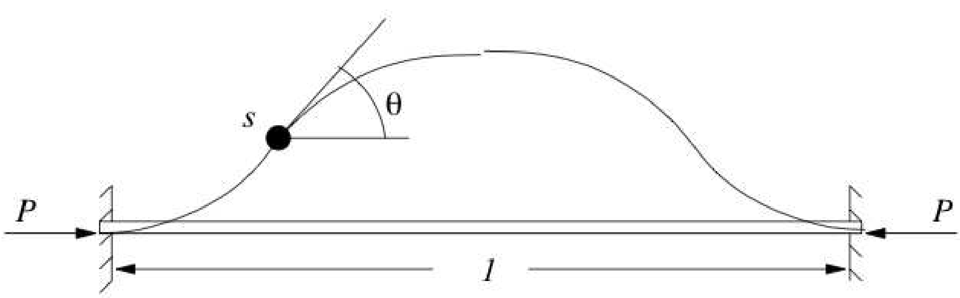
PHYS3071/7073 - Due Friday, 23rd of May, 2014 by 11am

Assignment 10 —- The Elastica Problem

Note: It is highly recommend that you have a go at worksheet 18/19 first before working on the assignment problem. Not only will this help you with the advanced part, finite differences will also play a crucial role for the solution of partial differential equations to be discussed in weeks 11 and 12.

Theory

Consider a thin, elastic, and inextensional rod subjected to an axial loading and clamped at its ends as shown in the following figure:



The transverse deformation of the rod can be described by the following differential equation:

$$\frac{d^2\theta}{ds^2} + P\sin\theta = 0 \quad \text{for} \quad 0 \leq s \leq 1 \tag{1}$$

with the boundary values $\theta(0) = \theta(1) = 0$ and $y(0) = y(1) = 0$. Here P is the external pressure applied to the rod and θ is the angle that the deformed rod makes with the x axis. $\theta(s) = 0$ is a possible solution to the differential equation. However this solution becomes increasingly unstable as P increases until the rod bends into a deformed shape as shown in the above picture. Symmetry considerations imply $\theta(s = 0.5) = 0$, so we only need to integrate the above differential equation between $s = 0$ and $s = 0.5$.

Once $\theta(s)$ has been determined, the Cartesian coordinates of a point on the deformed rod can be determined as the solution of the initial value problems

$$\frac{dx}{ds} = \cos\theta \quad \text{and} \quad \frac{dy}{ds} = \sin\theta \tag{2}$$

with initial conditions $x(s = 0) = 0$ and $y(s = 0) = 0$.

1 Problem 1/4

Write a C program `as10-surname-studentid-01.c` that asks the user for the load P and the stepsize Δs and the name of the file used for data output. Your program should then solve the boundary value problem of eq. 1 using the midpoint method together with the bisection method for root finding. You can use $d\theta_1/ds = 0.5$ and $d\theta_2/ds = 30.0$ as the starting values for the bisection method. Use a constant stepsize $\Delta s = 10^{-3}$ for all integrations.

Once $\theta(s)$ has been determined your program should integrate the two equations in eq. 2 to determine $x(s)$ and $y(s)$. Your program should also determine the maximum elongation y_{max} and print it to the screen. The output of your program should be a file containing s, $\theta(s)$, $x(s)$ and $y(s)$ for 100 values of s equally spaced between $s = 0$ and 1.

2 Problem 2/4

Take the program you wrote in Problem 1 and run it with the following P values:

1. P = 40.0
2. P = 60.0
3. P = 100.0

For each of these P values, copy and paste the input/output of your program into a file called `as10-output-surname-studentid.dat`.

3 Problem 3/4

Create a plot that shows the curves x vs. y for the three cases above. Make sure that the axes and curves in the plot are properly labeled, that the title of the plot is set to AS10 SURNAME STUDENTID and name the plot `as10-plot-surname-studentid.ps`

4 Problem 4/4 (Advanced)

For the same problem as before, write a program `as10-surname-studentid-02.c` that uses a finite difference method to calculate $\theta(s)$. The input for this program should be the load P and the number of steps N_{Step}. Note that this is a non-linear differential equation, so you need to use Newton-Raphson iteration together with finite differencing. As the initial guess for $\theta^{(0)}$ you can use $\theta(s) = \sin 2\pi s$. You can again use the `tridiag` function from the worksheet problem to invert your matrix equation. Use 1000 equally spaced points between $s = 0$ and $s = 1$ to solve for $\theta(s)$. Thin about a good criterion which determines if the Newton-Raphson method has converged to a stable solution and stop the iteration if it does not converge. The output of your program should again be a file containing s, $\theta(s)$, $x(s)$ and $y(s)$ for 1000 values of s equally spaced between $s = 0$ and 1. Make sure that for the same P values you get the same output as with the shooting method. Then use your `as10-surname-studentid-02.c` program and answer the following question: What is the minimum P value (to within ± 1) for which non-trivial solutions exist, i.e. what is the minimum load when the rod will start to bend? Add the answer to this question to your file `as10-output-surname-studentid.dat`.

Submission

The full set of files you will have when you are done should be:

- `as10-surname-studentid-1.c`
- `as10-plot-surname-studentid.ps`
- `as10-output-surname-studentid.dat`
- `as10-surname-studentid-02.c`

As always, to submit your assignment, mount your directory on the submission server, create a subdirectory called `as10` under your student ID, eg. `s1234567/as10/` and copy your files into this subdirectory. You must submit your files into this subdirectory. Be sure to follow the above submission instructions, otherwise you make the marking of the assignments unnecessary complicated for us.

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Grading Sheet – Assignment 10 The Elastica Problem

A: /30% **Function**: Does the program run and produce the correct output?

B: /10% **Usability**: Is the program easy to use? Are the input requirements and output formatting easy to understand?

C: /10% **Readability**: Is the program easy to read and comprehend? Is it well-commented? If the code is sufficiently complex, has it been broken up into manageable subroutines, each of which is well-documented?

D: /10% **Efficiency**: Does the program run efficiently? Is the coding clunky or unnecessarily complicated?

E: /10% **Analysis**: Were correct answers given to the questions asked in the assignment, and was the process used to obtain them reasonable and clearly explained?

F: /10% **Presentation (Plots)**: Do the plots clearly convey the results? Does each plot have an appropriate title? Are the axes and the plot items clearly labeled? Was the correct style (points or lines) used for each item?

G: /20% **Advanced part**: Grading for this part follows the same rules as detailed under points A to E.

Total Points: /100