VIRGINIA TECH

Lecture 3

Electrical Design

DC and AC Resistances

January 29, 2025

Reminders and Announcements

- Office hours tomorrow, 3:00pm – 4:00pm
- Quiz #1 on lectures 1-2 on Canvas
  - Due by 11:59pm (midnight) on Wednesday, Jan. 29th
  - 5 multiple choice questions, 15 minute limit, open book/notes, you cannot discuss the questions or answers with other students
- Homework #1 will be assigned on Friday
- Download and install ANSYS Electronics Desktop and LTspice and check that you can open them before Feb. 3rd

Course Topics

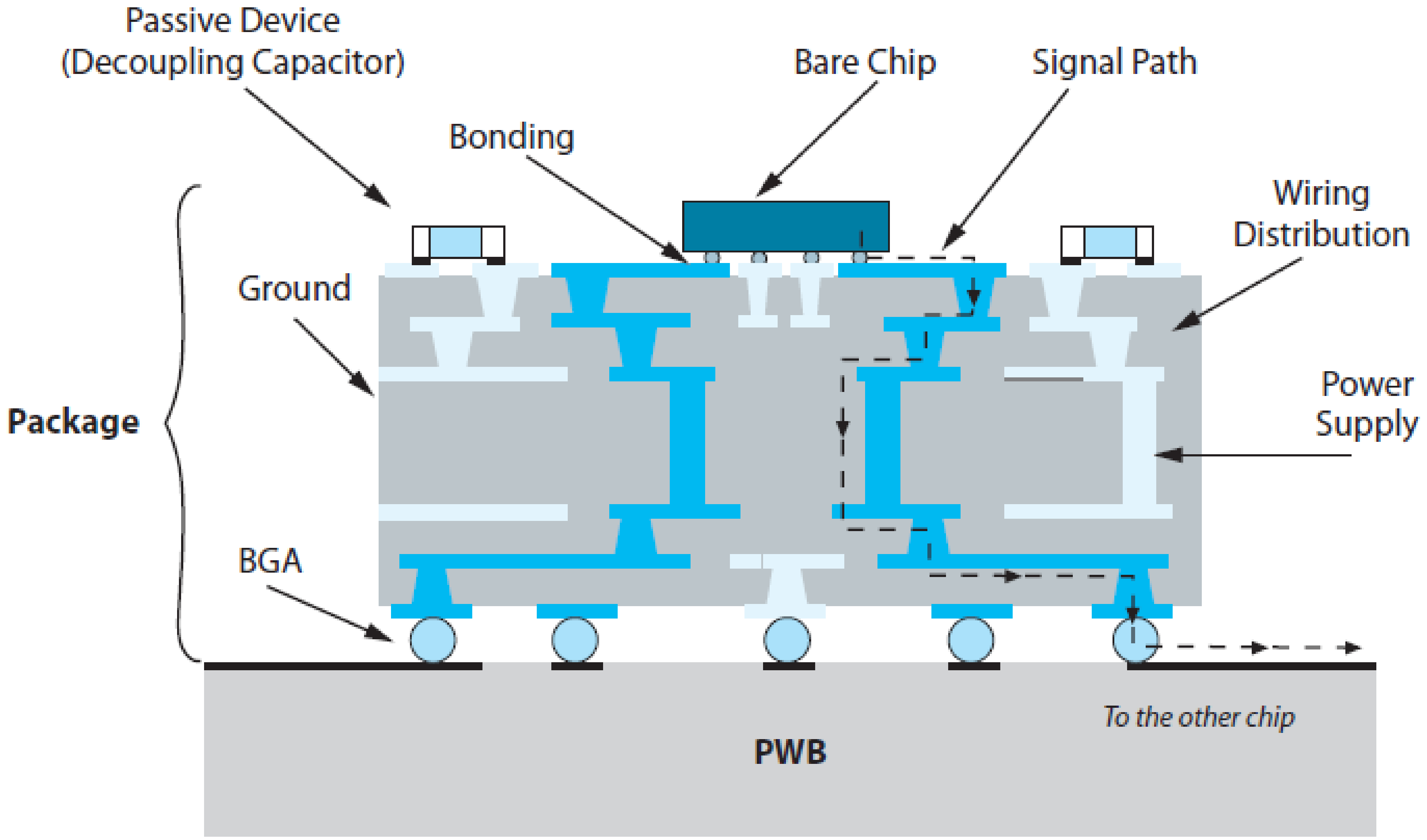
1. Introduction
2. **Electrical**
3. Thermal
4. Materials and Processes
5. Characterization and Testing
6. Reliability and Ruggedness
7. Emerging Technologies

Electrical Design

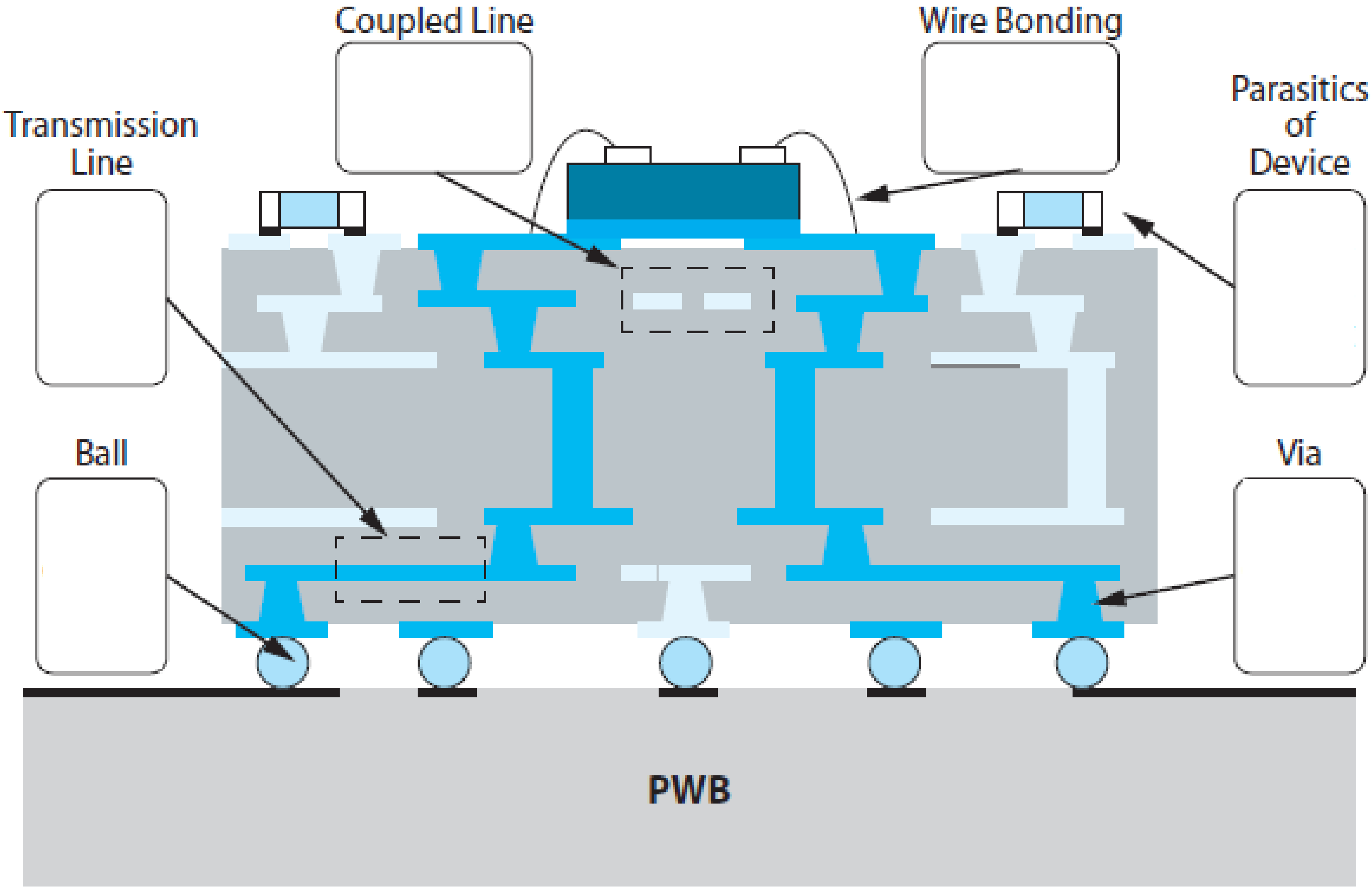
- Objectives
  - **Signal distribution & integrity**
  - **Power distribution & integrity**
- Main package elements
  - **Interconnects**
    - Electrical connections *within the package*, from *die to substrate*
  - **Substrate**
    - Electrical traces *within the package*, from *interconnects to terminals*
  - **Terminals/leads**
    - Electrical connections from inside the package to *outside* the package, from the *substrate to the external PCB/PWB*

Tummala, 2.2.

Example Package



Example Package



Electrical Design

- Challenges
  - Parasitic resistance, inductance, and capacitance
    - Delays, distortion, noise, voltage drop, reflection
  - Dielectric loss
  - Electrical insulation
- Solutions
  - Short signal lengths
  - Separated signal and power
  - High electrical conductivity
  - Low dielectric constant
- Consequences
  - Increased heat flux
  - Higher thermal resistance
  - Reduced reliability

Electrical Design Topics

- Parasitic elements
  - Resistance (DC and AC)
  - Inductance
  - Capacitance
- Transmission lines
- Electrical insulation

Package Equivalent Circuit

Physical Package Structure → Equivalent Electrical Circuit

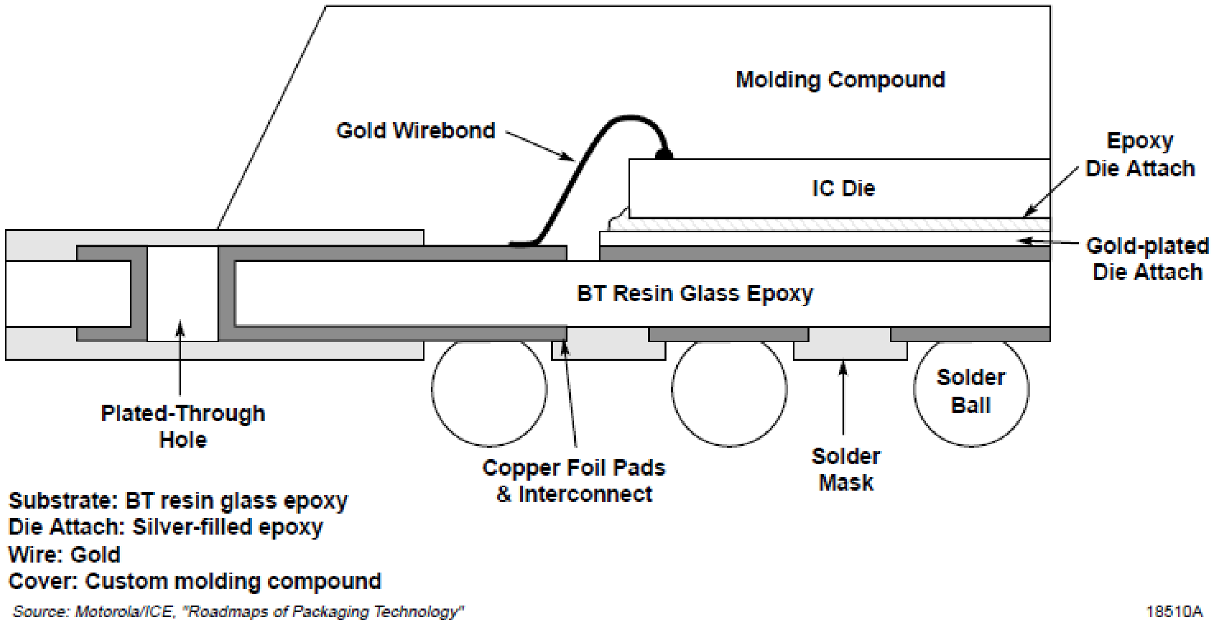


Figure 7-3. OMPAC Ball Grid Array From Motorola

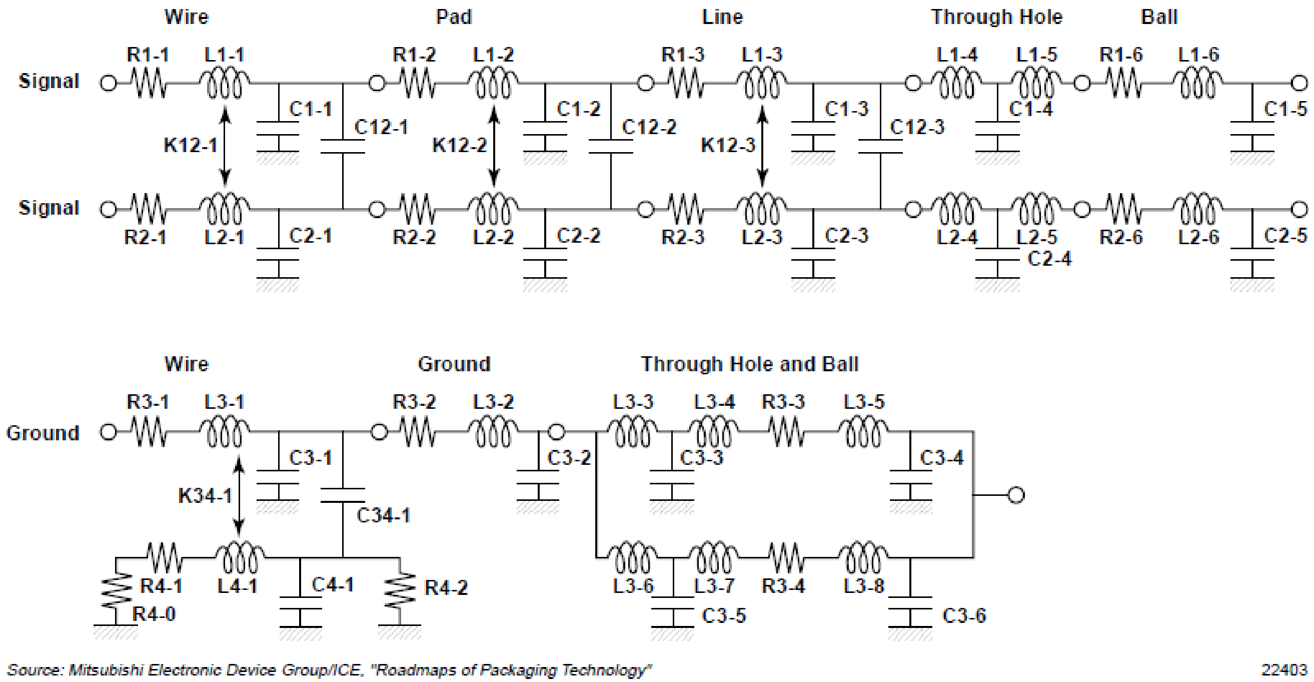
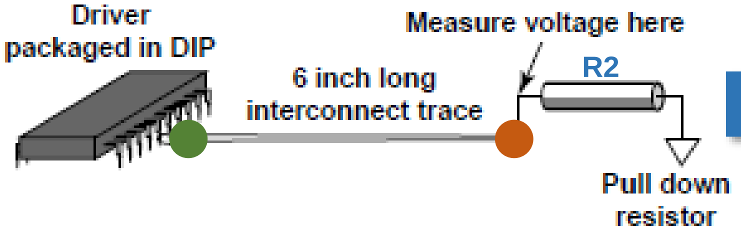


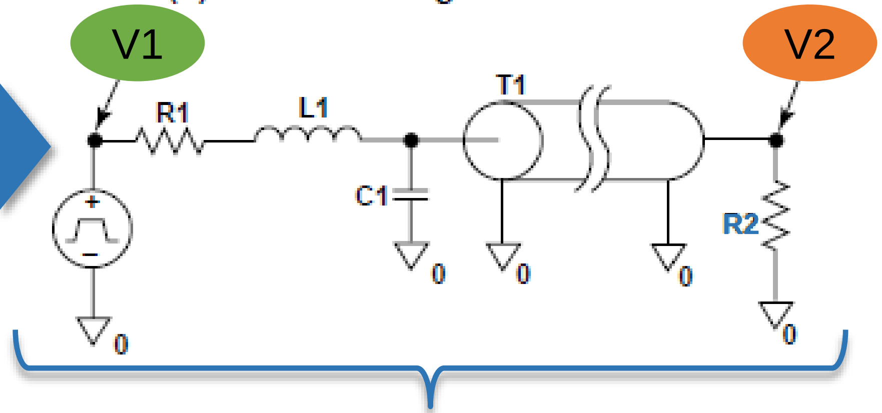
Figure 7-4. The Equivalent Schematic for a BGA Package for Adjacent Signal to Signal Lines and for Ground Lines

Simulation of Packaged Driver and Interconnect

(a) Physical Drawing of the Structure Being Simulated



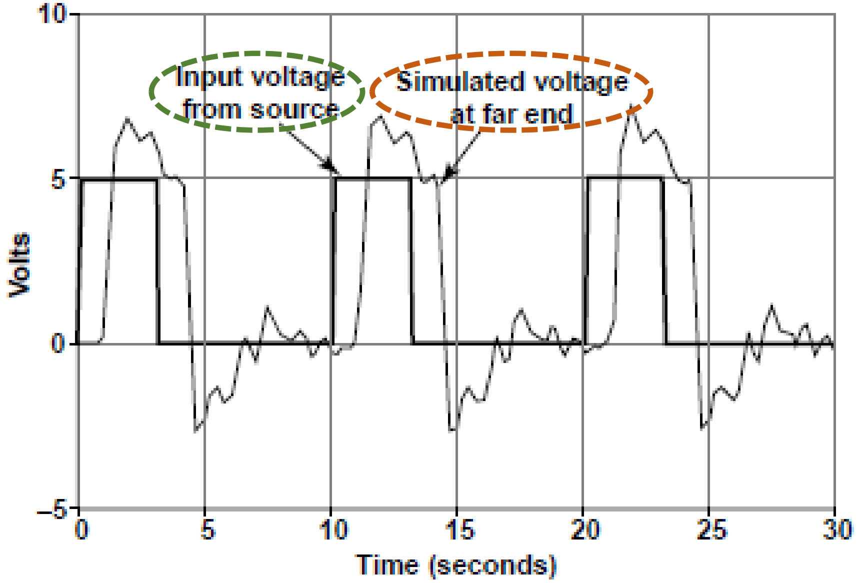
(b) Schematic Diagram of the Circuit



(c) SPICE Netlist of the Circuit

```
** Analysis setup **

.tran .02n 30n

R1 1 2 25
L1 2 3 7n
C1 3 0 5 p
T1 5 0 6 0 Z0=50 TD=1n
R2 6 0 1k

V1 0 1 +PULSE 0 5 0 .1n .1n 3n 10n

.probe

.end
```

(d) Simulated Far End Voltage



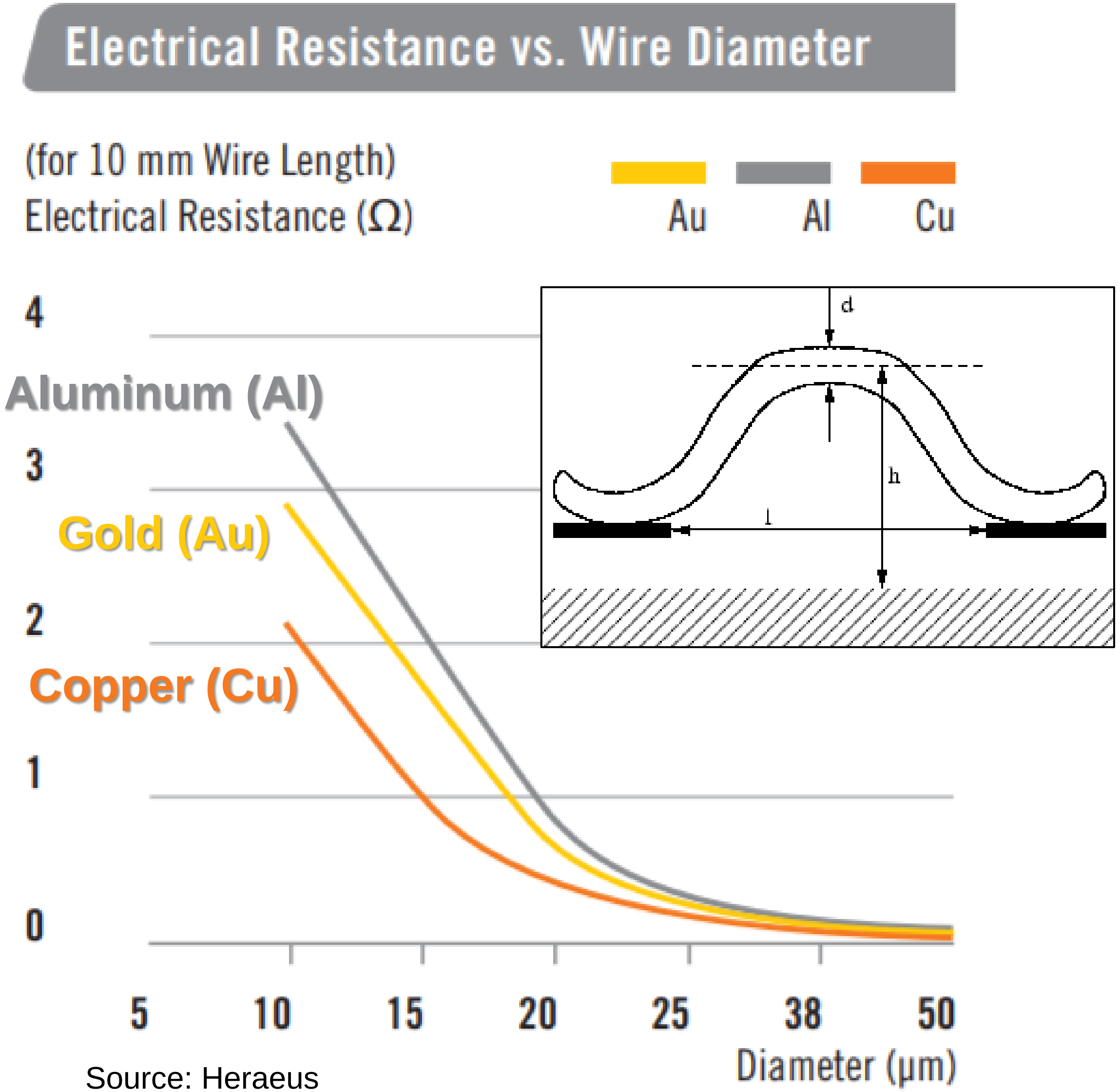
Source: ICE, "Roadmaps of Packaging Technology"

R, C, L Overview

- Resistance, R
  - Unit: Ohms, Ω
  - Effects: damping, voltage drop, loss ($P = I^2R$)
  - Types: DC & AC (skin effect)
  - Ohm's Law: $V = IR$
- Capacitance, C
  - Unit: Farad, F
  - Definition: amount of charge stored per volt
  - Effects: coupling, resonance
  - $I = C\frac{dV}{dt}$
- Inductance, L
  - Unit: Henry, H
  - Definition: ratio of magnetic flux linked by a loop of current to the current
  - Effects: coupling, resonance, voltage drop/overshoot
  - $V = L\frac{dI}{dt}$
- Every current-carrying conductor has some R and L

DC Resistance

- Critical components
  - Interconnects
  - Substrate traces/vias
  - Terminals
- $V = I\boldsymbol{R}$, $P = I^2\boldsymbol{R}$
- $R = \dfrac{\rho l}{A_c}$
  - ρ = resistivity, Ω·m
  - $\sigma = 1/\rho$ = conductivity, S/m
  - l = length, m
  - A_c = cross-sectional area, m^2



Source: Heraeus

Electrical Resistivity, ρ

- Definition: measure of resisting power of a material to flow of an electric current
- Determined by the scattering of electrons
- More scattering = higher resistivity
- $\rho = 1/\sigma = (m_\mathrm{e}\, v) / (n_e\, e^2\, \lambda)$
  - m_e = electron mass
  - v = electron velocity
  - n_e = electron density (number of electrons/volume)
  - e^2 = electron charge
  - λ = mean free path

Electrical Resistivity, ρ

- Silver: $\rho = 1.6 \times 10^{-8}\ \Omega\cdot m$
- Copper: $\rho = 1.7 \times 10^{-8}\ \Omega\cdot m$
- Gold: $\rho = 2.2 \times 10^{-8}\ \Omega\cdot m$
- Aluminum: $\rho = 2.7 \times 10^{-8}\ \Omega\cdot m$
- Platinum: $\rho = 10.6 \times 10^{-8}\ \Omega\cdot m$

*These are the resistivity values at *room temperature* (25 °C)

Example: DC Resistance of Wire Bond

- 50 µm (2 mil*) gold wire bond with 10 mm length
- $\rho_{gold@25\mathrm{C}}$= 2.2 x 10^{-8} Ω·m
- What is the resistance of the wire bond at room temperature?

$$\boldsymbol{R_{25\mathrm{C}} = \rho_{25\mathrm{C}} l / A_c}$$

$$l = 10 \text{ mm} = 0.01 \text{ m}$$

$$A_c = \pi r^2 = \pi(2.5 \times 10^{-5} \text{ m})^2$$

$$R_{25\mathrm{C}} = (2.2 \times 10^{-8}\ \Omega\cdot\text{m}) \cdot (0.01 \text{ m}) / (\pi(2.5 \times 10^{-5} \text{ m})^2) = \mathbf{0.11\ \Omega}$$

- What if you have 5 wire bonds in parallel?

$$R_{eq} = R / 5 = 0.11\ \Omega / 5 = 0.022\ \Omega = \mathbf{22\ m\Omega}$$

*1 mil = 0.001 inch

Example: DC Resistance of PCB Trace

- 200-mm-long, 0.1-mm-wide PCB trace with 1 oz* copper
- $\rho_{copper@25C}$ = 1.7 x 10^{-8} Ω·m
- 1 oz copper = 1.37 mils = 0.00137 in = 0.034798 mm
- What is the resistance of the copper trace at room temperature?

$$\boldsymbol{R_{25C} = \rho_{25C} l / A_c}$$

$$l = 200 \text{ mm} = 0.2 \text{ m}$$

$$A_c = t \cdot w = (3.5 \times 10^{-5} \text{ m}) \cdot (1 \times 10^{-4} \text{ m}) = 3.5 \times 10^{-9} \text{ m}^2$$

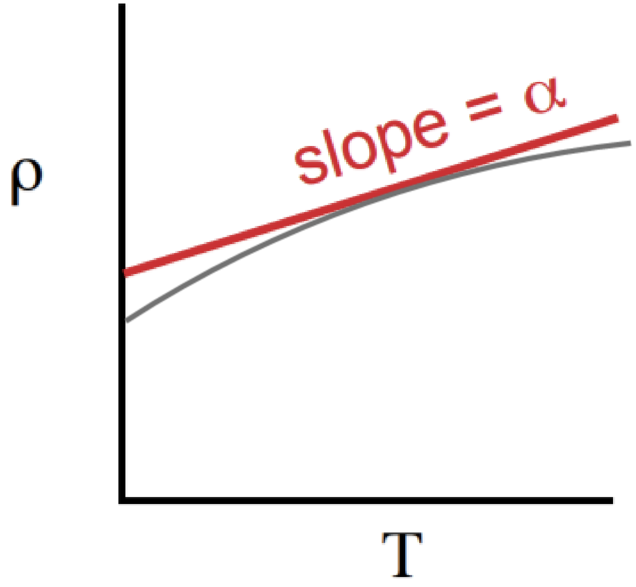
$$R_{25C} = (1.7 \times 10^{-8}\ \Omega\cdot\text{m}) \cdot (0.2 \text{ m}) / (3.5 \times 10^{-9} \text{ m}^2) = \mathbf{0.97\ \Omega}$$

… at room temperature

*1 oz of copper rolled out over 1 sq. ft.

Temperature Dependence of DC Resistance

- Resistance of conductors increases with temperature
- The fractional change in resistance is proportional to the change in temperature:
- $R_1 = R_0 \cdot [1 + \alpha(T_1 - T_0)]$
  - R_0 = resistance at T_0, Ω
  - T_1 = temperature of interest, K or °C
  - α = temperature coefficient, 1/K or 1/°C
- This linear approximation can be used if α does not change much with T and $\alpha\Delta T \ll 1$
- When T_0 is room temperature,



$$R_1 = \rho_{25\mathrm{C}} l / A_c \cdot [1 + \alpha(T_1 - 25°\mathrm{C})]$$

Temperature Coefficient of Resistance, α

$$R_1 = R_0 \cdot [1 + \alpha(T_1 - T_0)]$$

- Gold: $\alpha = 3.4 \times 10^{-3}$ /°C
- Silver: $\alpha = 3.8 \times 10^{-3}$ /°C
- Aluminum: $\alpha = 3.9 \times 10^{-3}$ /°C
- Platinum: $\alpha = 3.9 \times 10^{-3}$ /°C
- Copper: $\alpha = 4.0 \times 10^{-3}$ /°C

Example: DC Resistance of PCB Trace at 100 °C

- 200-mm-long, 0.1-mm-wide PCB trace with 1 oz copper

$$\boldsymbol{R_1 = \rho_{25C} l/A \cdot [1 + \alpha(T_1 - 25°C)]}$$

$$R_{25C} = \frac{\rho_{25C} l}{A} = (1.7 \text{ x } 10{-}8\ \Omega\cdot\text{m}) \cdot (0.2\text{ m}) / (3.5 \text{ x } 10{-}9\ \text{m}^2) = \mathbf{0.97\ \Omega}$$

- What is the resistance of the copper trace *at 100°C*?

$$\alpha_{copper} = 4.0 \text{ x } 10^{-3}\ /°\text{C}$$

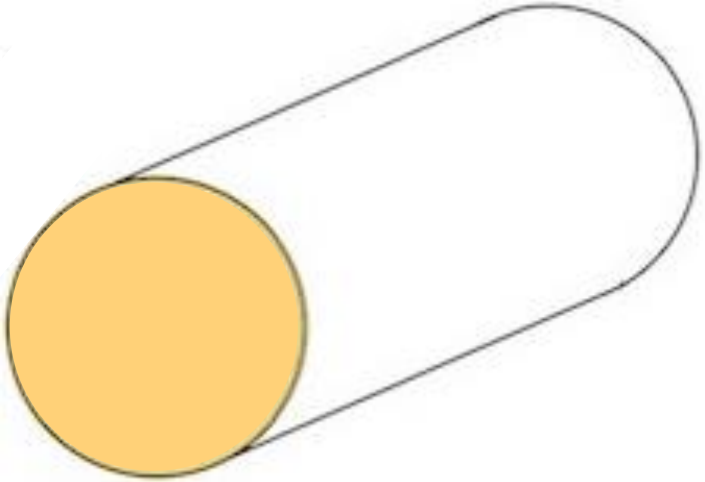
$$R_{100C} = R_{25C} \cdot [1 + (4.0 \text{ x } 10^{-3}\ /°\text{C})\ (100°\text{C} – 25°\text{C})] = \mathbf{1.25\ \Omega}$$

➢ **30 % increase!**

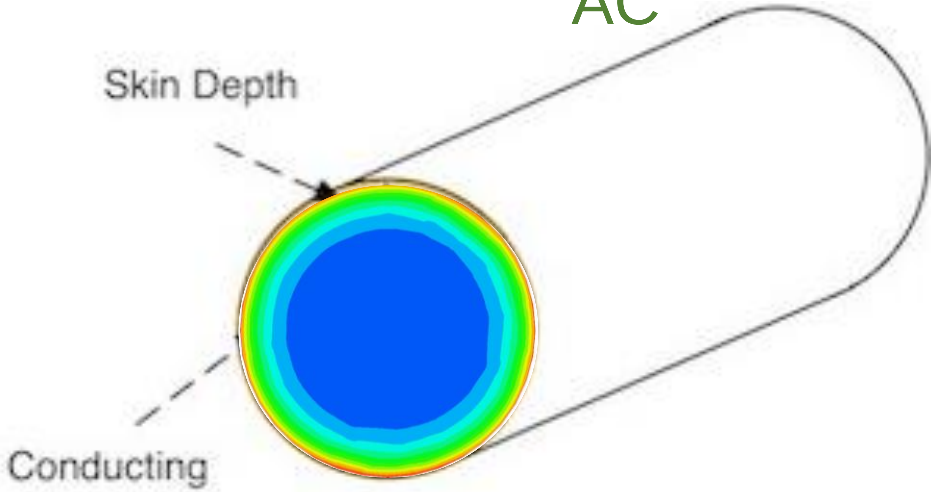
DC and AC Resistance

- Direct current (DC) flows uniformly in a conductor
- With high-frequency alternating current (AC), current crowds along the conductor surface
- Current density = flow of charge per unit area [Amps / m^2]

DC

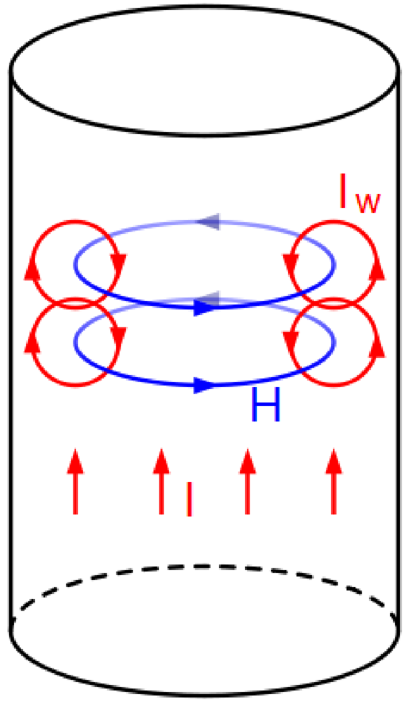


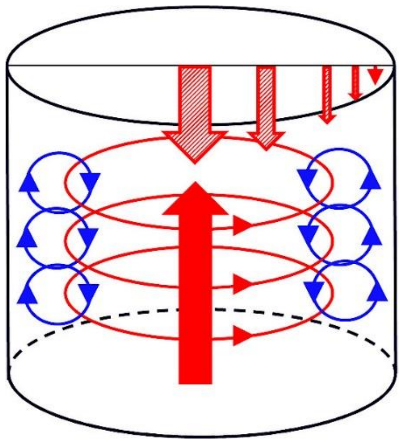
AC



Skin Effect

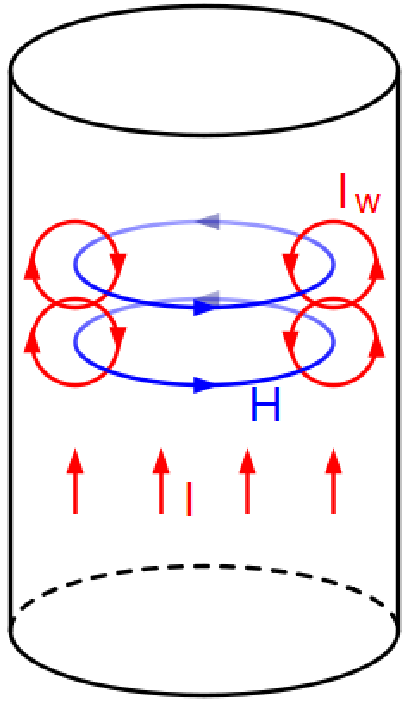
- AC produces an alternating magnetic field H
- When the current intensity changes, the magnetic field changes
- Change in magnetic field creates an electric field that opposes the change in current intensity
- The opposing electric field (back EMF) is strongest at the center of the conductor and forces the electrons to the outside of the conductor
- Eddy currents (from changing magnetic field) cancel the current flow in the conductor center and reinforce it in the skin (skin depth)

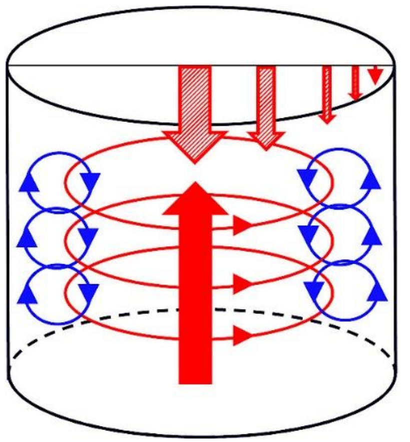




Eddy Currents

- Loops of current induced within conductors by a changing magnetic field
- Eddy currents flow in closed loops within conductors, in planes perpendicular to the magnetic field
- They can be induced within a nearby stationary conductor by a time-varying magnetic field (proximity effect)
- The magnitude of the current is proportional to the strength of the magnetic field, the area of the loop, and the rate of change of flux, and inversely proportional to the resistivity of the material



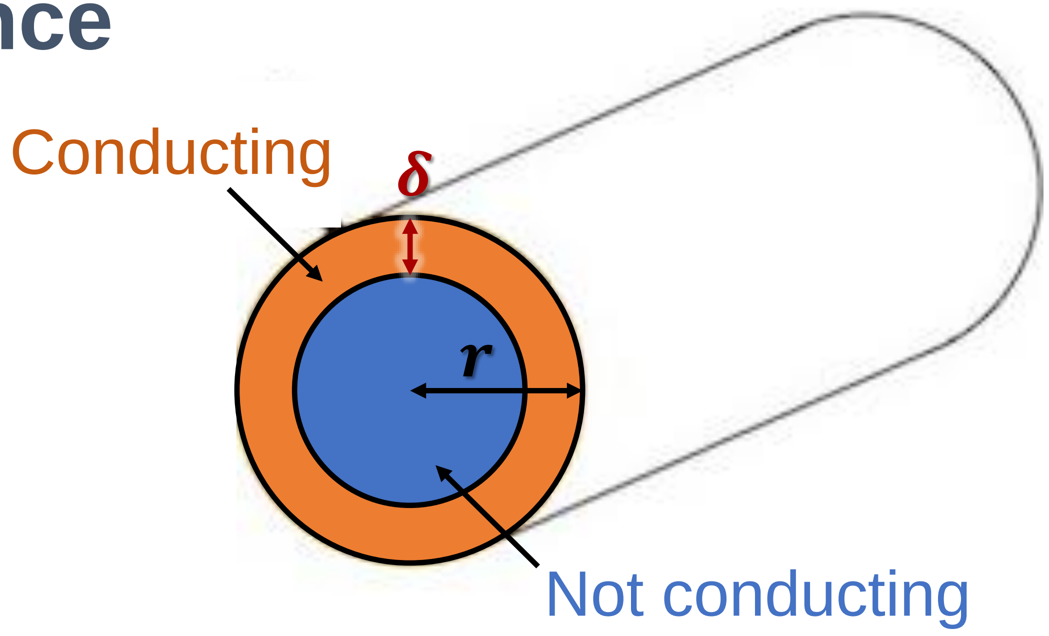


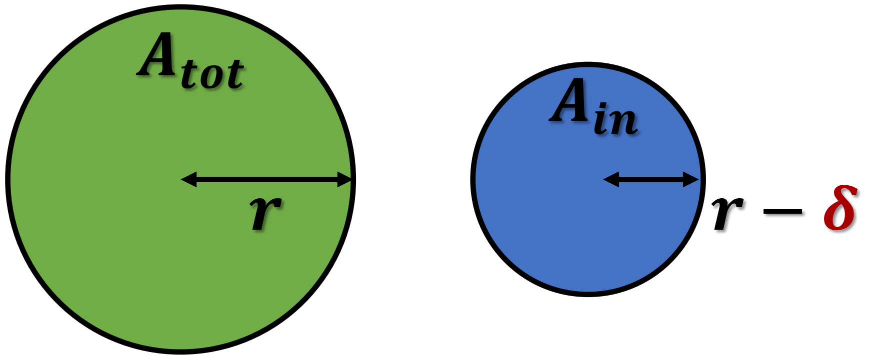
AC Resistance

- $\boldsymbol{R_{AC} = \rho l / A_{eff}}$
  - $A_{eff} =$ effective cross sectional area

If the skin depth is very pronounced:

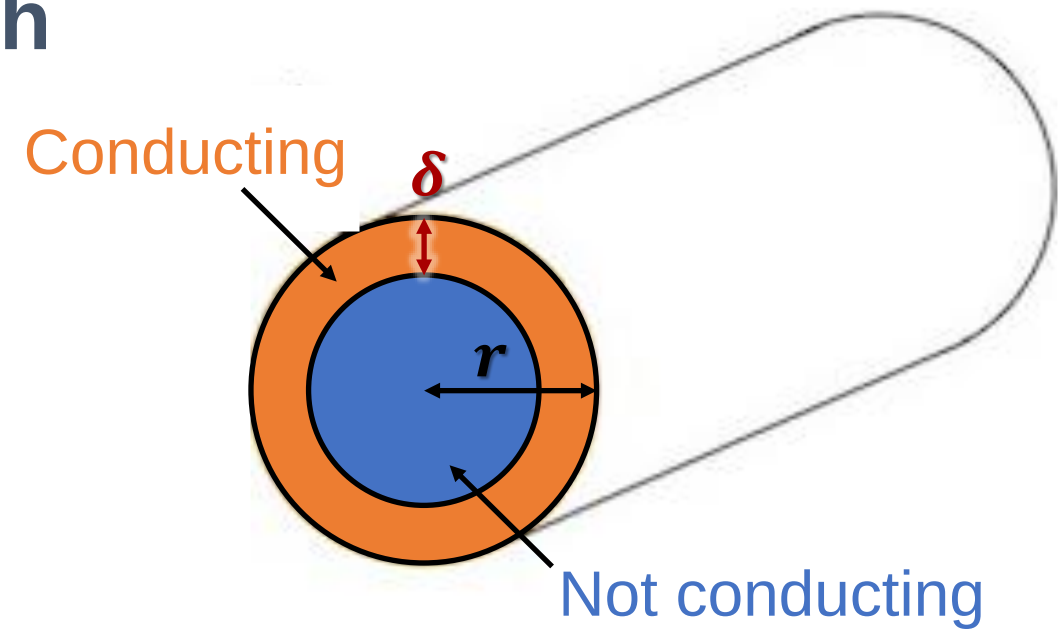
- $A_{eff} = \pi r^2 - \pi(r-\delta)^2$
  - $r =$ radius, m
  - $\delta =$ skin depth, m
- $A_{eff} = \pi(r^2 - r^2 + 2r\delta - \delta^2)$
- $A_{eff} = \pi(2r\delta - \delta^2)$
- **When** $\boldsymbol{\delta \ll r,\ A_{eff} = \pi(d\delta - \delta^2)}$





Skin Depth

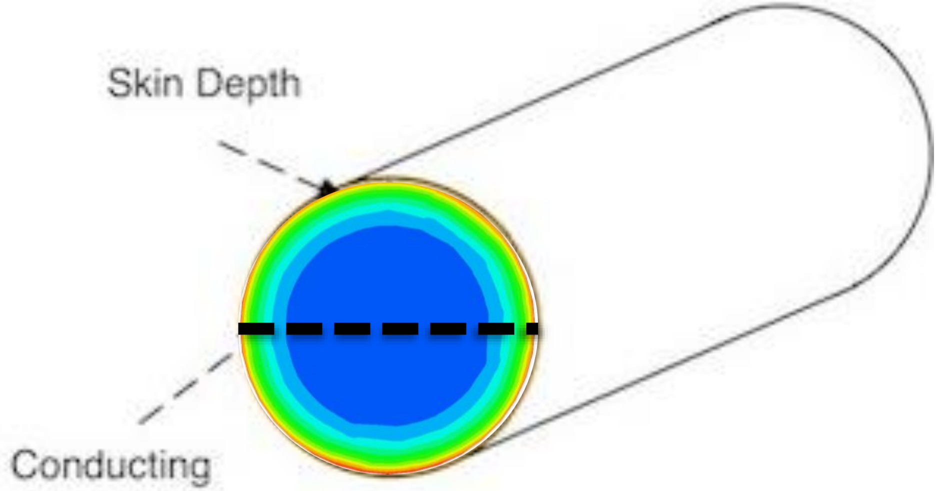
- $R_{AC} = \rho l / A_{eff}$
- $A_{eff} = \pi(d\,\boldsymbol{\delta} - \boldsymbol{\delta}^2)$, when $\delta \ll r$
  - A_{eff} = effective cross sectional area
  - d = diameter, m
  - δ = skin depth, m



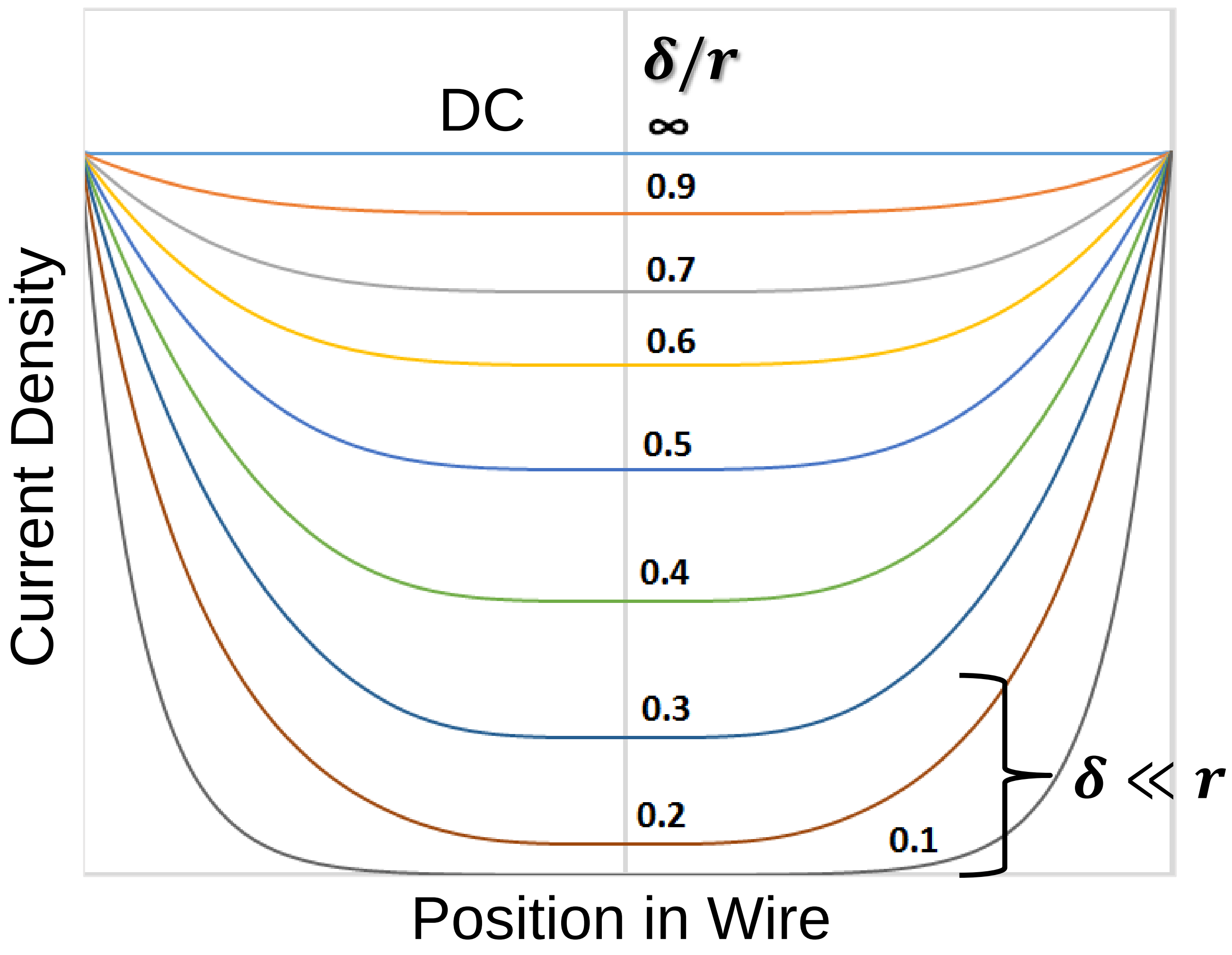
- $\delta = \sqrt{(\rho\,/\,(\pi\, f\, \mu))}$
  - ρ = resistivity, Ω·m
  - f = frequency, Hz
  - μ = permeability, H/m
    - μ_0 = permeability of free space ≈ 4π x 10^{-7} H/m
    - μ_r = relative permeability

$$\left.\begin{array}{l}\mu_0\\ \mu_r\end{array}\right\} \mu = \mu_0\mu_r$$

AC Resistance



Current Density for Different Skin Depths



Skin Depth and Electrical Resistivity

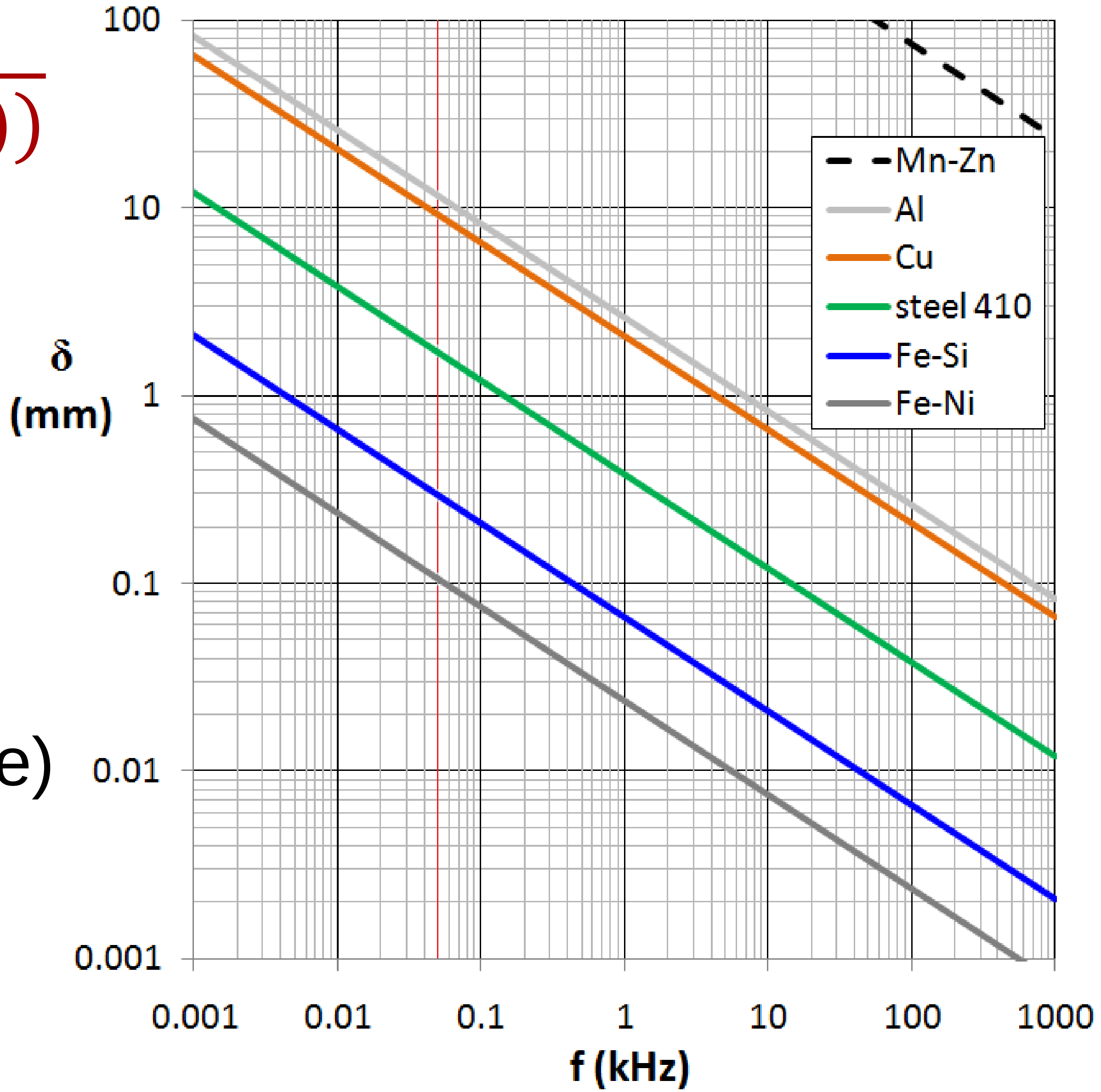
$$\delta = \sqrt{(\boldsymbol{\rho} \,/\, (\pi\, f\, \mu))}$$

Conductor	Resistivity (10^{-8} Ω·m)	Skin Depth at 10 GHz (μm)
Aluminum	2.7	0.83
Gold	2.2	0.75
Copper	1.7	0.65
Silver	1.6	0.64

- Materials with low resistivity ρ (high σ) have smaller skin depth δ
- R_{ac} increases as the resistivity increases $R_{AC} = \boldsymbol{\rho} l / A_{eff}$

Skin Depth and Frequency

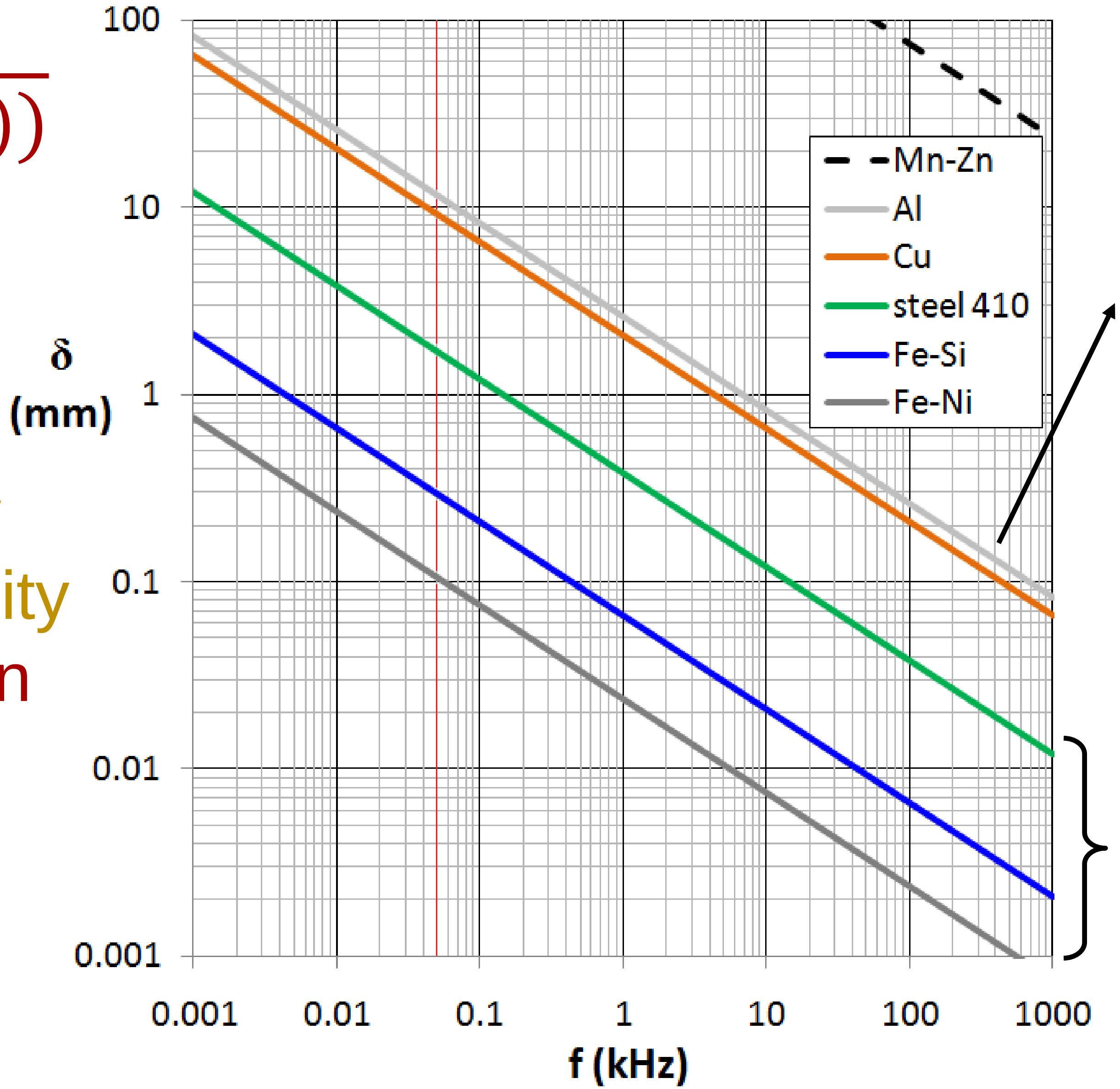
$$\delta = \sqrt{(\rho / (\pi \boldsymbol{f} \mu))}$$



- The skin depth decreases (more current crowds around the surface) as frequency $\boldsymbol{f}$ increases

Skin Depth and Permeability

$$\delta = \sqrt{(\rho / (\pi f \mu))}$$



- Materials with low relative permeability μ_r have larger skin depth for a given frequency

Conductors (Cu, Al, Au, Ag) with $\mu_r \approx 1$, have $\mu \approx 4\pi 10^{-7}$ H/m

Ferromagnetic materials (Fe, Ni) have $\mu_r \gg 1$, so $\mu \gg \mu_0$

Example: AC Resistance of Wire Bond

- 50 µm (2 mil*) gold wire bond with 10 mm length
- ρ_{gold}= 2.2 x 10^{-8} Ω·m
- What is the AC resistance of the wire bond at 150 MHz?

$$\boldsymbol{R_{AC} = \rho l / A_{eff}}$$

$$\boldsymbol{\delta = \sqrt{(\rho\ /\ (\pi\ f\ \mu))}}$$

$$= \sqrt{(2.2 \cdot 10^{-8}\ \Omega \cdot \mathrm{m} /\ (\pi\ (150\mathrm{MHz})(4\pi * 10^{-7}\mathrm{H/m})))} = \mathbf{6.095 \cdot 10^{-6} m}$$

Check $\boldsymbol{\delta} \ll \boldsymbol{r}$: 6 µm << 25 µm → skin depth is pronounced, can use approximation

$$\boldsymbol{A_{eff} = \pi(d\ \delta - \delta^2)}$$

$$= \pi\left(\left(5 \cdot 10^{-5}\mathrm{m}\right)6.095 \cdot 10^{-6}\mathrm{m} - \left(6.095 \cdot 10^{-6}\mathrm{m}\right)^2\right) = \mathbf{8.41 \cdot 10^{-10} m^2}$$

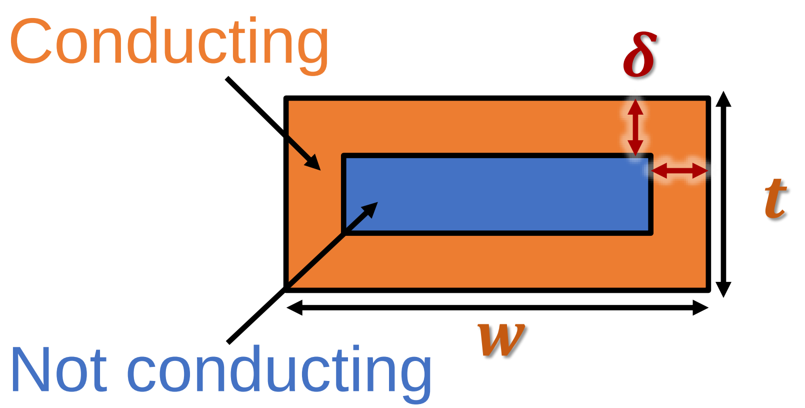
R = (2.2 x 10^{-8} Ω·m) · (0.01 m) / (8.41 x 10^{-10} m^2) = **0.26 Ω**

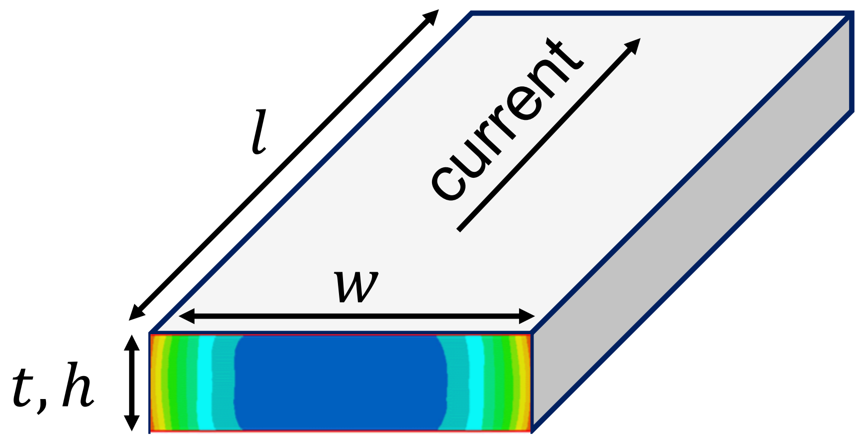
→ 2.3x higher than the DC resistance

*1 mil = 0.001 inch

AC Resistance of Rectangular Conductors
(First-Order Approximation)

- $R_{AC} = \rho l / A_{eff}$
- When $2\delta \ll$ w and t
  - $A_{eff} = wt - (w - 2\delta)(t - 2\delta)$
  - $A_{eff} = 2w\delta + 2t\delta - 4\delta^2$
  - $\boldsymbol{A_{eff} = 2\delta(w + t - 2\delta)}$
- $\delta = \sqrt{(\rho / (\pi f \mu))}$
- When $2\delta \geq w$ or t
  - $A_{eff} = wt$





Example: AC Resistance of PCB Trace

- 200-mm-long, 0.1-mm-wide PCB trace with 1 oz* copper
- ρ_{copper} = 1.7 x 10^{-8} Ω·m
- 1 oz copper = 1.37 mils = 0.00137 in = 0.034798 mm
- What is the AC resistance of the copper trace at 150 MHz?

$$\boldsymbol{R_{AC} = \rho l / A_{eff}}$$

$$\boldsymbol{\delta = \sqrt{(\rho \,/\, (\pi\, f\, \mu))}}$$

$$= \sqrt{(1.7 \cdot 10^{-8}\ \Omega \cdot \mathrm{m} / \ (\pi\ (150\mathrm{MHz})(4\pi * 10^{-7}\mathrm{H/m})))} = \mathbf{5.36 \cdot 10^{-6} m}$$

Check 2δ ≪ *w* and *t* : 10.7 µm << 100 µm and 35 µm

$$\boldsymbol{A_{eff} = 2\delta(w + t - 2\delta)}$$

$$= 2(5.36 \cdot 10^{-6}\mathrm{m})(1 \cdot 10^{-4}\mathrm{m} + 3.5 \cdot 10^{-5}\mathrm{m} - 2(5.36 \cdot 10^{-6}\mathrm{m}))$$

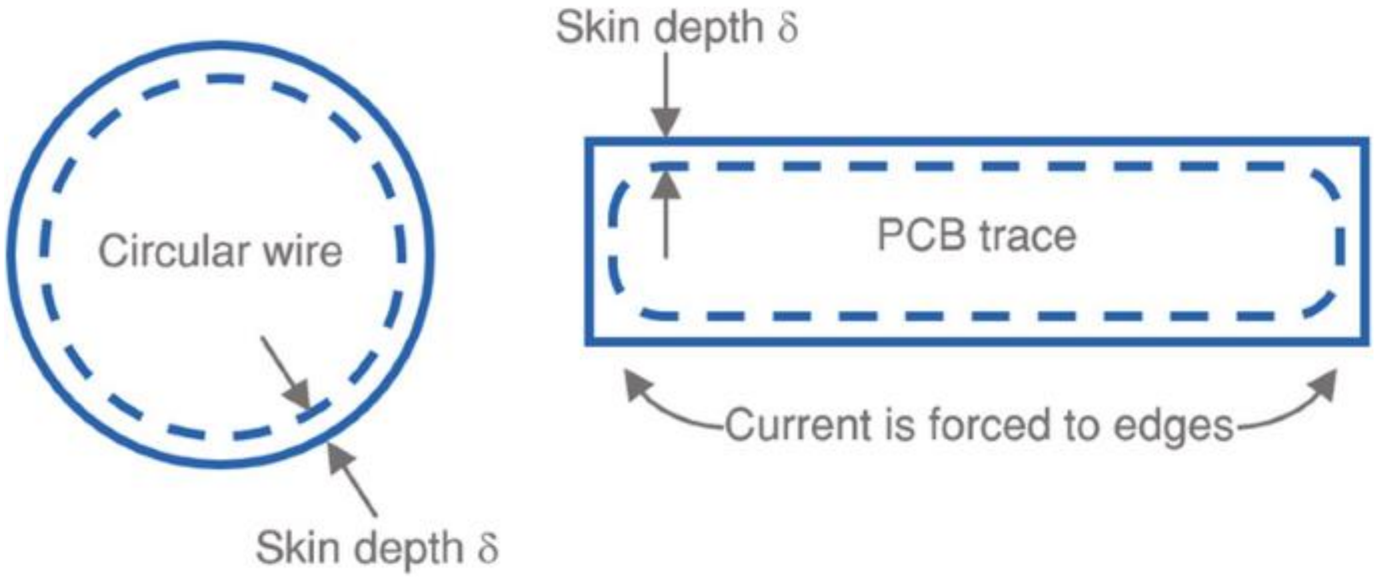
$$= \mathbf{1.33 \cdot 10^{-9} m^2}$$

R_{AC} = (1.7 x 10^{-8} Ω·m) · (0.2 m) / (1.33 x 10^{-9} m^2) = **2.56 Ω**

→ 2.6x higher than the DC resistance

AC Resistance: Circular vs Rectangular Cross-Sections

- Circles have less perimeter for a given cross-sectional area, so the skin effect is more pronounced
- Flat, wide conductors with rectangular cross-sections are preferred for high-frequency applications



Keller, R.B. (2023). Skin Effect. In: Design for Electromagnetic Compatibility--In a Nutshell. Springer, Cham. https://doi.org/10.1007/978-3-031-14186-7_10

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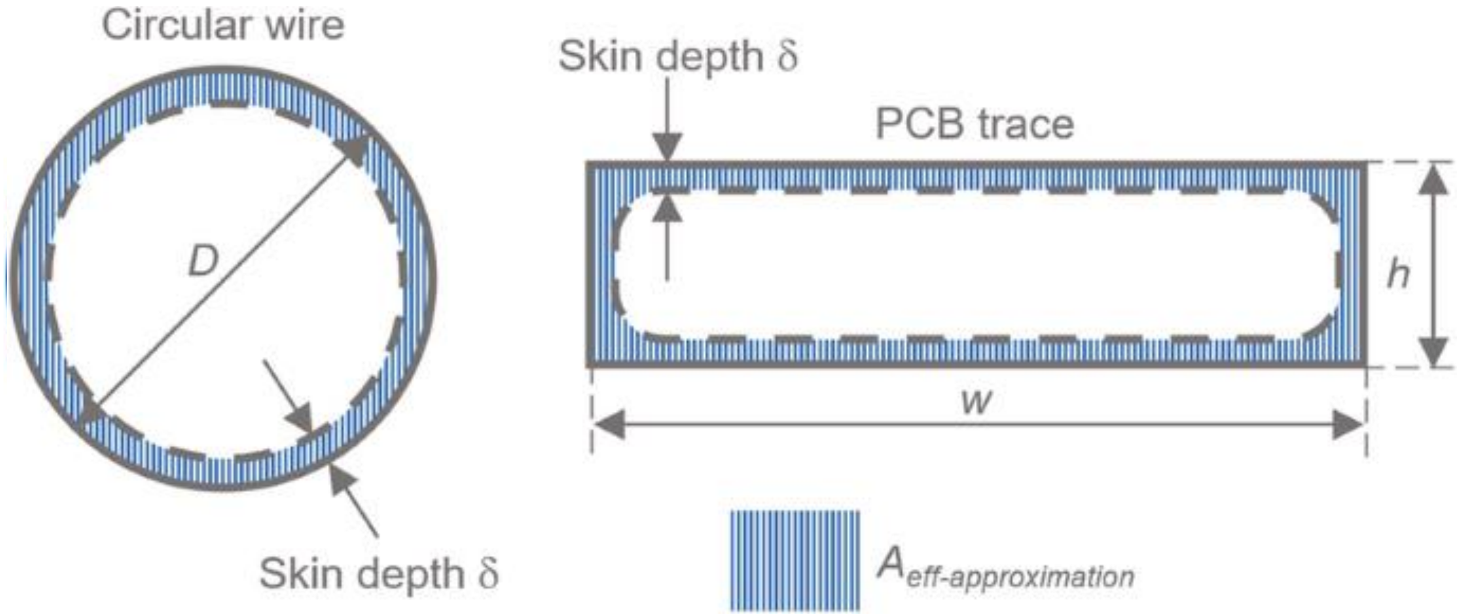

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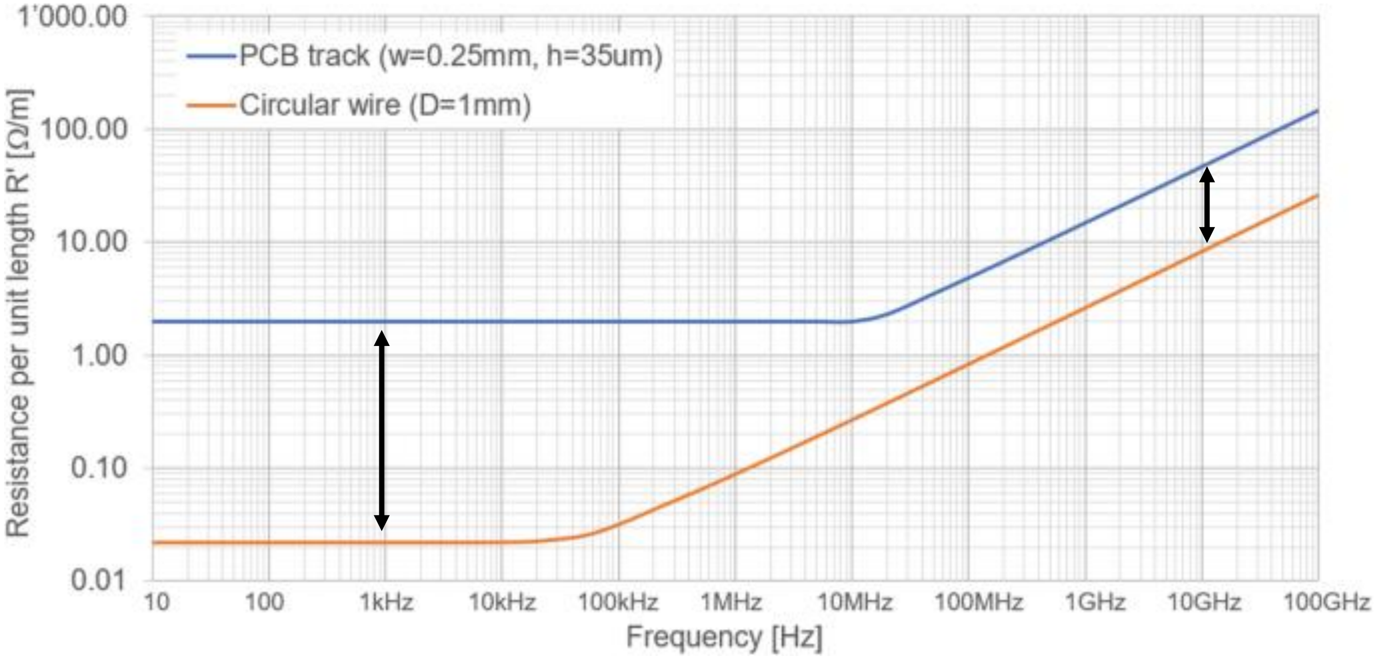

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Resistance Reduction Methods

DC Resistance

$$R_1 = \rho_0 l/A \cdot [1 + \alpha(T_1 - T_0)]$$

- Material
  - High electrical conductivity (low electrical resistivity)
- Geometry
  - Increase cross-sectional area
  - Decrease length
  - Parallel conductors
- Application
  - Decrease temperature
  - Decrease current

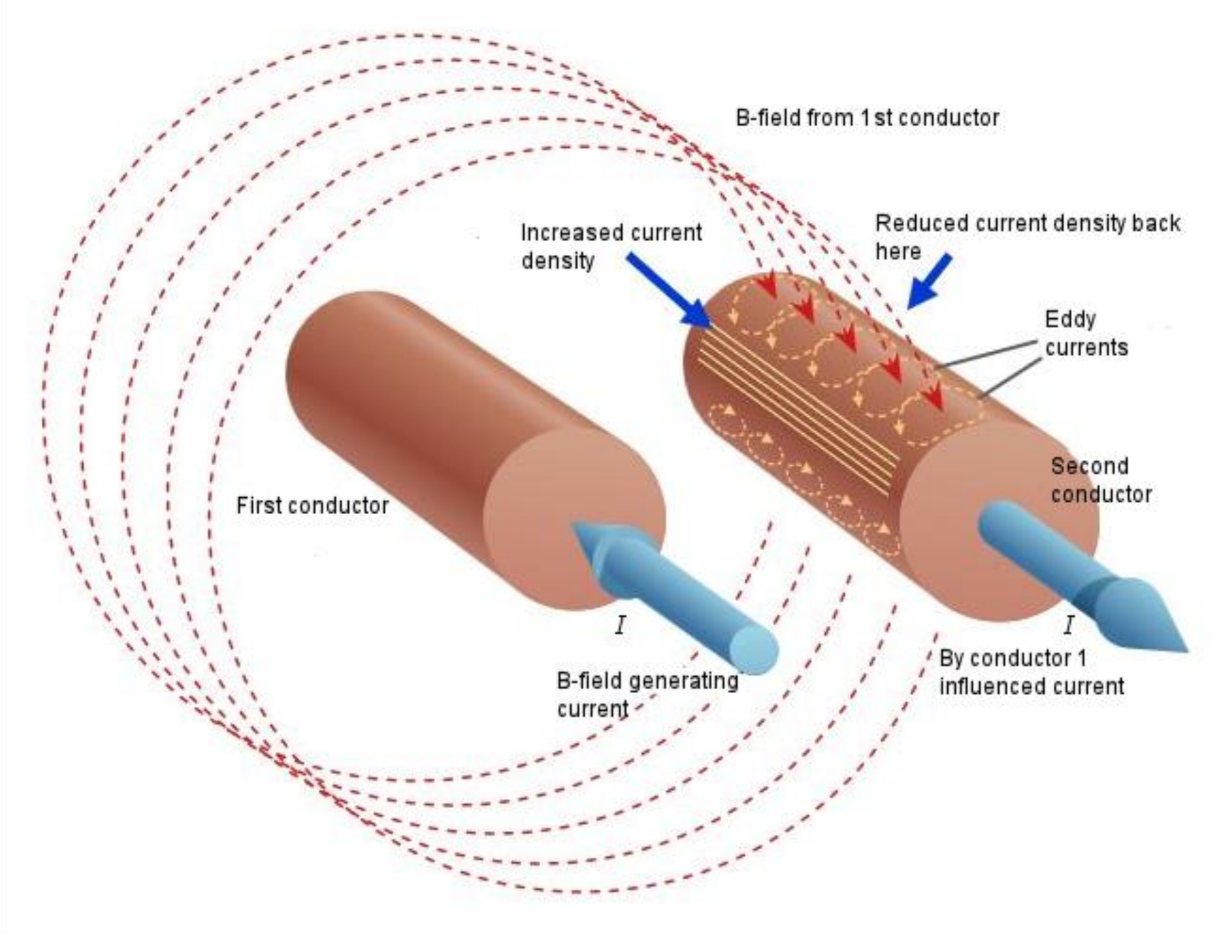
AC Resistance

$$\delta = \sqrt{(\rho \,/\, (\pi\, f\, \mu))}$$

- Material
  - Conductors with low permeability and low resistivity
- Geometry
  - Increase circumference/perimeter
  - Use wide flat conductor
  - Parallel smaller conductors
- Application
  - Decrease frequency

Skin and Proximity Effects

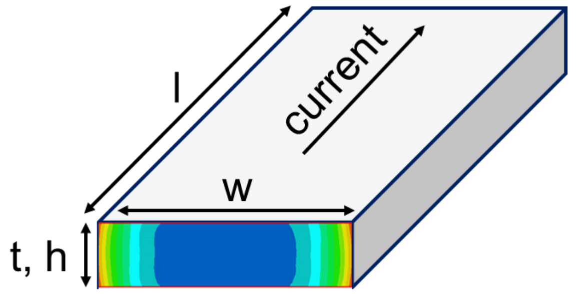
- Skin and proximity effects result from eddy currents
- When the magnetic field is generated by the conductor itself, this phenomena is called “skin effect”
- If the magnetic field is generated by an adjacent conductor, the phenomena is called “proximity effect”

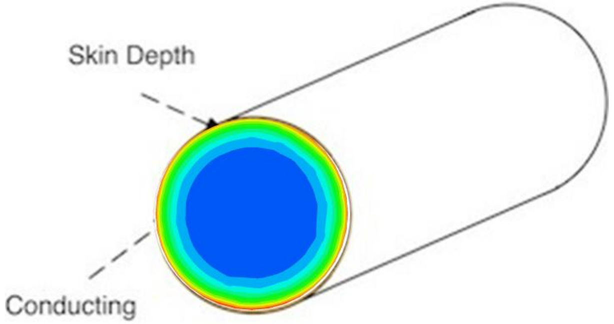


Proximity Effect

- Alternating current (AC) creates an alternating magnetic field around the conductor
- The alternating magnetic field induces eddy currents in adjacent conductors, changing the distribution of the current
- The result is current becoming concentrated in areas of the conductor closest to the first conductor
- Proximity effect significantly increases AC resistance of adjacent conductors
- The effect increases with frequency

Summary: Resistance





- DC resistance at room temperature: $R = \rho l / A$
- Temperature correction: $R = R_0 \cdot [1 + \alpha\,(T - T_0)]$
- AC resistance
  - Skin effect: $\delta = \sqrt{(\rho\,/\,(\pi\, f\, \mu))}$
    - AC current → alternating magnetic field → opposing EMF → opposes current flow in center of wire → majority of current flows through outer surface
  - Related to circumference, ρ, and μ of conductor, and frequency
  - Effective area for circular conductors: $A_{eff} = \pi(d\delta - \delta^2)$, when $\delta \ll r$
  - Effective area for rectangular conductors: $A_{eff} = 2\delta(w + t - 2\delta)$, when $2\delta \ll w$ and t

Next Class

- Electrical Design (Chapter 2)
  - Inductance