THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1 **ADVANCED MATHEMATICS 1**
(For Both Private and School Candidates)

Time: 3 Hours **Year: 2023**

Instructions

1. This paper consists of **ten (10)** questions.
2. Answer **all** the questions. Each question carries **ten (10)** marks.
3. All necessary working and answers of each question done must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. All writing must be in **blue** or **black** ink **except** drawing which must be in pencil.
6. Cellular phones and any unauthorised materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).

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1. (a) By using a scientific calculator, approximate the mean and standard deviation of the constants π, $\sqrt{2}$, e, $\sqrt{3}$, 1.414213, 2.718282, 3.1415, 1.732051 correct to six decimal places.

(b) Use a scientific calculator to compute the value of the following expressions correct to six significant figures:

(i) $$(21)\times\begin{vmatrix} 60 & 18 & 49 \\ 50 & 0 & -14 \\ 20 & -6 & -20 \end{vmatrix}\times\left(e^{\pi}\right)\times\left(\log_8 16\right)\times\left(2.7\times10^{-8}\right).$$

(ii) $$\left(\frac{\ln 612}{\ln 121+4\ln 2}\right)^{\frac{1}{4}}\ln\left(\frac{\ln\left(\frac{22}{7}\right)}{\log\left(\frac{22}{7}\right)}\right)^{\frac{1}{8}}.$$

2. (a) If $m\sinh x+n\cosh x=h$ has equal roots, express h in terms of n and m.

(b) Prove that $(\cosh x-\cosh y)^2-(\sinh x-\sinh y)^2=-4\sinh^2\left(\frac{x-y}{2}\right)$.

(c) Integrate the integrand $\frac{1}{\sqrt{x^2+2x+10}}$ with respect to x.

3. (a) A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make each ring and 30 minutes to make each chain. The maximum number of hours available per day is 16. The profit on the ring is shs. 3000 and that on the chain is 1900. If x and y are the numbers of rings and chains respectively, formulate the linear programming problem.

(b) A company owns two mines, A and B. Mine A produces 1 ton of high grade ore, 3 tons of medium grade ore and 5 tons of low grade ore each day while mine B produces 2 tons of each of the three grades of ore each day. The company needs 80 tons of high grade ore, 160 tons of medium grade ore and 200 tons of low grade ore. If it costs sh. 200,000/= per day to operate each mine, how many days should each mine be operated?

4. The masses of 36 stones in grams are as shown in the following table:

Mass (g)	50 - 100	100 - 150	150 - 200	200 - 250	250 - 300	300 - 350
Frequency	3	5	10	8	6	4

(a) If the assumed mean (A) is 225, use the deviation method to find the mean and variance of the distribution correct to 3 decimal places.

(b) Find the first and third quartiles of the distribution correct to three significant figures.

(c) Find the 90^{th} percentile of the distribution correct to 3 significant figures.

5. (a) By using the laws of algebra of sets, simplify the following sets' expressions:

(i) $(A \cap B') \cup (A' \cup B')$.

(ii) $(A \cup B)' \cap (A \cap B)'$.

(b) A survey of 500 students shows that 83 students study Economics and Geography, 63 study Geography and Mathematics, 217 study Economics and Mathematics, 295 study Mathematics, 186 study Geography and 329 study Economics. If every student studies at least one course among Economics, Geography and Mathematics, use Venn diagram to find the number of students who study;

(i) all three subjects,

(ii) Economics or Mathematics but not Geography.

6. (a) (i) Draw the curves of $f(x) = 3^x$ and $g(x) = \log_3 x$ on the same pair of axis.

(ii) How is $f(x)$ related to $g(x)$ in 6 (i)?

(b) Given that $f(x) = \dfrac{x^2 - 2x - 3}{x^2 - 4}$,

(i) find the vertical and horizontal asymptotes for $f(x)$,

(ii) sketch the graph of $f(x)$.

7. (a) The area enclosed between $x^2 + y^2 = 100$, $x \geq 0$ and $y \geq 0$ is divided into ten equal intervals. Use the Trapezoidal and Simpson's rules to approximate the value of π correct to three significant figures.

(b) If the actual value of π is 3.14, state which rule in part (a) gives a better approximation.

8. (a) Find the coordinates of the centre and radius of the circle $2x^2 + 2y^2 + 8x + 12y - 136 = 0$.

(b) Find the perpendicular distance from the centre of the circle in part (a) to the line $12x + 16y + 12 = 0$.

(c) The coordinates of points P and Q are (x_1, y_1) and (x_2, y_2) respectively. Find the coordinates of a point R which divides the line PQ internally in the ratio $m_1 : m_2$.

9. (a) Find the following indefinite integrals;

(i) $\displaystyle\int \frac{x}{\sqrt{x+1}} dx$,

(ii) $\displaystyle\int \theta^2 e^{2\theta} d\theta$.

(b) Find the volume of the solid generated by rotating about the x-axis the area enclosed by $y^2 - x^3 = 4$ and $y = 0$ between $x = 0$ and $x = 3$.

10. (a) Differentiate $y=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ with respect to x.

(b) Use the second derivative test to investigate the stationary values of the function $y=3xe^{-x}$.

(c) The air pollution index p in a certain city is determined by the amount of solid waste (x) and noxious gas (y) in the air. If the index is given by the equation $p=x^2+2xy+4xy^2$, obtain the following partial derivatives at $(10, 5)$:

(i) $\frac{\partial p}{\partial x}$,

(ii) $\frac{\partial p}{\partial y}$.

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142/2

ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours **Year : 2023**

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. Section A carries **sixty (60)** marks and section B carries **forty (40)** marks.
4. All work done in answering each question must be shown clearly.
5. NECTA's mathematical tables and non-programmable calculators may be used.
6. All writing must be in **blue** or **black** ink **except** drawing which must be in pencil.
7. Cellular phones and any unauthorised materials are **not** allowed in the examination room.
8. Write your **Examination Number** on every page of your answer booklet(s).

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SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) In a school garden, 15 percent of tomatoes are on average defective. Prepare the probability distribution table of obtaining 0, 1, 2, 3, 4 and 5 defective tomatoes in a random batch of 20 tomatoes using:
   (i) Binomial distribution,
   (ii) Poison distribution.

   (b) Compute the mean and standard deviation of the two cases in part (a).

   (c) The mean and standard deviation recorded for 100 students in a senior Mathematics contest examination for the year 2014 were 64 and 16 respectively. Suppose their marks are normally distributed, find the number of students who scored between 30% and 70% inclusive.

2. (a) By using laws of propositions of algebra, simplify $(P \rightarrow (Q \vee \sim R)) \rightarrow (P \wedge Q)$.

   (b) Use the truth table to verify whether $\sim(P \leftrightarrow Q) \equiv (P \wedge \sim Q)$ or not.

   (c) Test the validity of the argument: "Every time we celebrate my mother's birthday, I always bring her flowers. It is my mother's birthday or I wake up late. I did not bring her flowers. Therefore, I woke up late."

3. (a) Show that the position vectors $2\underline{i}-\underline{j}+\underline{k}$, $\underline{i}-3\underline{j}-5\underline{k}$ and $3\underline{i}-4\underline{j}-4\underline{k}$ mark the vertices of a right angled triangle.

   (b) The vector $\overrightarrow{PQ}$ has the magnitude of 5 units. If it is inclined at an angle of 150° to the $x-$axis, express $\overrightarrow{PQ}$ in the form $a\underline{i}+b\underline{j}$ where $a,b \in \Re$.

   (c) A certain particle is displaced by the forces $\underline{F}_1 = 2\underline{i}-5\underline{j}+6\underline{k}$ and $\underline{F}_2 = -\underline{i}-2\underline{j}-\underline{k}$ from point A to B. If the position vectors of points A and B are $4\underline{i}-3\underline{j}-2\underline{k}$ and $6\underline{i}-\underline{j}-3\underline{k}$ respectively, determine the work done by forces on the particle.

4. (a) If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, show that $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$. Hence, verify that $\left|\frac{z-2}{z+3i}\right| = 4$ represents a circle.

   (b) If α and β are two roots of the equation $Z^2+4Z+8=0$, without solving the equation $Z^2+4Z+8=0$, express $\frac{\alpha+\beta+4i}{\alpha\beta+8i}$ in its simplest form.

   (c) Find all the complex roots of $z^3 = 1$.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

5. (a) Factorize $\cos\theta - \cos 3\theta - \cos 5\theta + \cos 7\theta$.

(b) Show that $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B}$. Hence, find $\tan y$ if $\tan(2x+y)=2$ and $\tan 2x = \dfrac{3}{2}$.

(c) Given that $\sin x = \dfrac{3}{5}$ and $\cos y = \dfrac{24}{25}$, where angle x is obtuse and angle y is acute, find the exact values of $\cos(x+y)$ and $\cot(x-y)$.

(d) Express $3\sin x - 2\cos x$ in the form $R\sin(x-\beta)$.

(e) Use the results obtained in part (d) to solve the equation $3\sin x - 2\cos x = 1$.

6. (a) Use the Binomial theorem to expand $\dfrac{1}{(4-x)^2}$ in ascending powers of x up to the term containing x^3.

(b) Find the partial fractions of the expression $\dfrac{x^2-2x+1}{(x+1)^2}$.

(c) If α, β and μ are the roots of the equation $2x^3 - x^2 + 1 = 0$, find the equation whose roots are $\alpha+1$, $\beta+1$ and $\mu+1$.

(d) Without using a calculator, find the inverse of $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & -2 \end{pmatrix}$.

(e) Use the inverse obtained in part (d) to solve the system of the simultaneous equations
$$\begin{cases} x+y+z=2 \\ 2x-3y+4z=-4. \\ 3x-2y+4z=-9 \end{cases}$$

7. (a) Show that $y = Ae^{2x}\cos(3x+\varepsilon)$ is a solution of the differential equation $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 13y = 0$, where ε is an arbitrary constant.

(b) Find the general solution of the differential equation $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 6e^x + \sin x$.

(c) Solve the differential equation $(2x-1)\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} = 0$, given that when $x=0$, $y=2$ and $\dfrac{dy}{dx} = 3$.

(d) The rate of increase in the population of a certain village is proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in the year 1999 and 25,000 in the year 2004, what was the population of the village in 2009?

8. (a) A point moves so that its distance from the point $(3,2)$ is half its distance from the line $2x+3y=1$.

(i) Show that the locus of the point is a circle.

(ii) What is the centre and radius of the circle?

(b) Show that the equation of a normal at the point $(a\cos\theta, b\sin\theta)$ to the ellipse $b^2x^2+a^2y^2=a^2b^2$ is $ax\sin\theta - by\cos\theta = (a^2-b^2)\sin\theta\cos\theta$.

(c) If the normal at P in part (b) meets the x-axis at Q and the y-axis at R, find the greatest value of the area of the triangle OQR, where O is the origin.

(d) Sketch the graph of $r^2 = a^2\sin 2\theta$.