If a number $\frac{1}{2}g(\epsilon_F)k_BT$ electrons are thermally excited by $\simeq 2k_BT$

$$E(T) - E(0) \;\simeq\; \frac{1}{2}g(\epsilon_F)k_BT \times 2k_BT \;=\; g(\epsilon_F)k_B^2T^2$$

Therefore $C_V = \left(\frac{\partial E}{\partial T}\right)_V \simeq 2g(\epsilon_F)k_B^2T$

note density of states at ϵ_F enters C_V

We saw in the previous lecture that $g(\epsilon) = \alpha\epsilon^{1/2}$ and $N = \frac{2}{3}\alpha\epsilon_F^{3/2}$, so

$$\text{free electron gas:} \qquad g(\epsilon_F) = \frac{3N}{2\epsilon_F} = \frac{3N}{2k_BT_F}.$$

Therefore for one mole (i.e. $N = N_A$, Avagadro's number)

$$C_V \simeq 3N_Ak_B\left(\frac{T}{T_F}\right).$$

Note: a more exact calculation gives $C_V = (\pi^2/3)g(\epsilon_F)k_B^2T$.
Kittel presents a derivation with flaws. Ashcroft & Mermin better.