A Simulation Analysis on the Existence of Network Traffic Flow Equilibria

Shu Lin, *Member, IEEE*, Qing-Jie Kong, *Member, IEEE*, and Qingming Huang, *Senior Member, IEEE*

***Abstract*—A macroscopic fundamental diagram (MFD) can be applied to design simple and effective traffic network control strategies to prevent congestion in an urban traffic network. Network flow equilibria on MFDs are important properties for designing effective network controllers for urban traffic networks. In order to verify the existence of equilibria on MFDs, microscopic simulations are run for an urban traffic network with various network input traffic flows and traffic-signal control strategies. The simulation results show that the traffic network can reach its network flow equilibria if the traffic flows in the network are in the linear region (the free-flow region) of the MFD, although the network flows are heterogeneous; however, network flow equilibria do not exist for the nonlinear region (the non-free-flow region) of the MFD unless the traffic network flows are homogeneous. Properly designed traffic-signal control strategies can improve the degree of traffic flow homogeneity; thus, network flow equilibria are easy to be achieved under a bounded network input flow.**

***Index Terms*—Macroscopic fundamental diagram (MFD), traffic flow equilibrium, traffic network control.**

I. Introduction

A macroscopic fundamental diagram (MFD) is an inherent aggregate-level characteristic of traffic movement in cities, which illustrates the relationship between the aggregated average network traffic flow (travel production) and the number of vehicles in the network (network accumulation) [1], [2]. It is derived on the basis of the fundamental diagrams (FDs, e.g., the flow-density relationship) of single roads [3], [4]. In 2008, Daganzo and Geroliminis [1] gave a theoretical approximation to analyze the existence of MFDs for traffic networks with many intersections. At the same time, empirical results were generated by Geroliminis and Daganzo [2] through real traffic statistic data from Yokohama, Japan, which verified the existence of MFDs for urban traffic networks. In 2009, Helbing [5] gave another version of analysis for the existence of an urban FD. According to the work in [2], a well-defined MFD relationship holds for a traffic network and is irrelevant with time if the traffic flows homogeneously scatter in the traffic network and the network input traffic demands slowly change with time. In [6], the factors that influence the shape of MFDs were concluded through simulations, i.e., the structure of traffic networks, the heterogeneity of the networks, the traffic demands, and the control strategies. Moreover, Buisson and Ladier found that the heterogeneity of traffic networks was responsible for the hysteresis phenomena in MFDs [7]. As a result, a well-defined MFD curve only exists for a traffic network when the traffic flows are uniformly scattered in the traffic network; otherwise, hysteresis phenomena occur on the MFDs, e.g., in the case of congestion, for the sake of traffic heterogeneity.

The result that the MFD exists independently of the network traffic demands suggests that it is possible to predict the network flow trend without the knowledge of the detailed traveling behaviors of the vehicles inside the traffic network. Therefore, the MFD properties can be used to anticipate the effect of the traffic management policies for the traffic networks and can be further applied to design traffic network control strategies without the information for the uncertainties inside traffic networks. In this regard, the MFD has been used as a performance indicator for the estimation of the mobility for the signalized traffic networks under different control strategies [8]–[11]. In addition, control strategies have been proposed based on MFDs to regulate the output flow of the aggregated network and improve the network mobility by adjusting the ratio of the network inflow (i.e., adjusting the number of vehicles in the network). Geroliminis *et al.* proposed a model predictive control (MPC) approach for a two-region urban network based on the subnetwork MFD models [12], and Haddad and Geroliminis analyzed and proved the stability of the control approach for the two-region urban network [13]. Ekbatani *et al.* proposed an easy-to-implement feedback control strategy for regulating the inflow ratio of an urban network to prevent the protected urban region from being congested and to increase the traffic mobility within the region [14], [15]. In [16], a high-level traffic flow coordinator was designed for a multilevel urban traffic network controller based on the MFD features of the subnetworks.

MFD characteristics are important for designing high-level and efficient traffic network control strategies. Thus, it is necessary to further investigate the properties of traffic network MFDs. A traffic network equilibrium is a network traffic state where the average traffic flow reaches an equilibrium point and keeps on the equilibrium point if the network input flow

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S. Lin is with the School of Computer and Control Engineering, University of Chinese Academy of Sciences, Beijing 100049, China (e-mail: lisashulin@gmail.com).

Q.-J. Kong is with the State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China (e-mail: qingjie.kong@ia.ac.cn).

Q. Huang is with the School of Computer and Control Engineering, University of Chinese Academy of Sciences, Beijing 100049, China and also with the Institute of Computing Technology, Chinese Academy of Sciences, Beijing 100190, China (e-mail: qmhuang@ucas.ac.cn).

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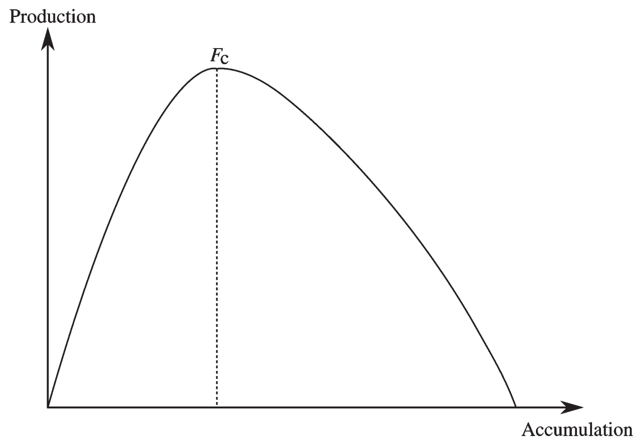

Fig. 1. Well-defined traffic network MFD.

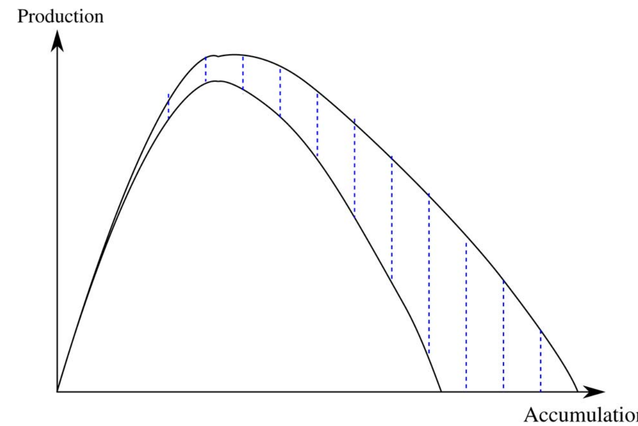

Fig. 2. MFD for the heterogeneous traffic network.

does not change. By regulating the network input flows, it is possible to regulate the average network traffic flow to a desired value and thus prevent the traffic network from falling into congestion.

In this paper, in order to verify the existence of equilibria on MFDs, microscopic simulations are run for an urban traffic network with various network input traffic flows to find the equilibrium and unequilibrium regions on the MFDs of the given traffic network. Since the homogeneity of traffic networks can influence the shape of the MFDs and may affect the existence of MFD equilibria, we select different traffic-signal control strategies within the network, i.e., fixed-time control, traffic-responsive control (TRC), and MPC, to create network traffic environments with different degrees of traffic flow homogeneity. Therefore, microscopic simulations are run with these signal control strategies on the same traffic network to analyze the existence of MFD equilibria in situations of homogeneous and heterogeneous traffic flows. The simulation analysis on the existence of the network traffic flow equilibria may help improve and perfect the design for MFD-based traffic network controllers.

This paper is organized as follows. Section II gives the definition of the MFD and the discussion on its equilibria. Section III provides the analysis on MFD equilibria by simulations with different traffic demands. Section IV further illustrates the MFD equilibria under different traffic-signal control strategies, and Section V concludes this paper.

II. MFD and Its Equilibria

An MFD illustrates an aggregated feature for traffic networks, which can describe how the aggregated network traffic flow (the travel production) changes with the aggregated network occupancy (or the network accumulation) [1], [2], as shown in Fig. 1. Network traffic accumulation $A(t)$ refers to the number of vehicles in a traffic network at a time instant t, i.e.,

$$A(t) = \sum_{i\in L} a_i(t) \tag{1}$$

where $a_i(t)$ is the number of vehicles on link i at time instant t, and L is the set of links in the traffic network. The network traffic accumulation can be also replaced by the average traffic density in the traffic network. The network travel production is actually the weighted network traffic flow, which represents the average traffic flow in a traffic network, i.e.,

$$F(t) = \frac{\sum\limits_{i\in L} l_i \cdot f_i(t)}{\sum\limits_{i\in L} l_i} \tag{2}$$

where $F(t)$ is the weighted average network flow rate, $f_i(t)$ is the traffic flow rate on link i at time instant t, and l_i is the length of link i.

However, the well-defined MFD curve in Fig. 1 only exists for an ideal situation, i.e., a well-defined MFD curve can be only realized when the traffic flows in a traffic network are always evenly scattered, i.e., homogeneous [7]. When the traffic in a traffic network is heterogeneous, hysteresis will occur, particularly in the saturated and oversaturated regions (i.e., the right-hand-side region of the MFD curve). Therefore, the MFD is not a well-defined curve anymore, and it could expand to a region, as shown in Fig. 2. During the elimination of spillbacks and gridlocks within the network, hysteresis phenomena appear in this region because of the traffic flow heterogeneity. Since the MFD shapes are different for homogeneous and heterogeneous traffic networks, the existence of MFD equilibria needs to be separately considered for both cases.

Specifically, a traffic network is homogeneous if the traffic flows in the traffic network are scattered evenly, i.e., the traffic densities on all the links are all the same. On the contrary, a traffic network is heterogeneous if the traffic flows in the traffic network are scattered unevenly, i.e., the traffic densities on some links are different from each other.

If a traffic network is homogeneous, we first investigate the equilibria on the well-defined MFD, as shown in Fig. 1.

If $f_{\text{out}}(t) = f_{\text{in}}(t)$ is satisfied, i.e., network output traffic flow $f_{\text{out}}(t)$ is equal to network input flow $f_{\text{in}}(t)$, and the number of vehicles in the traffic network does not vary with time, then the average traffic flow in the traffic network reaches the network equilibrium. The condition can be further relaxed into

$$\bar{f}_{\text{out}}(t) = \bar{f}_{\text{in}}(t) \tag{3}$$

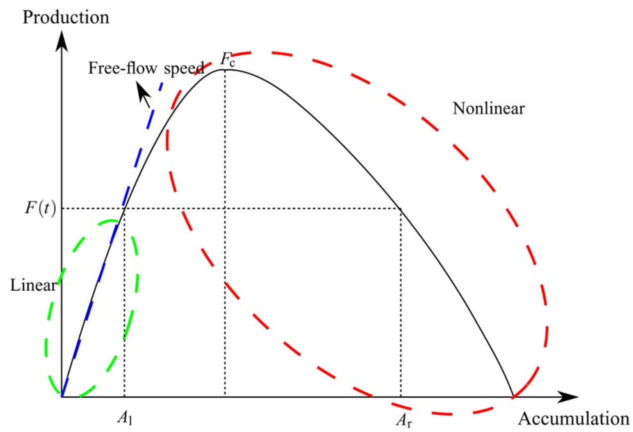

Fig. 3. Illustration for the equilibria in homogeneous traffic networks.

where $\bar{f}_{\rm in}(t)$ and $\bar{f}_{\rm out}(t)$ are the average network input flow and the average network output flow over a period of time from time instant t, respectively. The condition can be interpreted as the network input flow and the network output flow generally varying in similar steps, and both network flows slowly change with time.

Since the relationship between the weighted average network flow and the average network output flow is

$$F(t) = r\bar{f}_{\rm out} \tag{4}$$

where r is a coefficient that satisfies $0 < r \leq 1$, $r \approx 1$ if the vehicles in the network output traffic flows have traveled throughout the whole network. The relationship in (4) has been verified in [2] through real traffic data.

Given an average network input flow $\bar{f}_{\rm in}(t)$ and letting it equal the average network output flow, then we have

$$\bar{f}_{\rm in}(t) = \bar{f}_{\rm out}(t) = \frac{1}{r}F(t) \leq \frac{1}{r}F_c. \tag{5}$$

Consequently, traffic flow equilibria exist for a traffic network under the bounded average network input flow $\bar{f}_{\rm in}(t) \in [0,\ (1/r)F_c]$, and two network equilibria $(A_l(t),\ F(t))$ and $(A_r(t),\ F(t))$ exist for any weighted average network flow rate $F(t) = r\bar{f}_{\rm in}(t)$.

That is, for homogeneous traffic networks, there exist two equilibrium points on both sides of the MFD curve, as shown in Fig. 3, under bounded network input flow $F(t) = r\bar{f}_{\rm in}(t)$ and $\bar{f}_{\rm in}(t) \in [0,\ (1/r)F_c]$.

However, most of the time, the traffic flows in a traffic network are heterogeneously scattered. The homogeneity of a traffic network is just an ideal situation, which cannot be achieved in reality. If the traffic flows are heterogeneously scattered in a traffic network, then how would the MFD equilibria vary?

For a heterogeneous traffic network, supposing the traffic flows on all the links stay on the linear part of their FD curves (i.e., the free-flow region on the left-hand side of their FD), the speeds of the traffic flows on all the links will be the same (i.e., the free-flow speed). Since the speeds of the traffic flows on all the links in the network are equal to the free-flow speed, the MFD curve, which is an aggregated result of the FDs for all the links, also demonstrates the linear property in this region. The linearity in this region of the MFD is irrelevant to the degree of the traffic flow homogeneity in the traffic network. Therefore, in the linear region of the MFD, since the traffic flow speeds on all the links are equal to the free-flow speed (i.e., $v_i(t) = v_{\rm free}\ \forall\, i \in L$), the following equation still holds:

$$F(t) = \frac{\sum\limits_{i\in L} l_i \cdot \rho_i(t) \cdot v_i(t)}{\sum\limits_{i\in L} l_i} = v_{\rm free} \cdot \bar{\rho}(t) = \frac{v_{\rm free}}{C} \cdot A(t) \tag{6}$$

although the traffic flows within the traffic network are not homogeneous. Consequently, $(A_l(t), F(t))$ is the unique solution for

$$F(t) = r\bar{f}_{\rm in}(t) = \frac{v_{\rm free}}{C} \cdot A(t), \quad \bar{f}_{\rm in}(t) \in \left[0,\quad \frac{1}{r}F_c\right] \tag{7}$$

which is a network traffic flow equilibrium in a heterogeneous traffic network.

However, for a heterogeneous traffic network, if the traffic flows on all the links stay on the nonlinear part (i.e., the non-free-flow region of their FDs), then the speeds of the traffic flows on the links are different from one another. Thus, the previous equation does not hold anymore, i.e., $F(t) \neq (v_{\rm free}/C) \cdot A(t)$; that is, in such a situation, the nonlinear region of the network MFD should not be the curve that we expected before. Since the traffic flow densities on the links in a heterogeneous network are different from each other, let us define the maximum traffic flow density among the links in the traffic network as $\rho_{\max}$ and the minimum traffic flow density as $\rho_{\min}$. Thus, we can deduce

$$\frac{\rho_{\min}(t)\sum\limits_{i\in L} l_i \cdot v_i(t)}{\sum\limits_{i\in L} l_i} \leq \frac{\sum\limits_{i\in L} l_i \cdot \rho_i(t) \cdot v_i(t)}{\sum\limits_{i\in L} l_i} \leq \frac{\rho_{\max}(t)\sum\limits_{i\in L} l_i \cdot v_i(t)}{\sum\limits_{i\in L} l_i}. \tag{8}$$

And then, we have

$$\frac{\bar{v}(t)}{C} \cdot A_{\min}(t) \leq F(t) \leq \frac{\bar{v}(t)}{C} \cdot A_{\max}(t) \tag{9}$$

where $A_{\min}(t)$ and $A_{\max}(t)$ are the corresponding vehicle accumulations (the number of vehicles) in the traffic network derived from the maximum and minimum traffic flow densities. Let

$$F_{\min}(t) = \frac{\bar{v}(t)}{C} \cdot A_{\min}(t) \tag{10}$$

$$F_{\max}(t) = \frac{\bar{v}(t)}{C} \cdot A_{\max}(t) \tag{11}$$

in the nonlinear region of the MFD; due to the heterogeneity of the traffic network, the MFD trajectory can be an arbitrary curve within the area surrounded by (10) and (11). It also verifies that the MFD of a heterogeneous traffic network is not a single well-defined curve for the nonlinear region but an area (as shown in Fig. 4). In such a situation, network traffic flow equilibrium $(A_r(t),\ F(t))$ does not exist anymore.

In a word, within a heterogeneous traffic network, for any average network input flow $\bar{f}_{\rm in}(t) \in [0,\ (1/r)F_c]$, there exists only one network flow equilibrium, i.e., $(A_l(t),\ F(t))$, that satisfies $F(t) = r\bar{f}_{\rm in}(t)$.

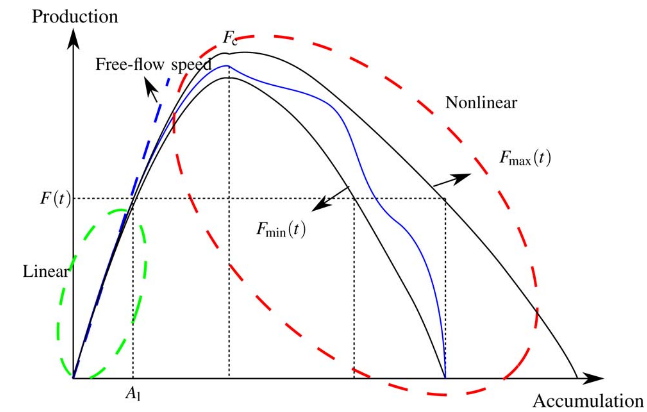

Fig. 4. Illustration for the equilibria in heterogeneous traffic networks.

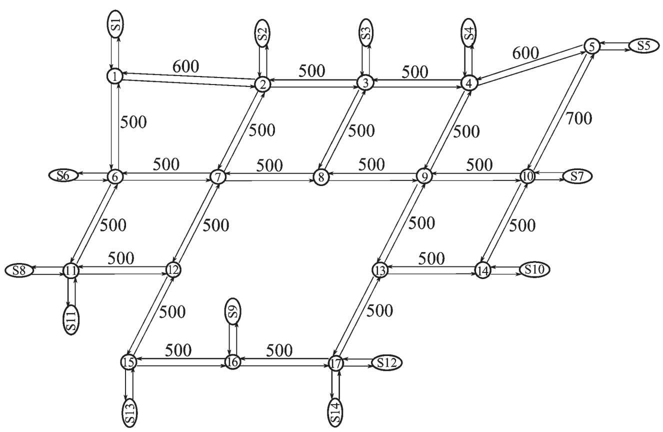

Fig. 5. Urban traffic network.

III. Simulation Analysis on the MFD Equilibria

In order to verify the existence of the network traffic flow equilibria on the MFDs, we can provide different input flows with constant values to a traffic network and see how the weighted average network flows vary with time. It is difficult to realize this experiment in real-world traffic because it is almost impossible to fix the network input flow to a specific value and keep the flow value enough for a long time in reality. Therefore, we use the microscopic traffic simulation software Corridor Simulation (CORSIM), which is developed in [17], to establish an urban traffic network. This urban traffic network is considered a real traffic environment, and experiments are carried out on the network by specifying different traffic demands and giving different traffic control strategies.

The layout of the simulated traffic network and the link lengths (in meters) are shown in Fig. 5, and each link has three lanes. In CORSIM, the turning ratios in intersections are specified instead of specifying route choices. The turning rates of traffic flows in a link are considered constant, i.e., 1/3 for the left, right, and through turning movements. The free-flow speed is 30 km/h, and the saturation flow rate is 2000 vehicles per hour (veh/h) per lane. The network input flow rates are set equal to each other and constant in time, and they are 500, 1000, 1500, and 2000 veh/h, respectively. A fixed-time control strategy is executed in the traffic network. The signal timings are predesigned offline for the saturated traffic scenario, i.e., the signal timings of an intersection are allocated according to the saturation flow rates of every link. The value of the saturation flow rate in a link depends on the physical structure of the link, i.e., the number of lanes. For more details on the fixed-time controller design, see [11].

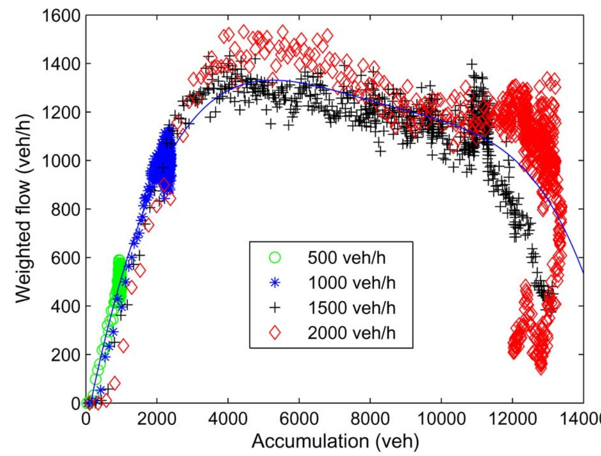

Fig. 6. MFD under the fixed-time controller.

The simulation is executed on the traffic network in Fig. 5 for a time period of 6 h. During the simulations, the MFDs are drawn by marking the network status at each time instant on the accumulation and weighted flow plane. As shown in Fig. 6, the network states are illustrated by different marks for the network input flows of 500, 1000, 1500, and 2000 veh/h. For the network input flows of 500 and 1000 veh/h, the network states are all kept in the linear region on the left-hand side of the MFDs. This linear region is the free-flow region, i.e., the vehicles in the traffic network run with free-flow speed on all the links, although they are heterogeneously scattered in the network. For the network input flows of 1500 and 2000 veh/h, the network states experience all the network scenarios in the time period of 6 h, from the undersaturated, saturated, and oversaturated scenarios, to the congested scenario. When the traffic density of the network grows, the speed of the traffic flows is not free-flow speed anymore, and it begins to reduce. Consequently, the MFD curves cannot keep being linear anymore. In addition, due to the heterogeneity of the traffic network, the nonlinear part of the MFD expands to a region, instead of sticking to a well-defined curve. This result verifies the analysis in Section II.

In order to illustrate the existence of network equilibria, we further draw the figure to show how the weighted average network flow changes with time. Fig. 7 gives the curves of the weighted average network flows in 6 h under different network input flows. As shown in Fig. 7, it is obvious that the weighted average network flows reach their equilibria when the network input flows are 500 and 1000 veh/h. Moreover, the equilibria of the weighted average network flows approximate the corresponding values of the network input flows. However, when the network input flows are 1500 and 2000 veh/h, the weighted average network flows lose their equilibria and are reduced to values that are much less than their network input flows; thus, the traffic network is totally congested at the end of the simulations. This result verifies that the traffic network can reach its network flow equilibria if the traffic flows in the network are in the free-flow region (the linear region), although

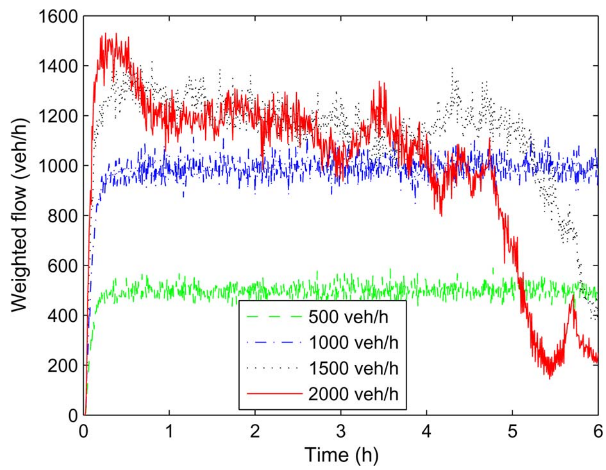

Fig. 7. Weighted average network flow under the fixed-time controller.

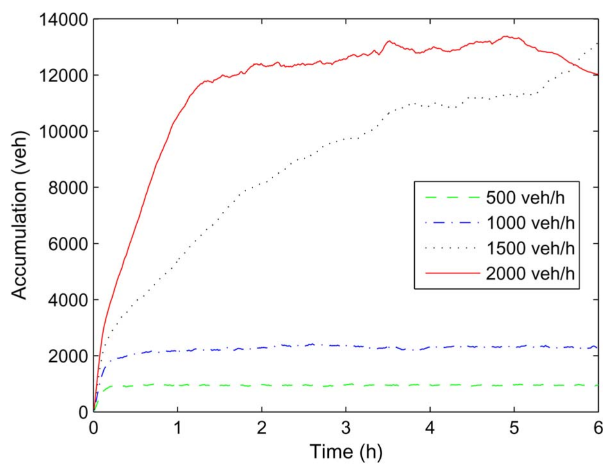

Fig. 8. Accumulation under the fixed-time controller.

the network is heterogeneous; however, network flow equilibria do not exist for the non-free-flow region (the nonlinear region). Similar conclusions can be derived from the curves of the accumulations varying with time under different network input flows, as shown in Fig. 8.

IV. Network Equilibrium Analysis Under Different Traffic Controllers

The equilibria are verified existing for the traffic network flow under the fixed-time controller. Do the equilibria also exist under other traffic control strategies? How could the control strategies influence the status of the equilibria, and why?

To answer these questions, we further run the simulations under two other control strategies, i.e., the TRC and the MPC.

The traffic-responsive controller adjusts the traffic-signal timings according to the real-time measured traffic states. The traffic states, i.e., the number of waiting vehicles in links, are measured by the two rows of loop detectors that are implemented at the stop line and at the upstream of the coming flows. The number of waiting vehicles on a link $n(k)$ is calculated every sampling time interval Δg. Therefore, we can express the green time increment of Phase $p \in P$ at time instant k as

$$\Delta g_p(k) = \begin{cases} \Delta g, & \mathrm{n(k)} \neq 0 \\ 0, & n(k) = 0 \end{cases} \tag{12}$$

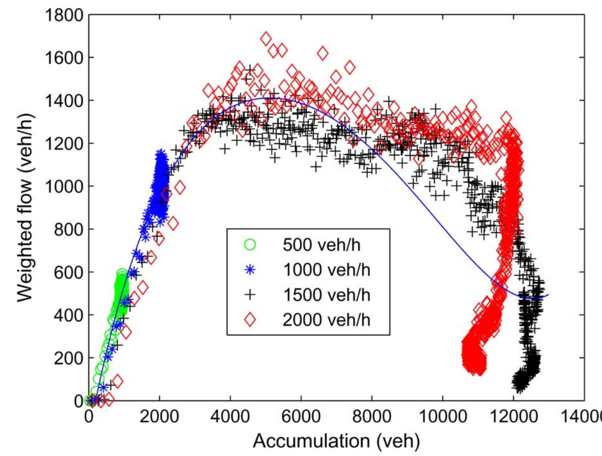

Fig. 9. MFD under the traffic-responsive controller.

where $\Delta g_p(k)$ is the green time increment of Phase p, P is the set of traffic-signal phases, and $n(k)$ is the number of waiting vehicles at time k. It means that, if there are vehicles waiting in the link, then the green time of the corresponding phase will be extended by Δg; if not, the green time will end. Thus, the green time of Phase p can be calculated as

$$g_p = \sum_{k \in K} \Delta g_p(k) \tag{13}$$

where K is the number of time intervals in a cycle time. The sum of the green times for all the phases is equal to cycle time c, i.e.,

$$\sum_{p \in P} g_p = c. \tag{14}$$

The model predictive controller not only can adapt the control actions with the real-time measured traffic states but can also predict the future traffic tendency and make optimizations based on the prediction model. In each control step k_c, an optimal control problem is solved over a prediction horizon to search for the best control sequence, but only the first control action of the sequence is transferred to the real traffic network to implement. When arriving at the next control step $k_c + 1$, the whole time horizon is shifted one step forward, and the real-time measured traffic states are collected to update the prediction model to reduce the prediction errors. Then, the optimization over the new prediction horizon starts over based on the adapted prediction model, and so forth. This rolling horizon scheme closes the control loop, enables the system to get feedback from the real traffic network, and makes the MPC controller adaptive to the uncertainties of the real traffic environment. For more details on the MPC controller design, see [11] and [18].

The simulations are executed again on the traffic network in Fig. 5 under the traffic-responsive controller and the model predictive controller for a time period of 6 h. In Figs. 9 and 10, the outline of the MFDs are marked by the network states that were generated from the simulations under two control strategies, i.e., the TRC and the MPC, with different network input flows of 500, 1000, 1500, and 2000 veh/h. Comparing Figs. 9 and 10 with Fig. 6, the network traffic states still stay in the linear region, i.e., the free-flow region of the network MFD, for the network input flows of 500 and 1000 veh/h. Under the

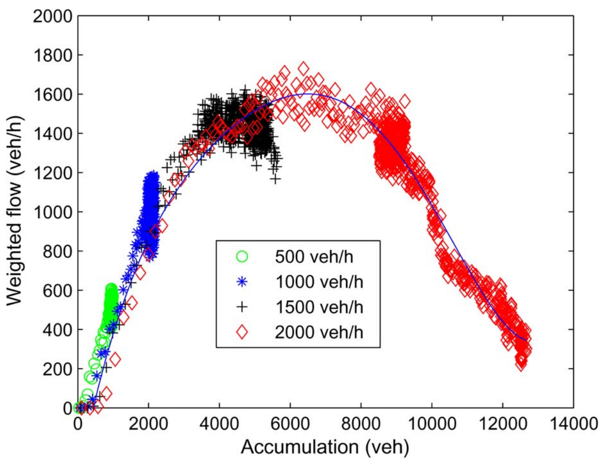

Fig. 10. MFD under the model predictive controller.

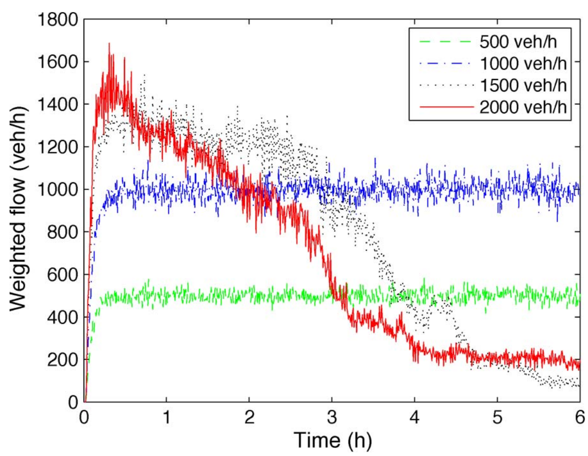

Fig. 11. Weighted average network flow under the traffic-responsive controller.

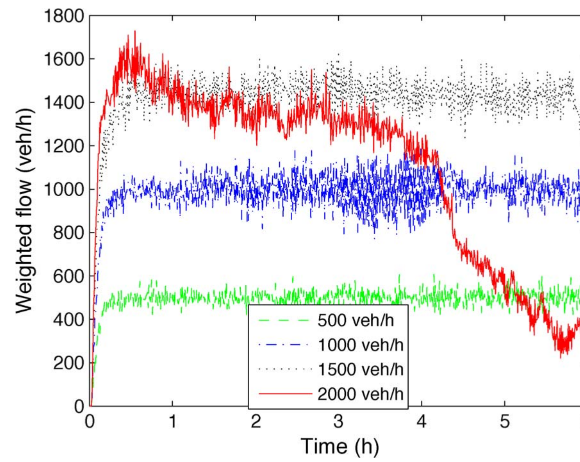

Fig. 12. Weighted average network flow under the model predictive controller.

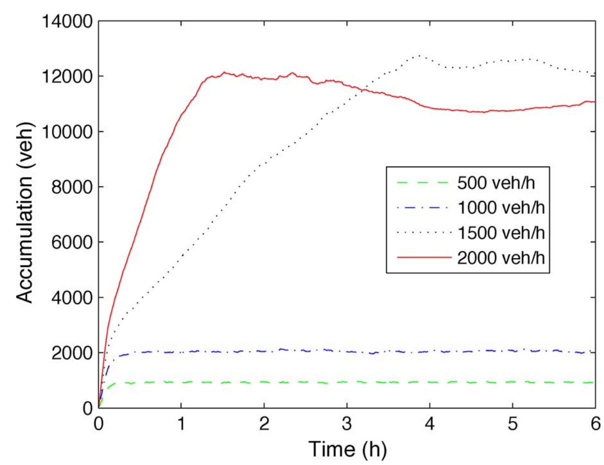

Fig. 13. Accumulation under the traffic-responsive controller.

TRC, as the simulation time grows, the network states get into the nonlinear region of the MFD and reach the congestion in the end for both the network input flows of 1500 and 2000 veh/h. However, under the MPC, only the network states with the network input flow of 2000 veh/h get into congestion in the end, and the network states with the network input flow of 1500 veh/h keep in the uncongested region. Comparing the three control strategies, the linear regions of the MFDs are extremely similar to one another, but the nonlinear regions of the MFDs are quite different from one another. As aforementioned, the nonlinear part of the MFD should be a region for a heterogeneous situation, instead of a well-defined curve. This MFD region under the fixed-time controller and the traffic-responsive controller is broader than that under the model predictive controller. These results illustrate that the traffic flows under the model predictive controller more homogeneously scatter in the traffic network than the traffic flows under the fixed-time and traffic-responsive controllers. Consequently, due to the higher degree of homogeneity, the network states under the model predictive controller are able to keep from falling into congestion.

In Figs. 11 and 12, the evolution of the weighted average network flows are further shown for the traffic-responsive controller and the model predictive controller. As shown, just as the result under the fixed-time controller, the weighted average network flows reach their equilibria under both the traffic-responsive controller and the model predictive controller when the network input flows are 500 and 1000 veh/h, respectively. In addition, the equilibria of the weighted average network flows approximate the corresponding values of the network input flows. When the network input flows are 1500 and 2000 veh/h, the weighted average network flow under the traffic-responsive controller loses the equilibrium and gets into congestion at the end of the simulation. However, when the network input flow is 1500 veh/h, the weighted average network flow under the model predictive controller can stay around the value of its theoretical equilibrium for the sake of the lower degree of network heterogeneity. However, when the network input flow increases to 2000 veh/h, the weighted average network flow under the model predictive controller exceeds the capacity of the network (i.e., the maximum value of the network flow) and thus has no equilibrium. This result verifies that the traffic network can reach its network flow equilibria if the traffic flows in the network are in the free-flow region (the linear region), although the network is heterogeneous; however the network flow equilibria do not exist for the non-free-flow region (the nonlinear region) unless the traffic network is homogeneous. Similar conclusions can be derived from the evolutions of the accumulations under the two controllers, as shown in Figs. 13 and 14. However, one exception is that the results for the MPC under the input flow of 1500 veh/h are neither on the linear part of

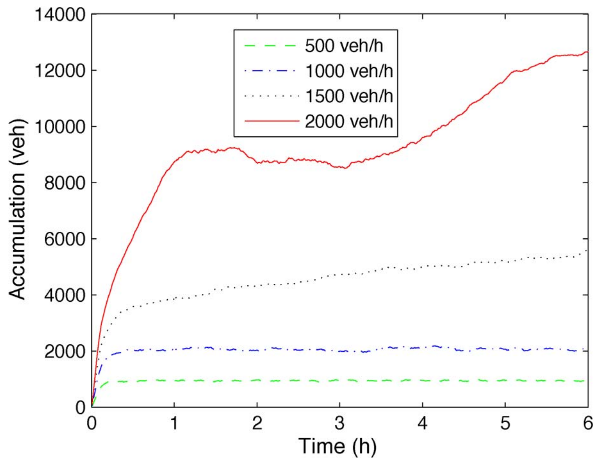

Fig. 14. Accumulation under the model predictive controller.

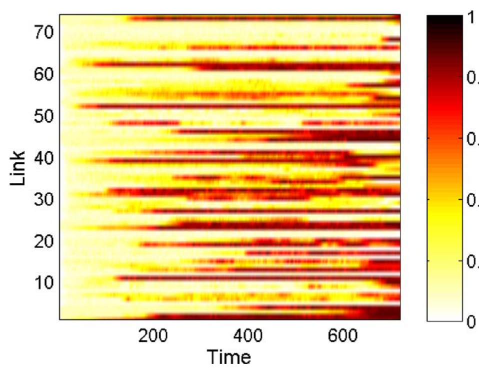

Fig. 15. Spatiotemporal density of the traffic network under the fixed-time controller.

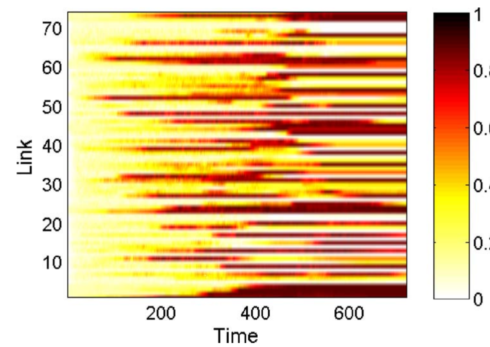

Fig. 16. Spatiotemporal density of the traffic network under the traffic-responsive controller.

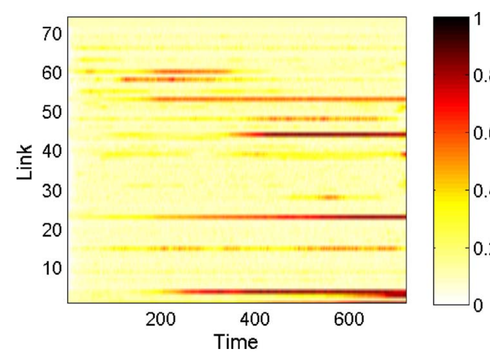

Fig. 17. Spatiotemporal density of the traffic network under the model predictive controller.

the MFD nor deteriorating to the totally congested region (see Fig. 10). The reason the weighted network average traffic flow can linger around the theoretical equilibrium on the MFD is that the traffic flows are less heterogeneously regulated (i.e., more approximating homogeneity) in the traffic network with the help of the model predictive controller.

Furthermore, in order to verify the aforementioned results, we show the spatiotemporal network density maps for the network input flow of 1500 veh/h under the three controllers (see Figs. 15–17). On the spatiotemporal network density map, "time" is on the horizontal axis and "link" is on the vertical axis, where the number on the vertical axis represents the number of links in the traffic network. The color on the density map represents the value of the density at its coordinate, and the density value varies from 0 to 1. As expected, when the network input flow is 1500 veh/h, the density map unevenly distributed under the fixed-time and traffic-responsive controllers, which means that the traffic densities in the traffic network are scattered more heterogeneously; on the contrary, the density map much more uniformly distributed under the model predictive controller, which reveals that the traffic densities in the traffic network are less heterogeneously scattered than those of the other controllers. Therefore, the result reveals the trend that a more homogeneous traffic network tends to have a more well-defined MFD curve, and in a homogeneous situation, it is possible to maintain the network average flow at its equilibrium on the nonlinear region of the MFD.

V. Conclusion

An MFD is an aggregated feature of traffic networks describing the relationship between the network traffic flow and the network accumulation. Equilibria are theoretically proved to exist on MFDs of urban traffic networks. In order to verify the existence of equilibria on MFDs, microscopic simulations are run for an urban traffic network with various network input traffic flows. The simulation results are also compared for different traffic-signal control strategies, i.e., fixed-time control, TRC, and MPC, to verify the conclusions of the MFD equilibria in situations of homogeneous and heterogeneous traffic flows.

The simulations verifies that the equilibria exist on the linear region (the free-flow region) of the MFDs under all the traffic-signal control strategies, although the network flows are scattered heterogeneously. However, equilibria do not exist on the nonlinear region (the non-free-flow region) of the MFDs in general cases where the traffic flows are heterogeneous in the traffic network. However, in ideal cases where the traffic flows are homogeneously scattered in the network, the equilibria could appear on both sides of the MFD curves. Properly designed traffic-signal control strategies can improve the degree of homogeneity for traffic flows, which can increase the stability of the network traffic flows, and thus can result in a traffic network with high mobility and sustainability.

In the future, the existence of the MFD equilibria will be further investigated through real traffic data, and the derived MFD properties will be considered to design traffic network controllers or regulators.

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Shu Lin (M'07) received the M.Sc. degree in control science and engineering from Shandong University, Jinan, China, in 2006 and the Ph.D. degree from Delft University of Technology, Delft, The Netherlands, in 2011.

From 2011 to 2013, she was a Postdoctoral Researcher with the Department of Automation, School of Electronic Information and Electrical Engineering, Shanghai Jiao Tong University, Shanghai, China. She is currently an Assistant Professor with the School of Computer and Control Engineering, University of Chinese Academy of Sciences, Beijing, China. Her research interests include modeling, optimization, and control for large-scale traffic networks.

Qing-Jie Kong (M'07) received the Ph.D. degree in pattern recognition and intelligent systems from Shanghai Jiao Tong University, Shanghai, China, in 2010.

From 2008 to 2009, he was a Visiting Scholar with the Beckman Institute for Advanced Science and Technology, Department of Electrical and Computer Engineering, College of Engineering, University of Illinois at Urbana–Champaign, Urbana, IL, USA. From 2010 to 2012, he was a Postdoctoral Scientist with the Department of Automation, School of Electronic Information and Electrical Engineering, Shanghai Jiao Tong University. Since 2012, he has been with the State Key Laboratory for Management and Control of Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China, where he is now an Associate Professor. His research interests include traffic data mining and fusion, traffic network modeling and analysis, parallel transportation management and control, and video object detection and recognition.

Qingming Huang (SM'08) received the B.S. degree in computer science and the Ph.D. degree in computer engineering from Harbin Institute of Technology, Harbin, China, in 1988 and 1994, respectively.

He is currently a Professor with the School of Computer and Control Engineering, University of Chinese Academy of Sciences, Beijing, China, and an Adjunct Research Professor with the Institute of Computing Technology, Chinese Academy of Sciences, Beijing, China. He is the author or coauthor of more than 200 academic papers in prestigious international journals and conferences. His research interests include multimedia computing, image processing, computer vision, and pattern recognition.

He has served as an organization committee member and a technical program committee member for various well-known conferences, including Association for Computing Machinery Multimedia, Computer Vision and Pattern Recognition (CVPR), the International Conference on Computer Vision, and the International Conference on Multimedia and Expo. He was a recipient of the China National Funds for Distinguished Young Scientists.