Math 530-630 Midterm: Exercises

Part 1: Probability

1. The following table shows the cumulative distribution function of a discrete random variable. Find the probability mass function.

k	$F(k)$
0	0
1	.1
2	.3
3	.7
4	.8
5	1.0

2. The probability density function of a random variable X is given by:

$$f(x) = \begin{cases} cx, & 0 < x < 4 \\ 0, & \text{otherwise} \end{cases}$$

   a) find c
   b) find the cumulative distribution function $F(x)$.
   c) Compute $P(1 < X < 3)$

3. The random variable X has a cumulative distribution function (cdf):

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^3}{(2+x^2)} & x > 0 \end{cases}$$

   Find the probability density function (pdf) of X.

4. The joint probability of the continuous random variable (X, Y) is given by:

$$f(x,y) = \begin{cases} \frac{1}{28}(4x + 2y + 1) & 0 \leq x < 2, \quad 0 \leq y < 2 \\ 0 & \text{otherwise.} \end{cases}$$

   a) Find $E(XY)$
   b) Find $Cov(X, Y)$
   c) Find the correlation coefficient ρ_{XY}

Part 2: Sampling Distributions

5. (This problem references the following slides: http://cslu.ohsu.edu/~presmane/courses/MATH630/Math_530-630_Class_6_Simulating_Sampling_Dists.html) Examine the behavior of a lognormal random variable with parameters 0.2938933 and 1.268636. Transform this variable linearly so that we have a new variable Y mean of 100 and a standard deviation of 15. Take 100,000 means based on samples of size 25 from the transformed lognormal distribution. Examine the population, sample, and sampling distributions- what did you expect to see? What do you actually see? What is the mean/standard deviation of this simulated sampling distribution? Do the same for an exponential distribution with mean and standard deviation of 1. Overall, what conclusions do you make about the applicability of the Central Limit Theorem given what we have demonstrated with variables from the binomial distribution (in class), the normal distribution (warm-up), the uniform distribution (warm-up), the lognormal (on your own), and the exponential (on your own).

Part 3: Problems from your peers!

6. You attend a party where there are already 20 guests in the room. Unbeknownst to you, 5 guests are zombies, and 7 are vampires.

   a) One person approaches you and buys you a drink. What is the probability that this person is a vampire?

   b) Two people approach you and ask your opinion on the host's outfit. What is the probability that they are **both** zombies?

   c) Three people approach you and ask you to be the fourth player in their Texas hold'em game. What is the probability that they are all normal humans?

7. Bud only goes out trick-or-treating when there are clear skies (not too dark or too wet) and there is no full moon (he's superstitious). There is a full moon every 27.32 days.
   Assuming it is a random Halloween – i.e. we are not aware of any weather forecast or pattern, nor recent moon phases – and the probability of clear skies on a random October 31 is 0.6, what is the probability that Bud will go trick-or-treating?