Quantum theory of the electronic heat capacity 2

Argument number 2: slightly more careful [not examinable]

Our previous argument neglects the density of states – there are proportionately more electrons with greater energies.

$$\frac{\partial f(\epsilon)}{\partial \epsilon} = \frac{\partial}{\partial \epsilon}\frac{1}{e^{(\epsilon-\mu)/k_BT}+1} = -\frac{1}{k_BT}\frac{e^{(\epsilon-\mu)/k_BT}}{(e^{(\epsilon-\mu)/k_BT}+1)^2} \qquad \Rightarrow \qquad \left.\frac{\partial f(\epsilon)}{\partial \epsilon}\right|_{\epsilon=\mu} = -\frac{1}{4k_BT}$$

The slope of the Fermi-Dirac distribution at μ means $f(\epsilon)$ drops from 1 to zero over a range $\simeq 4k_BT$. Therefore approximately (see figure) $\frac{1}{2}\times 2k_BT \times \frac{1}{2}g(\epsilon_F) = \frac{1}{2}g(\epsilon_F)k_BT$ states go from below ϵ_F to above.