MATH 630 Midterm

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Midterm: Simple Linear Regression

Overview

```
Teams <- Lahman::Teams
Salaries <- Lahman::Salaries
```

HLO Lahman

```
str(Teams)
str(Salaries)
glimpse(Teams)
glimpse(Salaries)
head(Teams)
head(Salaries)
tail(Teams)
tail(Salaries)
names(Teams)
names(Salaries)
ncol(Teams)
ncol(Salaries)
length(Teams)
length(Salaries)
head(rownames(Teams))
head(rownames(Salaries))
dim(Teams)
dim(Salaries)
nrow(Teams)
nrow(Salaries)
summary(Teams)
summary(Salaries)
?Teams
?Salaries
```

Are they data.frames, matrices, vectors, lists?

> data.frames

What is the unit of analysis in the dataset?

> Teams: Yearly statistics and standings for teams.
> Salaries: Player salary data.

How many variables/columns?

Teams: 48
Salaries: 5

How many rows/observations?

Teams: 2775
Salaries: 24758

Find the variables for games won, team, year, and salary.

W: Wins
teamID: Team; a factor
yearID: Year
salary: Salary

Which variables are continuous?

In theory, salary could be continuous but in practice, salary looks like it's rounded to the nearest thousand.

Which variables are discrete?

W
teamID
yearID
salary

Which variables are categorical?

teamID

How many levels do they have?

149

What about missing data for any variables?

```
unique(is.na(Teams))
unique(is.na(Salaries))
```

Teams: Missing data in several columns
Salaries: No missing data reported

Data wrangling in dplyr

Create a new dataset that includes total yearly payroll for each team in the Salaries dataframe.

```r
typ <- Salaries %>% group_by(yearID,teamID) %>% summarise(payroll = sum(salary))
head(typ)
```

```
Source: local data frame [6 x 3]
Groups: yearID [1]

  yearID teamID  payroll
   (int) (fctr)    (int)
1   1985    ATL 14807000
2   1985    BAL 11560712
3   1985    BOS 10897560
4   1985    CAL 14427894
5   1985    CHA  9846178
6   1985    CHN 12702917
```

Add this payroll column to the Teams dataframe.

```r
teams_pay <- inner_join(Teams,typ,c("yearID","teamID"))
head(teams_pay)
```

```
  yearID lgID teamID franchID divID Rank   G Ghome  W  L DivWin WCWin
1   1985   NL    ATL      ATL     W    5 162    81 66 96      N  <NA>
2   1985   AL    BAL      BAL     E    4 161    81 83 78      N  <NA>
3   1985   AL    BOS      BOS     E    5 163    81 81 81      N  <NA>
4   1985   AL    CAL      ANA     W    2 162    79 90 72      N  <NA>
5   1985   AL    CHA      CHW     W    3 163    81 85 77      N  <NA>
6   1985   NL    CHN      CHC     E    4 162    81 77 84      N  <NA>
  LgWin WSWin   R   AB    H X2B X3B  HR  BB  SO  SB CS HBP SF  RA  ER  ERA
1     N     N 632 5526 1359 213  28 126 553 849  72 52  NA NA 781 678 4.19
2     N     N 818 5517 1451 234  22 214 604 908  69 43  NA NA 764 694 4.38
3     N     N 800 5720 1615 292  31 162 562 816  66 27  NA NA 720 659 4.06
4     N     N 732 5442 1364 215  31 153 648 902 106 51  NA NA 703 633 3.91
5     N     N 736 5470 1386 247  37 146 471 843 108 56  NA NA 720 656 4.07
6     N     N 686 5492 1397 239  28 150 562 937 182 49  NA NA 729 667 4.16
  CG SHO SV IPouts   HA HRA BBA  SOA   E  DP   FP              name
1  9   9 29   4371 1512 134 642  776 159 197 0.97    Atlanta Braves
2 32   6 33   4281 1480 160 568  793 115 168 0.98 Baltimore Orioles
3 35   8 29   4383 1487 130 540  913 145 161 0.97    Boston Red Sox
4 22   8 41   4371 1453 171 514  767 112 202 0.98 California Angels
5 20   8 39   4353 1411 161 569 1023 111 152 0.98 Chicago White Sox
6 20   8 42   4326 1492 156 519  820 134 150 0.97      Chicago Cubs
                           park attendance BPF PPF teamIDBR teamIDlahman45
1 Atlanta-Fulton County Stadium    1350137 105 106      ATL            ATL
2              Memorial Stadium    2132387  97  97      BAL            BAL
3                Fenway Park II    1786633 104 104      BOS            BOS
4               Anaheim Stadium    2567427 100 100      CAL            CAL
5                 Comiskey Park    1669888 104 104      CHW            CHA
6                 Wrigley Field    2161534 110 110      CHC            CHN
  teamIDretro  payroll
1         ATL 14807000
2         BAL 11560712
3         BOS 10897560
```

```
4          CAL 14427894
5          CHA  9846178
6          CHN 12702917
```

We'll focus on the years 2000 - 2014. Use dplyr to filter() the dataset you created with the Teams data plus the payroll column for just those years.

```r
recent_tpay <- filter(teams_pay, yearID >= 2000, yearID <= 2014)
```

Gift

```r
bat_stats <-
  battingStats(data = Lahman::Batting,
               idvars = c("playerID",
                          "yearID",
                          "stint",
                          "teamID",
                          "lgID"), cbind = TRUE)
```

Write a dplyr expression to create a new dataframe that contains means for each of these three new variables for each team and year from 2000 - 2014 (rather than for each player).

```r
bat_avgs <- bat_stats %>%
  filter(yearID >= 2000, yearID <= 2014) %>%
  group_by(yearID,teamID) %>%
  summarise(ob_perc = mean(OBP,na.rm = TRUE),
            slug_perc = mean(SlugPct,na.rm = TRUE),
            ops = mean(OPS,na.rm = TRUE))
head(bat_avgs)
```

```
Source: local data frame [6 x 5]
Groups: yearID [1]

  yearID teamID   ob_perc slug_perc       ops
   (int) (fctr)     (dbl)     (dbl)     (dbl)
1   2000    ANA 0.3385926 0.3756667 0.7142593
2   2000    ARI 0.3016216 0.3618378 0.6634595
3   2000    ATL 0.2430000 0.3009167 0.5439167
4   2000    BAL 0.2474000 0.3088571 0.5562571
5   2000    BOS 0.2644571 0.3063429 0.5708000
6   2000    CHA 0.2937600 0.3396800 0.6334400
```

Adds these new batting statistic columns to your current dataframe

```r
# warning seems ok based on http://goo.gl/9QH3fo
teams_bat <- inner_join(bat_avgs,recent_tpay,c("yearID","teamID"))
head(teams_bat)
```

```
Source: local data frame [6 x 52]
Groups: yearID [1]
```

```
  yearID teamID   ob_perc slug_perc       ops   lgID franchID divID  Rank
   (int)  (chr)     (dbl)     (dbl)     (dbl) (fctr)   (fctr) (chr) (int)
1   2000    ANA 0.3385926 0.3756667 0.7142593     AL      ANA     W     3
2   2000    ARI 0.3016216 0.3618378 0.6634595     NL      ARI     W     3
3   2000    ATL 0.2430000 0.3009167 0.5439167     NL      ATL     E     1
4   2000    BAL 0.2474000 0.3088571 0.5562571     AL      BAL     E     4
5   2000    BOS 0.2644571 0.3063429 0.5708000     AL      BOS     E     2
6   2000    CHA 0.2937600 0.3396800 0.6334400     AL      CHW     C     1
Variables not shown: G (int), Ghome (int), W (int), L (int), DivWin (chr),
  WCWin (chr), LgWin (chr), WSWin (chr), R (int), AB (int), H (int), X2B
  (int), X3B (int), HR (int), BB (int), SO (int), SB (int), CS (int), HBP
  (int), SF (int), RA (int), ER (int), ERA (dbl), CG (int), SHO (int), SV
  (int), IPouts (int), HA (int), HRA (int), BBA (int), SOA (int), E (int),
  DP (int), FP (dbl), name (chr), park (chr), attendance (int), BPF (int),
  PPF (int), teamIDBR (chr), teamIDlahman45 (chr), teamIDretro (chr),
  payroll (int)
```

Univariate EDA (+ more wrangling)

```r
teams_bat %>%
  group_by(teamID) %>%
  tally() %>% arrange(n) %>%
  print(n = 33)
```

```
Source: local data frame [33 x 2]

   teamID     n
    (chr) (int)
1     MIA     3
2     ANA     5
3     MON     5
4     LAA    10
5     WAS    10
6     FLO    12
7     ARI    15
8     ATL    15
9     BAL    15
10    BOS    15
11    CHA    15
12    CHN    15
13    CIN    15
14    CLE    15
15    COL    15
16    DET    15
17    HOU    15
18    KCA    15
19    LAN    15
20    MIL    15
21    MIN    15
22    NYA    15
23    NYN    15
24    OAK    15
```

```
25    PHI    15
26    PIT    15
27    SDN    15
28    SEA    15
29    SFN    15
30    SLN    15
31    TBA    15
32    TEX    15
33    TOR    15
```

How many teams are there?

> 33

Which teams have data for the least number of seasons?

> MIA, ANA, and MON

Which have the most seasons?

> ARI,ATL,BAL,BOS,CHA,CHN,CIN,CLE,COL,
> DET,HOU,KCA,LAN,MIL,MIN,NYA,NYN,OAK,
> PHI,PIT,SDN,SEA,SFN,SLN,TBA,TEX,TOR all have 15

```r
teams_bat <- teams_bat %>%
  filter(!(teamID %in% c("ANA", "MIA", "MON"))) # you should understand what this does
```

```r
teams_bat %>%
  group_by(yearID) %>%
  select(G) %>%
  summarise_each(funs(min,max,mean,median))
```

```
Source: local data frame [15 x 5]

   yearID   min   max     mean median
    (int) (int) (int)    (dbl)  (dbl)
1    2000   161   163 161.9286    162
2    2001   161   162 161.9286    162
3    2002   161   162 161.7143    162
4    2003   161   163 162.0000    162
5    2004   161   162 161.8571    162
6    2005   162   163 162.0667    162
7    2006   161   162 161.9333    162
8    2007   162   163 162.0667    162
9    2008   161   163 161.8667    162
10   2009   161   163 162.0000    162
11   2010   162   162 162.0000    162
12   2011   161   162 161.9333    162
13   2012   162   162 162.0000    162
14   2013   162   163 162.0690    162
15   2014   162   162 162.0000    162
```

Is there a lot of variability in number of games played per season across teams?

No

What is the range of games played by teams per season?

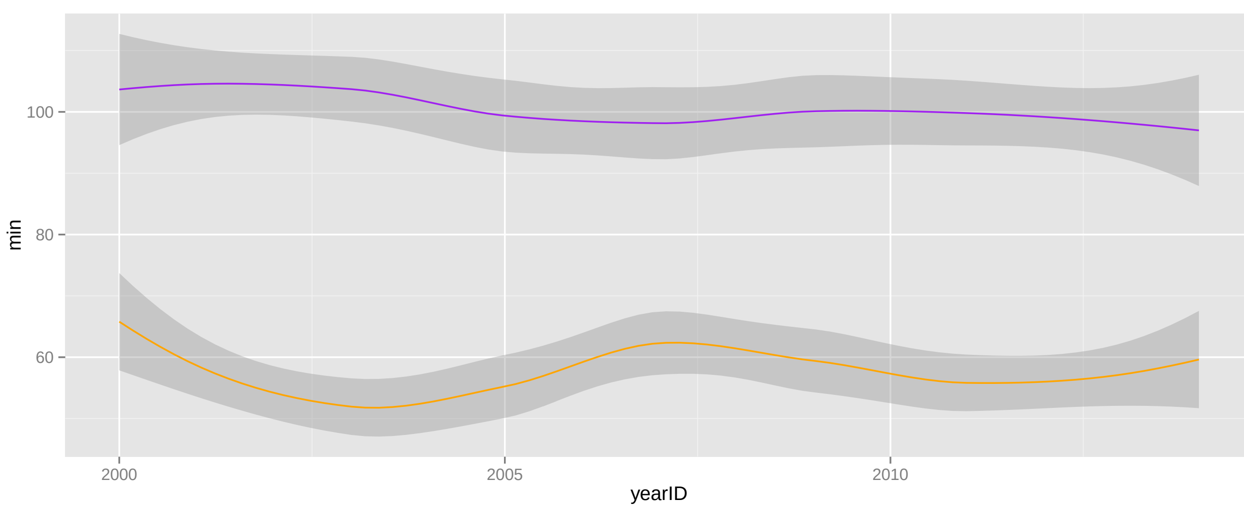
2 (163 - 161)

Number of games won

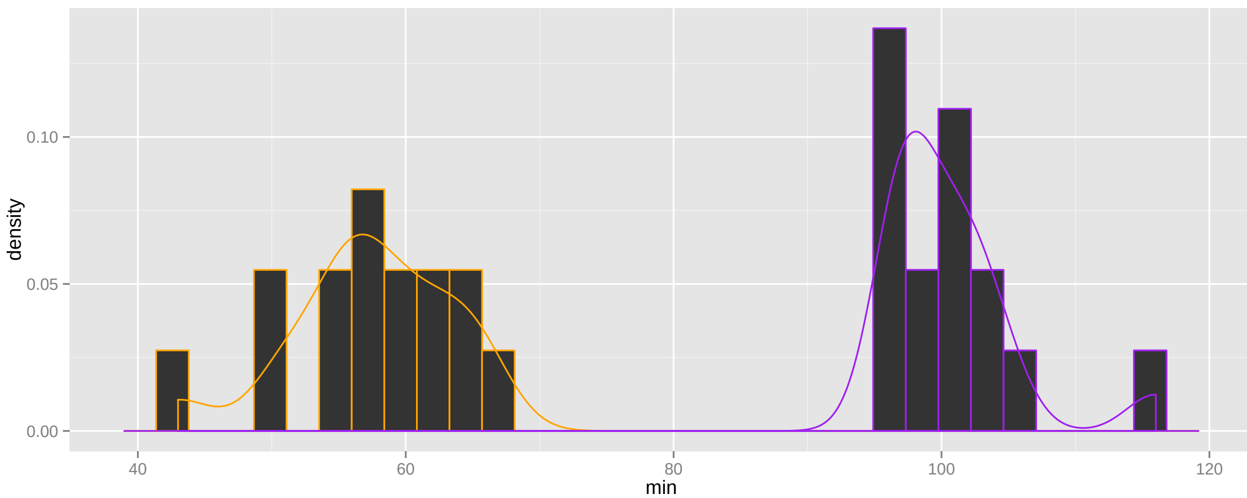
```
teams_bat %>%
  group_by(yearID) %>%
  select(W) %>%
  summarise_each(funs(min,max)) %>% head()
```

```
Source: local data frame [6 x 3]

  yearID   min   max
   (int) (int) (int)
1   2000    65    97
2   2001    62   116
3   2002    55   103
4   2003    43   101
5   2004    51   105
6   2005    56   100
```

```
teams_bat %>%
  group_by(yearID) %>%
  select(W) %>%
  summarise_each(funs(min,max)) %>% ggplot() +
  geom_smooth(aes(x=yearID,y=min),color="orange") +
  geom_smooth(aes(x=yearID,y=max),color="purple")
```



```
teams_bat %>%
  group_by(yearID) %>%
  select(W) %>%
  summarise_each(funs(min,max)) %>% ggplot() +
  geom_histogram(aes(min,y=..density..),color="orange") +
  geom_density(aes(min),color="orange") +
  geom_histogram(aes(max,y=..density..),color="purple") +
  geom_density(aes(max),color="purple")
```

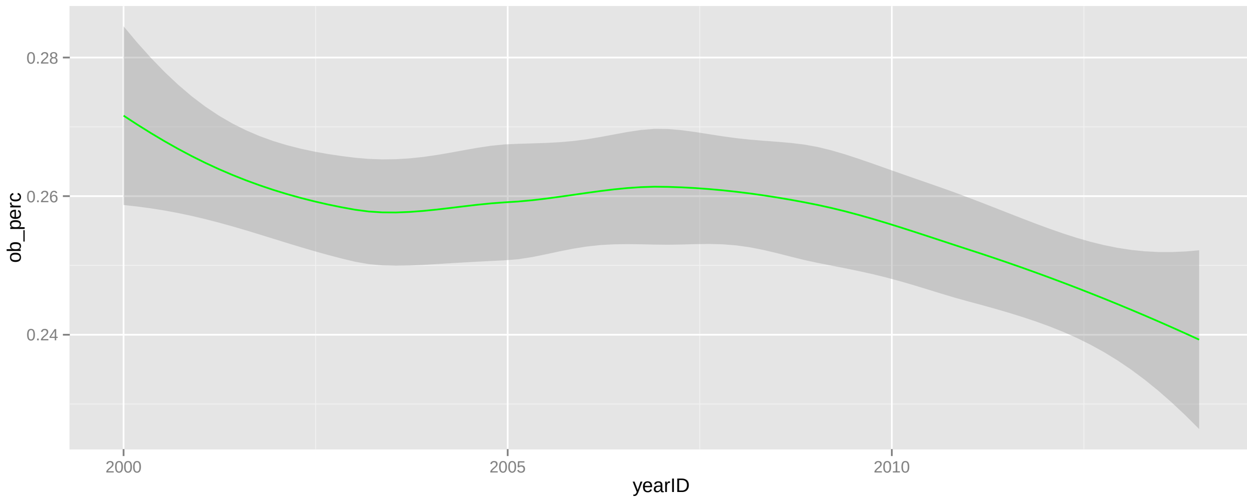


Mean on-base percentage

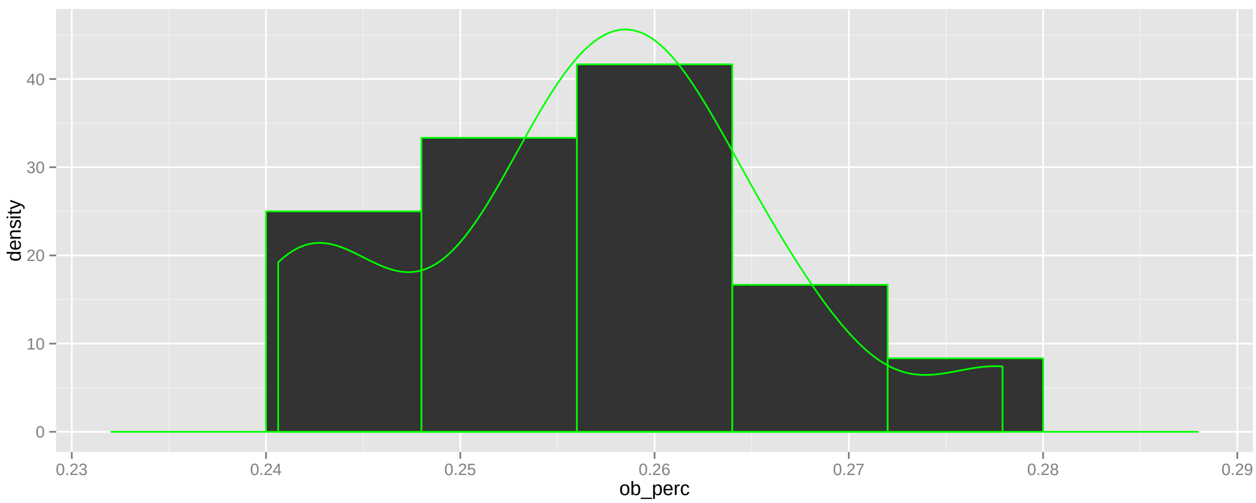
```
teams_bat %>%
  group_by(yearID) %>%
  select(ob_perc) %>%
  summarise_each(funs(mean)) %>% head()
```

```
Source: local data frame [6 x 2]

  yearID   ob_perc
   (int)     (dbl)
1   2000 0.2779122
2   2001 0.2554038
3   2002 0.2574950
4   2003 0.2679267
5   2004 0.2494640
6   2005 0.2587080
```

```
teams_bat %>%
  group_by(yearID) %>%
  select(ob_perc,slug_perc) %>%
  summarise_each(funs(mean)) %>% ggplot() +
  geom_smooth(aes(x=yearID,y=ob_perc),color="green")
```




```
teams_bat %>%
  group_by(yearID) %>%
  select(ob_perc,slug_perc) %>%
  summarise_each(funs(mean)) %>% ggplot() +
  geom_histogram(aes(ob_perc,y=..density..),color="green",binwidth=0.008) +
  geom_density(aes(ob_perc),color="green")
```



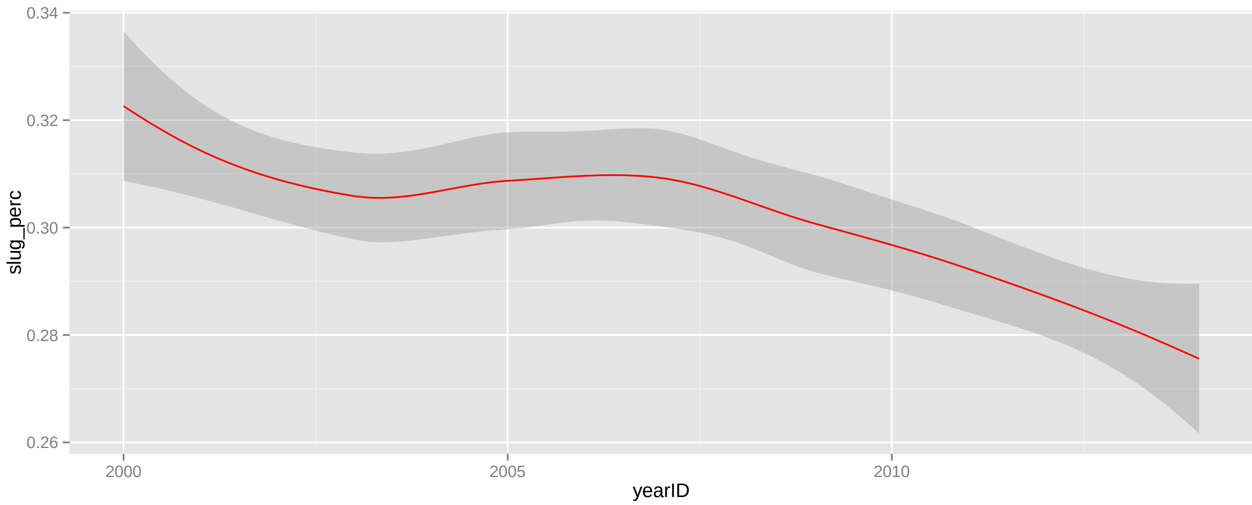
Mean slugging percentage

```
teams_bat %>%
  group_by(yearID) %>%
  select(slug_perc) %>%
  summarise_each(funs(mean)) %>% head()
```

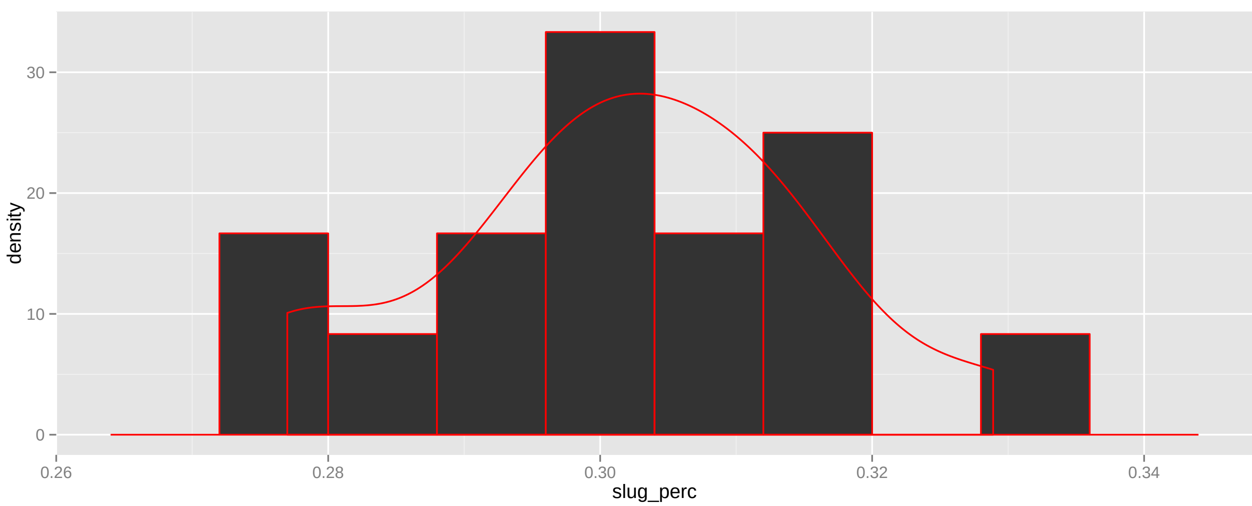
```
Source: local data frame [6 x 2]

  yearID slug_perc
   (int)     (dbl)
1   2000 0.3288996
2   2001 0.3080489
3   2002 0.3008938
```

```
4   2003 0.3138558
5   2004 0.2970384
6   2005 0.3076036
```

```
teams_bat %>%
  group_by(yearID) %>%
  select(ob_perc,slug_perc) %>%
  summarise_each(funs(mean)) %>% ggplot() +
  geom_smooth(aes(x=yearID,y=slug_perc),color="red")
```




```
teams_bat %>%
  group_by(yearID) %>%
  select(ob_perc,slug_perc) %>%
  summarise_each(funs(mean)) %>% ggplot() +
  geom_histogram(aes(slug_perc,y=..density..),color="red",binwidth=0.008) +
  geom_density(aes(slug_perc),color="red")
```

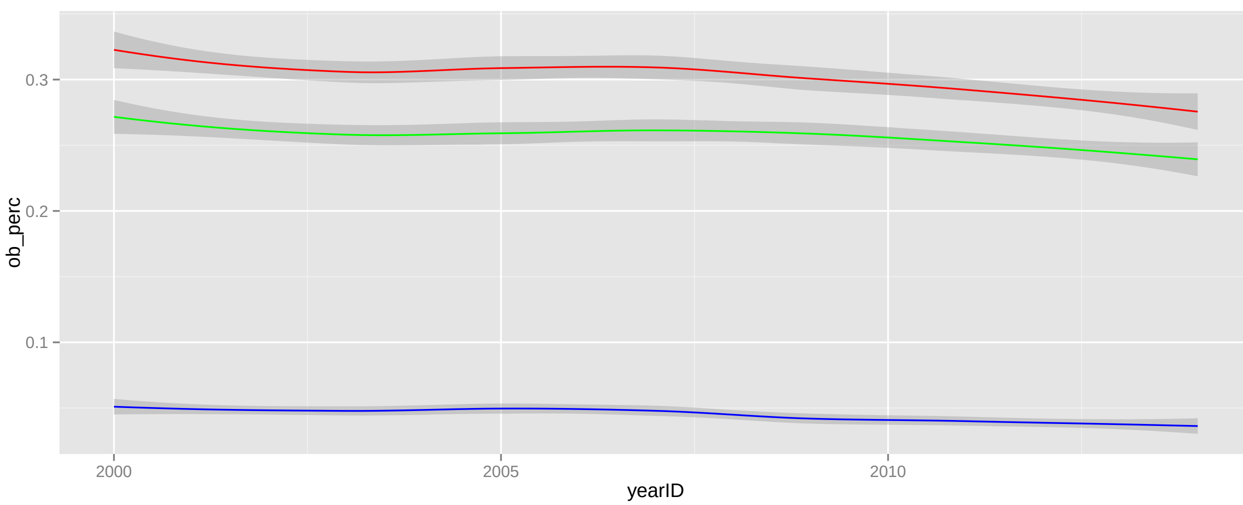


Mean on-base percentage + slugging

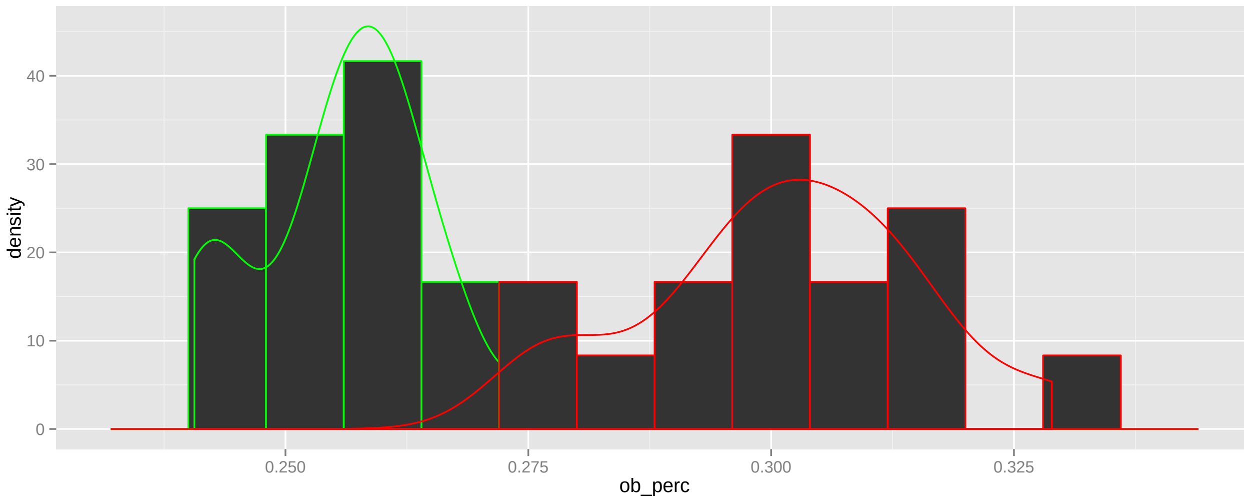
```
teams_bat %>%
  group_by(yearID) %>%
  select(ob_perc,slug_perc) %>%
  summarise_each(funs(mean)) %>% head()
```

```
Source: local data frame [6 x 3]

  yearID   ob_perc slug_perc
   (int)     (dbl)     (dbl)
1   2000 0.2779122 0.3288996
2   2001 0.2554038 0.3080489
3   2002 0.2574950 0.3008938
4   2003 0.2679267 0.3138558
5   2004 0.2494640 0.2970384
6   2005 0.2587080 0.3076036
```

```
teams_bat %>%
  group_by(yearID) %>%
  select(ob_perc,slug_perc) %>%
  summarise_each(funs(mean)) %>% ggplot() +
  geom_smooth(aes(x=yearID,y=ob_perc),color="green") +
  geom_smooth(aes(x=yearID,y=slug_perc),color="red") +
  geom_smooth(aes(x=yearID,y=(slug_perc - ob_perc)),color="blue")
```



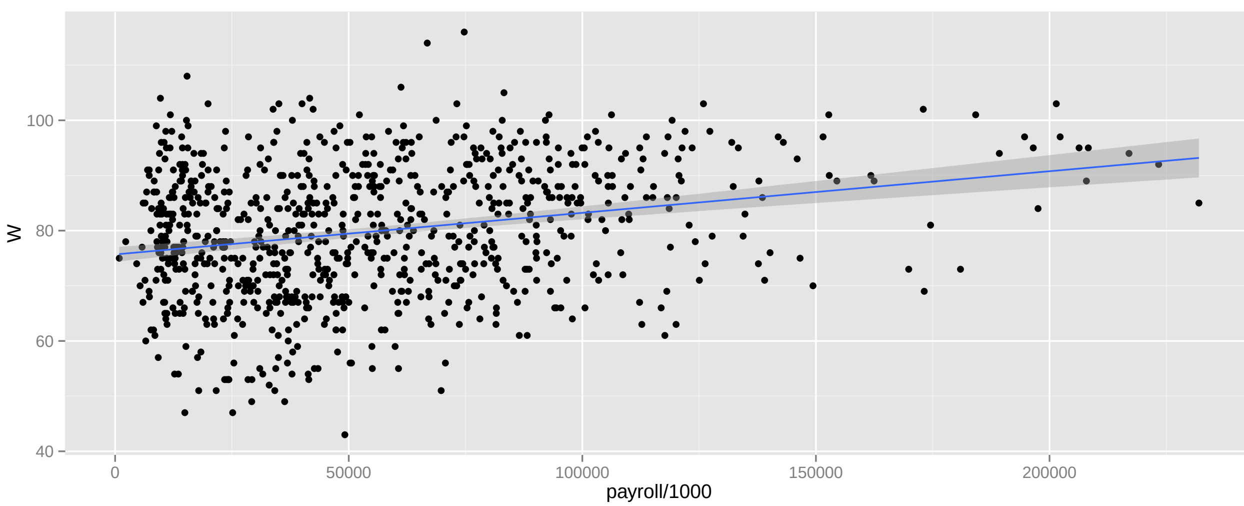

```
teams_bat %>%
  group_by(yearID) %>%
  select(ob_perc,slug_perc) %>%
  summarise_each(funs(mean)) %>% ggplot() +
  geom_histogram(aes(ob_perc,y=..density..),color="green",binwidth=0.008) +
  geom_density(aes(ob_perc),color="green") +
  geom_histogram(aes(slug_perc,y=..density..),color="red",binwidth=0.008) +
  geom_density(aes(slug_perc),color="red")
```



Bivariate EDA (+ even more wrangling)

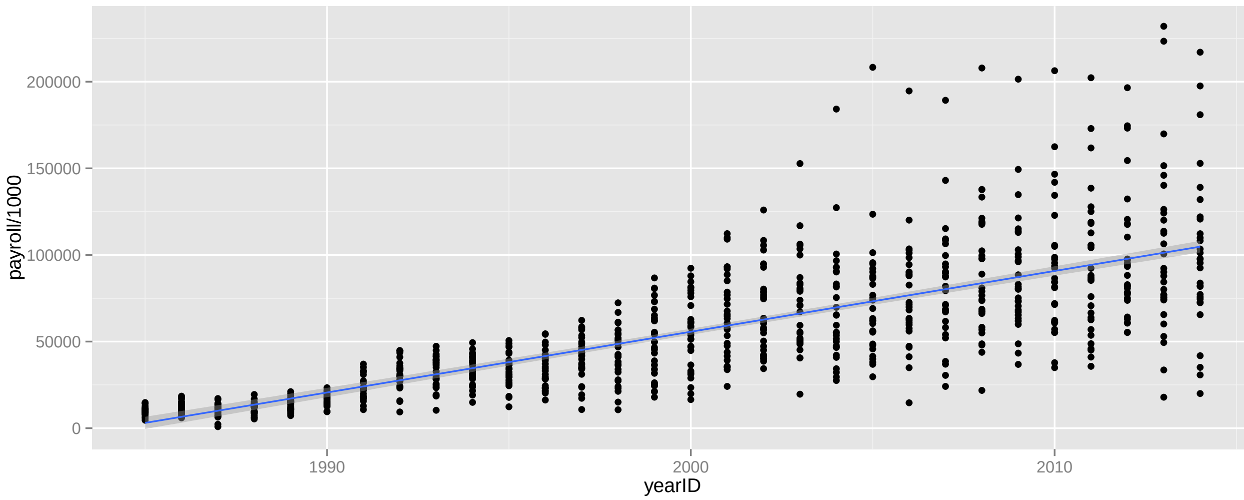
Use ggplot2 to create a scatterplot showing payroll (x-axis) and wins (y-axis) across all time periods and teams.

```
teams_pay %>%
  select(payroll,W) %>%
  ggplot() +
  geom_point(aes(x=payroll/1000,y=W)) +
  geom_smooth(aes(x=payroll/1000,y=W),method="lm")
```



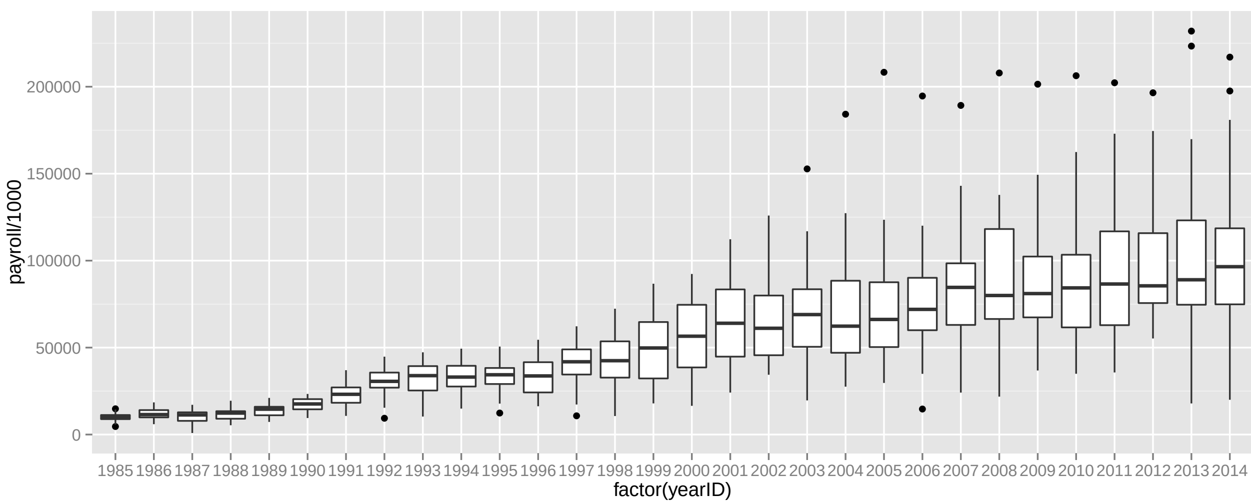
One variable we are not accounting for in this scatterplot is year. It is possible that payrolls increase from season to season. Check this out using the same ggplot code you just used above, but make this plot with year on the x-axis and payroll/1000 on the y-axis.

```
teams_pay %>%
  select(yearID,payroll) %>%
  ggplot() +
  geom_point(aes(x=yearID,y=payroll/1000)) +
  geom_smooth(aes(x=yearID,y=payroll/1000),method="lm")
```



A scatterplot may not be the best way to look at this pattern, since year is a discrete variable. So also try making boxplots stratified by yearID.

```
teams_pay %>%
  select(yearID,payroll) %>%
  ggplot() +
  geom_boxplot(aes(x=factor(yearID),y=payroll/1000))
```



Create new variables for the average payroll and the standard deviation of payrolls each year across teams and add them to your dataframe.

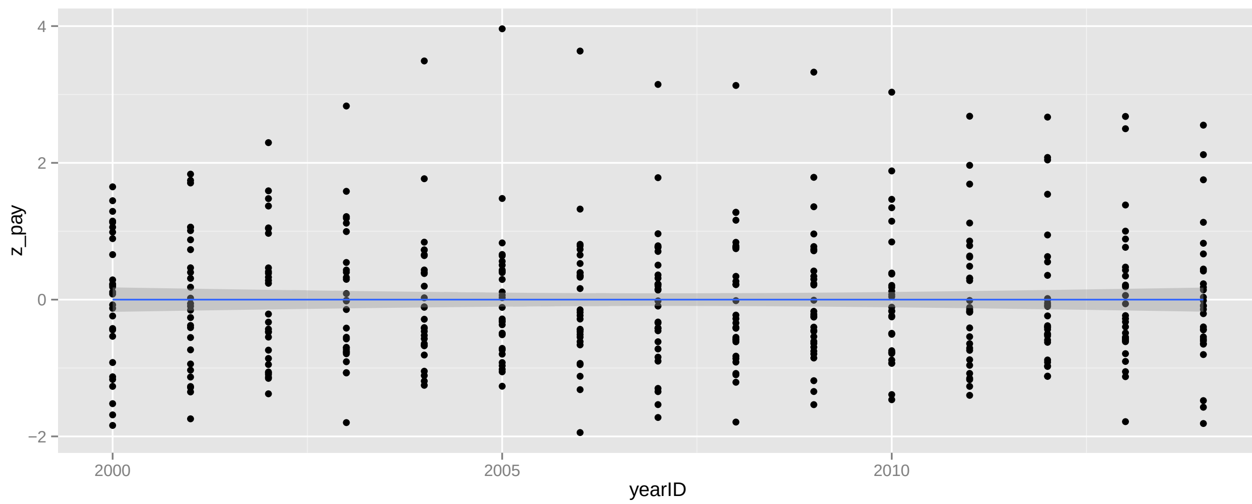
```
teams_bat <- teams_bat %>% group_by(yearID) %>%
  mutate(avg_pay = mean(payroll),
         std_pay = sd(payroll))
```

Add another variable to your dataset that is the z-score for each team for each year.

```
teams_bat <- teams_bat %>% group_by(yearID) %>%
  mutate(z_pay = (payroll - avg_pay) / std_pay)
```

Make a scatterplot in ggplot with year on the x-axis and payroll z-scores on the y-axis and two geoms: geom_point() and geom_smooth(method = "lm").

```
teams_bat %>%
  select(yearID,z_pay) %>%
  ggplot() +
  geom_point(aes(x=yearID,y=z_pay)) +
  geom_smooth(aes(x=yearID,y=z_pay),method="lm")
```



What do you see?

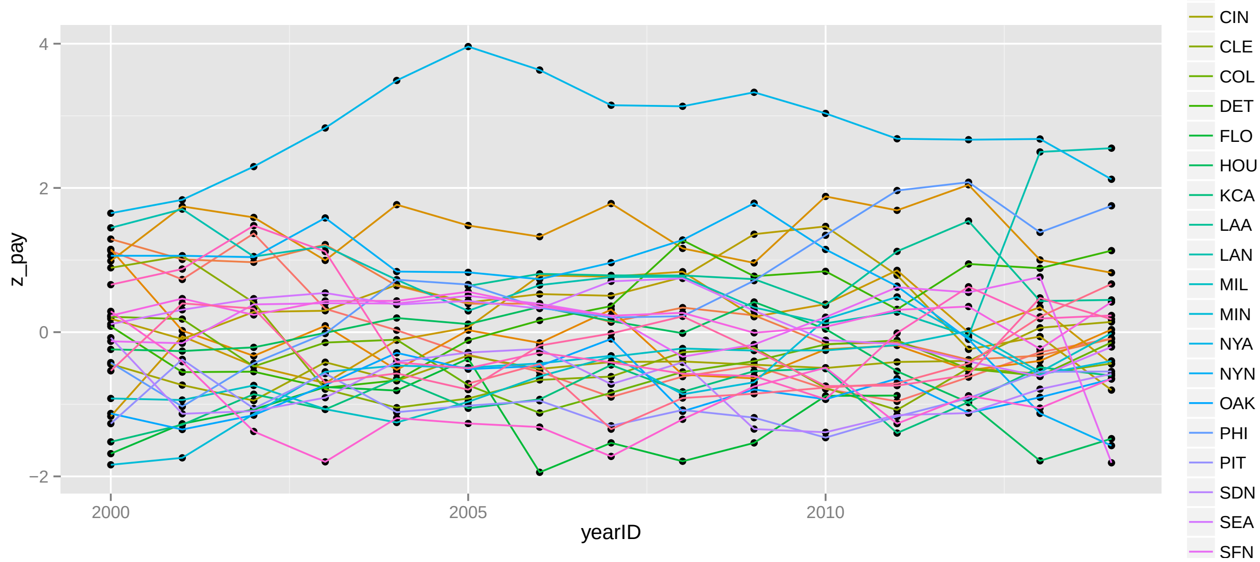
> I see a scatter plot centered about y = 0 with a z_pay spread that looks similar across year.

How is this plot different from the previous one with payroll/1000 on the y-axis (from #12)?

> The other plot showed payroll/1000 increasing with year. This plot shows z_pay steady across year (which, of course, makes perfect sense as it's relative to each year's mean payroll)

Make a new scatterplot (minus geom_smooth) in ggplot with year on the x-axis and payroll z-scores on the y-axis. This time, add an additional aesthetic to colour the points in the scatterplot with a different color for each teamID, and an additional geom called geom_line().

```
teams_bat %>%
  select(yearID,z_pay,teamID) %>%
  ggplot() +
  geom_point(aes(x=yearID,y=z_pay)) +
  geom_line(aes(x=yearID,y=z_pay,color=factor(teamID)))
```



What do you see?

> I see points indicating team payroll by year connected by colored lines indicating teamID.

What is not surprising here?

> Teams tend to move only slightly (relative to each other) each year. Teams at the top tend to stay toward the top, teams on the bottom tend to stay toward the bottom.

Use dplyr to create a new dataset … that includes two new variables: average payroll z-score and average number of wins. Both averages should be calculated for each team across all seasons.

```
teams_anl <- teams_bat %>%
  group_by(teamID) %>%
  summarise(avg_z_pay = mean(z_pay,na.rm=TRUE),
  avg_w_cnt = mean(W,na.rm=TRUE))
```

What will be the mean and standard deviation of this new variable across the 30 teams?

> mean: This will be the mean distance of the annual payroll from the annual mean payroll across teams.

> sd: This will be the spread of the distance of the annual payroll from the annual mean payroll across teams.

That is, is your new average payroll z-score also a z-score?

> yes

Are you surprised?

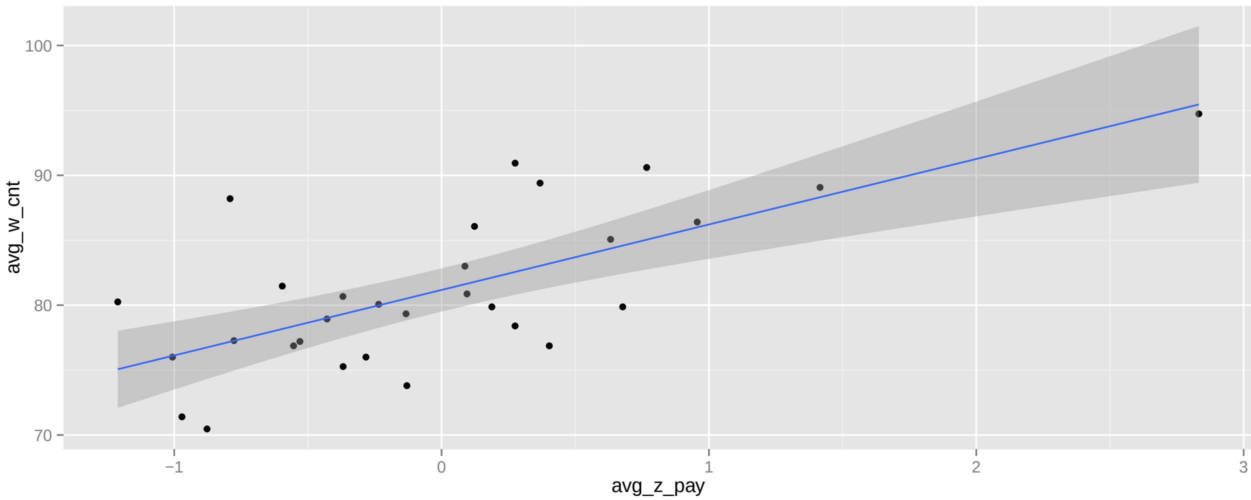
> Not too surprised.

Why or why not?

> If I think about a z-score has having a unit, z-scoreness?, it makes sense to me that an average of z-scores keeps its z-scoreness.

Now create a scatterplot to see the association between average payroll z-scores (x-axis) and average number of wins (y-axis).

```
teams_anl %>% ggplot() +
  geom_point(aes(x=avg_z_pay,y=avg_w_cnt)) +
  geom_smooth(aes(x=avg_z_pay,y=avg_w_cnt),method="lm")
```




```
teams_anl %>% lm(avg_w_cnt ~ avg_z_pay,.) %>% summary()
```

```

Call:
lm(formula = avg_w_cnt ~ avg_z_pay, data = .)

Residuals:
    Min      1Q  Median      3Q     Max 
-6.7168 -3.3719 -0.0858  1.3803 11.0187 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  81.1713     0.8122  99.937  < 2e-16 ***
avg_z_pay     5.0438     0.9974   5.057 2.37e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.449 on 28 degrees of freedom
Multiple R-squared:  0.4773,    Adjusted R-squared:  0.4587 
F-statistic: 25.57 on 1 and 28 DF,  p-value: 2.375e-05
```

```
cor(teams_anl$avg_z_pay,teams_anl$avg_w_cnt)
```

```
[1] 0.6908895
```

Use both the plot and the correlation statistics to evaluate (in words) the form (does the relationship look linear?) and strength of the association between these two variables.

> The form looks as if it can be approximated by a linear model. The correlation is slight (cor = 0.6908895, R-squared = 0.4773283) but significant (p-value: 2.375e-05).

Would you be comfortable using a linear model to predict the mean number of wins in a given season given their average relative payroll for that season?

> Given such a low R-squared (not close to 1), I'd be hesitant to predict the exact number of wins, though with such a low p-value, I might be comfortable making more general predictions, like predicting the mean number of wins is within some range etc.

Regression model

Build a simple linear regression model predicting mean wins from mean payroll z-scores across seasons.

```r
teams_anl_lm <- teams_anl %>% lm(avg_w_cnt ~ avg_z_pay,.)
teams_anl_lm %>% anova()
```

```
Analysis of Variance Table

Response: avg_w_cnt
          Df Sum Sq Mean Sq F value    Pr(>F)
avg_z_pay  1 506.05  506.05  25.571 2.375e-05 ***
Residuals 28 554.13   19.79
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
teams_anl_lm %>% summary()
```

```
Call:
lm(formula = avg_w_cnt ~ avg_z_pay, data = .)

Residuals:
    Min      1Q  Median      3Q     Max
-6.7168 -3.3719 -0.0858  1.3803 11.0187

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  81.1713     0.8122  99.937  < 2e-16 ***
avg_z_pay     5.0438     0.9974   5.057 2.37e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.449 on 28 degrees of freedom
Multiple R-squared:  0.4773,    Adjusted R-squared:  0.4587
F-statistic: 25.57 on 1 and 28 DF,  p-value: 2.375e-05
```

```r
teams_anl_lm %>% glance()
```

```
  r.squared adj.r.squared    sigma statistic      p.value df   logLik
1 0.4773283     0.4586614 4.448623  25.57091 2.374621e-05  2 -86.3111
       AIC      BIC deviance df.residual
1 178.6222 182.8258 554.1268          28
```

What are the total, model, and residual sums of squares for this simple linear regression?

> model: 506.05 residual: 554.13

What percent of the variation in mean wins is "explained" by variation in mean payroll z-scores?

> 47.73283% (R-squared)

Write up a summary of your findings.

> The results indicate that payroll predicted wins (b1 = 5.0438), with 47.73283% of the variance in wins accounted for by payroll levels. Each standard deviation payroll was from the annual mean was associated with an increase of 5.0434 wins. The OLS regression equation for predicting wins is of the form

$$\text{wins}_i = 81.1713 + 5.0438\text{standard deviations from mean payroll}_i + \epsilon_i$$

What is the average of all ŷ i values (in any simple linear regression model) equal to?

> The mean response (intercept).

What is the variance of the residuals in your regression model?

> 19.79

The standard error?

> 4.449

Compare the variance of the residuals to sample variance of mean wins overall, and to your model R2.

> variance of residuals: 19.79 variance of mean wins overall: 36.55798 R-squared: 0.4773 Adj. R-squared = 0.4587

$$1 - \frac{19.79}{36.55798} \approx 0.4587$$

How are these three statistics related (in any simple linear regression model)?

$$1 - \frac{\text{variance of residuals}}{\text{variance of mean wins overall}} = \text{Adj. R-squared}$$

Obviously, the book and movie about the Oakland A's suggests that this team may be an outlier in terms of the predicting wins from payroll. Look specifically at this team:

What is the observed number of mean wins?

```r
teams_anl %>% filter(teamID == "OAK")
```

```
Source: local data frame [1 x 3]

  teamID  avg_z_pay avg_w_cnt
   (chr)      (dbl)     (dbl)
1    OAK -0.7910688      88.2
```

88.2

What is the predicted?

$$81.1713 + 5.0438 \times -0.7910688 \approx 77.18$$

What is the residual?

$$88.2 - 77.18 = 11.02$$

How many standard deviations above/below the residual mean is the Oakland A's residual value?

```r
augment(teams_anl_lm) %>% filter(avg_w_cnt == 88.2)
```

```
  avg_w_cnt  avg_z_pay  .fitted  .se.fit   .resid       .hat   .sigma
1      88.2 -0.7910688 77.18132 1.128597 11.01868 0.06436152 3.964489
    .cooksd .std.resid
1 0.2255216   2.560649
```

2.56 above

Are there any other teams with a residual value as extreme or more extreme than the Oakland A's?

```r
augment(teams_anl_lm) %>% arrange(.std.resid) %>% tail()
```

```
   avg_w_cnt  avg_z_pay  .fitted   .se.fit    .resid       .hat   .sigma
25  86.06667  0.1232494 81.79296 0.8222835  4.273711 0.03416583 4.452282
26  80.25000 -1.2111056 75.06274 1.4511598  5.187262 0.10640924 4.405447
27  90.60000  0.7672145 85.04099 1.1196456  5.559013 0.06334466 4.393321
28  89.40000  0.3683245 83.02907 0.8936759  6.370934 0.04035608 4.353930
29  90.93333  0.2751527 82.55913 0.8590704  8.374207 0.03729120 4.222002
30  88.20000 -0.7910688 77.18132 1.1285966 11.018681 0.06436152 3.964489
      .cooksd .std.resid
25 0.01690114  0.9775259
26 0.09059354  1.2335117
27 0.05637217  1.2911665
28 0.04493801  1.4619162
29 0.07128904  1.9185392
30 0.22552164  2.5606486
```

No. The next highest is SLN (St. Louis Cardinals) with 1.92

Create a bootstrap distribution for the correlation and the regression coefficients. Copy and paste the following code into your file, and annotate each line with a # to (briefly) explain what each line of code is doing.

```
orig_cor <- 0.6908895
orig_slp <- 5.0438
gt_cor_cnt <- 0
gt_slp_cnt <- 0
N <- 10^4 # storing 10000 as N
cor.boot <- numeric(N) # store vector of size 10000 as cor.boot
int.boot <- numeric(N) # store vector of size 10000 as int.boot
slope.boot <- numeric(N) # store vector of size 10000 as slope.boot
n <- 30 # number of observations here
for (i in 1:N){ # loop 10000 times, storing loop iteration as i
    index <- sample(n, replace = TRUE) # store a vector of size n
                                       # with values ranging from 1
                                       # to n as index
    team.boot <- teams_anl[index, ] # resampled data
    cor.boot[i] <- cor(team.boot$avg_z_pay, team.boot$avg_w_cnt)
    # what is x and y? The input & response variables, avg_z_pay and avg_w_cnt
    if(cor.boot[i] > orig_cor){
      gt_cor_cnt <- gt_cor_cnt + 1
    }
    # recalculate linear model estimates
    team.boot.lm <- lm(avg_w_cnt ~ avg_z_pay, data = team.boot)
    # what is x and y?
    int.boot[i] <- coef(team.boot.lm)[1] # new intercept
    slope.boot[i] <- coef(team.boot.lm)[2] # new slope
    if(slope.boot[i] > orig_slp){
      gt_slp_cnt <- gt_slp_cnt + 1
    }
  }

mean(cor.boot) #mean correlation of bootstrapped data
```

```
[1] 0.6775616
```

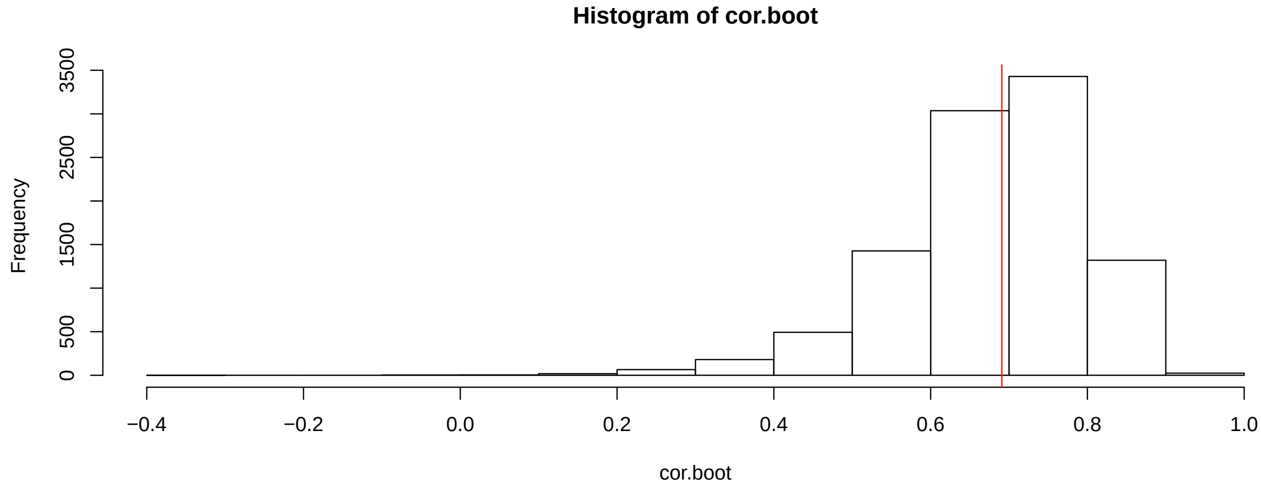
```
sd(cor.boot) #standard deviation of correlation of bootstrapped data
```

```
[1] 0.1186705
```

```
quantile(cor.boot, c(.025, .975)) #95% CI of correlation of bootstrapped data
```

```
     2.5%     97.5%
0.3920053 0.8547026
```

```
hist(cor.boot)
#create histogram of correlation of bootstrapped data
observed <- cor(teams_anl$avg_z_pay, teams_anl$avg_w_cnt)
# what is x and y? The input & response variables avg_z_pay and avg_w_cnt
abline(v = observed, col = "red") # add line at original sample correlation
```




```r
# do the same as above for slope.boot (don't worry about int.boot)
mean(slope.boot) #mean slope of bootstrapped data
```

```
[1] 5.10667
```

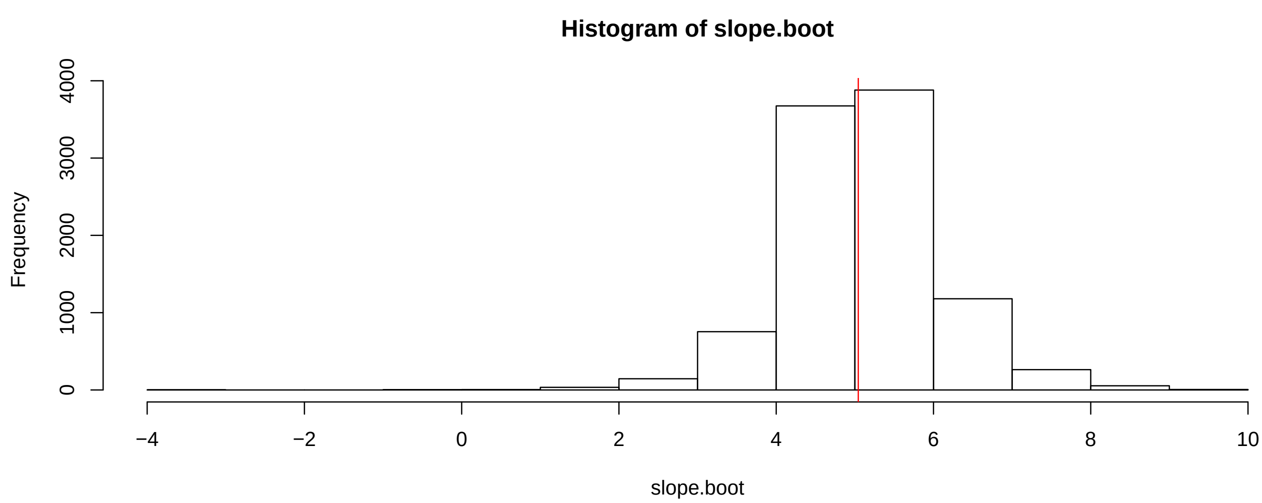
```r
sd(slope.boot) #standard deviation of slope of bootstrapped data
```

```
[1] 0.9633642
```

```r
quantile(slope.boot, c(.025, .975)) #95% CI of slope of bootstrapped data
```

```
    2.5%    97.5%
3.195022 7.154022
```

```r
hist(slope.boot) #create histogram of slope of bootstrapped data
observed <- summary(teams_anl_lm)$coefficients[2]
# what is x and y? The input & response variables avg_z_pay and avg_w_cnt
abline(v = observed, col = "red") # add line at original sample slope
```



```
gt_cor_cnt
```

```
[1] 5138
```

```
gt_slp_cnt
```

```
[1] 5169
```

Figure out how many bootstrap samples had a higher correlation than the one you observed as your original sample correlation.

> 5198 (51.98%)

How many bootstrap samples had a higher slope coefficient than the one you observed.

> 5065 (50.65%)

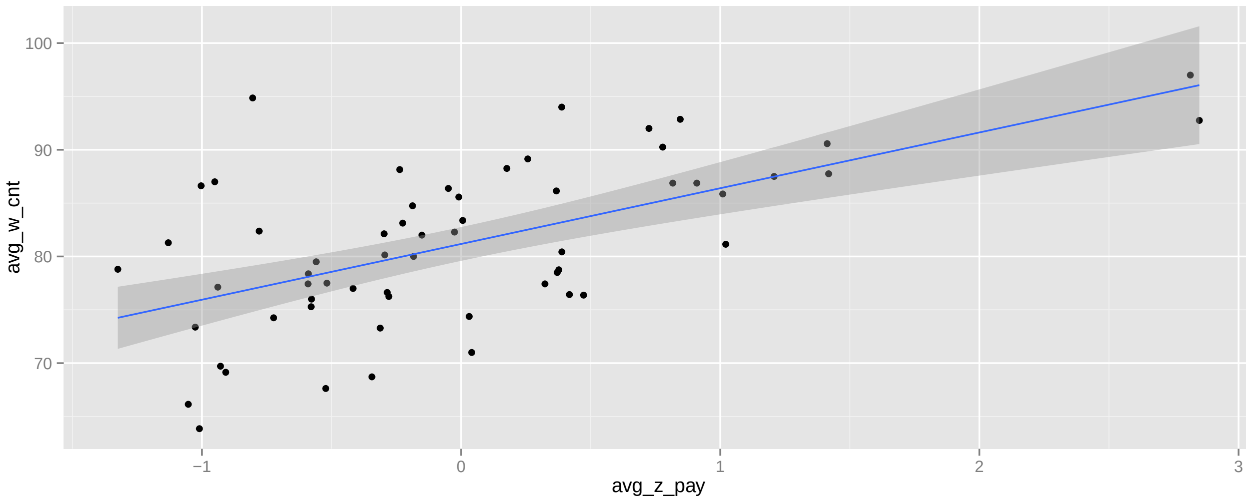
Use dplyr::mutate() with ifelse() to create a categorical variable that splits our yearID variable into two time intervals: 2000 - 2006 and 2007 - 2014. Then look at your work for question 14 and update to re-calculate average wins and average payroll z-scores separately for each team and time interval (hint: that means two variables in a dplyr::group_by() statement).

```
teams_bat_year_split <- teams_bat %>% # rename these dataframes as appropriate
  mutate(recent = ifelse(yearID < 2007, 0, 1))# rename variables as appropriate
table(teams_bat_year_split$recent, teams_bat_year_split$yearID) # trust but verify
```

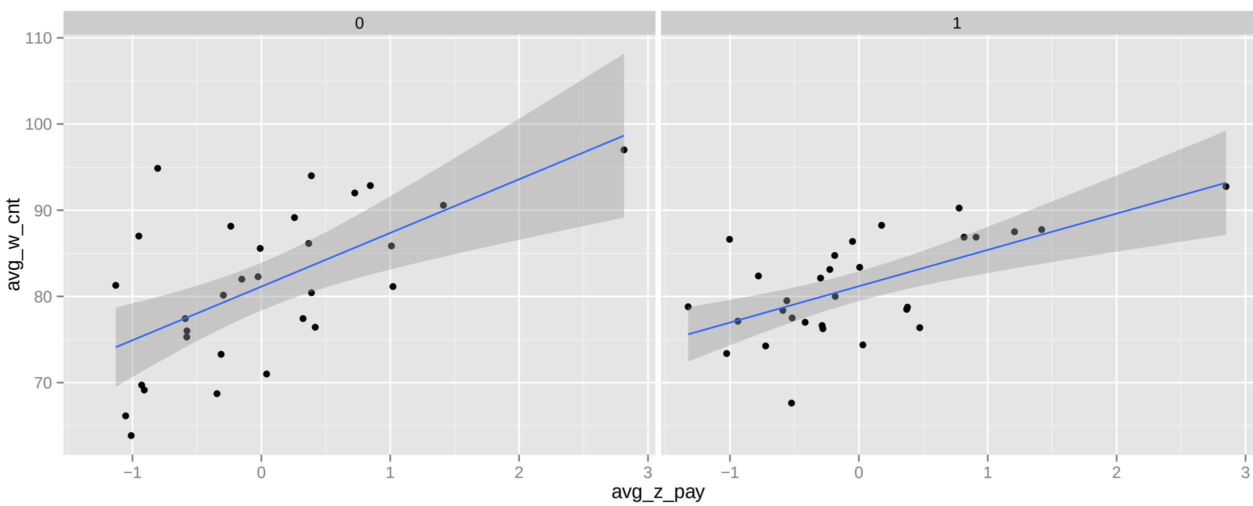
```

    2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013
  0   28   28   28   28   28   30   30    0    0    0    0    0    0    0
  1    0    0    0    0    0    0    0   30   30   30   30   30   29   29

    2014
  0    0
  1   29
```

```
teams_bat_year_split_anl <- teams_bat_year_split %>%
  group_by(teamID,recent) %>%
  summarise(avg_z_pay = mean(z_pay,na.rm=TRUE),
  avg_w_cnt = mean(W,na.rm=TRUE))
```

```
teams_bat_year_split_anl %>% group_by(teamID) %>%
  ggplot() +
  geom_point(aes(x=avg_z_pay,y=avg_w_cnt)) +
  geom_smooth(aes(x=avg_z_pay,y=avg_w_cnt),method="lm")
```



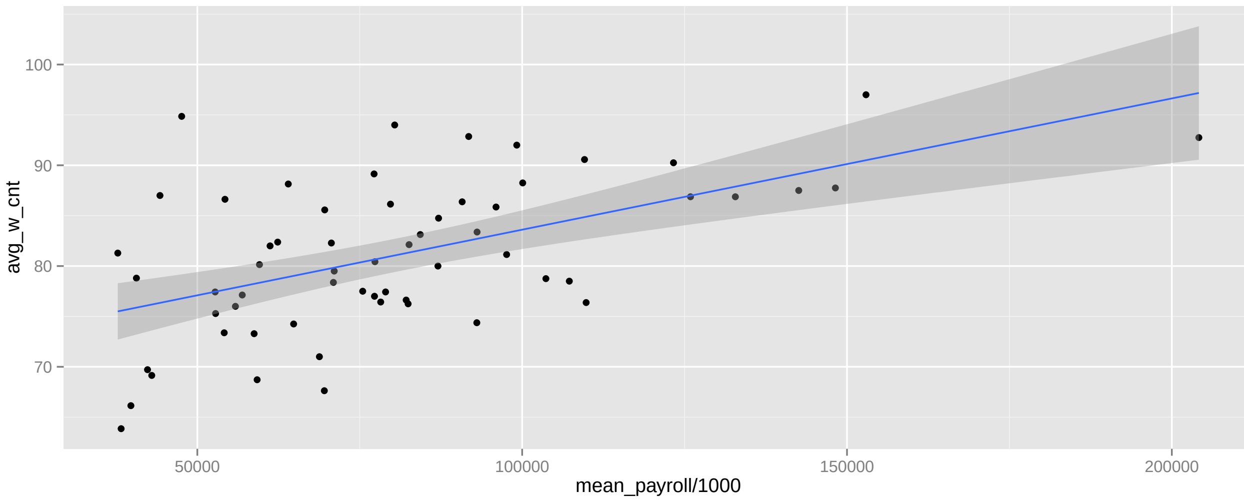

```
teams_bat_year_split_anl %>% group_by(teamID) %>%
  ggplot() +
  facet_wrap(~ recent) +
  geom_point(aes(x=avg_z_pay,y=avg_w_cnt)) +
  geom_smooth(aes(x=avg_z_pay,y=avg_w_cnt),method="lm")
```



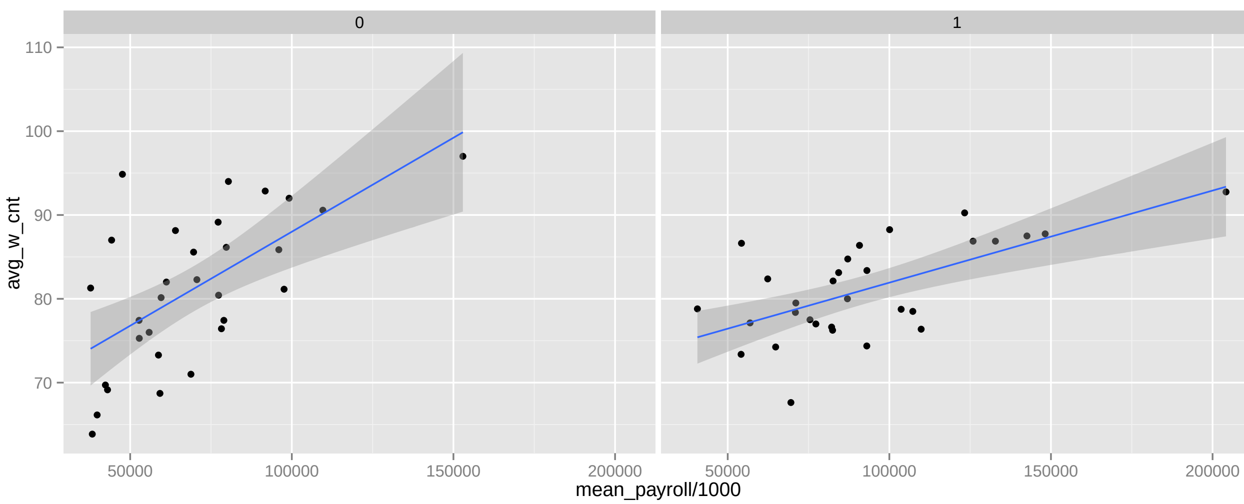
Using ggplot2, create one plot, with side-by-side scatterplots for each time interval, showing mean payroll (x-axis) and mean number of wins (y-axis) across all teams.

```
teams_bat_year_split_anl_w_mean_payroll <- teams_bat_year_split %>%
  group_by(teamID,recent) %>%
  summarise(mean_payroll = mean(payroll,na.rm=TRUE),
  avg_w_cnt = mean(W,na.rm=TRUE))
```

```
teams_bat_year_split_anl_w_mean_payroll %>% group_by(teamID) %>%
  ggplot() +
  geom_point(aes(x=mean_payroll/1000,y=avg_w_cnt)) +
  geom_smooth(aes(x=mean_payroll/1000,y=avg_w_cnt),method="lm")
```




```
teams_bat_year_split_anl_w_mean_payroll %>% group_by(teamID,recent) %>%
  ggplot() +
  facet_wrap(~ recent) +
  geom_point(aes(x=mean_payroll/1000,y=avg_w_cnt)) +
  geom_smooth(aes(x=mean_payroll/1000,y=avg_w_cnt),method="lm")
```



Comment on differences you see between these two plots, and compare to your previous scatterplot across all seasons.

> It appears the slope of the regression line was steeper before 2007, indicating wins were cheaper. The scatterplot across all seasons hides the fact that these two quite models exist and simply draws a regression line over all the datapoints.

Now, run two linear regression analyses (as shown in class), one for each time interval, using dplyr::group_by() %>% do() and broom::tidy()/glance()/augment().

```
models <- teams_bat_year_split_anl %>%
  group_by(recent) %>%
  do(mod = lm(avg_w_cnt ~ avg_z_pay, data = .))

models %>% tidy(mod) #coefs
```

```
Source: local data frame [4 x 6]
Groups: recent [2]

  recent        term  estimate std.error statistic      p.value
   (dbl)       (chr)     (dbl)     (dbl)     (dbl)        (dbl)
1      0 (Intercept) 81.140032 1.3542026 59.917201 4.145641e-31
2      0   avg_z_pay  6.225057 1.5798253  3.940345 4.929825e-04
3      1 (Intercept) 81.183945 0.8403026 96.612750 6.856896e-37
4      1   avg_z_pay  4.213821 0.9859629  4.273813 2.008604e-04
```

```
models %>% augment(mod) %>%
  group_by(recent) %>%
  summarize(tot_ss = sum((avg_w_cnt - mean(avg_w_cnt))^2),
            res_ss = sum((avg_w_cnt - .fitted)^2))
```

```
Source: local data frame [2 x 3]

  recent    tot_ss    res_ss
   (dbl)     (dbl)     (dbl)
1      0 2394.6014 1540.4205
2      1  979.6834  592.9072
```

```
models %>% glance(mod)
```

```
Source: local data frame [2 x 12]
Groups: recent [2]

  recent r.squared adj.r.squared    sigma statistic      p.value    df
   (dbl)     (dbl)         (dbl)    (dbl)     (dbl)        (dbl) (int)
1      0 0.3567111     0.3337365 7.417211  15.52632 0.0004929825     2
2      1 0.3947971     0.3731827 4.601658  18.26548 0.0002008604     2
Variables not shown: logLik (dbl), AIC (dbl), BIC (dbl), deviance (dbl),
  df.residual (int)
```

Compare the coefficient estimates to each other, and to your original model.

> The intercepts of both models are similar (81.14 and 81.18) but the slopes are not (6.22 and 4.21). The coefficients of the original model are between these two models' coefficients (original intercept = 81.17 and original slope = 5.04).

Take the Oakland A's team as a specific case:

Which of your three model/time interval regression models (model 1: across all seasons; model 2: 2000 - 2006; model 3: 2007 - 2014) was better at predicting mean wins for them specifically?

```
#Oakland A's actual data
avg_w_cnt = 88.2
avg_z_pay = -0.7910688

#model 1
(81.17 + 5.04 * avg_z_pay) - avg_w_cnt
```

```
[1] -11.01699
```

```r
#model 2
(81.14 + 6.22 * avg_z_pay) - avg_w_cnt
```

```
[1] -11.98045
```

```r
#model 3
(81.18 + 4.21 * avg_z_pay) - avg_w_cnt
```

```
[1] -10.3504
```

> model 3

Which model overall accounted for the most variability in mean wins overall across all teams?

> model 1. It had an R-squared of 0.4773 while model 2 and 3 had R-squared values of 0.3567 and 0.3947, respectively.

How is the R2 estimate related to the plain old correlation between average wins and average payroll z-scores for each time interval?

$$0.4773 = 0.690869^2$$
$$0.3567 = 0.5972437^2$$
$$0.3947 = 0.6282515^2$$

And in general in any simple linear regression model?

> R-squared is the square of the correlation.

Midterm: Exercises

Part 1: Probability

The following table shows the cumulative distribution function of a discrete random variable. Find the probability mass function.

```r
k = c(0,1,2,3,4,5)
F.k = c(0,.1,.3,.7,.8,1.0)
df = data.frame(k,F.k)
kable(df)
```

k	F.k
0	0.0
1	0.1
2	0.3
3	0.7
4	0.8
5	1.0

$$\text{cdf: } F(x) = \sum_{t \le x} f(t)$$

$$\text{pmf: } f(t) = F(x) - F(x-1) = \sum_{t \le x} f(t) - \sum_{t \le x-1} f(t)$$

$$f(0) = F(0) = 0$$
$$f(1) = F(1) - F(0) = .1 - 0 = .1$$
$$f(2) = F(2) - F(1) = .3 - .1 = .2$$
$$f(3) = F(3) - F(2) = .7 - .3 = .4$$
$$f(4) = F(4) - F(3) = .8 - .7 = .1$$
$$f(5) = F(5) - F(4) = 1.0 - .8 = .2$$

```
k = c(0,1,2,3,4,5)
F.k = c(0,.1,.3,.7,.8,1.0)
f.k = c(0,.1,.2,.4,.1,.2)
df = data.frame(k,F.k,f.k)
kable(df)
```

k	F.k	f.k
0	0.0	0.0
1	0.1	0.1
2	0.3	0.2
3	0.7	0.4
4	0.8	0.1
5	1.0	0.2

The probability density function of a random variable X is given by:

$$f(x) = \begin{cases} cx, & 0 < x < 4 \\ 0, & \text{otherwise} \end{cases}$$

a) find c

$$\int_{-\infty}^{\infty} f_x(x)\, dx = 1$$

$$
\begin{aligned}
1 &= \int_0^4 cx\,dx \\
&= c \times \left[\frac{x^2}{2}\right]_0^4 \\
&= c \times \left[\frac{4^2}{2} - 0\right] \\
&= c \times \left[\frac{16}{2}\right] \\
&= 8c
\end{aligned}
$$

$$
c = \frac{1}{8}
$$

b) find the cumulative distribution function F(x).

$$
F_x(x) = \int_{-\infty}^{x} f_x(y)\,dy = P(X \leq x)
$$

$$
\begin{aligned}
F_x(x) &= \int_0^x \frac{1}{8}(y)\,dy \text{ when } 0 \leq x \leq 4 \\
&= \frac{1}{8}\left[\frac{y^2}{2}\right]_0^x \\
&= \frac{1}{8}\left[\frac{x^2}{2} - 0\right] \\
&= \frac{x^2}{16}
\end{aligned}
$$

$$
F_x(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{16}, & 0 \leq x \leq 4 \\ 1, & x > 4 \end{cases}
$$

c) Compute P(1 < X < 3)

$$
\begin{aligned}
P(1 < X < 3) &= F_x(3) - F_x(1) \\
&= \frac{3^2}{16} - \frac{1^2}{16} \\
&= \frac{9}{16} - \frac{1}{16} \\
&= \frac{8}{16} \\
&= \frac{1}{2} \\
&= 0.5
\end{aligned}
$$

The random variable X has a cumulative distribution function (cdf):

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x^3}{2+x^2}, & x > 0 \end{cases}$$

Find the probability density function (pdf) of X.

$$\begin{aligned} pdf(x) = f_x(x) &= F'(x) \\ &= \frac{x^3}{2+x^2}' \text{ when } x > 0 \\ &= \frac{(2+x^2) \times 3x^2 - x^3 \times 2x}{(2+x^2)^2} \text{ (quotient rule)} \\ &= \frac{6x^2 + 3x^4 - 2x^4}{(2+x^2)^2} \\ &= \frac{6x^2 + x^4}{(2+x^2)^2} \\ &= \frac{x^2(6+x^2)}{(2+x^2)^2} \end{aligned}$$

$$f_x(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x^2(6+x^2)}{(2+x^2)^2}, & x > 0 \end{cases}$$

The joint probability of the continuous random variable (X, Y) is given by:

$$f(x,y) = \begin{cases} \frac{1}{28}(4x + 2y + 1), & 0 \leq x < 2,\ 0 \leq y < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find E(XY)

$$\begin{aligned} E\big[XY\big] &= \int_0^2 \int_0^2 xy\Big[\frac{1}{28}(4x + 2y + 1)\Big]\, dxdy \\ &= \frac{1}{28} \int_0^2 \int_0^2 4x^2y + 2xy^2 + xy\, dxdy \end{aligned}$$

$$
\begin{aligned}
(\text{expr. } 1) &= \frac{1}{28}\int_0^2 4x^2y + 2xy^2 + xy\,dx \\
&= \frac{1}{28}\Big[4\frac{x^3}{3}y + 2\frac{x^2}{2}y^2 + \frac{x^2}{2}y\Big]_{x=0}^{x=2} \\
&= \frac{1}{28}\Big[4\frac{8}{3}y + 2 \times 2y^2 + 2y\Big] \\
&= \frac{1}{28}\Big[\frac{32y}{3} + 4y^2 + 2y\Big] \\
&= \frac{1}{28}\Big[\frac{38y}{3} + 4y^2\Big] \\
&= \frac{38y}{84} + \frac{4y^2}{28} \\
&= \frac{19y}{42} + \frac{y^2}{7}
\end{aligned}
$$

$$
\begin{aligned}
E\big[XY\big] &= \int_0^2 (\text{expr. } 1)\,dy \\
&= \int_0^2 \frac{19y}{42} + \frac{y^2}{7}\,dy \\
&= \Big[\frac{19}{42} \times \frac{y^2}{2} + \frac{1}{7} \times \frac{y^3}{3}\Big]_0^2 \\
&= \Big[\frac{19}{42} \times 2 + \frac{1}{7} \times \frac{8}{3}\Big] - 0 \\
&\approx 1.286
\end{aligned}
$$

Find Cov(X, Y)

Marginal Probabilities:

$$
\begin{aligned}
f_x(x) &= \int_{-\infty}^{\infty} f(x,y)dy \\
f_y(y) &= \int_{-\infty}^{\infty} f(x,y)dx
\end{aligned}
$$

$$
\begin{aligned}
f_x(x) &= \int_0^2 \frac{1}{28}(4x + 2y + 1)\,dy \\
&= \frac{1}{28}\Big[4xy + 2\frac{y^2}{2} + y\Big]_0^2 \\
&= \frac{1}{28}\Big[8x + 4 + 2\Big] \\
&= \frac{8x}{28} + \frac{6}{28} \\
&= \frac{2x}{y} + \frac{3}{14}
\end{aligned}
$$

$$\begin{aligned}
f_y(y) &= \int_0^2 \frac{1}{28}(4x + 2y + 1)\,dx \\
&= \frac{1}{28}\Big[4\frac{x^2}{2} + 2yx + x\Big]_0^2 \\
&= \frac{1}{28}\Big[8 + 4y + 2\Big] \\
&= \frac{4y}{28} + \frac{10}{28} \\
&= \frac{y}{7} + \frac{5}{14}
\end{aligned}$$

$$\begin{aligned}
E\big[X\big] &= \int_{-\infty}^{infty} x f_x(x)\,dx \\
&= \int_0^2 x\Big[\frac{2x}{7} + frac314\Big]\,dx \\
&= \int_0^2 \frac{2x^2}{7} + \frac{3x}{14}\,dx \\
&= \Big[\frac{2x^3}{21} + \frac{3x^2}{28}\Big]_0^2 \\
&= \Big[\frac{16}{21} + \frac{12}{28}\Big] - 0 \\
&\approx 1.190
\end{aligned}$$

$$\begin{aligned}
E\big[Y\big] &= \int_{-\infty}^{infty} y f_y(y)\,dy \\
&= \int_0^2 y\Big[\frac{y}{7} + \frac{5}{14}\Big]\,dy \\
&= \int_0^2 \frac{y^2}{7} + \frac{5y}{14}\,dy \\
&= \Big[\frac{y^3}{21} + \frac{5y^2}{28}\Big]_0^2 \\
&= \Big[\frac{8}{21} + \frac{20}{28}\Big] - 0 \\
&\approx 1.095
\end{aligned}$$

$$\begin{aligned}
\text{Cov(X,Y) = E[XY] - E[X]E[Y]} \\
&\approx 1.286 - 1.095 \times 1.190 \\
&\approx 1.286 - 1.30305 \\
&\approx -0.01705
\end{aligned}$$

Find the correlation coefficient ρ_{XY}

$$
\begin{aligned}
Var(X) &= \int_{-\infty}^{\infty} (X - \mu x)^2 f(x)\, dx \\
&= \int_0^2 (X - 1.190)^2 \times \left[\frac{2x}{7} + \frac{3}{14}\right] dx \\
&\text{used wolfram alpha solver} \\
&\approx 0.297
\end{aligned}
$$

$$
\begin{aligned}
Var(Y) &= \int_{-\infty}^{\infty} (Y - \mu y)^2 f(y)\, dy \\
&= \int_0^2 (Y - 1.095)^2 \times \left[\frac{y}{7} + \frac{5}{14}\right] dy \\
&\text{used wolfram alpha solver} \\
&\approx 0.324
\end{aligned}
$$

$$
\begin{aligned}
Cor(X,Y) &= \frac{Cov(X,Y)}{\sqrt{Var(X) \times Var(Y)}} \\
&\approx \frac{-0.017}{\sqrt{0.297 \times 0.324}} \\
&\approx -0.055
\end{aligned}
$$

Part 2: Sampling Distributions

Examine the behavior of a lognormal random variable with parameters 0.2938933 and 1.268636.

```r
set.seed(12345)
logn_1samp <- rlnorm(1e+07, 0.2938933, 1.268636)
mean(logn_1samp)
```

```
[1] 3.002705
```

```r
sd(logn_1samp)
```

```
[1] 6.036241
```

Transform this variable linearly so that we have a new variable Y mean of 100 and a standard deviation of 15.

```r
set.seed(12345)
logn_1samp <- 2.5 * rlnorm(1e+07, 0.2938933, 1.268636) + 92.5
mean(logn_1samp)
```

```
[1] 100.0068
```

```r
sd(logn_1samp)
```

```
[1] 15.0906
```

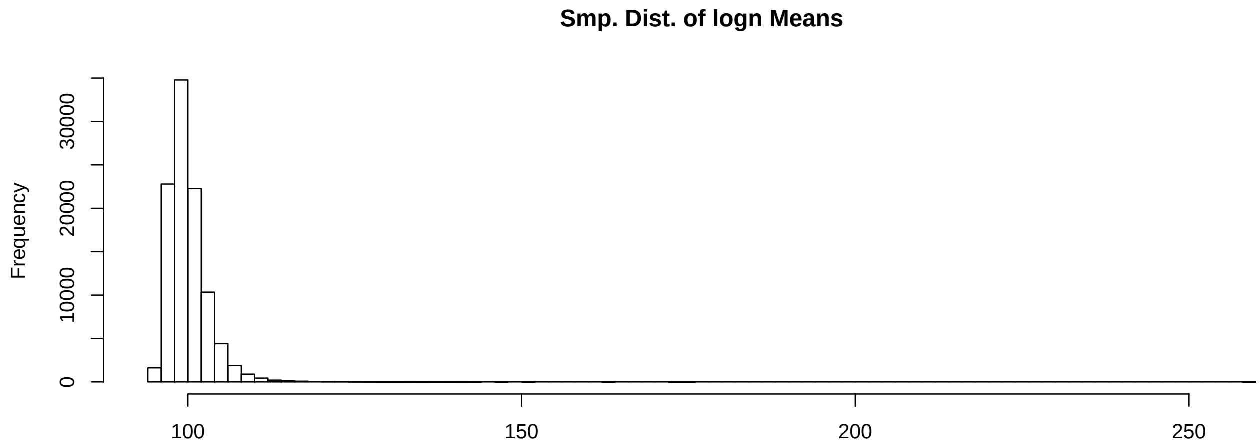
Take 100,000 means based on samples of size 25 from the transformed lognormal distribution.

```r
set.seed(12345)
N = 100000
logn_means <- numeric(N)
for (i in 1:N) {
    x <- 2.5 * rlnorm(25, 0.2938933, 1.268636) + 92.5
    logn_means[i] <- mean(x)
}
head(logn_means)
```

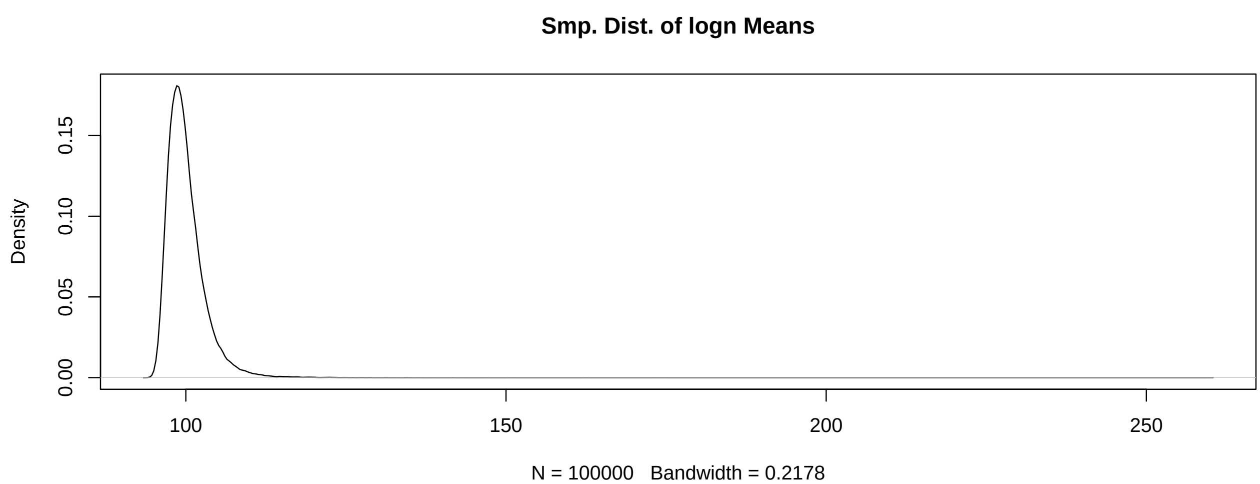
```
[1]  98.69147 104.86052 104.36481 105.31974 101.71476 101.00076
```

Examine the population, sample, and sampling distributions.

```r
logn_means %>% hist(breaks=100,main="Smp. Dist. of logn Means")
```




```r
plot(density(logn_means),main="Smp. Dist. of logn Means")
```



What did you expect to see?

I suppose I expected the means to be evenly distributed above and below 100.

What do you actually see?

> A high peak around 100 with a long tail to the right.

What is the mean/standard deviation of this simulated sampling distribution?

```
mean(logn_means)
```

```
[1] 100.0029
```

```
sd(logn_means)
```

```
[1] 3.023481
```

> mean: 100.0029 sd: 3.023481, odd. didn't we transform this to 15?

Do the same for an exponential distribution with mean and standard deviation of 1.

```
set.seed(12345)
exp_1samp <- rexp(1e+07,1)
mean(exp_1samp)
```

```
[1] 0.9997628
```

```
sd(exp_1samp)
```

```
[1] 0.9995945
```

```
set.seed(12345)
exp_1samp <- 15 * rexp(1e+07,1) + 92.5
mean(exp_1samp)
```

```
[1] 107.4964
```

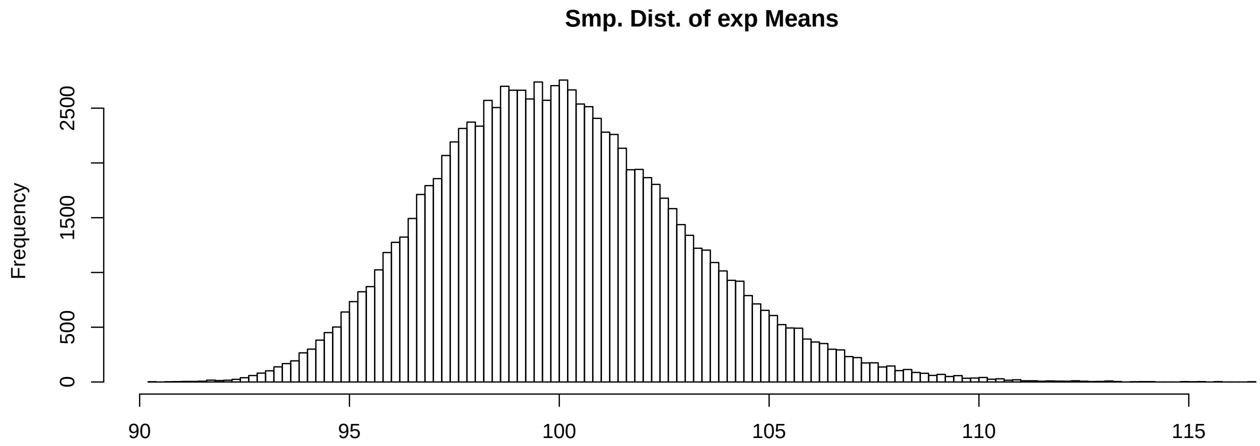
```
sd(exp_1samp)
```

```
[1] 14.99392
```

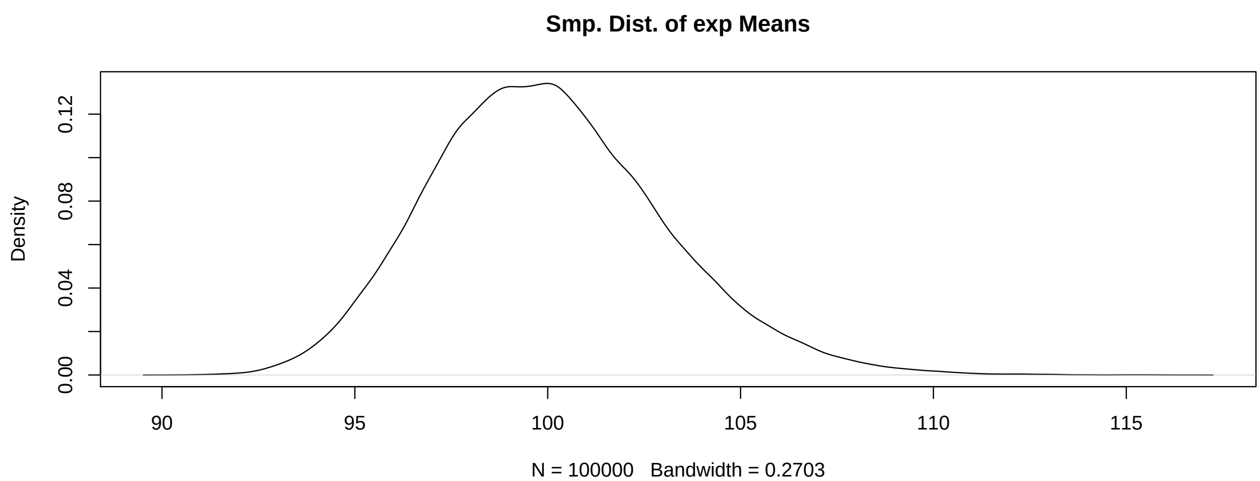
```
set.seed(12345)
N = 100000
exp_means <- numeric(N)
for (i in 1:N) {
    x <- 15 * rexp(25, 1) + 85 #transform so mean = 100, sd = 15
    exp_means[i] <- mean(x)
}
head(exp_means)
```

```
[1] 103.28723 100.75480  98.79687  97.14471  99.90882  95.07226
```

```r
exp_means %>% hist(breaks=100,main="Smp. Dist. of exp Means")
```




```r
plot(density(exp_means),main="Smp. Dist. of exp Means")
```




```r
mean(exp_means)
```

```
[1] 100.004
```

```r
sd(exp_means)
```

```
[1] 3.00933
```

> This distribution has a longer tail on the right than on the left but is not as extreme as the lognormal distribution. The sd looks like it's near 3, just like the lognormal distribution above, though it was transformed.

Overall, what conclusions do you make about the applicability of the Central Limit Theorem given what we have demonstrated with variables from:

The binomial distribution (in class). The normal distribution (warm-up). The uniform distribution (warm-up). The lognormal (on your own). The exponential (on your own).

With enough iterations, sample distributions for each of the above distributions converge to the same distribution: the normal distribution.

Part 3: Problems from your peers!

You attend a party where there are already 20 guests in the room. Unbeknownst to you, 5 guests are zombies, and 7 are vampires.

One person approaches you and buys you a drink. What is the probability that this person is a vampire?

Assuming a vampire is equally as likely approach me and buy me a drink as any other guest, (Probably a naive assumption)

$$\begin{aligned} P(V) &= \frac{7}{20} \\ &= 0.35 \end{aligned}$$

Two people approach you and ask your opinion on the host's outfit. What is the probability that they are both zombies?

Assuming a zombie is equally as likely to approach me and ask my opinion on the host's outfit as any other guest and that these two events are independent, (Again, probably a naive assumption; it is a well known fact that zombies perform a complex flocking phenomenon.)

$$\begin{aligned} P(Z_1) &= \frac{5}{20} = 0.25 \\ P(Z_2) &= \frac{4}{19} \approx 0.21 \\ P(Z_1) \cap P(Z_2) &= P(Z_1)P(Z_2) \text{ from Def. 2.4.2 (p. 73)} \\ &\approx 0.25 \times 0.21 \\ &\approx 0.0525 \end{aligned}$$

Three people approach you and ask you to be the fourth player in their Texas hold'em game. What is the probability that they are all normal humans?

Assuming all the guests at the party who are not vampires or zombies are normal humans and assuming the aforementioned bits about equal likelyhood and independence,

$$\begin{aligned} P(H_1) &= \frac{8}{20} = 0.4 \\ P(H_2) &= \frac{7}{19} \approx 0.368 \\ P(H_3) &= \frac{6}{18} \approx 0.333 \\ P(H_1) \cap P(H_2) \cap P(H_3) &= P(H_1)P(H_2)P(H_3) \text{ from Def. 2.4.2 (p. 73)} \\ &\approx 0.4 \times 0.368 \times 0.333 \\ &\approx 0.0490176 \end{aligned}$$

Bud only goes out trick-or-treating when there are clear skies (not too dark or too wet) and there is no full moon (he's superstitious). There is a full moon every 27.32 days. Assuming it is a random Halloween – i.e. we are not aware of any weather forecast or pattern, nor recent moon phases – and the probability of clear skies on a random October 31 is 0.6, what is the probability that Bud will go trick-or-treating?

M = full moon
C = clear skies
T = Bud goes out trick-or-treating

$$P(M) = \frac{1}{27.32} \approx 0.037$$
$$P(C) = .06(\text{given})$$
$$P(T) = P(C)P(M^c)$$

$$P(M^c) = 1 - P(M) \text{ from Property 1 (p. 58)}$$

$$P(T) = P(C)(1 - P(M))$$
$$P(T) \approx 0.6 \times (1 - 0.037)$$
$$P(T) \approx 0.6 \times 0.963$$
$$\approx 0.5778$$