PHYS3071/7073 - Assignment 4 - Due Friday 4th April 2014 by 11am

ODEs, Part 1 —- Rabbits and foxes

Theory

The Lotka-Volterra equations, also known as the *predator-prey equations*, are a pair of first-order differential equations frequently used to describe the dynamics of biological systems in which two species interact, one as a predator and the other as prey. The populations change with time according to the pair of equations:

$$\begin{aligned} \frac{dx}{dt} &= x(\alpha - \beta y) \\ \frac{dy}{dt} &= -y(\gamma - \delta x) \end{aligned} \tag{1}$$

where

- x is the number of prey (for example rabbits)
- y is the number of predators (for example foxes)
- α, β, γ and δ are constants that describe the interaction of the two species

1 Problem 1/4

Write a C program that solves the two differential equations given in equation (1) using the Euler method. Your program should ask the user for

- The initial numbers of animals x_0 and y_0[1]
- The time step size, Δt, of the Euler integrator
- A time step size, Δt_{out}, when output is created
- And the final time t_{fin};

The program should then write the solution in the form of a table with the time in the first column and the value of $x(t)$ and $y(t)$ in the second and third columns into an output file. It should also print the final values for $x(t)$ and $y(t)$ to the screen. The values of α, β, γ and δ should be defined as constants using the C `#define` statement and you should use the following values for them: $\alpha = 1.5$, $\beta = 0.1$, $\gamma = 3.0$ and $\delta = 0.1$.

[1] Treat the numbers of animals as floating point numbers.

2 Problem 2/4

Use the program you wrote in Problem 1 to compute and plot the solution to equation (1) for the case where there are 10 animals of each species initially, $\Delta t = 0.0001$, $\Delta t_{out} = 0.1$, and $t_{fin} = 10.0$. Produce two plots. The first plot, which should be named `as04-plot1-surname-studentid.ps` should plot $x(t)$ and $y(t)$ as a function of time. The second plot, named `as04-plot2-surname-studentid.ps` should plot $x(t)$ vs. $y(t)$. Make sure that the graphs are properly labeled and have appropriate range of values for the x and y axes.

Question:

- What do you notice about the solution? Explain how the behavior of $x(t)$ and $y(t)$ can be understood from the differential equations.

3 Problem 3/4 (Advanced)

A closer look at eq. (1) shows that there are two stationary points that lead to $dx/dt = 0$ and $dy/dt = 0$ at all times. How large are they? We are now going to modify the program such that it iteratively finds one of these stationary points. Your program should now ask the user only for initial values x_0 and y_0, the time step size Δt and an end time t_{fin}. It should not print out values for x and y to a file during the integration. While integrating the differential equation between 0 and t_{fin}, your program should record minimum and maximum values of both x and y. After each integration these should be printed to the screen together with the iteration number. The minimum and maximum values should also be used as starting values for x_0 and y_0 in the next iteration according to $x_0 = (x_{min} + x_{max})/2.0$ and $y_0 = (y_{min} + y_{max})/2.0$. Your program should iterate until the differences $|x_{min} - x_{max}|$ and $|y_{min} - y_{max}|$ are both smaller than a threshold value $\Delta = 10^{-5}$. If this condition is not reached after 30 iterations, your program should stop with an error message. Otherwise it should print the values of x and y for the stable point it has found to the screen and stop.

4 Problem 4/4 (Advanced)

Run the program developed under problem 3 for the same starting values as in problem 2. If a solution is found, write down the values for x_{stat} and y_{stat} that you get. Do they agree with the theoretical answer? Write your answers to the questions in problems 2 to 4 in a text file named `as04-answers-surname-studentid.txt`.

The full set of files you will have when you are finished should be:

- `as04-problem12-surname-studentid.c`
- `as04-problem34-surname-studentid.c`
- `as04-plot1-surname-studentid.ps`
- `as04-plot2-surname-studentid.ps`
- `as04-answers-surname-studentid.txt`

Submission

To submit your assignment, upload the above files to your directory, eg. s1234567/as04/. Name the new directory `as04` and submit the files into this subdirectory. Upload the C programs and the text file with COURSENUMBER AS04 SURNAME STUDENTID (case doesn't matter) at the top. Your postscript figures should have the same information in their titles, i.e. use `set title <COURSENUMBER> AS04 <SURNAME> <STUDENTID>`. **It is important that you follow these instructions since otherwise marking your assignment becomes a mess and we reserve the right to deduct marks if this happens.**

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Grading Sheet – Assignment 4: ODEs, Part 1 —- Rabbits and foxes

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A: /30% **Function**: Does the program run and produce the correct output?

B: /10% **Usability**: Is the program easy to use? Are the input requirements and output formatting easy to understand?

C: /10% **Readability**: Is the program easy to read and comprehend? Is it well-commented? If the code is sufficiently complex, has it been broken up into manageable subroutines, each of which is well-documented?

D: /10% **Efficiency**: Does the program run efficiently? Is the coding clunky or unnecessarily complicated?

E: /10% **Presentation (Plots)**: Do the plots clearly convey the results? Does each plot have an appropriate title? Are the axes and the plot items clearly labeled? Was the correct style (points or lines) used for each item?

F: /10% **Analysis**: Were correct answers given to the questions asked in the assignment, and was the process used to obtain them reasonable and clearly explained?

G: /20% **Advanced part**: Grading for this part follows the same rules as detailed under points A to F.

Total Points: /100