Dynamics of the dissipative four-state system

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Abstract

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Dedication

To mum and dad

Declaration

I declare that..

Acknowledgements

I want to thank...

Contents

1 Introduction

This thesis will discuss the dynamics of a quantum mechanical four-state system, which is coupled to a photonic mode in an optical cavity.

2 Theoretical foundations

The bath correlation function in this case takes the form

$$Q(t) = S(t) + iR(t) = \int_0^\infty \mathrm{d}\omega \frac{J(\omega)}{\omega^2} \coth \frac{\hbar\omega\beta}{2} \Big[(1 - \cos\omega t) + i \sin\omega t\Big] \psi\mathcal{E}$$

3 The doublet-doublet system

In this chapter, we will consider the system of a single particle with mass $\mathcal{M}$, position operator $\mathbf{q}$ and momentum operator $\mathbf{p}$. The particle is placed in a double-well potential, giving the Hamiltionian

$$\mathbf{H}_{\mathrm{DW}} = \frac{\mathbf{p}^2}{2\mathcal{M}} + \frac{\mathcal{M}^2\omega_0^4}{64\Delta U}\mathbf{q}^4 - \frac{\mathcal{M}\omega_0^2}{4}\mathbf{q}^2 - \mathbf{q}\epsilon.$$

Here, ΔU is the barrier height, ω_0 is the classical oscillation frequency and ϵ is the bias factor of the double-well potential. In addition, we introduce a single cavity mode, which couples linearly to the doublet-doublet system and is described by

$$\mathbf{H}_{\mathrm{C,int}} = \Omega\mathbf{a}^\dagger\mathbf{a} + g\mathbf{q}\left(\mathbf{a} + \mathbf{a}^\dagger\right).$$

Next, the cavity mode is coupled to a bath consisting of simple harmonic oscillators. For now, we will neglect the direct coupling of the DW-system to a bath:

$$\mathbf{H}_{\mathrm{B,int}} = \left(\mathbf{a} + \mathbf{a}^\dagger\right)\sum_k \nu_k\left(\mathbf{b}_k + \mathbf{b}_k^\dagger\right) + \sum_k \omega_k\mathbf{b}_k^\dagger\mathbf{b}_k.$$

For the cavity mode, we choose Ohmic damping. In the continuous limit, this means the spectral density is given by

$$J_{Ohm}(\omega) = \sum_k \nu_k^2\delta(\omega - \omega_k) = \kappa\omega e^{-\omega/\omega_c},$$

where κ is the cavity damping constant and ω_c is the cut-off frequency.
According to Garg, Onuchic and Ambegaokar, this model can be mapped to a double-well coupling to a bath with a peaked spectral density with the coupling term

$$\mathbf{H}_{\mathrm{B,DW}} = \mathbf{q}\sum_k \lambda_k\left(\tilde{\mathbf{a}}_k + \tilde{\mathbf{a}}_k^\dagger\right) + \sum_k \tilde{\omega}_k\tilde{\mathbf{a}}_k^\dagger\tilde{\mathbf{a}}_k.$$

and the effective bath spectral density

$$J(\omega) = \sum_k \lambda_k^2\delta(\omega - \tilde{\omega}_k) = \frac{2\alpha\omega\Omega^4}{\left(\Omega^2 - \omega^2\right)^2 + \left(2\pi\kappa\omega\Omega\right)^2},$$

where $\alpha = 8\kappa\frac{g^2}{\Omega^2}$ is the effective low-frequency damping constant. In the low-frequence limit, i.e. $\omega \to 0$, the effective bath spectral density reduces to $J(\omega) \to 2\alpha\omega$.

4 Chapter 4

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5 Conclusion

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A Appendix

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