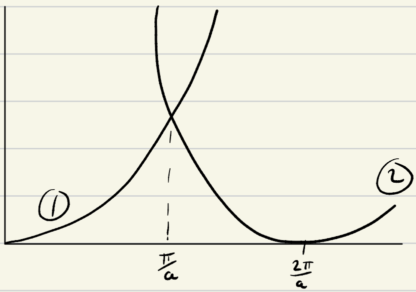
All waves are taken to Bloch

$\left.\begin{array}{l} k \\ k+\frac{2\pi}{a} \end{array}\right\} \rightarrow$ Periodic $\rightarrow$ $\exp\left(\frac{\pi}{a}\right)\ldots\ldots$

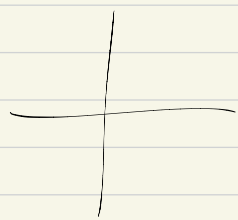


① $\varepsilon_k = \frac{\hbar^2 k^2}{2m}$, $\varphi_k(x) = \exp(ikx)$ @ $k = \frac{\pi}{a}$, $\varepsilon_{\frac{\pi}{a}} = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2$, $\varphi_{\frac{\pi}{a}}(x) = \exp\left(i\frac{\pi}{a}x\right)$

② $\varepsilon_k = \frac{\hbar^2 \left(k - \frac{2\pi}{a}\right)}{2m}$, $\varphi_k(x) = \exp\left(i\left[k - \frac{2\pi}{a}\right]x\right)$ @ $k = \frac{\pi}{a}$, $\varepsilon_{\frac{\pi}{a}} = \frac{\hbar^2}{2m}\left(-\frac{\pi}{a}\right)^2$, $\varphi_{\frac{\pi}{a}}(x) = \exp\left(i\left[\frac{\pi}{a} - \frac{2\pi}{a}\right]x\right)$

$$= \underbrace{\exp\left(-i\frac{\pi}{a}x\right)}_{\text{This is a backwards travelling wave.}}$$

Reduced wavelength.



So: ◦ Same ε for ① & ② @ $k = \frac{\pi}{a}$.

◦ Taking linear combinations:

$$\varphi_+(x) = \exp\left(i\frac{\pi}{a}x\right) + \exp\left(-i\frac{\pi}{a}x\right) = \cos\left(\frac{\pi}{a}x\right)$$

$$\varphi_-(x) = \exp\left(i\frac{\pi}{a}x\right) - \exp\left(-i\frac{\pi}{a}x\right) = 2i\sin\left(\frac{\pi}{a}x\right)$$