THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1 **ADVANCED MATHEMATICS 1**

(For Both Private and School Candidates)

Time: 3 Hours **Year: 2022**

Instructions

1. This paper consists of **ten (10)** questions.
2. Answer **all** questions.
3. Each question carries **ten (10)** marks.
4. All necessary working and answers of each question done must be shown clearly.
5. NECTA's mathematical tables and non-programmable calculators may be used.
6. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).

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1. (a) By using a non-programmable calculator, find the value of the expression $\frac{\sqrt{\pi^{\cos 60^\circ}}}{3.14} \times \frac{\left(e^{2.15} + \tan^{-1}(\ln 0.25)\right)}{\sqrt{\pi d^{n\pi}}}$ correct to 3 decimal places.

(b) If $h = 3$, $p = 500$, $g = 10$ and $m = 0.25$, use a non-programmable calculator to find the value of $T = \sqrt{\frac{2hp}{g(p - mg)^{\frac{1}{3}}}}$ correctly to nine significant figures.

(c) By using the statistical functions of a non-programmable calculator and the following frequency distribution table, find the mean, variance and standard deviation correctly to three decimal places.

Values	250	230	210	190	170	150	130	110	90	70	50	30
Frequency	4	11	5	6	21	40	26	4	8	35	28	12

2. (a) If $2\cosh 2x + 10\sinh 2x = 5$, obtain the value of x in logarithmic form.

(b) If $\theta = \ln(\tan\phi)$, show that $\tanh\theta = -\cos 2\phi$.

(c) By using the integration by parts technique, evaluate the integral $\int_0^1 x\sin 2x dx$ correct to 7 decimal places.

3. (a) Mr. Safari wants 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5 units of A, 2 units of B and 1 unit of C per jar and each jar is sold at 3,000/=. On the other hand a dry product contains 1 unit of A, 2 units of B and 4 units of C per carton and each carton is sold at 2,000/=. If x and y are the number of jars of liquid products and cartons of dry products respectively, formulate a linear programming problem to minimize the cost.

(b) A cement dealer has two depots; D_1 and D_2 holding 180 tons and 250 tons of cement respectively. The customers C_1 and C_2 have ordered 200 and 150 tons respectively. The transport cost per ton from each depot to each customer are as shown in the following table:

	Customer	
From	**C_1**	**C_2**
Depot D_1	1,000/=	1,500/=
Depot D_2	2,000/=	1,800/=

(i) How many tons of cement should be delivered to each customer in order to minimize the transport cost?

(ii) After meeting the orders, how many tons of cement will remain at D_2?

4. The masses of a sample of new potatoes were measured to the nearest gram and are summarized in the following table:

Mass (g)	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 89	90 - 99
Frequency	2	14	21	73	42	13	9	4	2

Determine the following measures of dispersion correct to three decimal places:

(a) first and third quartiles.

(b) semi-interquartile range.

(c) seventh decile.

(d) 80^{th} percentile.

(e) variance and standard deviation.

5. (a) If sets A and B are defined by $A=\{x\in\Re:-1\leq x\leq 2\}$ and $B=\{x\in\Re:2\leq x<5\}$, find $A-B$ in the same set notation.

(b) Use laws of algebra of sets to simplify $[(A-B)\cup(A\cap B)]-[A\cup(A\cap B)]$.

(c) In a class of 17 girls and 15 boys, 22 play handball, 16 play basketball, 12 of the boys play handball, 11 of the boys play basketball, 10 of the boys play both basketball and handball, 3 of the girls play neither of the two games. Use Venn diagram to determine;

(i) the number of girls who play both games and

(ii) the number of participants who play at least one game.

6. (a) Given that $f(x)=2^x$ and $g(x)=\log_2 x$,

(i) find the domain and range of $f(x)$ and $g(x)$.

(ii) draw the graphs of $f(x)$ and $g(x)$ on the same axes. Comment on the resulting graphs.

(b) Given that $f(x)=\dfrac{2x-x^2}{x^2-2x-3}$,

(i) write the values of x for which $f(x)=0$ and the values of x for which $f(x)>0$.

(ii) show that $f(x)\to -1$ as $x\to\pm\infty$.

(iii) sketch the graph of $f(x)$, showing particularly where the curve crosses the x-axis and how it approaches its asymptotes.

7. (a) Verify that $x^2 - 2x - 1 = 0$ has a root in the range $2 \le x \le 3$.

(b) By using the Secant method, perform four iterations to obtain an approximation for the root of the equation in 7 (a) correct to three decimal places.

(c) (i) Using the Trapezoidal rule with 5 strips, obtain an approximation to $\int_0^1 x^2 e^x dx$ correct to four significant figures.

(ii) By using the integration by parts technique, evaluate the exact value of the definite integral $\int_0^1 x^2 e^x dx$ correct to four significant figures.

(iii) Use the results obtained in (c) (i) and (ii) to find an absolute error in the value of $\int_0^1 x^2 e^x dx$.

8. (a) If the perpendicular distance of point $(1, k)$ to the line $6x + 9 = 8y$ is 5.5, determine the possible value of k.

(b) The curves $y = 2x^2 - 3$ and $y = x^2 - 5x + 3$ intersect at points U and W, of which W is in the fourth quadrant. Find the tangent of the acute angle between these curves at W.

(c) Find the equation of a circle which passes through points $A(0, 1)$, $B(4, 3)$ and $C(1, -1)$. (Write your answer in the form $x^2 + y^2 + 2gx + 2fy + c = 0$).

9. (a) If $I_m = \int \sin^m x\, dx$, show that $I_m = \frac{1}{m}\left(-\cos x \sin^{m-1} x + (m-1) I_{m-2}\right)$, hence find $\int \sin^3 x dx$.

(b) Evaluate $\int_0^1 \frac{1}{(x+1)(x^2+2x+2)} dx$ correct to three decimal places.

10. (a) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x.

(b) (i) Use Maclaurin's theorem to find the series expansion of $\ln(1+x)$ up to the term containing x^4.

(ii) By using the results obtained in (b) (i), compute $\ln(1.02)$ to four decimal places.

(c) If $f(x, y) = \sin xy$, find $\frac{\partial^2 f}{\partial x \partial y}$.

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142/2 **ADVANCED MATHEMATICS 2**

(For Both School and Private Candidates)

Time: 3 Hours **Year: 2022**

Instructions

1. This paper consists sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. Section A carries **sixty (60)** marks and section B carries **forty (40)** marks.
4. All work done in answering each question must be shown clearly.
5. NECTA's Mathematical tables and non-programmable calculators may be used.
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SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) The time taken by John to deliver milk to the High Street is normally distributed with mean 12 minutes and standard deviation 2 minutes. If he delivers milk every day, estimate the number of days during the year when he takes longer than 17 minutes. (1 year = 365 days)

   (b) Suppose that a group of people in a village attending hospital has been categorized according to the incidence of two diseases

Sex	**Malaria**	**Typhoid**
Male	16	12
Female	12	10

   Find the probability that the person chosen is a female given that the person is suffering from malaria.

   (c) In how many ways can a hand of 4 cards be chosen from an ordinary pack of 52 playing cards?

2. (a) Write the converse and inverse of the statement "If you score an A grade in a logic test, then I will buy you a new car" in words and symbolic form.

   (b) Using a truth table, examine whether $[(\sim p)\rightarrow(\sim q)]\wedge(p\rightarrow q)$ is equivalent to $(q\rightarrow p)\wedge(p\rightarrow q)$.

   (c) Use laws of algebra of propositions to simplify $[p\wedge(p\vee q)]\vee[q\wedge(\sim(p\wedge q))]$.

3. (a) Find the work done by force $\underline{F}=\underline{i}+2\underline{j}+\underline{k}$ moving an object at a distance of 7 m in the direction of the vector $\underline{r}=3\underline{i}+2\underline{j}+4\underline{k}$.

   (b) If P and Q are points $P(3,-4,6)$ and $Q(1,-3,8)$ respectively, find a unit vector parallel to the displacement vector $\overrightarrow{PQ}$.

   (c) The position vectors of points A and B are $\underline{a}$ and $\underline{b}$ respectively. If point C divides $\overrightarrow{AB}$ internally in the ratio of $2:1$, D divides $\overrightarrow{AB}$ externally in the ratio of $1:4$ and E divides $\overrightarrow{CD}$ internally in the ratio of $2:1$, find the position vectors of C, D and E in terms of $\underline{a}$ and $\underline{b}$.

4. (a) Express $\sqrt{1+i}$ in polar form.

   (b) Using the results in part (a), show that $\tan\frac{\pi}{8}=\sqrt{2}-1$.

   (c) If $z_1=r_1(\cos\theta_1+i\sin\theta_1)$ and $z_2=r_2(\cos\theta_2+i\sin\theta_2)$; Prove that

   $$Arg\left(\frac{z_1}{z_2}\right)=Arg(z_1)-Arg(z_2).$$

   (d) The complex numbers $z_1=\frac{c}{1+i}$ and $z_2=\frac{d}{1+2i}$ where $c,d\in\mathbb{R}$ are such that $z_1+z_2=1$, find the values of c and d.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

5. (a) For all values of α show that $\dfrac{\sin 3\alpha}{\sin\alpha} - \dfrac{\cos 3\alpha}{\cos\alpha} = 2$.

   (b) Prove that $\dfrac{\sin x + \sin 2x + \sin 3x}{\cos x + \cos 2x + \cos 3x} = \tan 2x$.

   (c) Solve for β in the trigonometric equation $\tan^{-1}\left(\dfrac{\beta-1}{\beta-2}\right) + \tan^{-1}\left(\dfrac{\beta+1}{\beta+2}\right) = \dfrac{\pi}{4}$.

   (d) Rewrite $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta-\alpha)$, hence solve the equation $4\cos\theta + 3\sin\theta = \dfrac{5\sqrt{2}}{2}$ in the interval $\dfrac{\pi}{2} \le \theta \le 2\pi$.

6. (a) If the coefficients of x and x^2 in the expansion of $\dfrac{1+px+qx^2}{(1-x)^2}$ are zero, find the numerical values of p and q

   (b) Use the principle of mathematical induction to prove that for every positive integer, $3^{2n-2} + 2^{6n}$ is divisible by 5.

   (c) Given that $P(x) = 2x^3 + 7x^2 - 5$, use the synthetic method to find the quotient and remainder when $P(x)$ is divided by $x+3$.

   (d) Find the determinant and inverse of the matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 5 & 2 \\ 1 & -1 & 1 \end{pmatrix}$, hence solve the simultaneous equations $\begin{cases} 2x + y = 4 \\ x + 5y + 2z = 7 \\ x - y + z = 1 \end{cases}$.

7. (a) (i) Determine the most general function $M(x,y)$ such that the differential equation $M(x,y)dx + \left(2x^2y^3 + x^4y\right)dy = 0$ is exact.

   (ii) By separating the variables, solve the differential equation $(xy + x)dx - \left(x^2y^2 + x^2 + y^2 + 1\right)dy = 0$

   (b) Find the general solution of the differential equation $\cos x \dfrac{d^2y}{dx^2} - \sin x \dfrac{dy}{dx} = 0$.

   (c) A liquid of $72\,^0C$ placed in a room at $25\,^0C$ has a temperature of $65\,^0C$ after 5 minutes. Find its temperature after further 10 minute.

   (d) Formulate a differential equation of a circle which passes through the origin and whose centre lies on the y-axis.

8. (a) Find the coordinates of the foci, the vertices, the eccentricity and the length of the latus rectum of the hyperbola $16x^2 - 9y^2 = 576$.

(b) (i) Determine the equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $P(a\cos\theta, b\sin\theta)$.

(ii) If the normal in part (b) (i) meets the x-axis at A and the y-axis at B, find the area of the triangle AOB where O is the origin.

(c) Show that the equation of the tangent to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is $x - ty + at^2 = 0$.

(d) (i) Change the Cartesian equation $(x^2 + y^2)^3 - 2xy(x^2 - y^2)$ into a polar equation.

(ii) Sketch the graph of $r = 1 - 2\cos\theta$ from $\theta = 0$ to $\theta = 2\pi$.