Lecture 9

Electrical Design

Transmission Line Effects

February 27, 2025

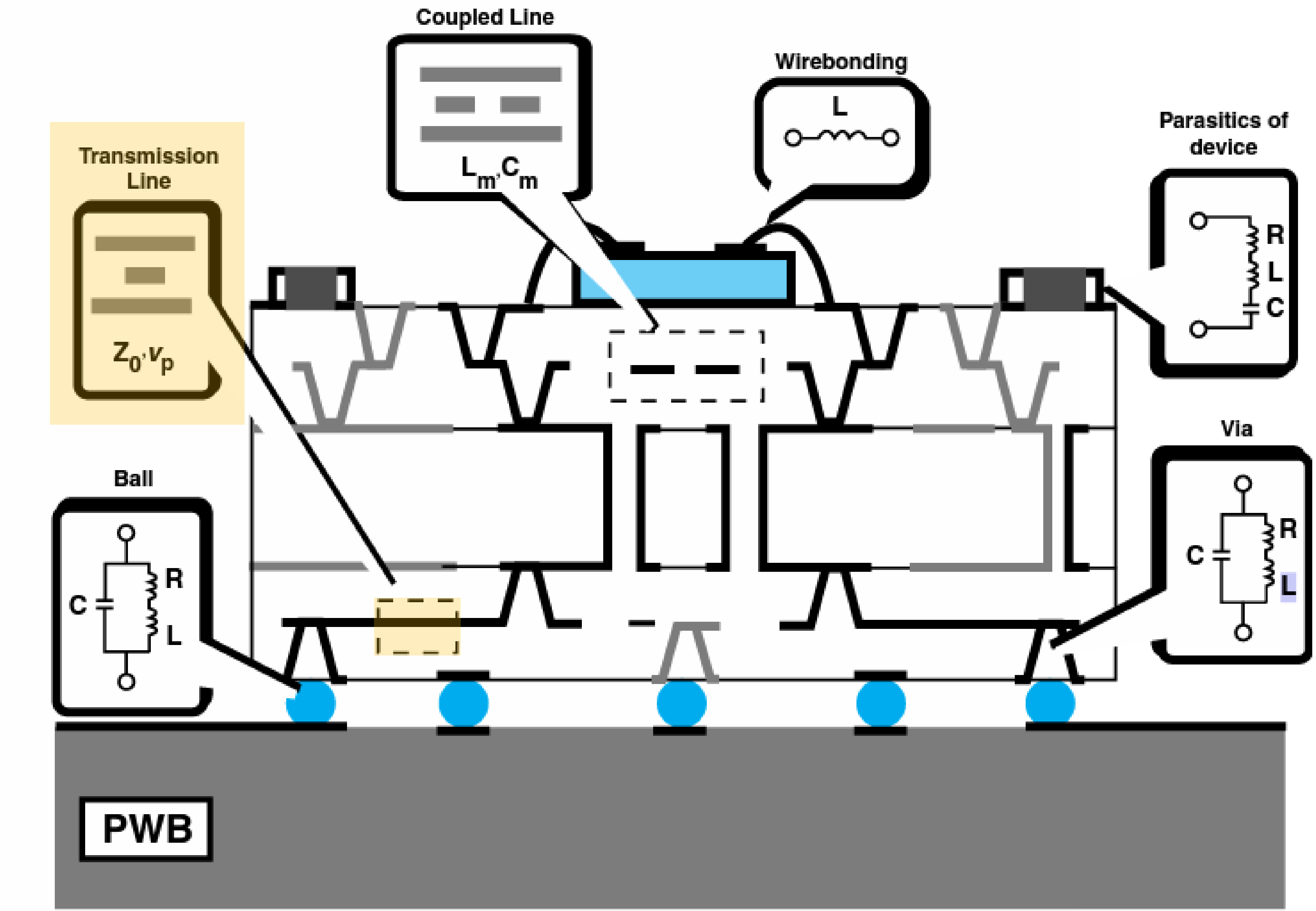
Reminders and Announcements

- Office hours: Monday, 3:00pm – 4:30pm

- Homework #2 due Wednesday, Feb. 26th, by 11:59pm (midnight)
  - Requires ANSYS Q3D

- Small Group Work #2 grades and answers on Canvas

Package Parasitics



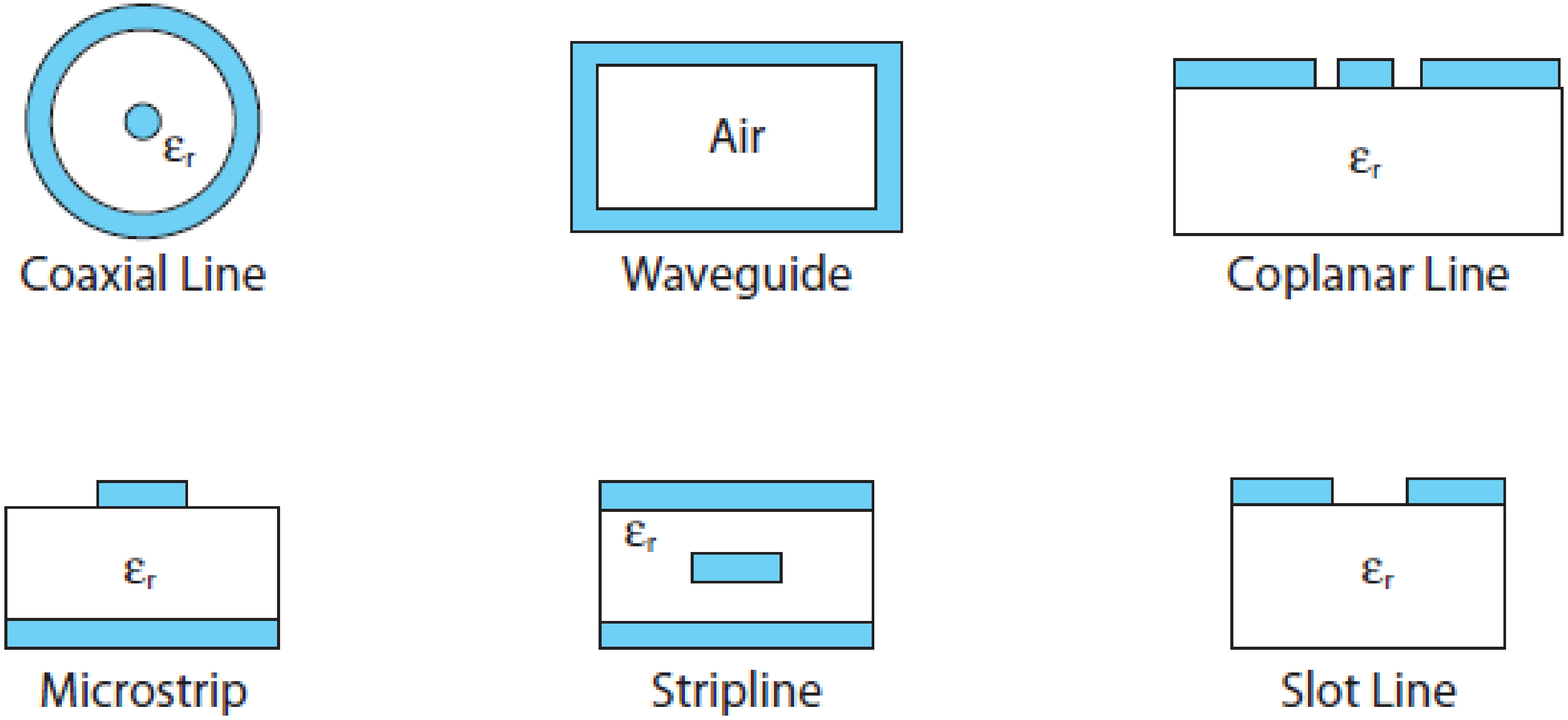
Transit Time of Electrical Signal

- Electrical signals propagate at the speed of light (3×10^8 m/s in air)
- Kirchhoff's laws neglect the finite velocity of electrical signals, and therefore fail when the time delay or phase shift due to that finite velocity becomes significant
- E.g., in air, there is ~1 ns time delay per 1 ft of travel
  - This is significant if the clock rate of the circuit is 1 GHz
- Transmission line theory accounts for this delay
- Speed of light is slower in dielectric packaging materials than in air

Transmission Lines

- Wires, cables, phone lines, PCB traces, and connector pins are transmission lines.
- If the wavelength is much larger than the total length of a conductor (low frequency), the signal/voltage/current, are the same everywhere in the conductor (i.e., there are no spatial variations and traditional lumped circuit modeling can be used).
- If the wavelength is *comparable or smaller than* the length of the conductor (*high* frequency), *variations* in voltage/current occur throughout the length of the conductor.
- The transmission line model accounts for these spatial variations.

Types of Transmission Lines



Definitions

- Propagation velocity, v_p – the speed at which an electrical signal can propagate through a medium. Unit: meters / second.
- Propagation delay – the amount of time taken for a signal to travel through a medium. Reciprocal of propagation velocity. Unit: seconds / meter
- Time delay – the delay from the start of the interconnect to the end of the interconnect. It is the propagation delay multiplied by the total length of the interconnect. Unit: seconds
- Wavelength, λ – the distance between consecutive corresponding points of the same phase on a wave. Unit: meters
- Characteristic impedance, Z_0 – the input impedance of a transmission line when its length is infinite. Unit: ohms.

Propagation Delay & Time Delay

- The propagation velocity of any electrical signal in a material is

$$v_p = \frac{c}{\sqrt{\varepsilon_r \mu_r}} \quad [\mathrm{m/s}]$$

where $c = 2.998 \times 10^8\ \mathrm{m/s}$ (speed of light in a vacuum) and $\frac{1}{\sqrt{\varepsilon_r \mu_r}}$ is the velocity factor; for non-magnetic media, the expression simplifies to $\frac{c}{\sqrt{\varepsilon_r}}$

- The propagation delay is $\frac{\sqrt{\varepsilon_r \mu_r}}{c} \quad [\mathrm{s/m}]$
- The time delay is the propagation delay times the length: $\frac{\sqrt{\varepsilon_r \mu_r}}{c} \times l \quad [\mathrm{s}]$
- Wavelength: $\lambda = c/f$ in air, where f is the frequency in Hertz
- The wavelength λ of a single-frequency signal in a medium with parameters ε_r and μ_r:

$$\lambda = \frac{c}{f\sqrt{\varepsilon_r \mu_r}} \quad [\mathrm{m}]$$

Example: Time Delay

Find the time delay of a signal propagating 1 ft in air.

- $\varepsilon_r = 1$ and $\mu_r = 1$
- $v_p = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{c}{1} = 2.998 \times 10^8 \text{ m/s}$
- $t_{pd} = \frac{\sqrt{\varepsilon_r \mu_r}}{c} = \frac{1}{c} = 33.3 \text{ ps/cm}$
- $1 \text{ foot} = 30.48 \text{ cm}$
- $(33.3 \text{ ps/cm})(30.48 \text{ cm}) = 1015 \text{ ps} \approx \mathbf{1\ ns}$

Find the time delay of a signal propagating 1 ft in a dielectric medium with $\varepsilon_r = 4$.

- $v_p = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{c}{\sqrt{4}} = 1.499 \times 10^8 \text{m/s}$
- $t_{pd} = \frac{\sqrt{\varepsilon_r \mu_r}}{c} = \frac{1}{v_p} = \frac{1}{1.499 \times 10^8 \text{m/s}} = 66.7 \text{ ps/cm}$
- $t_{delay} = (66.7 \text{ ps/cm})(30.48 \text{ cm}) = \mathbf{2\ ns}$

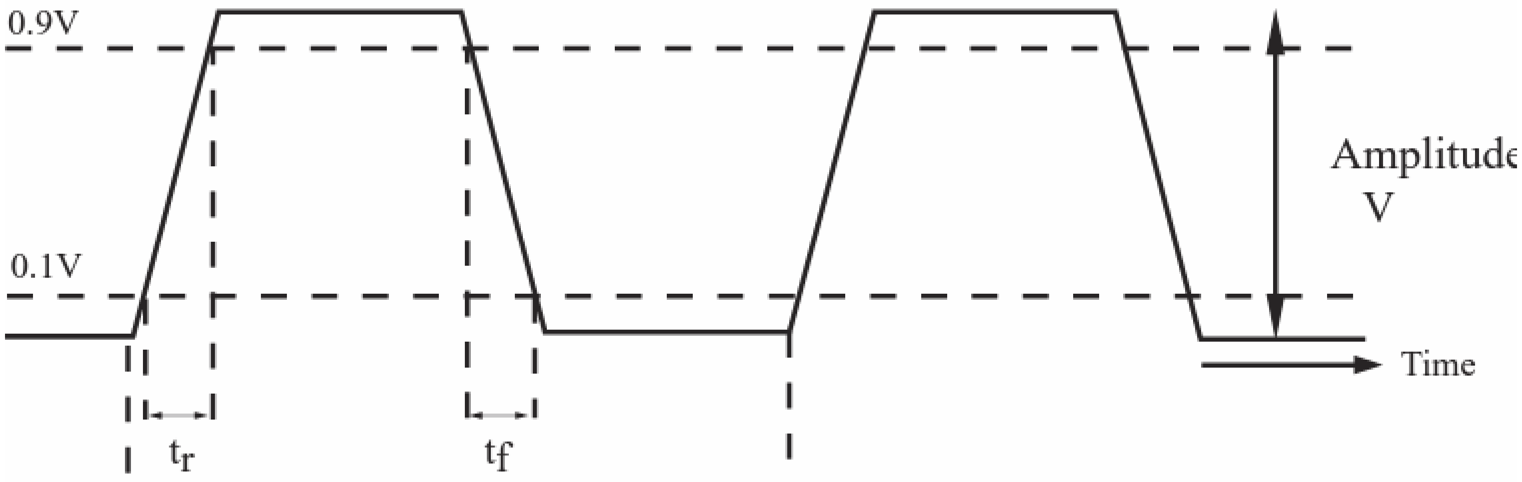
	ε_r	Prop. Delay (ps/cm)
FR4	4.9	73.8
lucite	2.6	53.7
mica	6.0	81.6
nylon	3.5	62.4
plexiglass	2.6	53.7
polyethylene	2.3	50.6
polyimide	3.5	62.4
polystyrene	2.6	53.7
quartz	3.5	62.4
Rexolite 1422	2.5	52.7
silicon	11.8	114.5
silicon dioxide	3.9	65.8
teflon	2.1	48.3

Transmission Line Consideration

- The propagation delay for package interconnects may not be negligible if the signal rise times t_r are *fast*
- Transmission line effects should be considered if the time delay of the interconnect is *greater* than the rise time of the signal
- Another way to think about it: the wavelength λ of the signal should be *greater* than the length of the interconnect l (the length the signal needs to travel)
- Check for transmission line effects by comparing:

Time delay to rise time t_r or Wavelength λ to length l

Consider Transmission-Line Effects When…



Waveforms are "fast" or Interconnects are "long"

$$t_r \leq (33.3\ \text{ps/cm})\sqrt{\varepsilon_r} \times 2l$$

round-trip length

$$l > \frac{0.5\ t_r}{(33.3\ \text{ps/cm})\sqrt{\varepsilon_r}}$$

where t_r = signal rise time (ps) ; l = interconnect length (cm)

- For an interconnect to behave as a transmission line, t_r has to be **less than** the **round-trip (2l) time delay** of the interconnect.

Example: Transmission Line Check

For 500 MHz clock with a rise time of 200 ps and 2 cm interconnect in $\varepsilon_r = 4$, should the transmission line effects be considered?

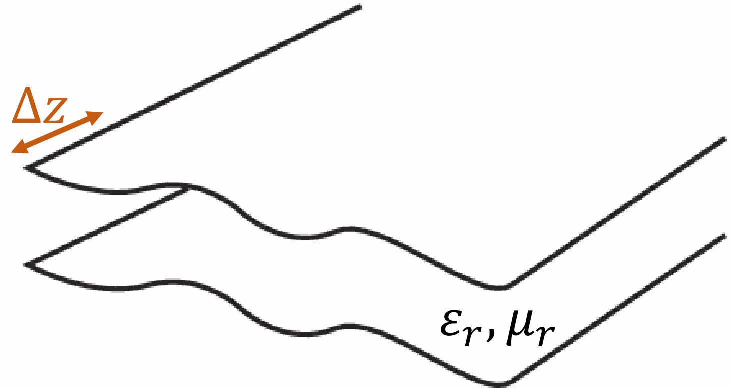
- Time approach: $t_r \leq (33.3\ \mathrm{ps/cm})\sqrt{\varepsilon_r} \times 2l$
  - $t_r = 200$ ps
  - $t_r \leq 33.33\sqrt{4}$ (2)(2 cm) = **266 ps** → t_r **= 200 ps < 266 ps**

- Length approach: $l > \dfrac{0.5\ t_r}{(33.3\ ps/cm)\sqrt{\varepsilon_r}}$
  - $l > 0.5(200$ ps) / $(33.33)\sqrt{4}$ = **1.5 cm** → l **= 2 cm > 1.5 cm**

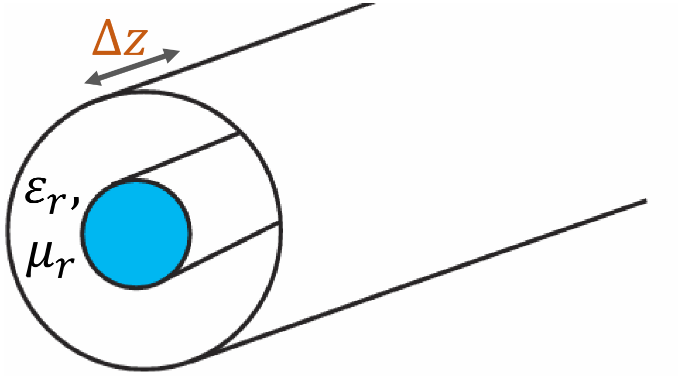
➢ Yes, transmission line effects should be considered!

Transmission Line Equivalent Circuit

Parallel Conducting Strips



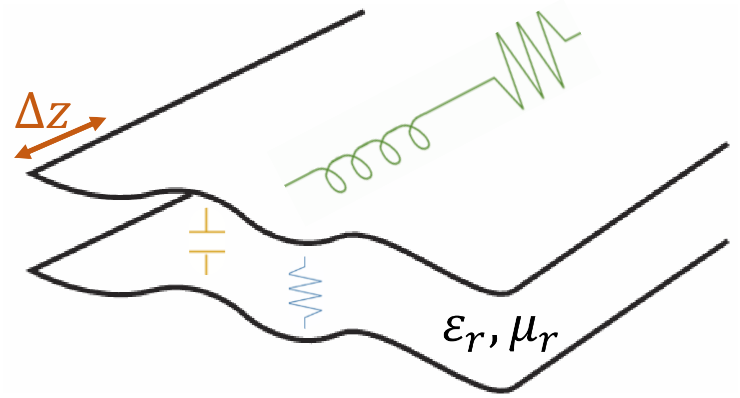
Coaxial Cable



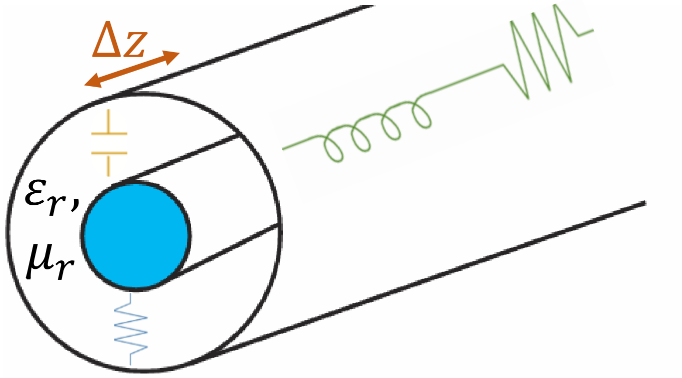
- We can draw the equivalent circuit for a short section Δz of the transmission line, where $\Delta z << \lambda$
- The equivalent circuit for section Δz will provide the time delay and phase shift
- Using circuit theory, we can assume that the Δz equivalent circuits provide a direct connection from one end to the other
- This equivalent circuit can be treated using KVL and KCL

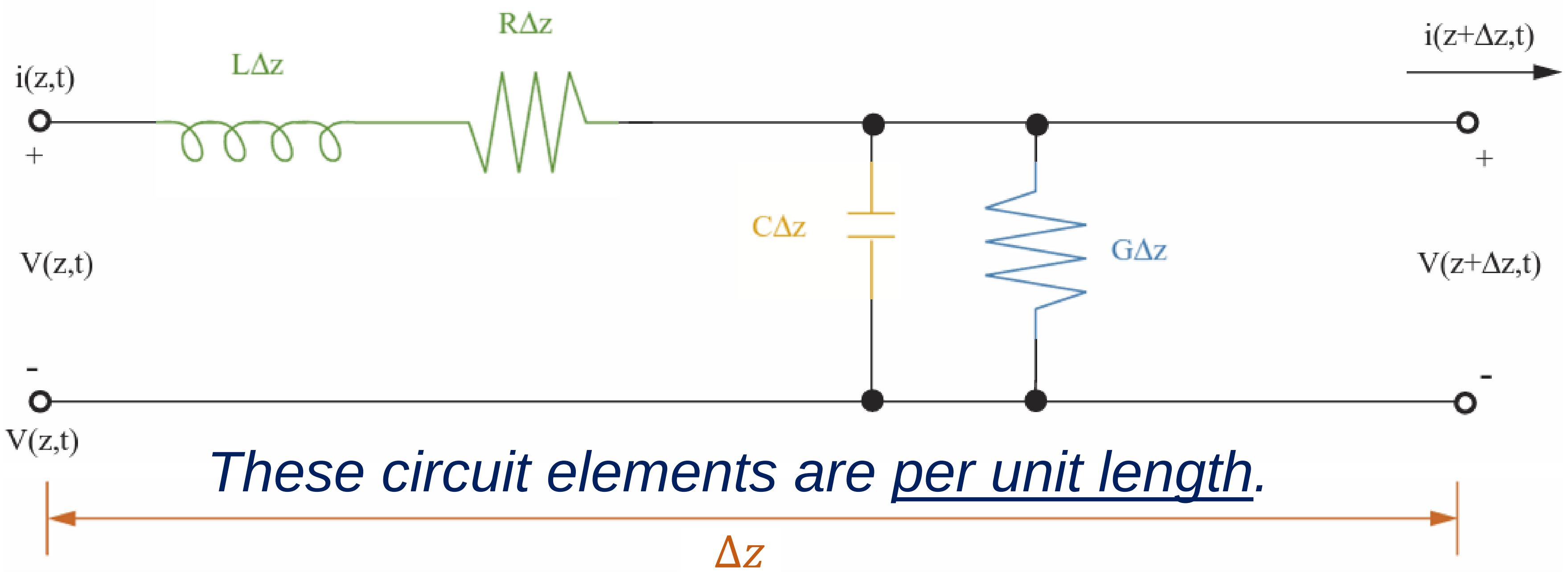
Transmission Line Equivalent Circuit

Parallel Conducting Strips



Coaxial Cable

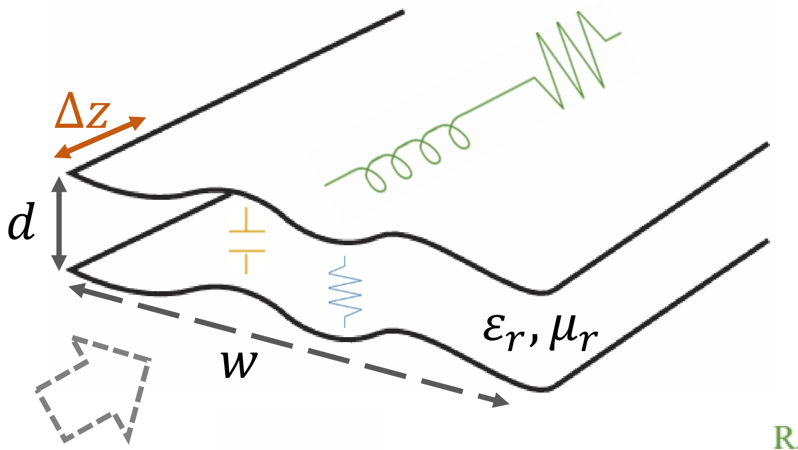




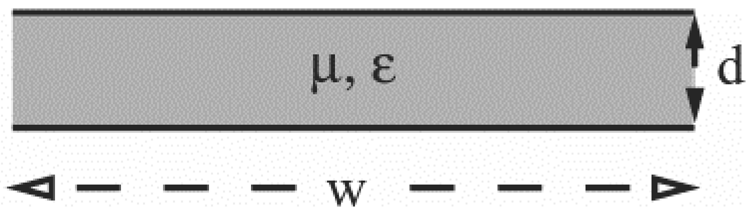
These circuit elements are <u>per unit length</u>.

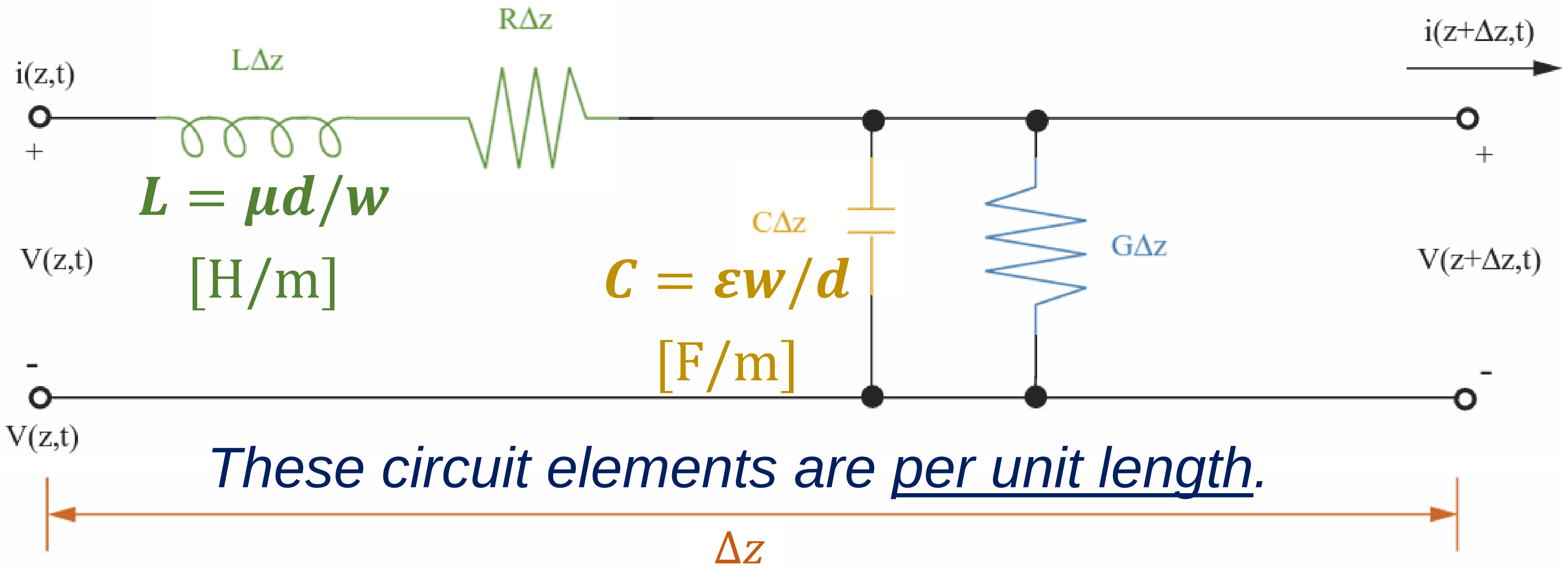
Transmission Line Equivalent Circuit

(Ideal) Parallel Conducting Strips



Front View





$$\boldsymbol{L = \mu d/w}$$

[H/m]

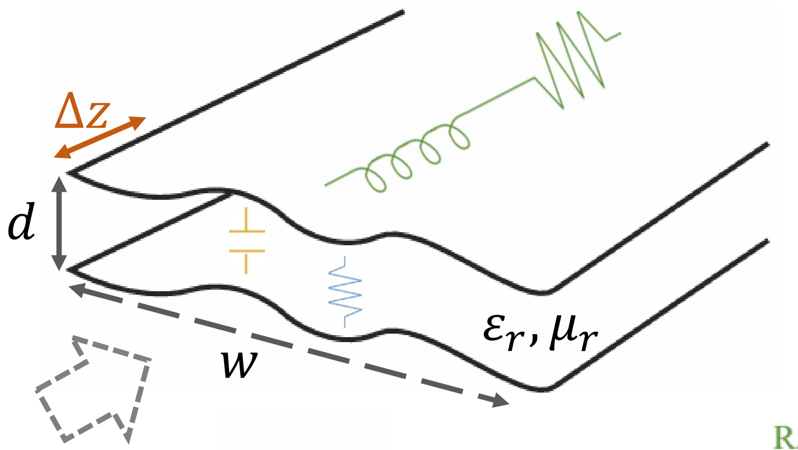
$$\boldsymbol{C = \varepsilon w/d}$$

[F/m]

These circuit elements are <u>per unit length</u>.

Example

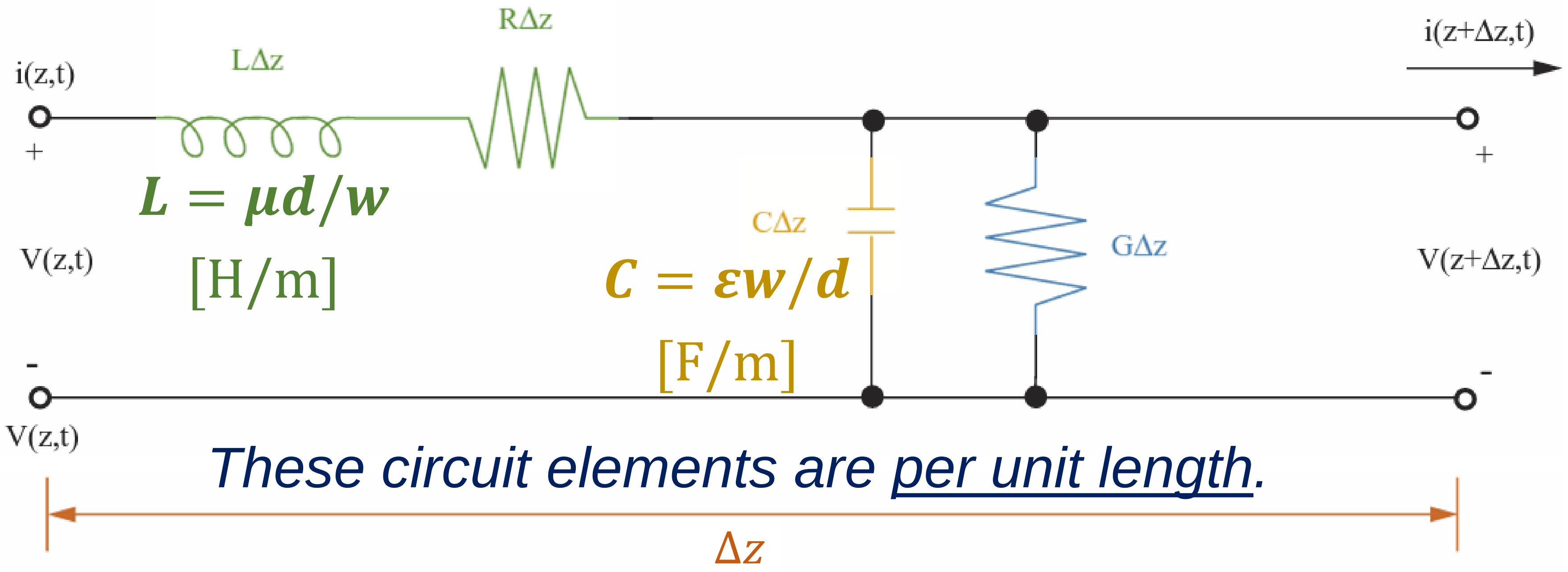
(Ideal) Parallel Conducting Strips



The total interconnect length is 1 cm, and Δz is 0.1 µm such that it satisfies $\Delta z << \lambda$.

Find the number of Δz segments.

- $l/\Delta z = 1\ \mathrm{cm}/0.1\mathrm{\mu m} = \mathbf{100,000\ segments}$



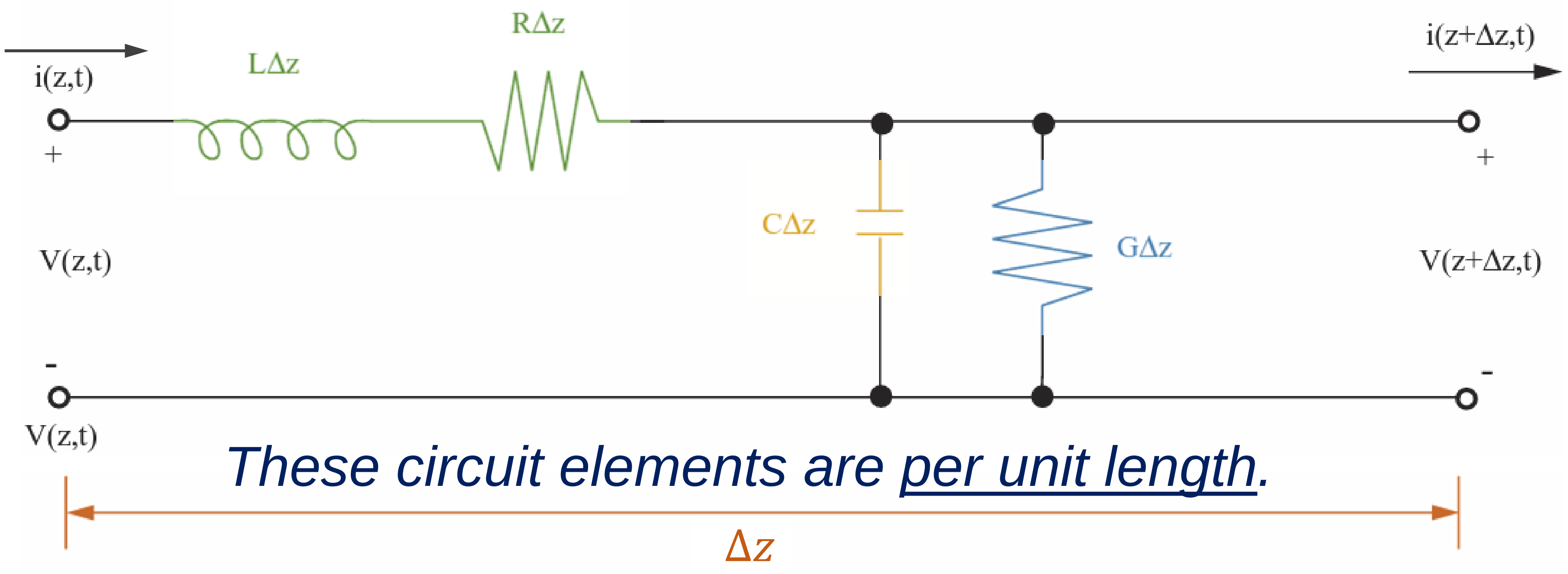
$L = \mu d/w$ [H/m]

$C = \varepsilon w/d$ [F/m]

These circuit elements are per unit length.

Apply Kirchhoff's Voltage Law

- $V(z,t) = (L\Delta z)\frac{\partial i}{\partial t} + (R\Delta z)i(z,t) + V(z+\Delta z,t)$
- Rearranging: $\{V(z+\Delta z,t) - V(z,t)\}/\Delta z = -Ri(z,t) - L\partial i/\partial t$
- As $\Delta z \rightarrow 0$: $\partial V/\partial z = -Ri(t) - L\partial i/\partial t$



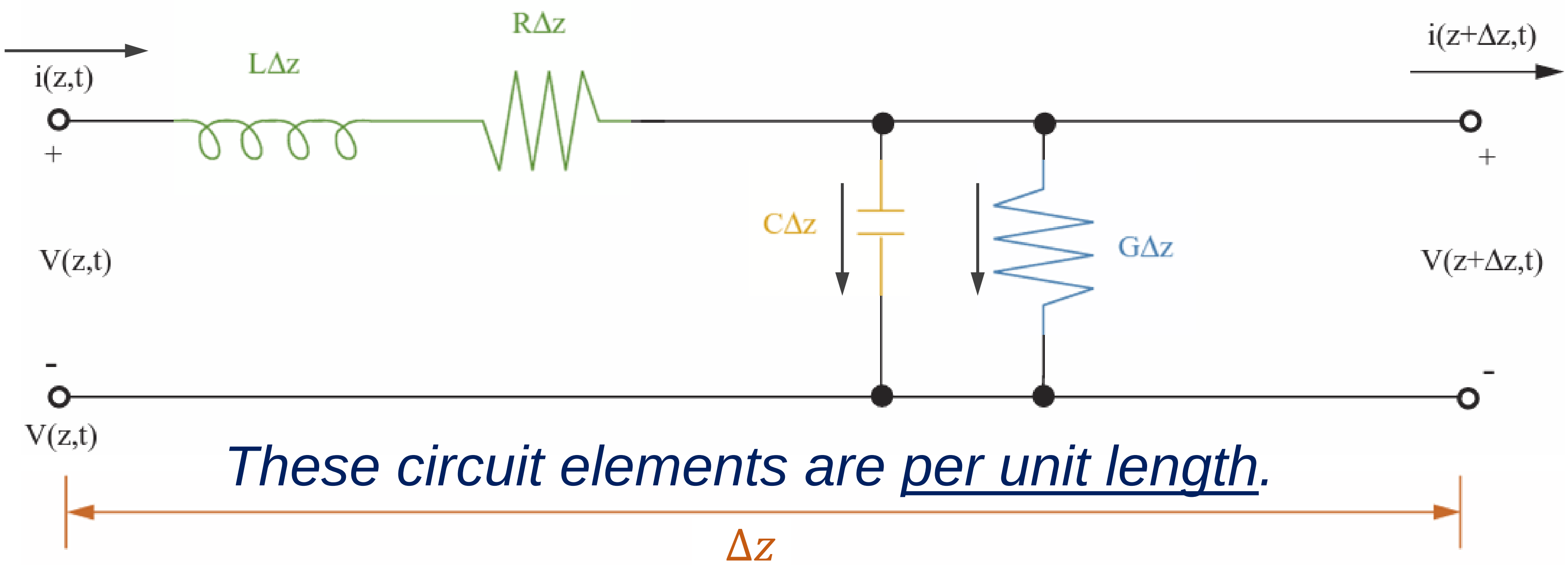
These circuit elements are per unit length.

Apply Kirchhoff's Current Law

- $i(z,t) - (G\Delta z)V(z+\Delta z,t) - (C\Delta z)\partial V/\partial t = i(z+\Delta z,t)$

Rearranging:

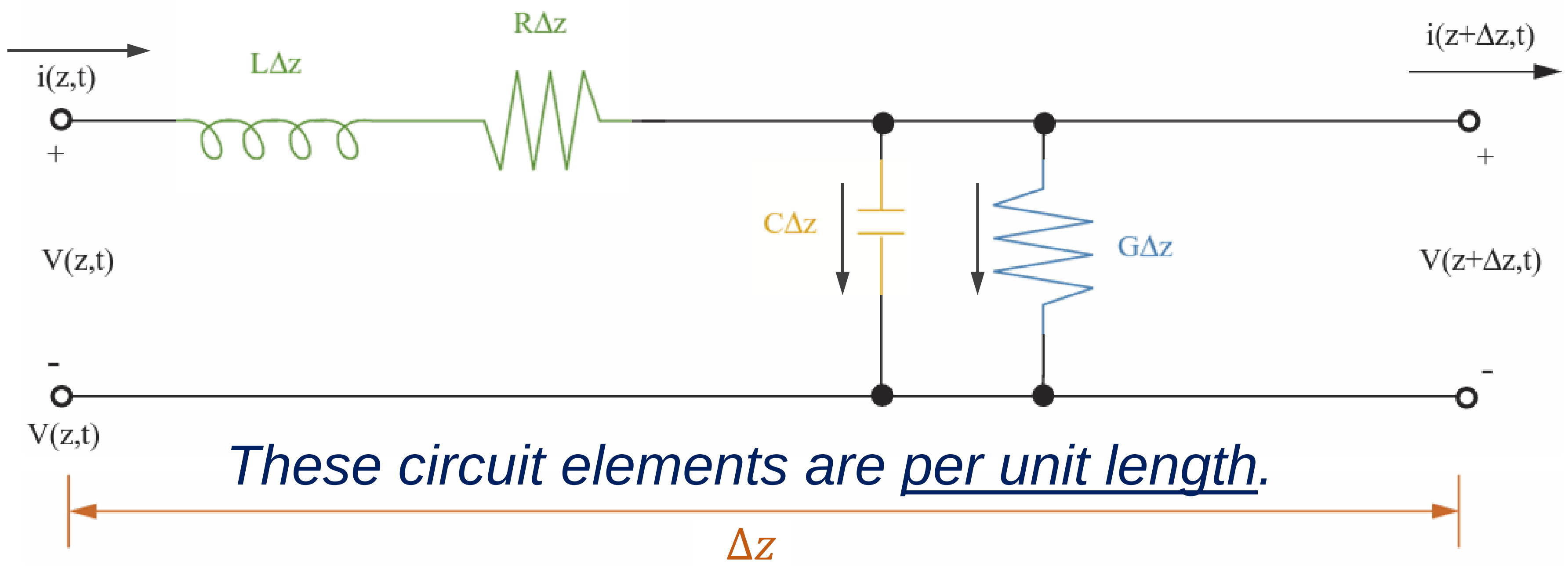
- $i(z+\Delta z,t) - i(z,t) = -(G\Delta z)V(z+\Delta z,t) - (C\Delta z)\partial V/\partial t$
- As $\Delta z \to 0$: $\partial i/\partial z = -GV - C\partial V/\partial t$



These circuit elements are per unit length.

Transmission Line Equations

- $\partial V/\partial z = -Ri(t) - L\partial i/\partial t$
- $\partial i/\partial z = -GV - C\partial V/\partial t$
- These equations are a coupled system with two PDEs in terms of $V(z,t)$ and $i(z,t)$



These circuit elements are <u>per unit length</u>.

One-Dimensional Wave Equation

- By differentiating the first w.r.t t and the second w.r.t z, we get

$$\frac{\partial^2 V}{\partial z^2} = LC\,\frac{\partial^2 V}{\partial t^2}$$

- This is known as the *one-dimensional wave equation*

- If instead you differentiate the first w.r.t z and the second w.r.t t, we get

$$\frac{\partial^2 i}{\partial z^2} = LC\,\frac{\partial^2 i}{\partial t^2}$$

➢ The voltage and current satisfy the same second-order differential equation

Derivation Summary

$$V(z+\Delta z,t) + (L\Delta z)\partial i/\partial t + (R\Delta z)i(z,t) = V(z,t)$$

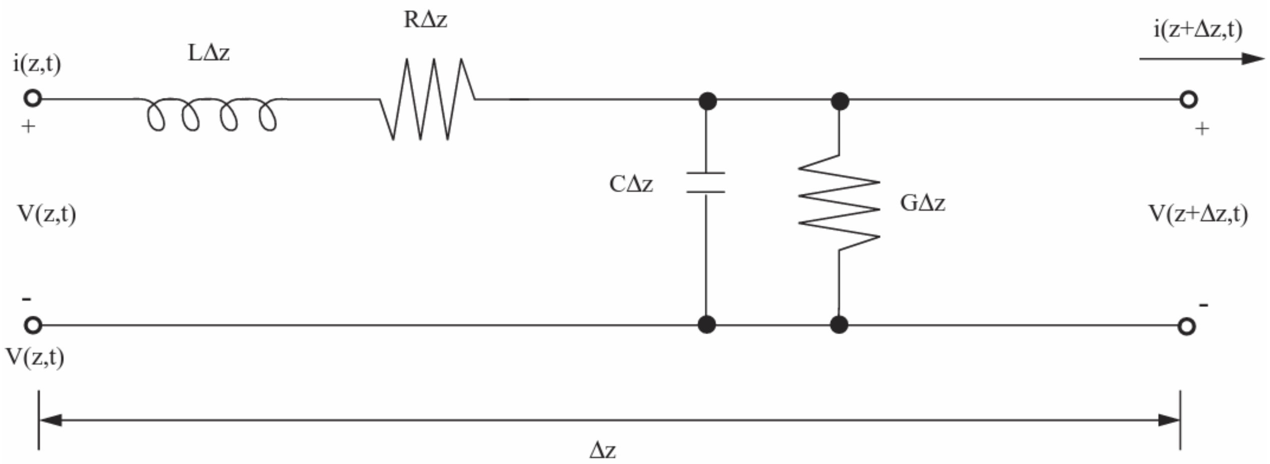
$$i(z+\Delta z,t) - i(z,t) = -(G\Delta z)V(z+\Delta z,t) - (C\Delta z)\partial V/\partial t$$

$$\partial V/\partial z = -Ri - L\partial i/\partial t \qquad \partial i/\partial z = -GV - C\partial V/\partial t$$

From boundary and initial conditions.

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} \longrightarrow V(z,t) = V^+ f\left(t - \frac{z}{v_p}\right) + V^- g\left(t + \frac{z}{v_p}\right)$$

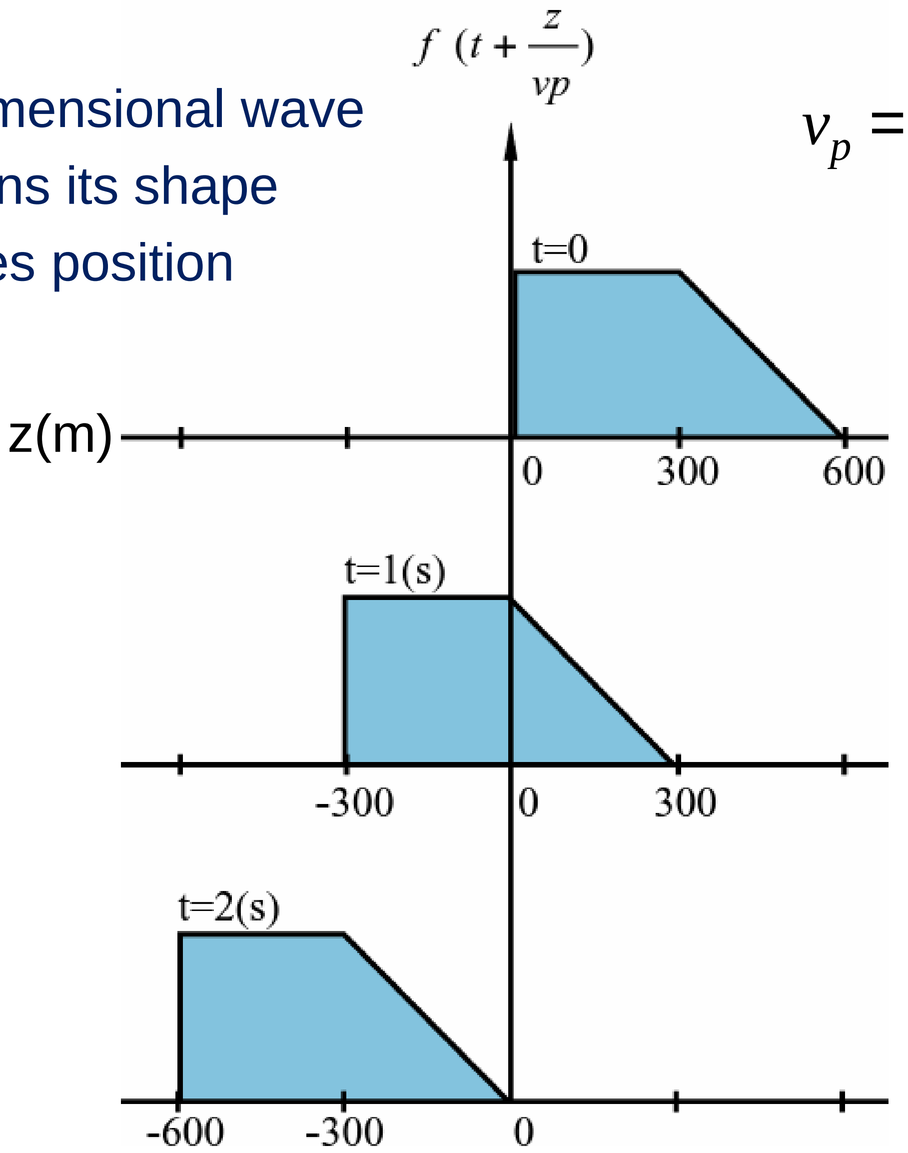
$$\frac{\partial^2 i}{\partial z^2} = LC\frac{\partial^2 i}{\partial t^2} \longrightarrow i(z,t) = \frac{V^+}{Z_0} f\left(t - \frac{z}{v_p}\right) - \frac{V^-}{Z_0} g\left(t + \frac{z}{v_p}\right)$$

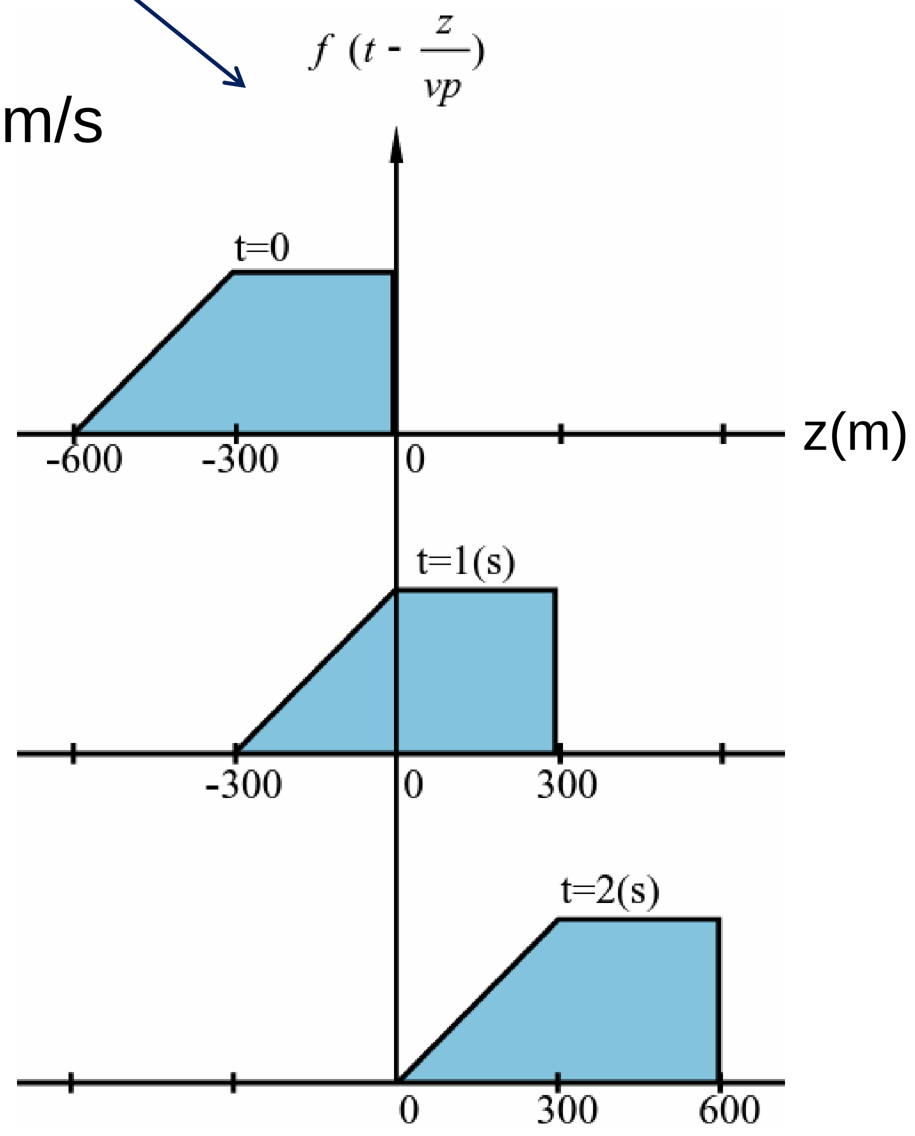


Example: Backward & Forward Traveling Waves

- One-dimensional wave
- Maintains its shape
- Changes position

v_p = 300 m/s



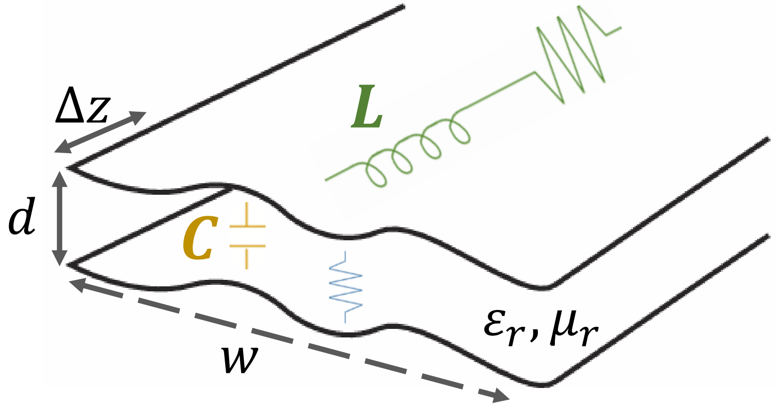


Tummala, 2.3.4 & 11.2.1

Propagation Velocity & Characteristic Impedance

- Propagation velocity, $v_p = \frac{1}{\sqrt{LC}}$
  - For (ideal) parallel strip: $v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}$
  - Independent of the line geometry
  - Depends on the material

(Ideal) Parallel Conducting Strips

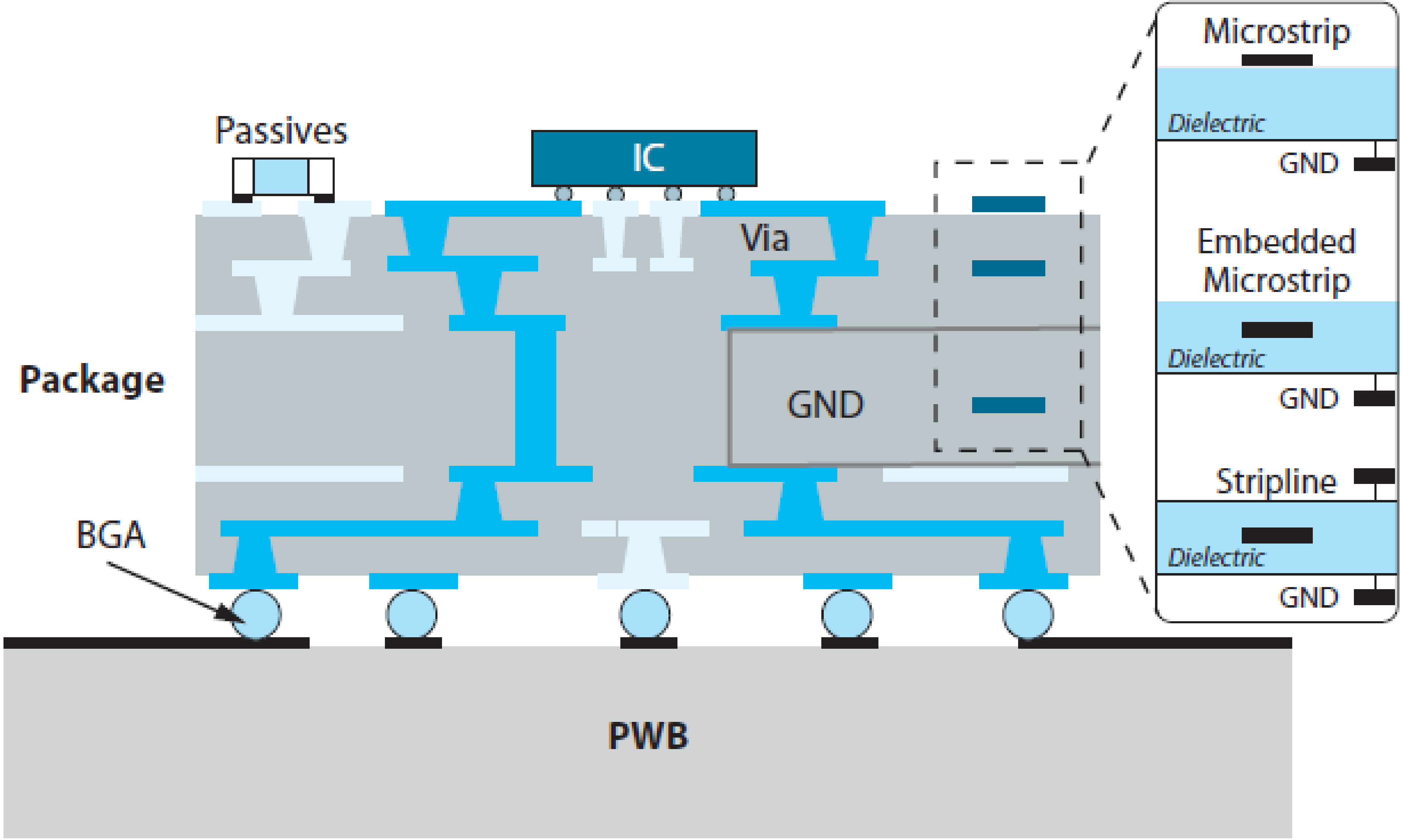


$\boldsymbol{C = \varepsilon w/d}$ [F/m]

$\boldsymbol{L = \mu d/w}$ [H/m]

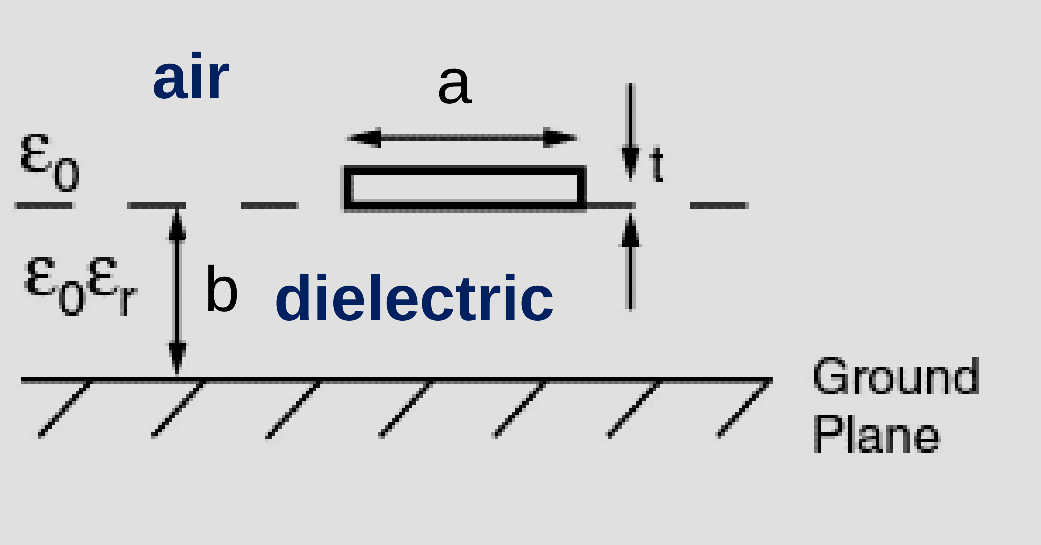
- Characteristic impedance, $Z_0 = \sqrt{\frac{L}{C}}$
  - For (ideal) parallel strip: $Z_0 = \sqrt{\frac{L}{C}} = \frac{d}{w}\sqrt{\frac{\mu}{\varepsilon}}$
  - Depends on geometry (cross-sectional dimensions) and the material
  - Can adjust the geometry to get a desired Z_0

Transmission Line Structures in Packaging



Microstrip Transmission Line

Microstrip Structure



Transmission Line Formulas

$$\varepsilon_{\text{eff}} = \varepsilon_0\left[\frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12b/a}}\right]$$

$$v_p = \frac{1}{\sqrt{\mu\varepsilon_{\text{eff}}}}$$

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon_{\text{eff}}}}\ln\left(\frac{8b}{a} + \frac{a}{4b}\right) \qquad a < b$$

$$Z_0 = \sqrt{\frac{\mu}{\varepsilon_{\text{eff}}}}\frac{1}{\frac{a}{b} + 1.393 + 0.667\ln\left(\frac{a}{b} + 1.444\right)} \qquad a > b$$

Effective permittivity, ε_{eff}

$\varepsilon = \varepsilon_0\varepsilon_r$, where ε_r = relative dielectric constant and $\varepsilon_0 = 8.854 \times 10^{-14}$ F/cm

$\mu = \mu_0\,\mu_r$, where μ_r = relative permeability and $\mu_0 = 4\pi \times 10^{-9}$ H/cm

Propagation velocity, v_p in cm/s

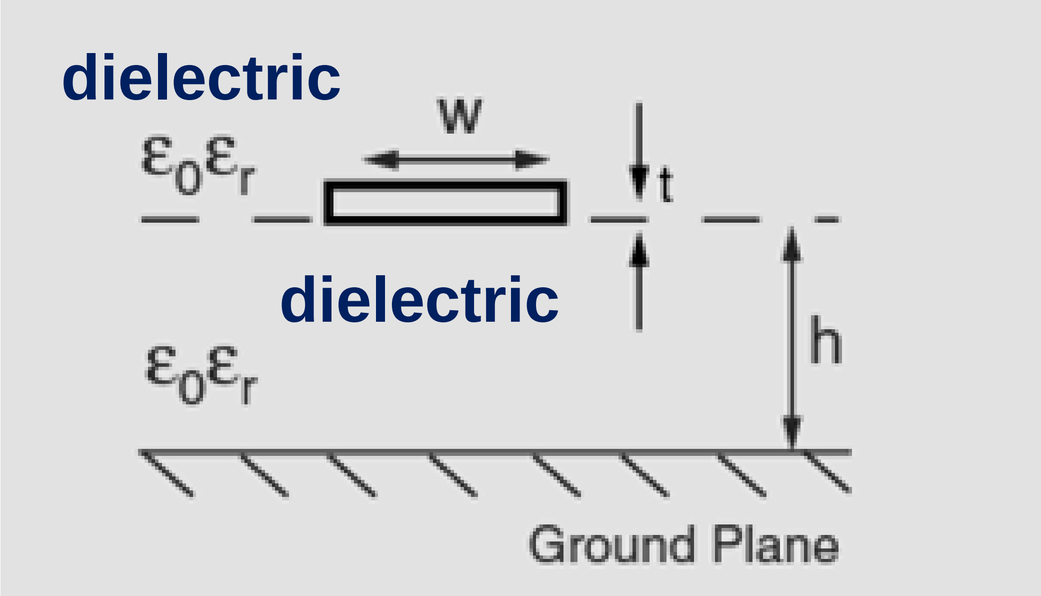
Characteristic impedance, Z_0 in Ω

a and *b* in cm

Tummala, 2.3.5

Embedded Microstrip Transmission Line

Embedded Microstrip Structure



Transmission Line Formulas

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}}$$

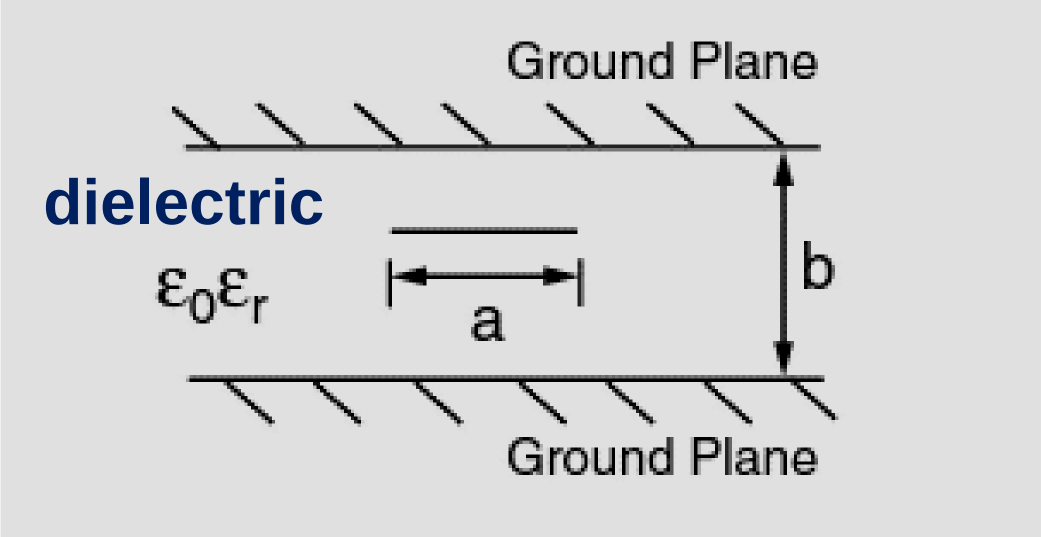
$$Z_0 = \frac{60}{\sqrt{\varepsilon_r + 1.41}} \ln\left(\frac{5.98h}{0.8w + t}\right)$$

$\varepsilon = \varepsilon_0\varepsilon_r$, where ε_r = relative dielectric constant and $\varepsilon_0 = 8.854 \times 10^{-14}$ F/cm
$\mu = \mu_0\,\mu_r$, where μ_r = relative permeability and $\mu_0 = 4\pi \times 10^{-9}$ H/cm
Propagation velocity, v_p in cm/s
Characteristic impedance, Z_0 in Ω
h, t, and *w* in cm

Tummala, 2.3.5

Stripline Transmission Line

Stripline Structure



Transmission Line Formulas

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$Z_0 = \frac{30\pi}{\sqrt{\varepsilon_r}} \frac{b}{a_{\text{eff}} + 0.441b}$$

$$a_{\text{eff}} = \begin{cases} a & a > 0.35b \\ a - \left(0.35 - \frac{a}{b}\right)^2 b & a < 0.35b \end{cases}$$

Effective dimension, a_{eff} in cm

$\varepsilon = \varepsilon_0\varepsilon_r$, where ε_r = relative dielectric constant and $\varepsilon_0 = 8.854 \times 10^{-14}$ F/cm

$\mu = \mu_0 \mu_r$, where μ_r = relative permeability and $\mu_0 = 4\pi \times 10^{-9}$ H/cm

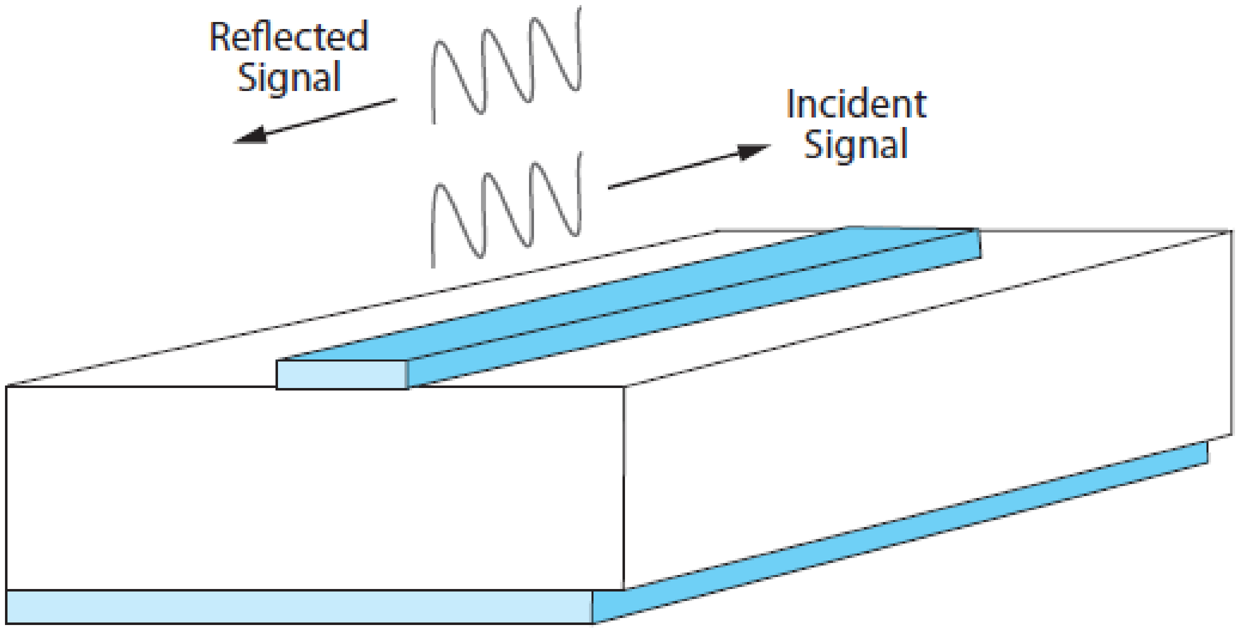
Propagation velocity, v_p in cm/s

Characteristic impedance, Z_0 in Ω

a and *b* in cm

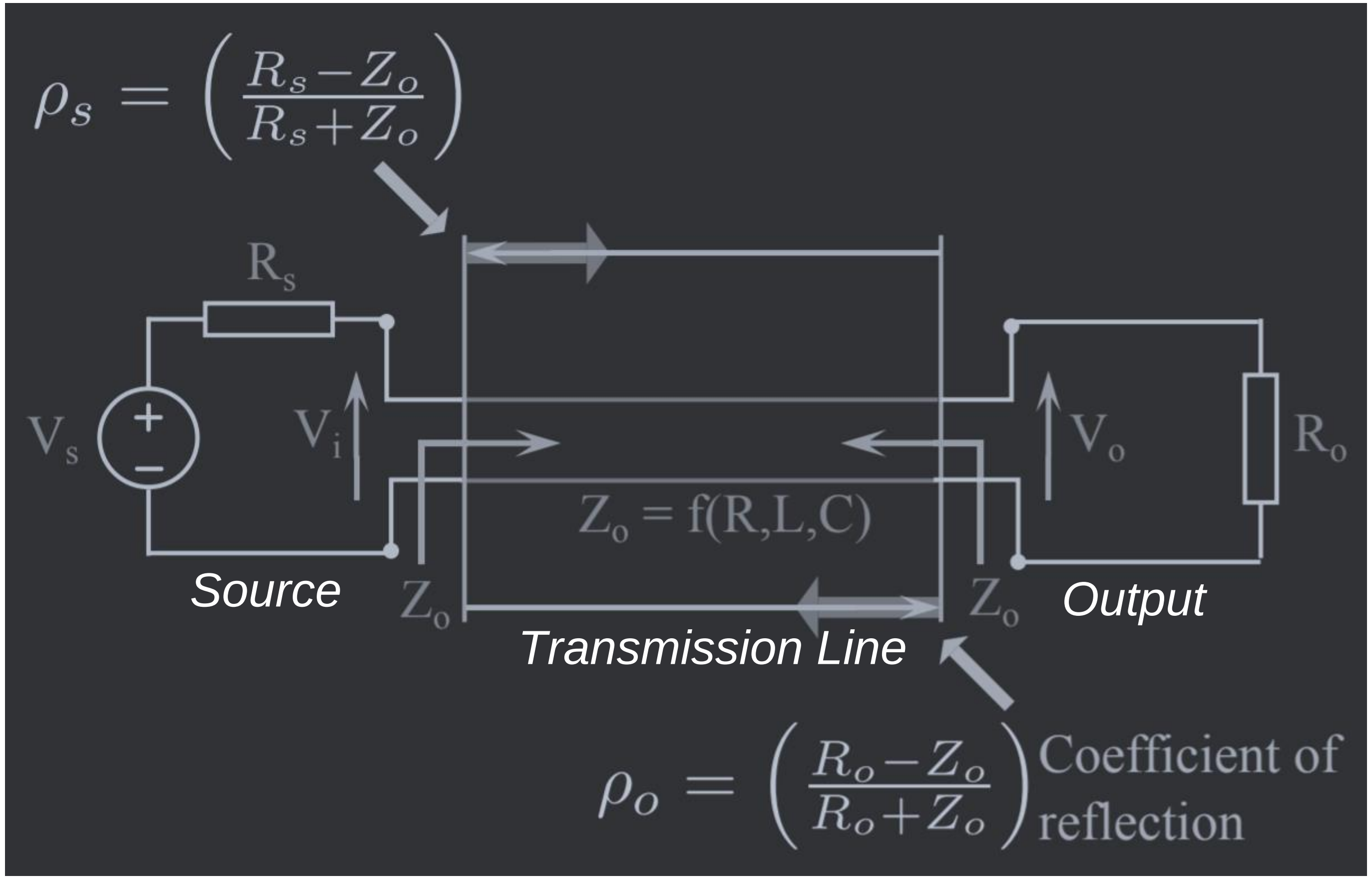
Reflection

- When a signal traveling in a transmission line encounters a <u>change in impedance</u>, a <u>reflected signal</u> is generated
- Any mismatch in impedance (e.g., from a termination) will generate a reflection
- For RF or microwave designs, reflections and standing waves are minimized by terminating the line with an impedance equal to the line wave impedance



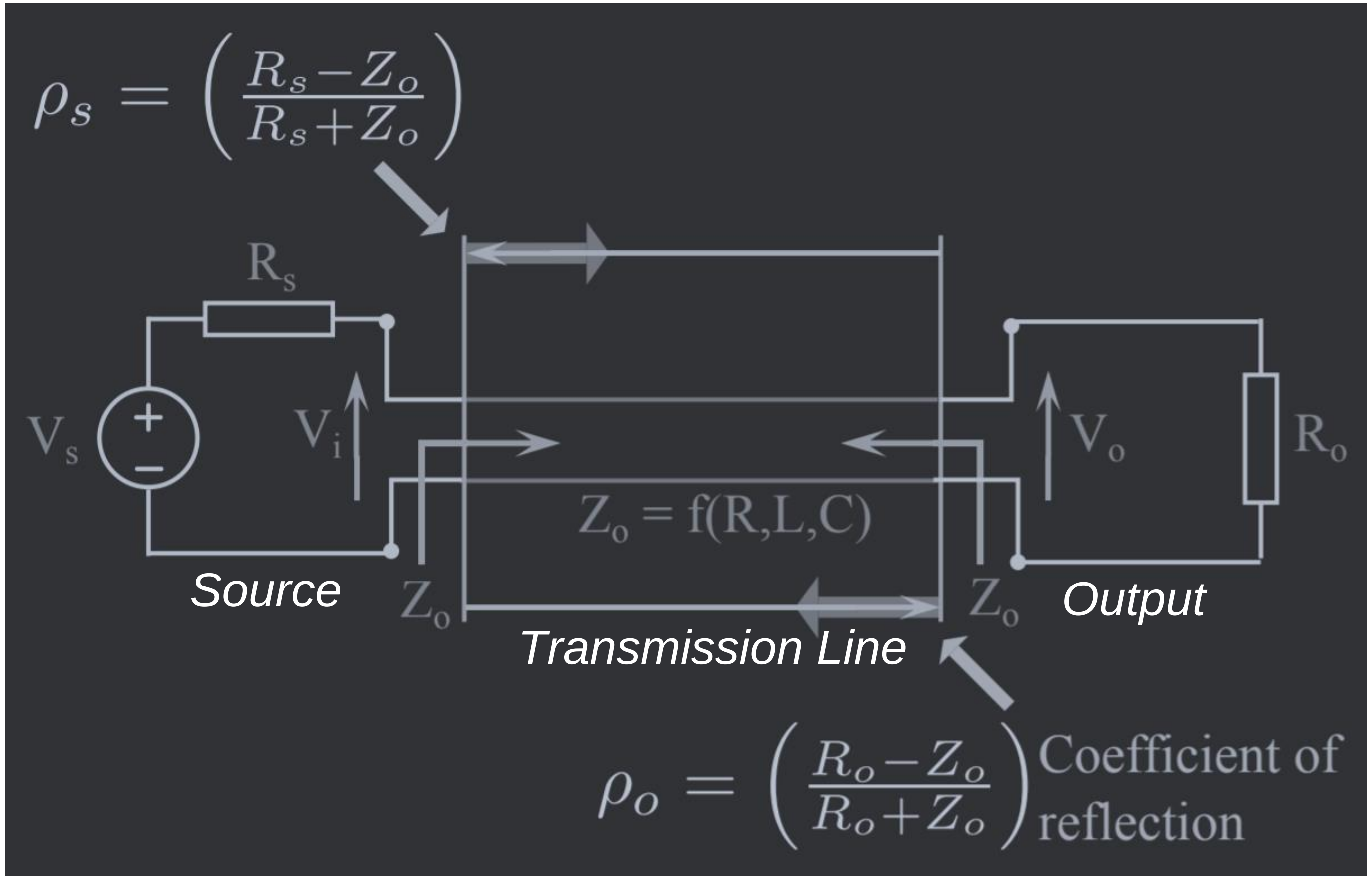
Reflection

- When a signal traveling in a transmission line encounters a change in impedance, a reflected signal is generated
- $\rho =$ reflection coefficient
- When $R_s = Z_0$, $\rho_s = 0$
- When $R_o = Z_0$, $\rho_o = 0$

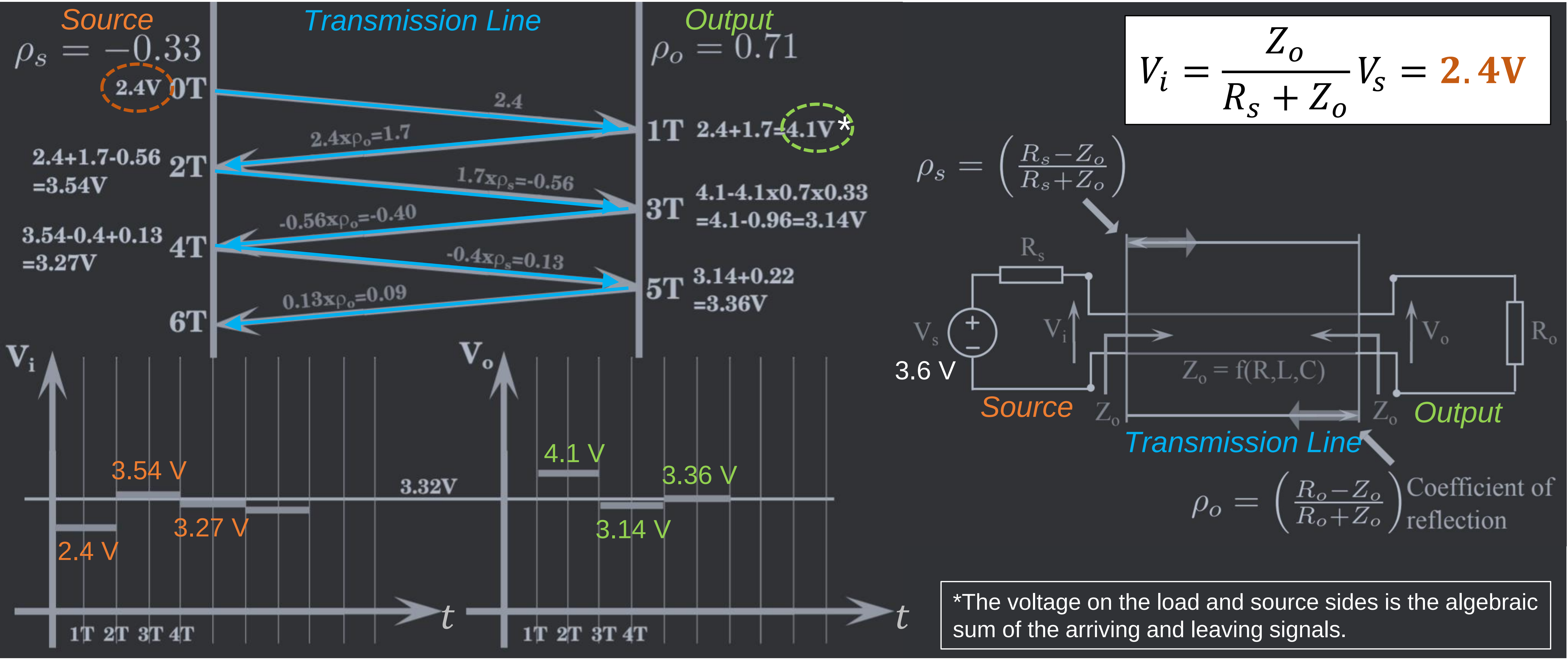


Example: Reflection

- $R_s = 25\ \Omega$
- $R_o = 300\ \Omega$
- $Z_o = 50\ \Omega$
- $V_s = 3.6\ \text{V}$
- $\rho_o = \frac{R_o - Z_o}{R_o + Z_o} = 0.71$
- $\rho_s = \frac{R_s - Z_o}{R_s + Z_o} = -0.33$
- $V_i = \frac{Z_o}{R_s + Z_o} V_s = \frac{50\Omega}{25\Omega + 50\Omega}(3.6\text{V}) = \mathbf{2.4V}$



Example: Reflection



T = one wave propagation delay time from source to load or vice versa

No transmission line effects: $V_i = V_o$ = 300 Ω / (300 Ω + 25 Ω) x 3.6 V = 300 Ω / 325 Ω x 3.6 V = **3.32 V**

Reducing Transmission Line Effects

- Slow down rise times such that $t_r > (33.3\ \mathrm{ps/cm})\sqrt{\varepsilon_r} \times 2l$
  - Minimizes impact of the line delay on the circuit performance
- Use materials with low dielectric constant ε_r
  - Increases propagation velocity/reduces delay $v_p = 1/\sqrt{LC}$
- Reduce length of the line such that $l < 0.5\, t_r/(33.3\ \mathrm{ps/cm})\sqrt{\varepsilon_r}$
  - Minimizes impact of the line delay on the circuit performance
  - Reduces transmission line losses
- Match impedances
  - Reduces reflection
  - Vary parasitic L and C by changing the line geometry $Z_0 = \sqrt{L/C}$

Additional References

- Sierra Circuits, "Losses in PCB Transmission Lines"
- NIST Technical Note 1520, "Dielectric Conductor-Loss Characterization and Measurements on Electronic Packaging Materials"
- A. Weisshaar, "Handbook of Engineering Electromagnetics"
  - Chapter 6: Transmission Lines