Project 1

CS 301 – Amar Raheja

Joshua Camacho
2-13-2017

Numerical Root Approximation

Joshua Camacho
California State Polytechnic University, Pomona
Numerical Methods

Amar Raheja
Professor of Computer Science
California State Polytechnic University, Pomona

I. Overview

Frequently, the solution to a scientific problem is a number about which we have little information other than that it satisfies some equation. Since every equation can be written so that a function stands on one side and zero on the other, the desired number must be a zero of the function. Thus, if we possess an arsenal of methods for locating zeros of functions, we shall be able to solve such problems.

II. Assignment

The assignment was to write programs for the following root approximation methods

- Bisection
- Newton-Raphson
- Secant
- False-Position
- Modified Secant

Use those programs to evaluate the roots for the following functions

a) $f(x) = 2x^3 - 11.7x^2 + 17.7x - 5$
b) $f(x) = e^{-x} - x$

Lastly, present graphs of the functions, graphs of the error evaluations, and discuss the results of each method.

III. Function Graphs

Looking at the graphs of the functions can help determine where the roots are located.

$$f(x) = 2x^3 - 11.7x^2 + 17.7x - 5$$

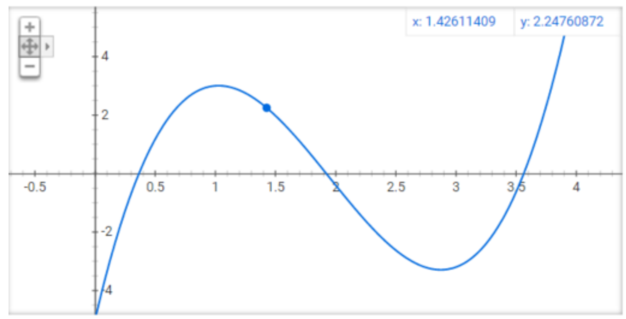


Figure 1: Function A

From the graph, there appears to be 3 roots. One on the interval (0,0.5), (1.5,2), and (3.5,4).

$$f(x) = e^{-x} - x$$

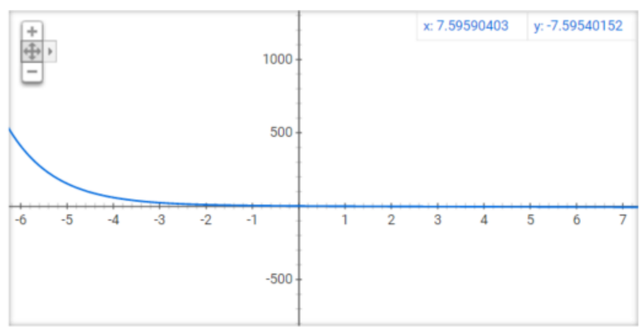


Figure 2: Function B

This graph was harder to see, but upon zoom there appears to be a root between (0,1). The true root for this function was also given, at x=0.56714329.

IV. Root Evaluation

Function A) $f(x) = 2x^3 - 11.7x^2 + 17.7x - 5$

This function had 3 roots on the intervals (0,0.5), (1.5,2), and (3.5,4)

Bisection Method – Function A

Bisection Method Root 1, using a=0, b=1

n = 0	a = 0.0	b = 1.0	c = 0.5	f(a) = -5.0	f(b) = 3.000001	f(c) = 1.1750002	Error = 1.0
n = 1	a = 0.0	b = 0.5	c = 0.25	f(a) = -5.0	f(b) = 1.1750002	f(c) = -1.2749999	Error = 0.33333334
n = 2	a = 0.25	b = 0.5	c = 0.375	f(a) = -1.2749999	f(b) = 1.1750002	f(c) = 0.09765673	Error = 0.2
n = 3	a = 0.25	b = 0.375	c = 0.3125	f(a) = -1.2749999	f(b) = 0.09765673	f(c) = -0.55029297	Error = 0.09090909
n = 4	a = 0.3125	b = 0.375	c = 0.34375	f(a) = -0.55029297	f(b) = 0.09765673	f(c) = -0.21690655	Error = 0.04347826
n = 5	a = 0.34375	b = 0.375	c = 0.359375	f(a) = -0.21690655	f(b) = 0.09765673	f(c) = -0.057294846	Error = 0.021276595
n = 6	a = 0.359375	b = 0.375	c = 0.3671875	f(a) = -0.057294846	f(b) = 0.09765673	f(c) = 0.020760536	Error = 0.010752688
n = 7	a = 0.359375	b = 0.3671875	c = 0.36328125	f(a) = -0.057294846	f(b) = 0.020760536	f(c) = -0.01812172	Error = 0.0053475937
n = 8	a = 0.36328125	b = 0.3671875	c = 0.36523438	f(a) = -0.01812172	f(b) = 0.020760536	f(c) = 0.001355648	

Figure 3: Function A, Bisection Method, Root 1

Method converges at c = 0.36523438 after 8 iterations

Bisection Method Root 2, using a=1, b=2

n = 0	a = 1.0	b = 2.0	c = 1.5	f(a) = 3.000001	f(b) = -0.3999977	f(c) = 1.9750023	Error = 0.14285715
n = 1	a = 1.5	b = 2.0	c = 1.75	f(a) = 1.9750023	f(b) = -0.3999977	f(c) = 0.86250305	Error = 0.06666667
n = 2	a = 1.75	b = 2.0	c = 1.875	f(a) = 0.86250305	f(b) = -0.3999977	f(c) = 0.23828125	Error = 0.032258064
n = 3	a = 1.875	b = 2.0	c = 1.9375	f(a) = 0.23828125	f(b) = -0.3999977	f(c) = -0.080566406	Error = 0.016393442
n = 4	a = 1.875	b = 1.9375	c = 1.90625	f(a) = 0.23828125	f(b) = -0.080566406	f(c) = 0.07911682	Error = 0.008130081
n = 5	a = 1.90625	b = 1.9375	c = 1.921875	f(a) = 0.07911682	f(b) = -0.080566406	f(c) = -6.828308E-4	

Figure 4: Function A, Bisection Method, Root 2

Method converges at c = 1.921875 after 5 iterations

Root 3, using a=3, b=4

n = 0	a = 3.0	b = 4.0	c = 3.5	f(a) = -3.1999931	f(b) = 6.600006	f(c) = -0.6249924	Error = 0.06666667
n = 1	a = 3.5	b = 4.0	c = 3.75	f(a) = -0.6249924	f(b) = 6.600006	f(c) = 2.3125	Error = 0.03448276
n = 2	a = 3.5	b = 3.75	c = 3.625	f(a) = -0.6249924	f(b) = 2.3125	f(c) = 0.6867218	Error = 0.01754386
n = 3	a = 3.5	b = 3.625	c = 3.5625	f(a) = -0.6249924	f(b) = 0.6867218	f(c) = -0.0069351196	Error = 0.008695652
n = 4	a = 3.5625	b = 3.625	c = 3.59375	f(a) = -0.0069351196	f(b) = 0.6867218	f(c) = 0.33026505	

Figure 5: Function A, Bisection Method, Root 3

Method converges at c = 3.59375 after 4 iterations

Newton Method – Function A

Newton Method Root 1, using x=1

n = 0	x = 1.0	f(x) = 3.000001	x+1 = -8.999965	error = 1.1111115
n = 1	x = -8.999965	f(x) = -2569.9749	x+1 = -5.402049	error = 0.66602796
n = 2	x = -5.402049	f(x) = -757.3339	x+1 = -3.0294547	error = 0.7831754
n = 3	x = -3.0294547	f(x) = -221.60545	x+1 = -1.4868301	error = 1.0375258
n = 4	x = -1.4868301	f(x) = -63.755424	x+1 = -0.5172516	error = 1.8744813
n = 5	x = -0.5172516	f(x) = -17.56246	x+1 = 0.041902423	error = 13.344193
n = 6	x = 0.041902423	f(x) = -4.278723	x+1 = 0.29765365	error = 0.8592242
n = 7	x = 0.29765365	f(x) = -0.7153802	x+1 = 0.3611499	error = 0.17581691
n = 8	x = 0.3611499	f(x) = -0.039459705	x+1 = 0.36508343	error = 0.010774302
n = 9	x = 0.36508343	f(x) = -1.4734268E-4	x+1 = 0.36509824	error = 4.056923E-5

Figure 6: Function A, Newton Method, Root 1

Method converges at x = 0.36509824 after 10 iterations

Newton Method Root 2, using x=2

n = 0	x = 2.0	f(x) = -0.3999977	x+1 = 1.9215691	error = 0.040816065
n = 1	x = 1.9215691	f(x) = 8.8119507E-4	x+1 = 1.9217416	error = 8.9760164E-5

Figure 7: Function A, Newton Method, Root 2

Method converges at x = 1.9217416 after 2 iterations

Newton Method Root 3, using x=3

n = 0	x = 3.0	f(x) = -3.1999931	x+1 = 5.1333237	error = 0.4155833
n = 1	x = 5.1333237	f(x) = 48.08954	x+1 = 4.269744	error = 0.20225564
n = 2	x = 4.269744	f(x) = 12.956093	x+1 = 3.792931	error = 0.12571092
n = 3	x = 3.792931	f(x) = 2.9475555	x+1 = 3.599818	error = 0.05364524
n = 4	x = 3.599818	f(x) = 0.39796448	x+1 = 3.5643375	error = 0.009954304

Figure 8: Function A, Newton Method, Root 3

Method converges at x = 3.5643375 after 5 iterations

Secant Method – Function A

Secant Method Root 1, using x_0=0, x=1

n = 0	x-1 = 1.0	x = 0.0	x+1 = 0.62499994	error = 1.0
n = 1	x-1 = 0.0	x = 0.62499994	x+1 = 0.4476776	error = 0.39609382
n = 2	x-1 = 0.62499994	x = 0.4476776	x+1 = 0.3376152	error = 0.32599962
n = 3	x-1 = 0.4476776	x = 0.3376152	x+1 = 0.3673572	error = 0.08096208
n = 4	x-1 = 0.3376152	x = 0.3673572	x+1 = 0.36515644	error = 0.0060268757

Convergence reached at x = 0.36515644

Secant Method Root 2, using x_0=1, x=2

```
n = 0     x-1 = 2.0          x = 1.0          x+1 = 1.8823535     error = 0.46875018
n = 1     x-1 = 1.0          x = 1.8823535    x+1 = 1.9456805     error = 0.03254746
n = 2     x-1 = 1.8823535    x = 1.9456805    x+1 = 1.9217039     error = 0.012476721
n = 3     x-1 = 1.9456805    x = 1.9217039    x+1 = 1.9217412     error = 1.941599E-5
Convergence reached at x = 1.9217412
```

Secant Method Root 2, using x_0=3, x=4

```
n = 0     x-1 = 4.0          x = 3.0          x+1 = 3.32653       error = 0.098159336
n = 1     x-1 = 3.0          x = 3.32653      x+1 = 3.8487203     error = 0.13567895
n = 2     x-1 = 3.32653      x = 3.8487203    x+1 = 3.5037088     error = 0.09847036
n = 3     x-1 = 3.8487203    x = 3.5037088    x+1 = 3.5497475     error = 0.01296955
n = 4     x-1 = 3.5037088    x = 3.5497475    x+1 = 3.5639358     error = 0.0039810734
Convergence reached at x = 3.5639358
```

False Position Method – Function A

False Position Method Root 1, using x_0=0, x=1

```
n = 0     a = 0.0          b = 1.0           c = 0.62499994     f(c) = 1.9804688      error = 0.10201422
n = 1     a = 0.0          b = 0.62499994    c = 0.44767764     f(c) = 0.75847864     error = 0.3960937
n = 2     a = 0.0          b = 0.44767764    c = 0.38871175     f(c) = 0.2298317      error = 0.15169567
n = 3     a = 0.0          b = 0.38871175    c = 0.3716293      f(c) = 0.064621925    error = 0.04596638
n = 4     a = 0.0          b = 0.3716293     c = 0.36688748     f(c) = 0.017784595    error = 0.012924447
n = 5     a = 0.0          b = 0.36688748    c = 0.36558712     f(c) = 0.0048656464   error = 0.0035569216
Convergence reached at c = 0.36558712
```

False Position Method Root 2, using x_0=1, x=2

```
n = 0     a = 1.0          b = 2.0           c = 1.8823535      f(c) = 0.2008934      error = 2.319009
n = 1     a = 1.8823535    b = 2.0           c = 1.9216857      f(c) = 2.861023E-4    error = 0.020467525
n = 2     a = 1.9216857    b = 2.0           c = 1.9217417      f(c) = 0.0            error = 2.9154993E-5
Convergence reached at c = 1.9217417
```

False Position Method Root 3, using x_0=3, x=4

```
n = 0     a = 3.0          b = 4.0           c = 3.32653        f(c) = -1.968853      error = 4.865414
n = 1     a = 3.32653      b = 4.0           c = 3.4812722      f(c) = -0.79590607    error = 0.04444991
n = 2     a = 3.4812722    b = 4.0           c = 3.5370946      f(c) = -0.26710892    error = 0.015781984
n = 3     a = 3.5370946    b = 4.0           c = 3.5551002      f(c) = -0.083992004   error = 0.0050647263
Convergence reached at c = 3.5551002
```

Modified Secant Method – Function A

Modified Secant Method Root 1, using x=1, delta=0.01

```
n = 0     x = 1.0            x+1 = -11.336192     f(x) = 3.000001        error = 1.0882131
n = 1     x = -11.336192     x+1 = -6.9877443     f(x) = -4622.8193      error = 0.62229633
n = 2     x = -6.9877443     x+1 = -4.095392      f(x) = -1382.3805      error = 0.7062454
n = 3     x = -4.095392      x+1 = -2.1890981     f(x) = -411.10144      error = 0.87081254
n = 4     x = -2.1890981     x+1 = -0.9592707     f(x) = -120.79617      error = 1.2820442
n = 5     x = -0.9592707     x+1 = -0.20634699    f(x) = -34.51088       error = 3.6488235
n = 6     x = -0.20634699    x+1 = 0.1955705      f(x) = -9.168089       error = 2.0551028
n = 7     x = 0.1955705      x+1 = 0.34339914     f(x) = -1.9709413      error = 0.43048635
n = 8     x = 0.34339914     x+1 = 0.36473054     f(x) = -0.22054434     error = 0.058485366
n = 9     x = 0.36473054     x+1 = 0.36509937     f(x) = -0.0036621094   error = 0.0010102278
Convergence reached at x = 0.36509937
```

Modified Secant Method Root 2, using x=2, delta=0.01

```
n = 0     x = 2.0            x+1 = 1.9214658      f(x) = -0.3999977      error = 0.040872052
n = 1     x = 1.9214658      x+1 = 1.9217411      f(x) = 0.0014076233    error = 1.4329373E-4
Convergence reached at x = 1.9217411
```

Modified Secant Method Root 3, using x=3, delta=0.01

```
n = 0     x = 3.0            x+1 = 4.8928747      f(x) = -3.1999931      error = 0.38686353
n = 1     x = 4.8928747      x+1 = 4.143112       f(x) = 35.7763         error = 0.18096602
n = 2     x = 4.143112       x+1 = 3.742388       f(x) = 9.734337        error = 0.10707713
n = 3     x = 3.742388       x+1 = 3.5910718      f(x) = 2.2040863       error = 0.04213677
n = 4     x = 3.5910718      x+1 = 3.5647001      f(x) = 0.30062866      error = 0.0073980186
Convergence reached at x = 3.5647001
```

Function B) f(x) = e-x -x

Bisection Method – Function B

Using a=0, b=1

```
n = 0    a = 0.0    b = 1.0      c = 0.5      f(a) = 1.0          f(b) = -0.63212055   f(c) = 0.10653067    Error = 0.3224172
n = 1    a = 0.5    b = 1.0      c = 0.75     f(a) = 0.10653067   f(b) = -0.63212055   f(c) = -0.27763346   Error = 0.102014326
n = 2    a = 0.5    b = 0.75     c = 0.625    f(a) = 0.10653067   f(b) = -0.27763346   f(c) = -0.08973855   Error = 0.008187105
n = 3    a = 0.5    b = 0.625    c = 0.5625   f(a) = 0.10653067   f(b) = -0.08973855   f(c) = 0.007282853
Convergence reached at c = 0.5625 f(c) = 0.007282853
```

Newton Method – Function B

Using x=3.0

```
n = 0     x = 3.0           f(x) = -2.950213     x+1 = 0.18970346     error = 0.6655105
n = 1     x = 0.18970346    f(x) = 0.63750094    x+1 = 0.5385976      error = 0.050332393
n = 2     x = 0.5385976     f(x) = 0.044968486   x+1 = 0.56699455     error = 2.622152E-4
Convergence reached at x = 0.56699455 f(x) = 0.044968486
```

Secant Method – Function B

Using x_0=1.0, x=0

```
n = 0     x-1 = 1.0          x = 0.0           x+1 = 0.6126998      error = 0.08032635
n = 1     x-1 = 0.0          x = 0.6126998     x+1 = 0.5721814      error = 0.008883368
Convergence reached at x = 0.5721814
```

False Position Method – Function B

Using a=0, b=1

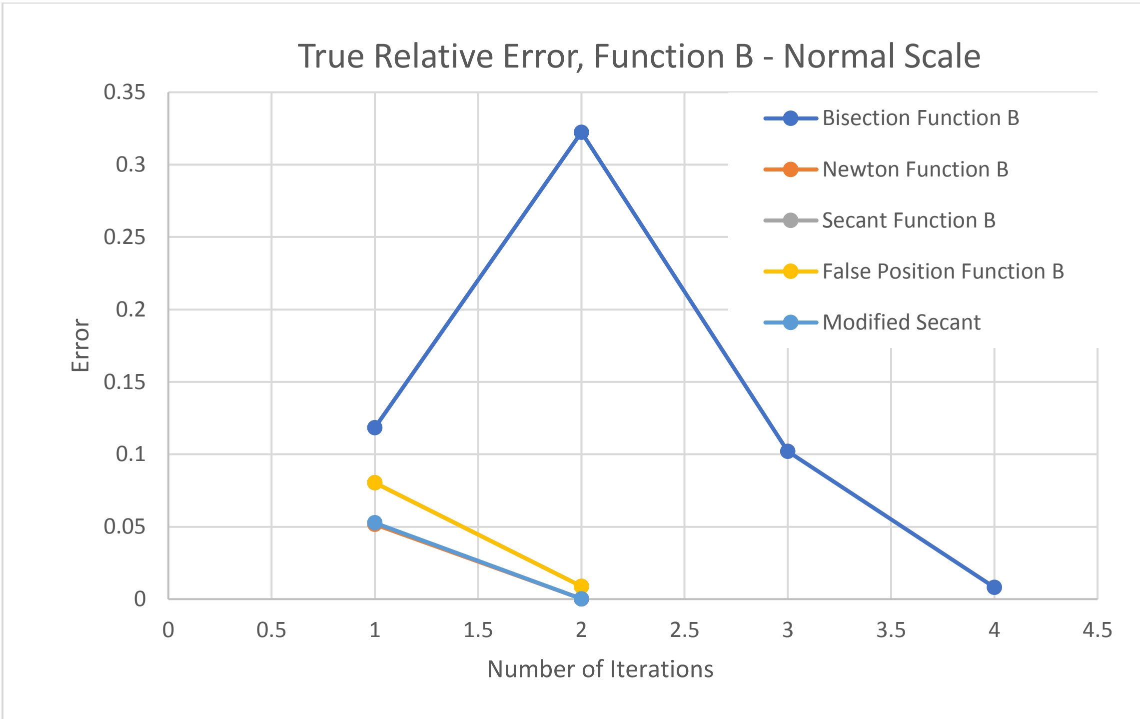
```
n = 0     a = 0.0          b = 1.0            c = 0.6126998      f(c) = -0.070813894     error = 0.08032635
n = 1     a = 0.0          b = 0.6126998      c = 0.5721814      f(c) = -0.0078882575    error = 0.008883368
Convergence reached at c = 0.5721814
```

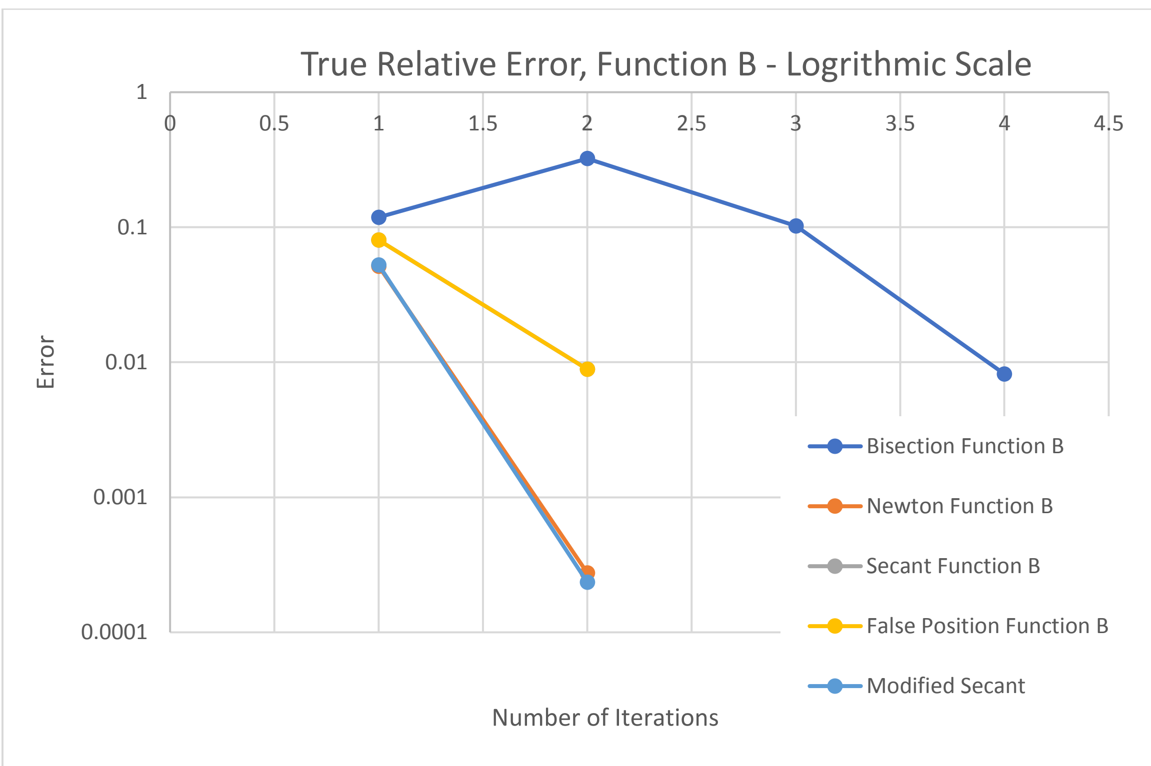
Modified Secant Method – Function B

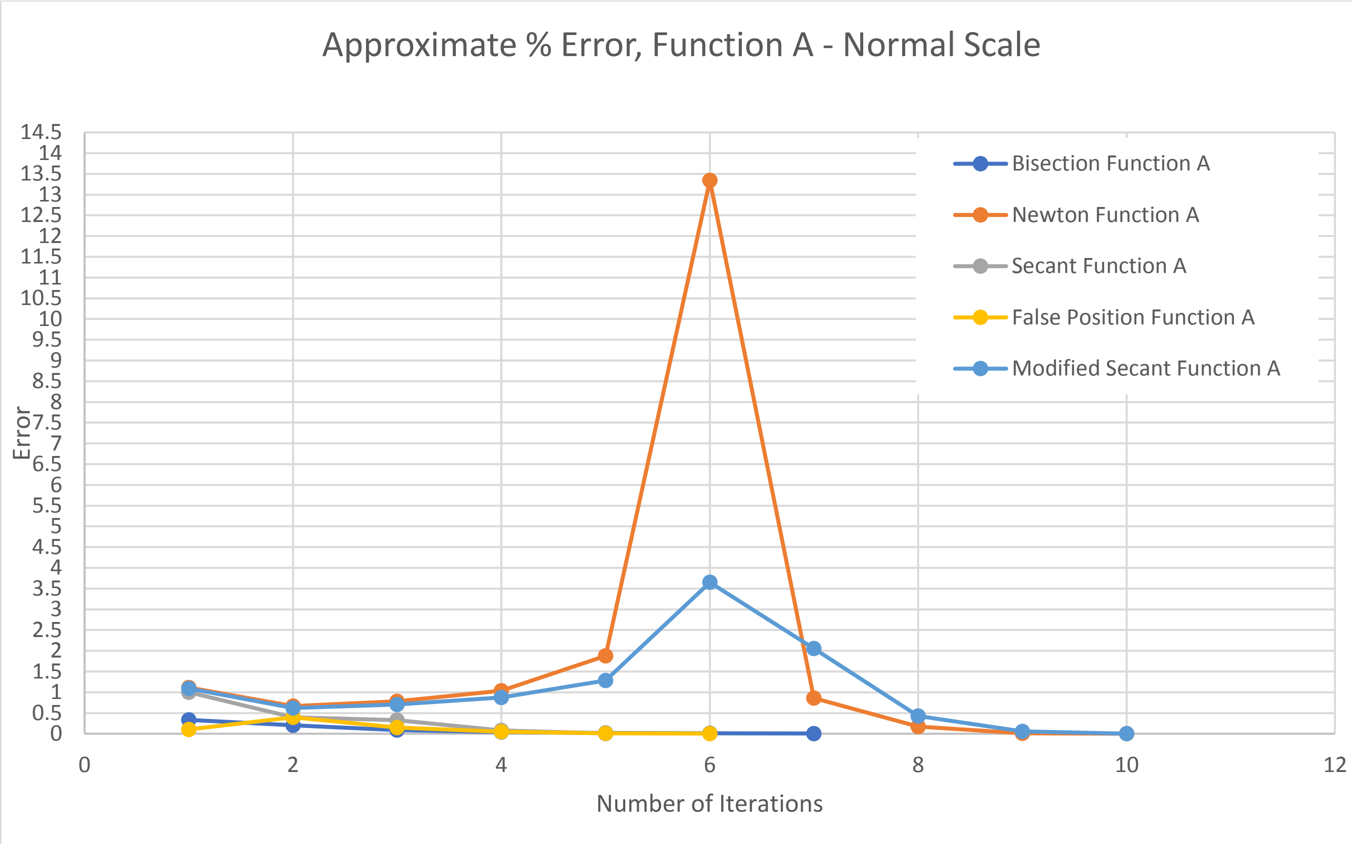
Using x=1, delta=0.01

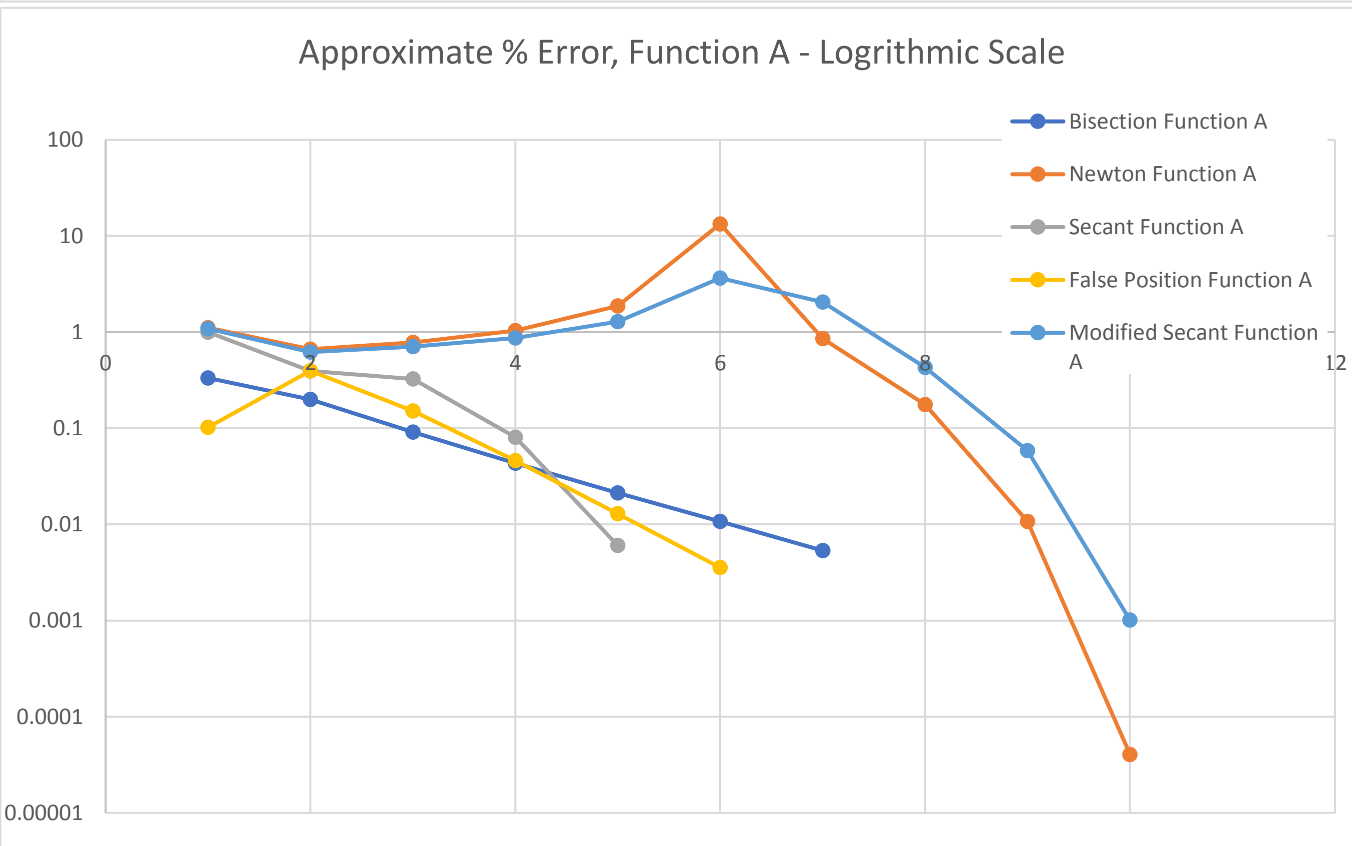
```
n = 0     x = 1.0           x+1 = 0.53726363     f(x) = -0.63212055     error = 0.05268445
n = 1     x = 0.53726363    x+1 = 0.56700975     f(x) = 0.04708141      error = 2.3541566E-4
Convergence reached at x = 0.56700975
```

V. ERROR GRAPHS






Approximate % Error, Function A - Normal Scale
Error
Number of Iterations
Bisection Function A
Newton Function A
Secant Function A
False Position Function A
Modified Secant Function A
14.5
14
13.5
13
12.5
12
11.5
11
10.5
10
9.5
9
8.5
8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12


Approximate % Error, Function A - Logrithmic Scale
Bisection Function A
Newton Function A
Secant Function A
False Position Function A
Modified Secant Function A
100
10
1
0.1
0.01
0.001
0.0001
0.00001
0
2
4
6
8
12

VI - DISCUSSION

Root Summary

Function A

	Bisection	*Newton*	*Secant*	*False Position*	*Modified Secant*
Root 1	0.36523438	0.36509824	0.365156	0.36558712	0.36509937
Root 2	1.921875	1.9217416	1.927412	1.9217417	1.9217411
Root 3	3.59375	3.5643375	3.56395	3.5551002	3.5647001

Function B

	Bisection	*Newton*	*Secant*	*False Position*	*Modified Secant*
Root	0.56726074	0.566987	0.5721814	0.5721814	0.56700975

On function A- Newton method and Modified took the longest to converge for the first point. Newton method was an outlier in approximate error, at one point shooting up to a very high error % before reaching convergence.

On function B – Modified Secant, Secant, and Newton method all converged extremely fast, with bisection method being the slowest to converge.

I used the float data types for the methods, since the significant digits of error can fit into a float data type.