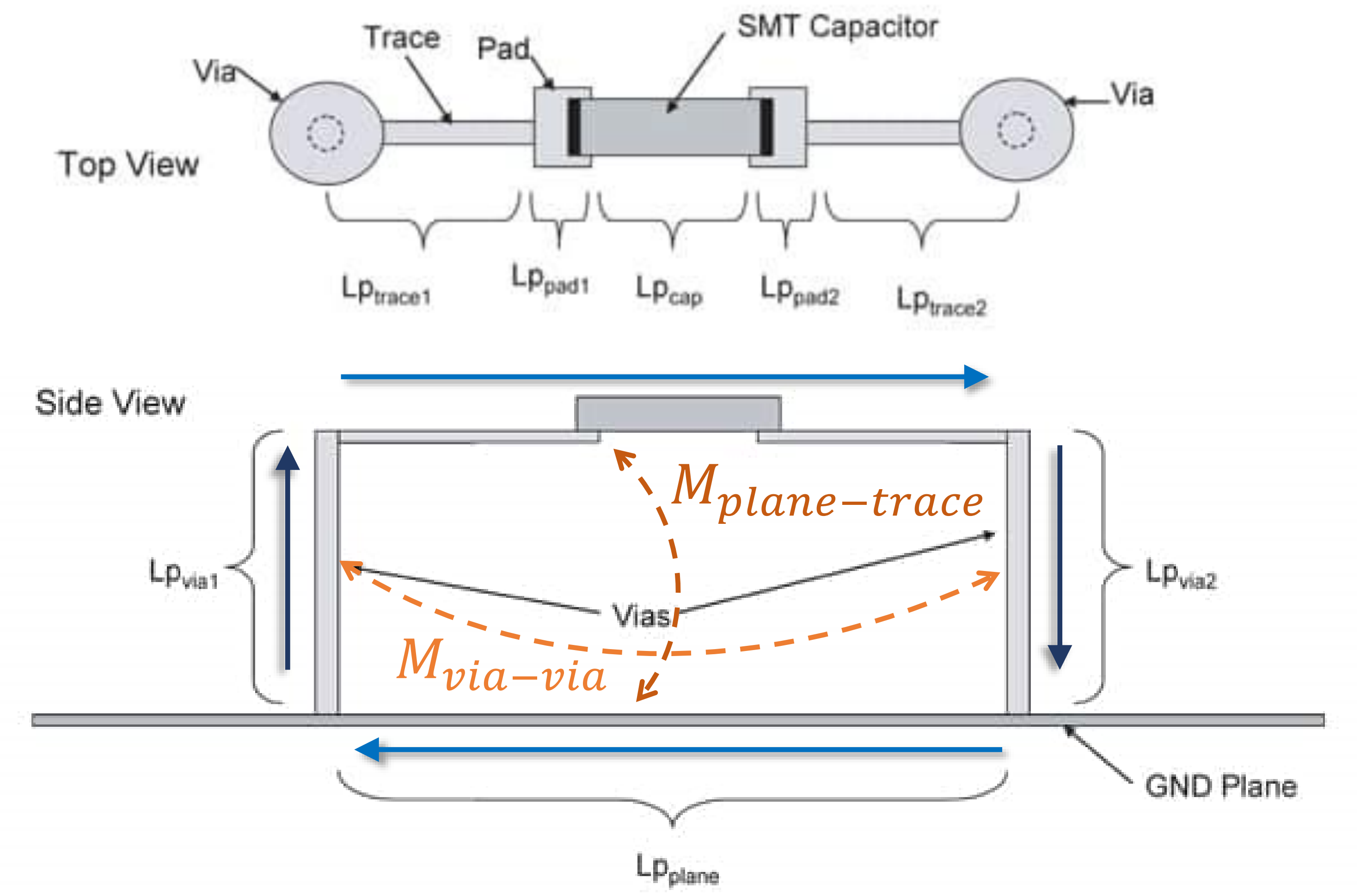
$$= 2(0.105\ \mu\text{H}) - 2(0.042\ \mu\text{H})$$

$$= \mathbf{0.126\ \mu H}$$

If s is very large such that M_{12} is negligible, then $L_{tot} = 2(0.105\ \mu\text{H})$

→ 40% reduction in L_{tot} due to M_{12}

Example: Capacitor Mounted to PCB



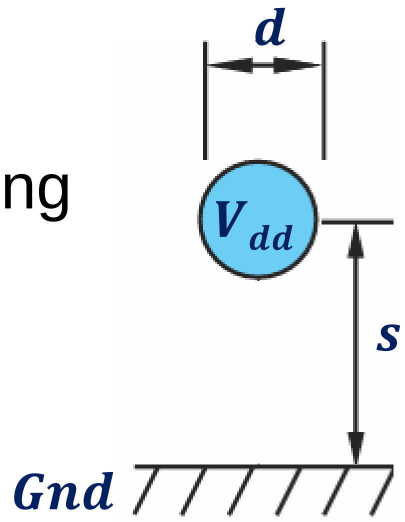
L_p = partial inductance
M = mutual inductance of parallel components

$$L_{total} = Lp_{trace1} + Lp_{pad1} + Lp_{cap} + Lp_{pad2} + Lp_{trace2} + Lp_{via2}$$
$$+ Lp_{plane} + Lp_{via1} - 2M_{via-via} - 2M_{plane-trace}$$

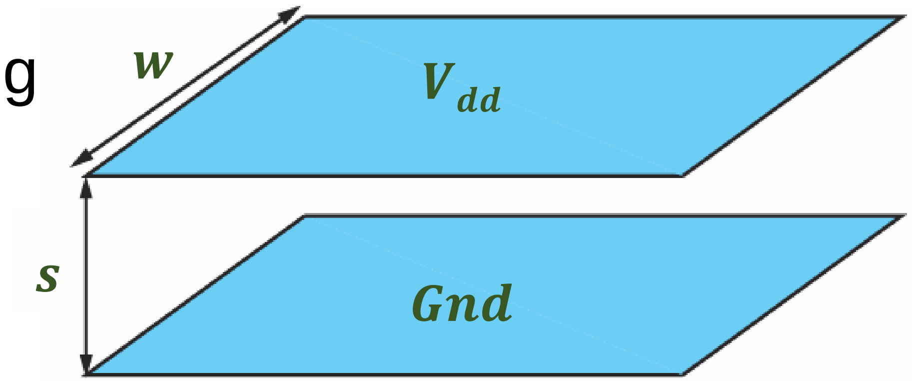
Effective Inductances for Different Structures

*Note: Tummala textbook 2nd Ed. shows 4π in the denominator, which is incorrect.

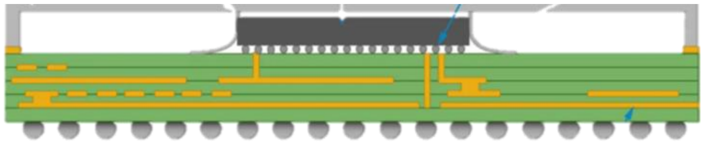
- Wire above a ground plane
  - $L_{eff} = \frac{\mu l}{2\pi^{*}} \cosh^{-1}\left(\frac{2s}{d}\right)$, where l= length, d= diameter, s= spacing
  - Package examples: TAB, QFP w/ ground plane
  - Typical inductance range: 1 – 10 nH
  - Assumes $s \ll l$, and $d \ll s$



- Parallel planes
  - $L_{eff} = \frac{\mu l s}{w}$, where l= length, w= width, s= spacing
  - Package examples: PGA, BGA
  - Typical inductance range: 0.25 – 1 nH
  - Assumes $s \ll l$

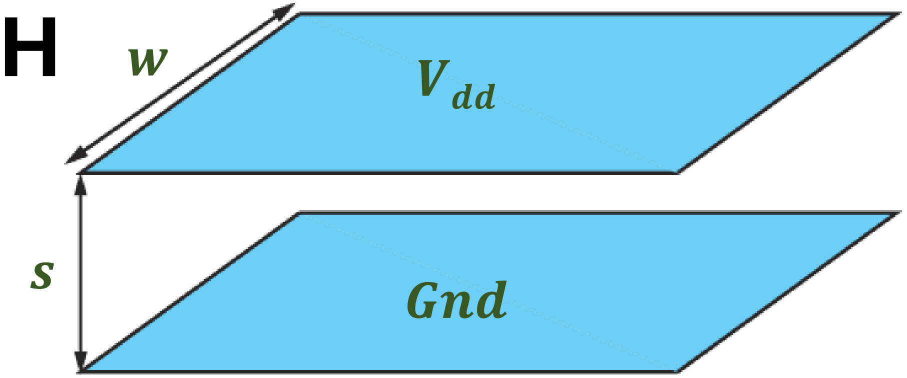


Example: Parallel Planes



A multi-layer ball grid array (BGA) package has plane layers used to supply both the V_{dd} and the ground (GND). Find the effective inductance for a pair of planes with dimensions of 1 cm by 1 cm, and a spacing of 6 mils.

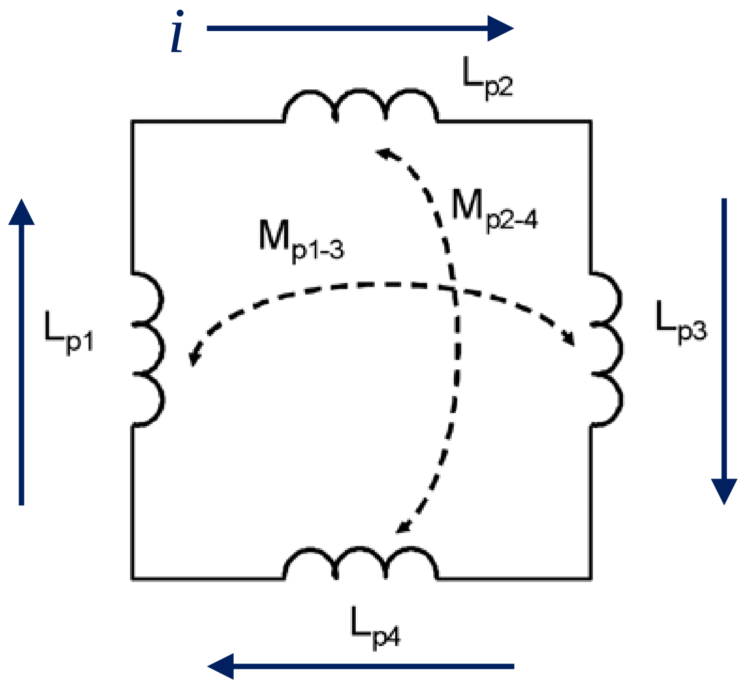
- $L_{eff} = \frac{\mu l s}{w}$, where $l = w = 1$ cm $= 0.01$ m, $s = 6$ mils $= 1.5$e-4 m
- $\mu = \mu_0 \mu_r \approx 4\pi \times 10^{-7} \mathrm{H/m}$ =1.26e-6 H/m
- L_{eff} =(1.26e-6 H/m)(1.5e-4 m) = **0.19 nH**



$$\boxed{V = L\, dI/dt}$$

Summary: Inductance

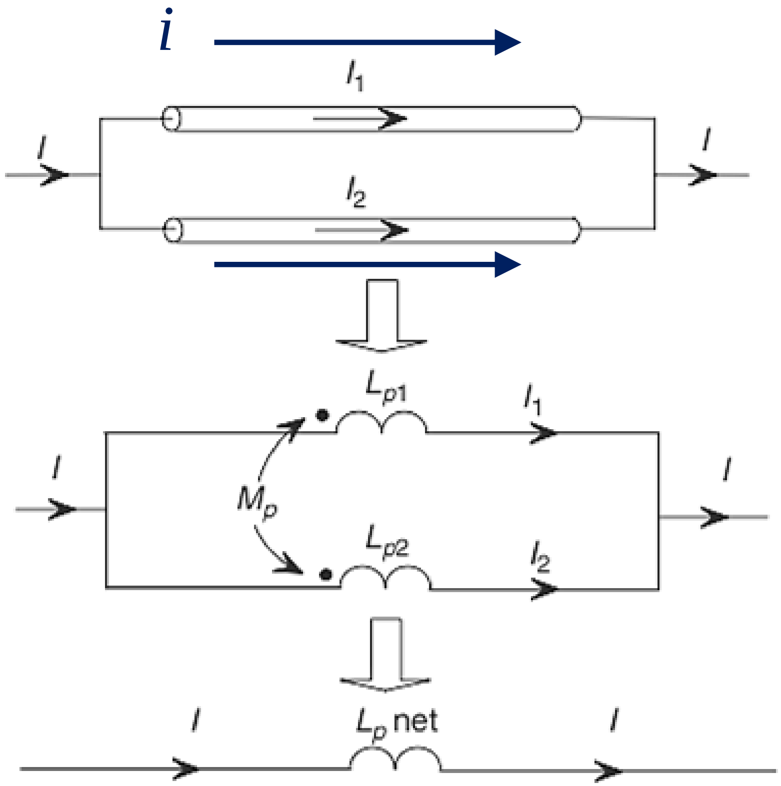
- Inductive delay: $\tau = L/R$
- Self inductance: L_p
  - Example: $L_{p1},\ L_{p2},\ L_{p3},\ L_{p4}$
- Mutual inductance: M
  - Example: $-M_{p1-3}\,, -M_{p2-4}$
  - Subtractive: current flowing in opposite directions (this example)
- Loop inductance: L_{total}
- Example: $L_{total} = L_{p1} + L_{p2} + L_{p3} + L_{p4} - 2M_{p1-3} - 2M_{p2-4}$



$V = L\, dI/dt$

Summary: Inductance

- Example 2: Additive
  - Two parallel wires with current in the same direction
  - Self inductance: $L_{p1},\ L_{p2}$
  - Mutual inductance: M
    - Positive
    - If $s \ll l$: $M = \frac{\mu l}{2\pi}\left[\ln\left(\frac{2l}{s}\right) - 1\right]$
  - Total inductance if $L_{p1} = L_{p2}$:
    - $L_{parallel} = (L_p + M)\ /\ 2$



References on Inductances

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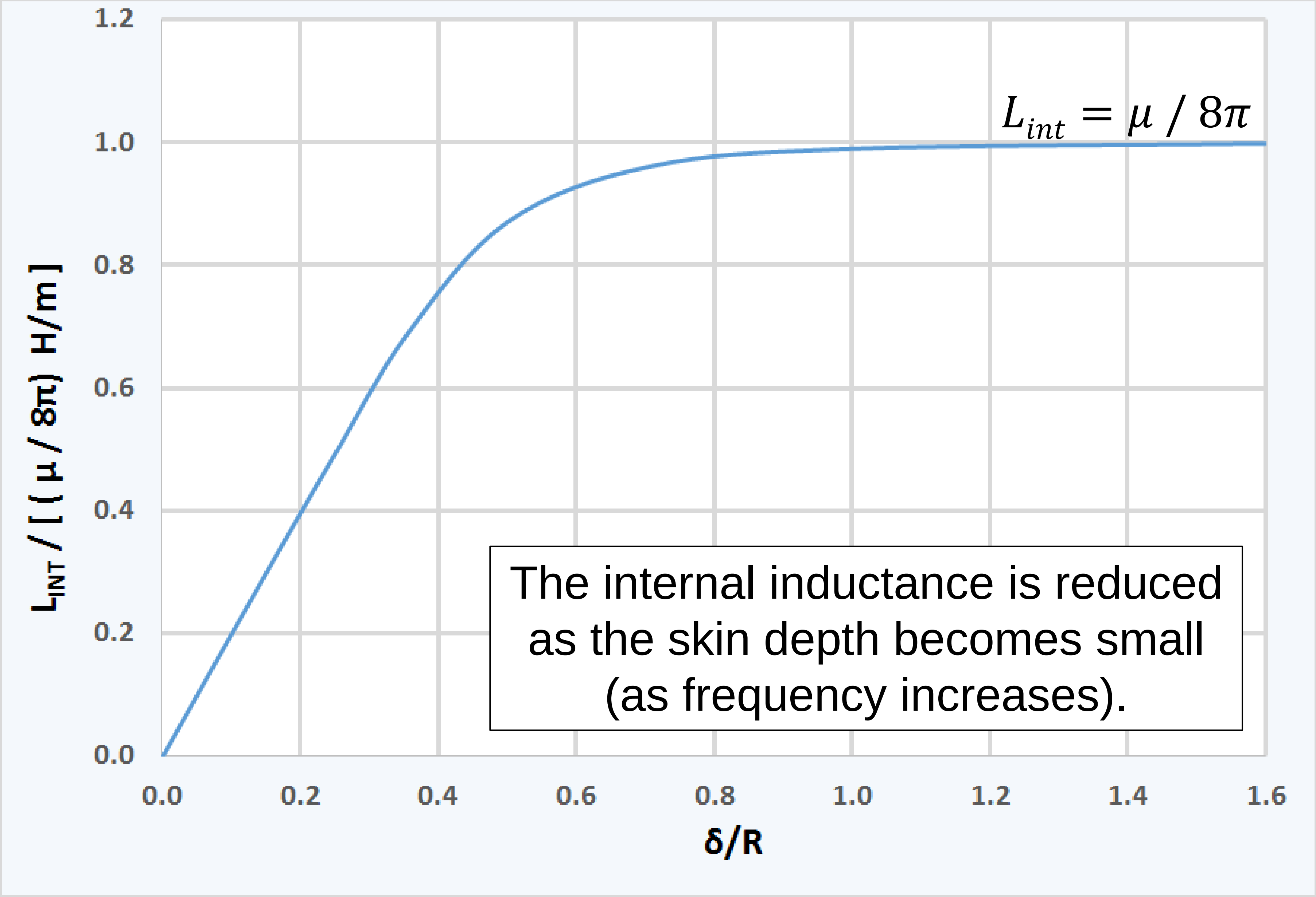
- Xiaoning Qi, “High frequency characterization and modeling of on-chip interconnects and RF IC wire bonds,” http://www-tcad.stanford.edu/tcad/pubs/theses/qi.pdf
- Clayton R. Paul, “Partial and internal inductance,” https://ieeexplore.ieee.org/document/6507331
- Eric Bogatin, “Roadmaps of packaging technology,” Chapter 7: Electrical Performance, ISBN: 1-877750-61-1, 1997.

Skin and Proximity Effects

- Skin effect reduces the *internal* wire inductance at high frequencies
- Internal inductance is due to the internal magnetic flux of a conductor
- Internal inductance is typically much less than the external inductance, which is due to external magnetic flux
- Proximity effect reduces the wire inductance by redistributing the currents to form a smaller current loop
- The skin and proximity effect eddy currents superimpose to form the total eddy current distribution

Internal Inductance vs. Skin Depth/Radius



Next Class

- Electrical Design (Chapter 2)
  - Inductance
  - Capacitance
  - Intro to Finite Element Analysis (FEA) and ANSYS Q3D