AE 551: Introduction to Optimal Control **(Due: 2020/05/15)**

Homework #7 Submission

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Problem 1: Augmented Lagrangian Method for Equality Constraints

$$\min \quad f(x_1, x_2) = \frac{1}{2}(x_1 - 1)^2 + 10(x_2 - 1)^2 \tag{1}$$

$$\text{subject to} \quad c(x_1, x_2) = (x_1 - 2)^2 + 2 - x_2 = 0 \tag{2}$$

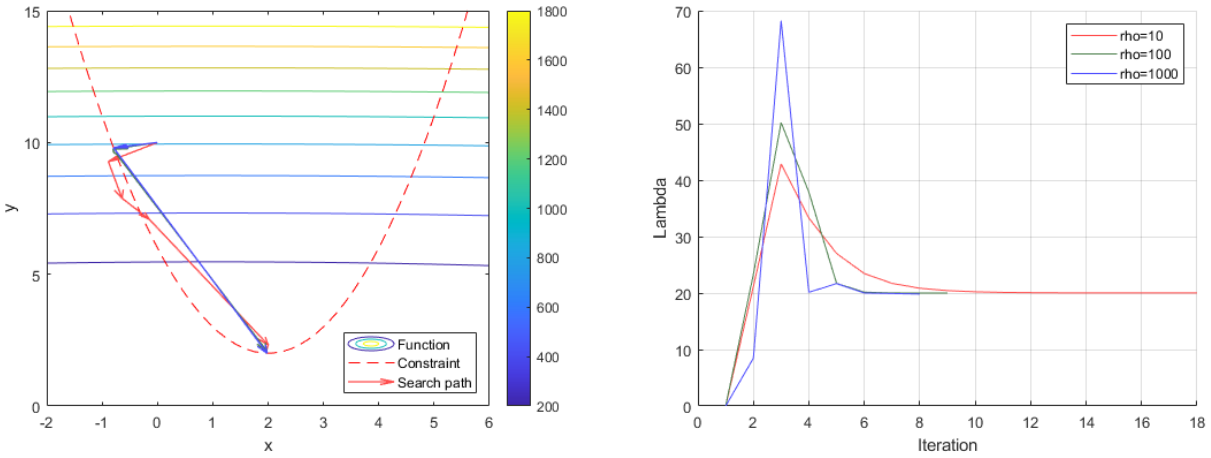
(Solution)

To solve the optimization problem on equation 1, Augmented Lagrangian method for equality constraint is used. It defines an augmented Lagrange function as

$$L_A(X, \lambda_k, \rho) = f(X) + \lambda_k^T c(X) + \rho c(X)^T c(X) \tag{3}$$

where $X = (x_1, x_2)$, $\rho = \{10, 100, 1000\}$, and $\lambda_k = \lambda_{k-1} + 2\rho c(\bar{X})$ with $\lambda_0 = 0$.

The result of the simulation is shown at figure 1. Figure 1a shows the path of the search which is initialized at $X_0 = (x_1, x_2)_0 = (0, 20)$. The search took 18, 9, and 8 iterations for $rho = 10, 100, 1000$ respectively. Figure 1b shows the value of λ_k at each iteration. Table 1 shows the value of X and $c(X)$ at each iterations.



(a) Search path

(b) λ_k value at each iteration

Figure 1: Simulation result of problem 1

Table 1: Simulation data of problem 1

(a) $\rho = 10$

Iteration	x_1	x_2	$c(X)$
1	0	10	-4
2	-0.8868	9.2751	1.058514
3	-0.6397	7.8836	1.084416
4	-0.1447	7.0784	-0.47866
5	2.0026	2.314	-0.31399
6	1.977	2.1782	-0.17767
7	1.9767	2.0875	-0.08696
8	1.9761	2.0436	-0.04303
9	1.976	2.0218	-0.02122
10	1.9758	2.011	-0.01041
11	1.9759	2.0058	-0.00522
12	1.9759	2.0032	-0.00262
13	1.9759	2.0019	-0.00132
14	1.9759	2.0013	-0.00072
15	1.9759	2.0009	-0.00032
16	1.9759	2.0008	-0.00022
17	1.9759	2.0007	-0.00012
18	1.9759	2.0007	-0.00012

(b) $\rho = 100$

x_1	x_2	$c(X)$
0	10	-4
-0.8042	9.7468	0.116738
-0.7846	9.6197	0.134297
-0.7368	9.5516	-0.06153
1.9782	2.0813	-0.08082
1.9753	2.0084	-0.00779
1.9755	2.0013	-0.0007
1.976	2.0006	-2.4E-05
1.976	2.0006	-2.4E-05

(c) $\rho = 1000$

x_1	x_2	$c(X)$
0	10	-4
-0.793	9.7968	0.004049
-0.7936	9.7745	0.029701
-0.7836	9.7725	-0.02407
1.9823	1.9995	0.000813
1.9826	2.0011	-0.0008
1.9828	2.0003	-4.2E-06
1.9828	2.0003	-4.2E-06

Problem 2: Augmented Lagrangian Method for Inequality Constraints

$$\min \quad f(x_1, x_2) = (1 - x_1)^2 + 100(x_2 - x_1^2)^2 \tag{4}$$

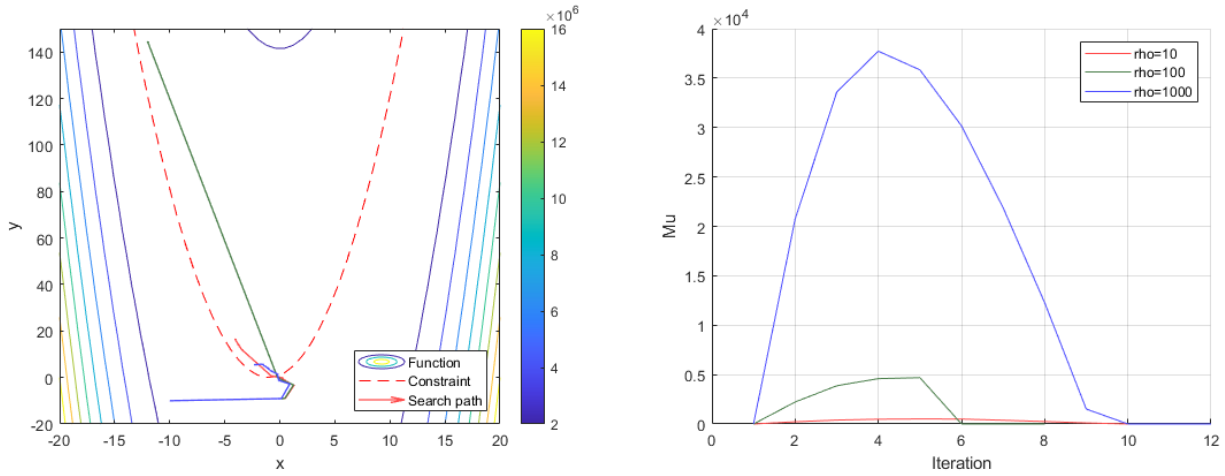
$$\text{subject to} \quad d(x_1, x_2) = (1 + x_1)^2 - x_2 \leq 0 \tag{5}$$

To solve the optimization problem on equation 4, Augmented Lagrangian method for inequality constraint is used. It defines an augmented Lagrange function as

$$L_A(X, \mu_k, \rho) = f(X) + \rho \sum_i \max^2 \left\{ d_i + \frac{\mu_{k,i}}{2\rho}, 0 \right\} - \sum_i \frac{\mu_{k,i}^2}{4\rho} \tag{6}$$

where $X = (x_1, x_2)$, $\rho = \{10, 100, 1000\}$, and $\mu_{k,i} = \max\left\{\mu_{k-1,i} + 2\rho d_i(\bar{X}), 0\right\}$ with $\mu_0 = 0$.

The result of the simulation is shown at figure 2. Figure 2a shows the path of the search which is initialized at $X_0 = (x_1, x_2)_0 = (-10, -10)$. The search took 11, 8, and 12 iterations for $rho = 10, 100, 1000$ respectively. Figure 2b shows the value of μ_k at each iteration. Table 2 shows the value of X and $d(X)$ at each iterations.



(a) Search path

(b) μ_k value at each iteration

Figure 2: Simulation result of problem 2

Table 2: Simulation data of problem 2

(a) $\rho = 10$

Iteration	x_1	x_2	$c(X)$
1	-10	-10	91
2	0.4953	-9.021	11.25692
3	1.2975	-3.3125	8.591006
4	0.6054	-1.7188	4.296109
5	0.1384	-0.0026	1.298555
6	-0.7683	0.6203	-0.56662
7	-3.4773	12.0751	-5.93808
8	-3.7711	14.1817	-6.5027
9	-4.0541	16.4001	-7.07257
10	-4.049	16.4006	-7.1042
11	-4.049	16.4006	-7.1042

(b) $\rho = 100$

x_1	x_2	$c(X)$
-10	-10	91
0.4434	-9.0412	11.12460356
1.248	-3.1612	8.214704
0.4189	-1.6031	3.61637721
-0.2192	0.1219	0.48774864
-11.9967	144.3666	-23.43918911
-12.015	144.3658	-23.035575
-12.015	144.3658	-23.035575

(c) $\rho = 1000$

x_1	x_2	$c(X)$
-10	-10	91
0.139	-9.0673	10.36462
0.8997	-2.8187	6.42756
-0.0957	-1.2534	2.071158
-0.2267	1.543	-0.94501
-0.8201	2.8653	-2.83294
-1.124	4.1552	-4.13982
-1.3992	4.97	-4.81064
-1.5834	5.7193	-5.37894
-2.3393	5.4793	-3.68558
-2.3392	5.4793	-3.68584
-2.3392	5.4793	-3.68584