AE 551: Introduction to Optimal Control **(Due: 2020/06/12)**

Homework #9 Submission

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Solve the four problems treated in the paper "Coevolutionary Augmented Lagrangian Methods for Constrained Optimization" by using CEALM and PSO. The paper is included in the ZIP file of cealm_v20. For each problem, run the codes of CEALM and PSO for at least 10 times to find the best and worst results of each method.

- CEALM: Use the CEALM code posted on KLMS.
- PSO:
  - You can use any PSO code but you need to explicitly state the source of the code (for example, URL or the reference papers)
  - Describe in detail the PSO algorithm you use in the homework report. (Do not use any code you don't understand the details.)
  - Attach the code to your report.

(Solution) Code for PSO method is attached at the Appendix section. However, the rest of the code can be accessed from `https://github.com/joshuadamanik/Homework-9`.

For the CEALM method [1], the code used in this homework is the one provided at cealm_v20_ae551.zip. However, the code for the PSO method is created based on proposed algorithm [2] by Zambrano-Bigiarini, et al.

Algorithm 1: Standard Particle Swarm Optimization

for *each particle:* $i \leftarrow 1, \ldots, N$ **do**
  initialize random particle's parameter: $X_i \in U(X_{min}, X_{max})$;
  initialize random particle's velocity: $V_i \in U(-(X_{max} - X_{min}), (X_{max} - X_{min}))$;
  initialize particle's best parameter: $\bar{X}_i \leftarrow x_i$;
  initialize swarm's best parameter: $X^* \leftarrow \arg\min f(\bar{X}_i)$;
end
while *termination criteria is not satisfied* **do**
  for *each particle:* $i \leftarrow 1, \ldots, N$ **do**
    for *each dimension:* $j \leftarrow 1, \ldots, M$ **do**
      initialize random numbers: $r_p, r_g \in U(0, 1)$;
      update particle's velocity: $V_{i,j} \leftarrow \omega V_{i,j} + c_1 r_p(\bar{X}_{i,j} - X_{i,j}) + c_2 r_g(X^*_j - X_{i,j})$;
    end
    update particle's parameter: $X_i \leftarrow X_i + v_i$;
    if $f(X_i) < f(\bar{X}_i)$ **then**
      update particle's best parameter: $\bar{X}_i \leftarrow X_i$;
      if $f(\bar{X}_i) < f(X^*)$ **then**
        update swarm's best parameter: $X^* \leftarrow \bar{X}_i$;
      end
    end
  end
end

However, the algorithm 1 is an unconstrained optimization method. For constrained optimization, the objective function $f(X)$ is replaced by augmented Lagrangian function $L_A(X, Y)$, defined as

$$L(X_i, \lambda_i, \mu_i) = f(X_i) + \rho \sum_{j=1}^{N_{ineq}} \max{}^2 \left\{ g_j(X_i) + \frac{\mu_{i,j}}{2\rho} \right\} + \sum_{j=1}^{N_{ineq}} \frac{\mu_{i,j}^2}{4\rho} + \lambda_i^T h(X_i) + \rho h(X_i)^T h(X_i) \tag{1}$$

where $g(X_i) \leq 0$ is set of inequality constraints and $h(X_i) = 0$ is set of equality constraints with their Lagrange multiplier μ_i and λ_i respectively. The multiplier μ_i and λ_i is then combined into a particle object Y_i with dimension $N_{ineq} + N_{eq}$.

By using augmented Lagrangian function (eq. 1), the optimization is done in an unconstrained fashion by solving both the X and Y particles. But, instead of searching for minimum value, we need to find the maximum value for particle Y. Then, the swarm's best parameter both for X and Y are selected by using security strategy [1].

In addition, because all of the problems defined in this homework have restricted search space, the softwall algorithm [1] is added into the Standard PSO (algorithm 1). While the search space of particle X is defined in each problem, the search space for particle Y is defined as

$$Y_{min} = \begin{cases} 0, & \text{inequality constraint} \\ -10, & \text{equality constraint} \end{cases} \tag{2}$$

$$Y_{max} = 10; \tag{3}$$

Problem 1

$$\min f(X) = 5x_1 + 5x_2 + 5x_3 + 5x_4 - 5\sum_{i=1}^{4} x_i^2 - \sum_{i=5}^{1} 3x_i \tag{4}$$

$$\begin{aligned} \text{subject to} \quad 2x_1 + 2x_2 + x_{10} + x_{11} &\leq 10 \\ 2x_1 + 2x_3 + x_{10} + x_{12} &\leq 10 \\ 2x_1 + 2x_3 + x_{10} + x_{12} &\leq 10 \\ -8x_1 + x_{10} &\leq 0 \\ -8x_2 + x_{11} &\leq 0 \\ -8x_3 + x_{12} &\leq 0 \\ -2x_4 - x_5 + x_{10} &\leq 0 \\ -2x_6 - x_7 + x_{11} &\leq 0 \\ -2x_8 - x_9 + x_{12} &\leq 0 \end{aligned}$$

$$\text{search space:} \quad 0 \leq x_i \leq 1, i = 1, \ldots, 9; \quad 0 \leq x_i \leq 10, i = 10, 11, 12; \quad 0 \leq x_{13} \leq 1$$

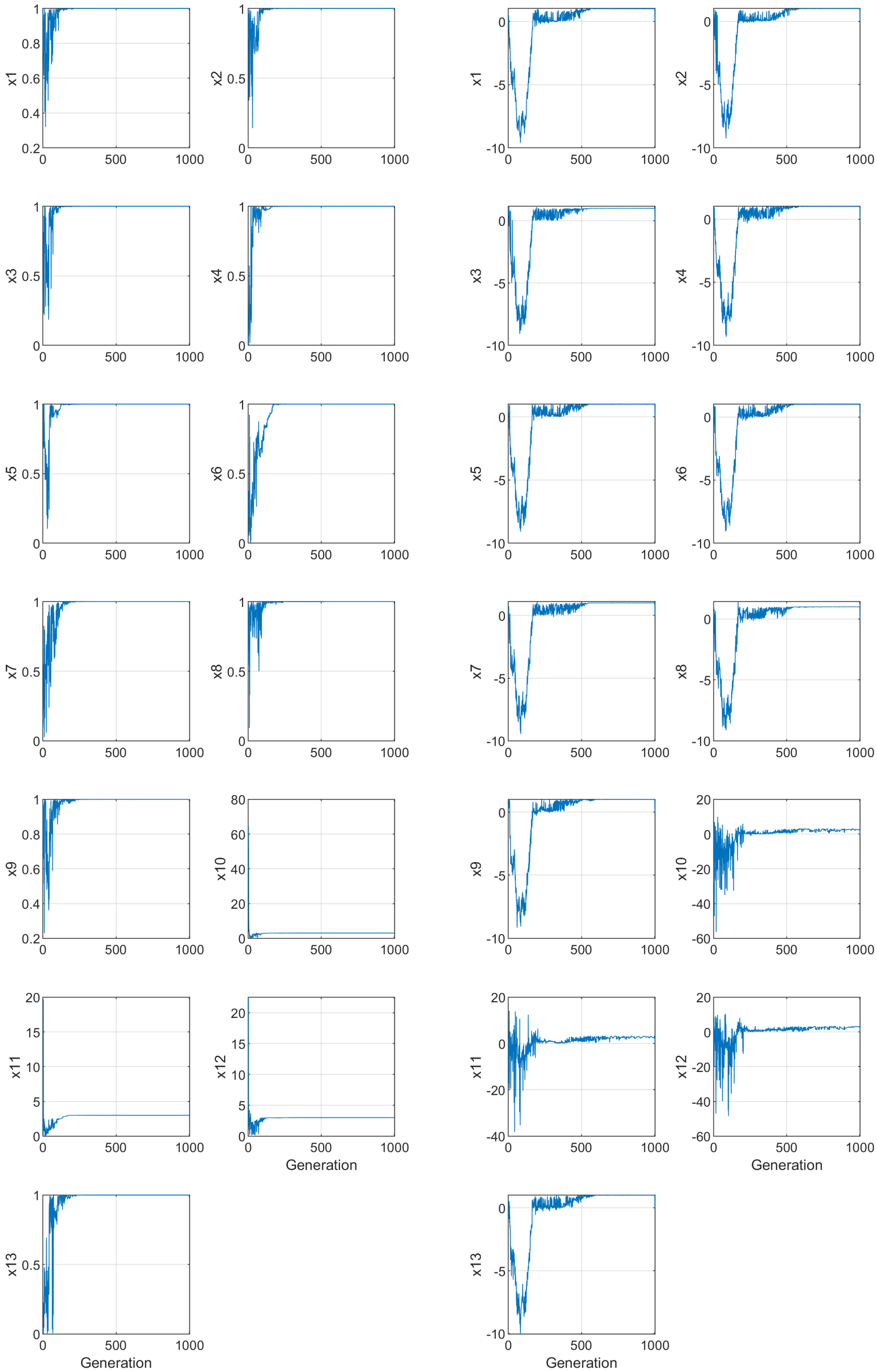
(Solution)

Table 1: Cost comparison of CEALM and PSO. Lowest cost is labelled with bold.

Run	CEALM	PSO	Analytical
1	-15.0000	-13.6296	-15.0000
2	-15.0000	**-14.5906**	
3	-15.0000	-4.7454	
4	-13.8281	-13.0326	
5	-15.0000	-14.1221	
6	-15.0000	-4.5065	
7	-13.8281	-13.1253	
8	-15.0000	-14.4504	
9	-15.0000	-3.8945	
10	**-15.0000**	-14.2518	

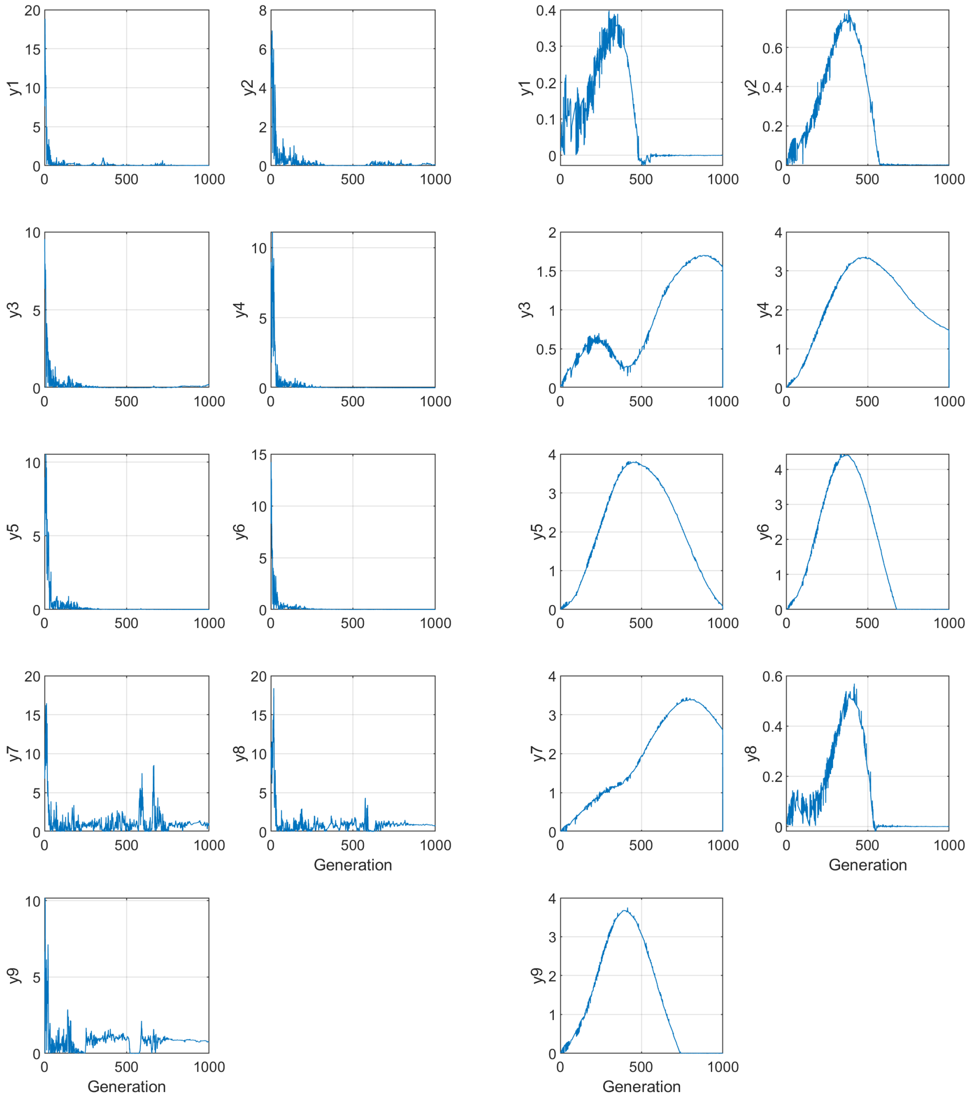
Table 2: Optimal parameter value of problem 1 with lowest cost

	CEALM	PSO		CEALM	PSO
x_1	1.0000	0.9993	y_1	0.0289	-0.0002
x_2	1.0000	0.9982	y_2	0.0754	-0.0004
x_3	1.0000	0.9995	y_3	0.1624	1.5536
x_4	1.0000	0.9999	y_4	0.0000	1.4803
x_5	1.0000	0.9991	y_5	0.0002	0.0763
x_6	1.0000	0.9999	y_6	0.0000	-0.0002
x_7	1.0000	0.9999	y_7	0.8247	2.6092
x_8	1.0000	0.9995	y_8	0.8140	-0.0005
x_9	1.0000	0.9990	y_9	0.7531	0.0017
x_{10}	3.0000	2.8896			
x_{11}	3.0000	2.7007			
x_{12}	3.0000	3.0186			
x_{13}	1.0000	0.9997			



(a) CEALM (b) PSO

Figure 1: Parameter X of problem 1 with lowest cost



(a) CEALM (b) PSO

Figure 2: Parameter Y of problem 1 with lowest cost

Problem 2

$$\begin{aligned}\min f(X) = x_1^2 + x_2^2 + x_1x_2 - 14x_1 - 16x_2 + (x_3-10)^2 + 4(x_4-5)^2 + (x_5-3)^2 + 2(x_6-1)^2 \\ + 5x_7^2 + 7(x_8-11)^2 + 2(x_9-10)^2 + (x_10-7)^2 + 45\end{aligned} \tag{5}$$

$$\begin{aligned}\text{subject to} \quad 105 - 4x_1 - 5x_2 + 3x_7 - 9x_8 \geq 0 \\ -3(x_1-2)^2 - 4(x_2-3)^2 - 2x_3^2 + 7x_4 + 120 \geq 0 \\ -10x_1 + 8x_2 + 17x_7 - 2x_8 \geq 0 \\ -x_1^2 - 2(x_2-2)^2 + 2x_1x_2 - 14x_5 + 6x_6 \geq 0 \\ 8x_1 - 2x_2 - 5x_9 + 2x_{10} + 12 \geq 0 \\ -5x_1^2 - 8x_2 - (x_3-6)^2 + 2x_4 + 40 \geq 0 \\ 3x_1 - 6x_2 - 12(x_9-8)^2 + 7x_{10} \geq 0 \\ -0.5(x_1-8)^2 - 2(x_2-4) - 3x_5^2 + x_6 + 30 \geq 0\end{aligned}$$

$$\text{search space:} \quad -10 \leq x_i \leq 10, i = 1, \ldots, 10$$

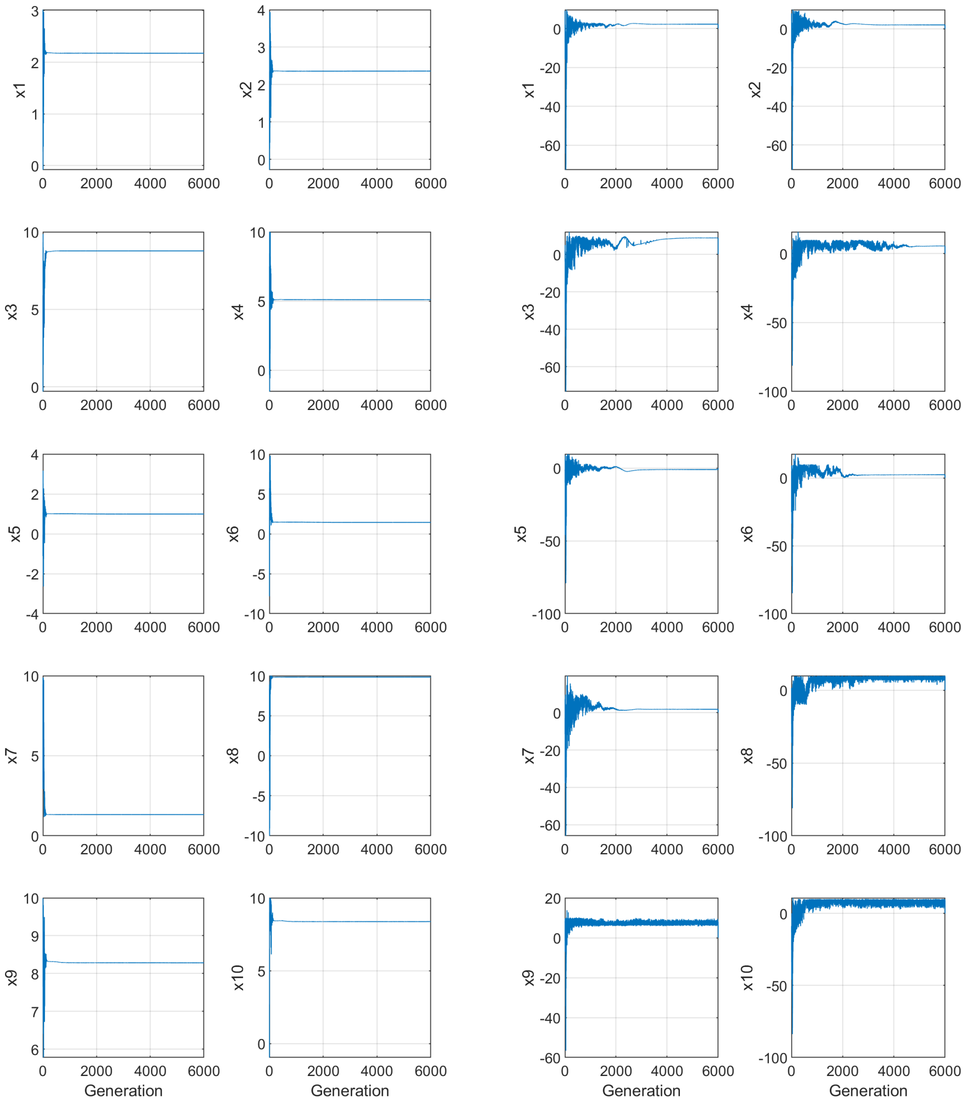
(Solution)

Table 3: Cost comparison of CEALM and PSO. Lowest cost is labelled with bold.

Run	CEALM	PSO	Analytical
1	24.4014	356.8283	24.3060
2	24.3097	107.9832	
3	24.4034	105.9014	
4	24.3591	270.3384	
5	24.3325	63.6385	
6	24.3077	203.3939	
7	24.3547	170.0315	
8	24.3079	93.1403	
9	24.3078	**61.1770**	
10	**24.3065**	70.9870	

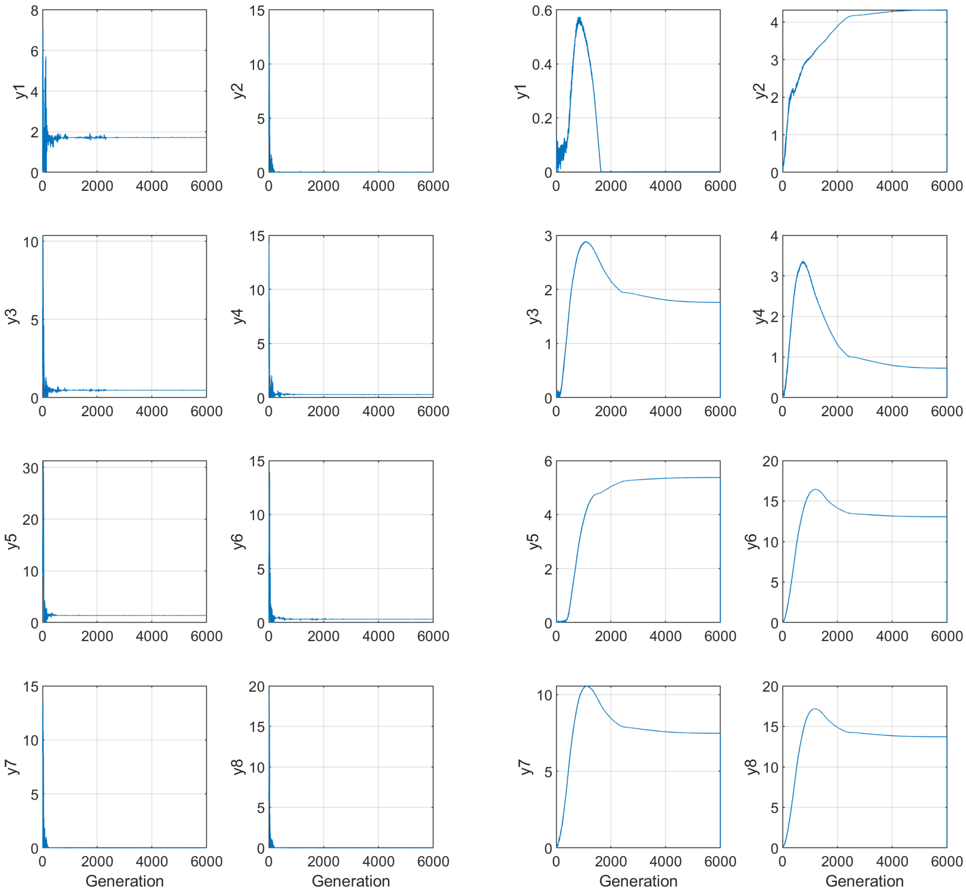
Table 4: Optimal parameter value of problem 2 with lowest cost

	CEALM	PSO		CEALM	PSO
x_1	2.1736	2.2521	y_1	1.7130	0.0000
x_2	2.3598	2.1415	y_2	0.0205	4.3195
x_3	8.7734	8.7517	y_3	0.4761	1.7603
x_4	5.0963	5.5805	y_4	0.2856	0.7250
x_5	0.9904	-0.9687	y_5	1.3778	5.3716
x_6	1.4318	2.3595	y_6	0.3054	13.0650
x_7	1.3247	1.7573	y_7	0.0001	7.4777
x_8	9.8312	9.4168	y_8	0.0004	13.7182
x_9	8.2833	8.0274			
x_{10}	8.3736	9.0898			



(a) CEALM (b) PSO

Figure 3: Parameter X of problem 2 with lowest cost



(a) CEALM (b) PSO

Figure 4: Parameter Y of problem 2 with lowest cost

Problem 3

$$\min f(X) = (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 + 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7 \quad (6)$$

$$\begin{aligned} \text{subject to} \quad 127 - 2x_1^2 - 3x_2^4 - x_3 - 4x_4^2 - 5x_5 &\geq 0 \\ 282 - 7x_1 - 3x_2 - 10x_3^2 - x_4 + x_5 &\geq 0 \\ 196 - 23x_1 - x_2^2 - 6x_6^2 + 8x_7 &\geq 0 \\ -4x_1^2 - x_2^2 + 3x_1x_2 - 2x_3^2 - 5x_6 + 11x_7 &\geq 0 \end{aligned}$$

$$\text{search space:} \quad -10 \leq x_i \leq 10, i = 1, \ldots, 7$$

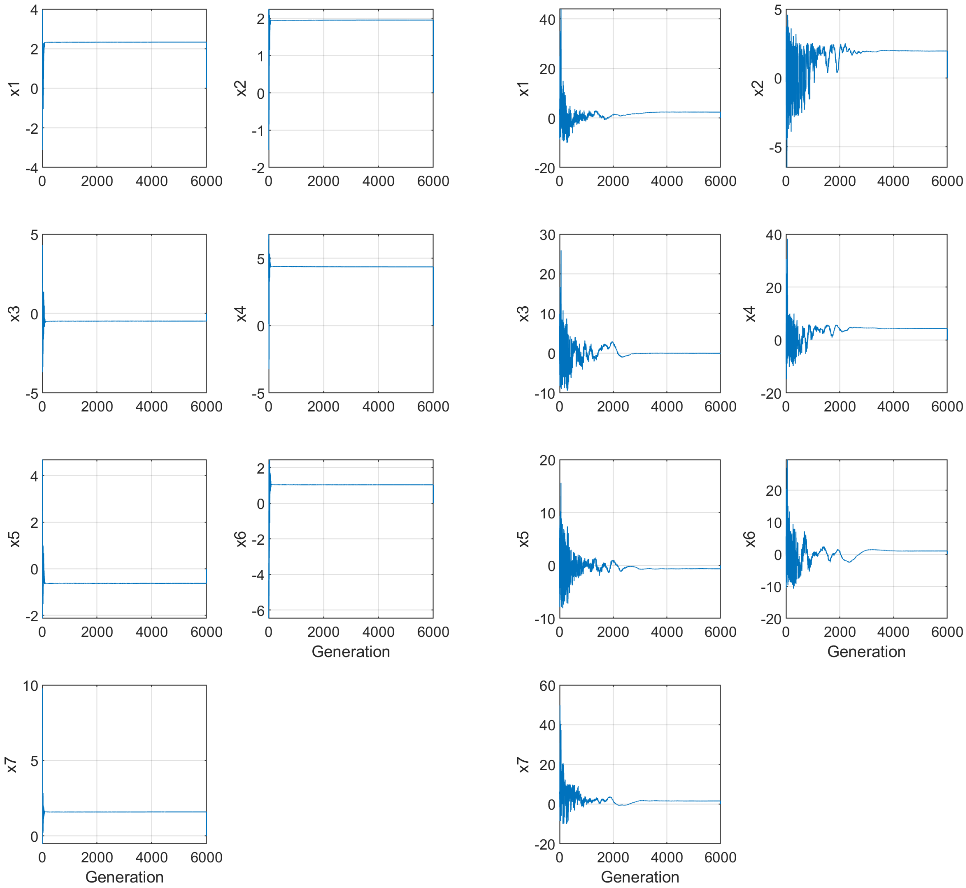
(Solution)

Table 5: Cost comparison of CEALM and PSO. Lowest cost is labelled with bold.

Run	CEALM	PSO	Analytical
1	680.6305	686.4466	680.6300
2	680.6300	693.6093	
3	680.6301	10007452.1252	
4	680.6301	838.4517	
5	680.6303	39618.8533	
6	**680.6300**	**698.7194**	
7	680.6314	681.0175	
8	680.6305	782.3090	
9	680.6300	803.8013	
10	680.6301	1307.3733	

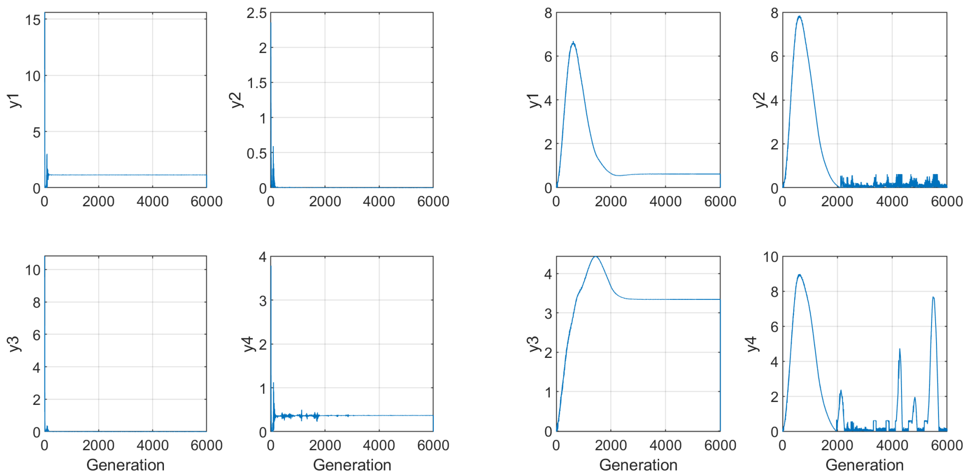
Table 6: Optimal parameter value of problem 3 with lowest cost

	CEALM	PSO		CEALM	PSO
x_1	2.3303	2.3486	y_1	1.1404	0.6142
x_2	1.9511	1.9554	y_2	0.0001	0.0531
x_3	-0.4777	-0.0253	y_3	0.0002	3.3404
x_4	4.3665	4.3385	y_4	0.3685	0.1580
x_5	-0.6245	-0.6265			
x_6	1.0382	1.0181			
x_7	1.5941	1.5908			



(a) CEALM (b) PSO

Figure 5: Parameter X of problem 3 with lowest cost



(a) CEALM (b) PSO

Figure 6: Parameter Y of problem 3 with lowest cost

Problem 4

$$\min f(X) = x_1 + x_2 + x_3 \tag{7}$$

$$\begin{aligned} \text{subject to} \quad 1 - 0.0025(x_4 + x_6) &\geq 0 \\ 1 - 0.0025(x_5 + x_7 - x_4) &\geq 0 \\ 1 - 0.01(x_8 - x_5) &\geq 0 \\ x_1 x_6 - 833.33252 x_4 - 100 x_1 + 83333.333 &\geq 0 \\ x_2 x_7 - 1250 x_5 - x_2 x_4 + 1250 x_4 &\geq 0 \\ x_3 x_8 - 1250000 - x_3 x_5 + 2500 x_5 &\geq 0 \end{aligned}$$

$$\text{search space:} \quad 100 \leq x_1 \leq 10000; \quad 1000 \leq x_i \leq 10000, i = 2, 3; \quad 10 \leq x_i \leq 1000, i = 4, \ldots, 8$$

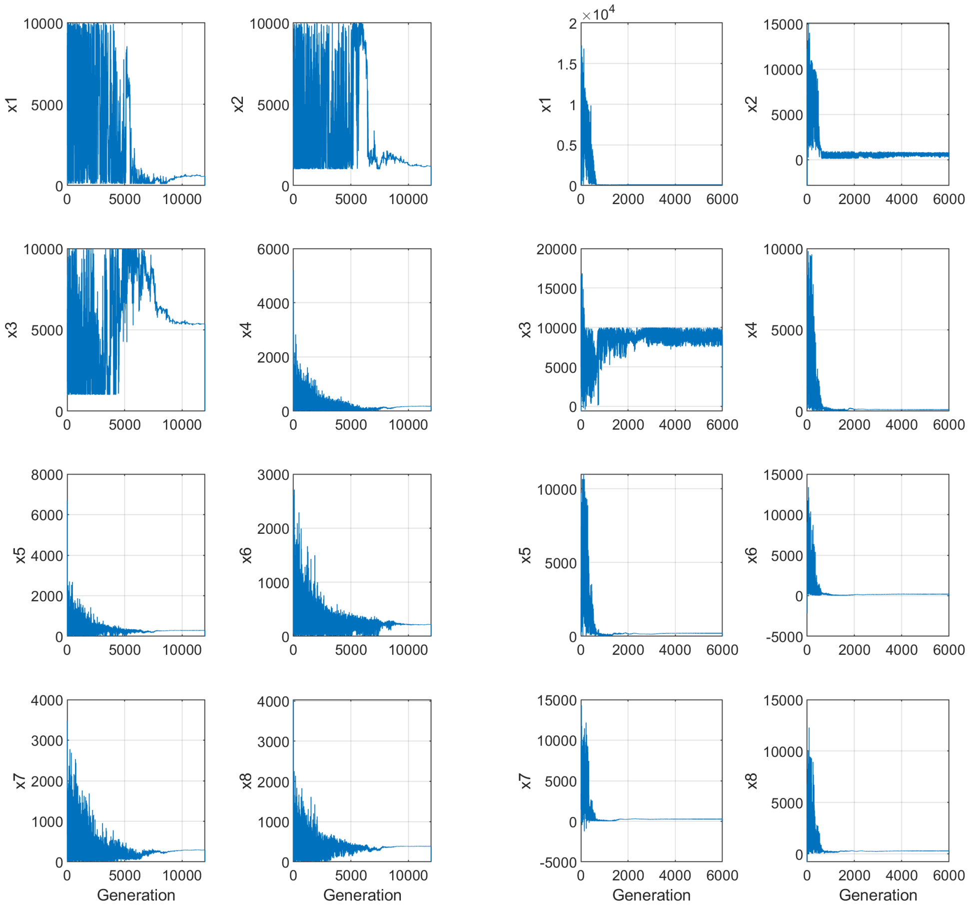
(Solution)

Table 7: Cost comparison of CEALM and PSO. Lowest cost is labelled with bold.

Run	CEALM	PSO	Analytical
1	**7107.6943**	Infeasible	7049.3310
2	7205.8859	Infeasible	
3	7122.9906	Infeasible	
4	7157.6723	Infeasible	
5	7140.7222	Infeasible	
6	7122.7595	Infeasible	
7	7145.1340	Infeasible	
8	7149.9819	Infeasible	
9	7156.5398	**10016.9597**	
10	7216.4679	Infeasible	

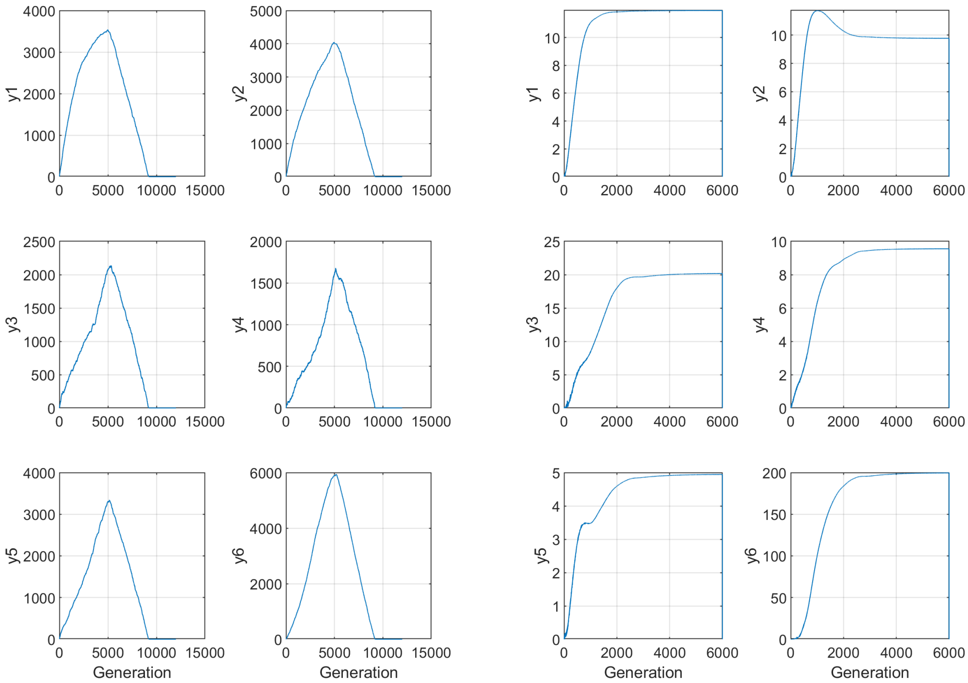
Table 8: Optimal parameter value of problem 4 with lowest cost

	CEALM	PSO		CEALM	PSO
x_1	572.9815	100.0000	y_1	0.0000	11.9362
x_2	1190.8736	836.9181	y_2	0.0002	9.7719
x_3	5343.8391	9080.0416	y_3	0.0000	20.1803
x_4	180.6481	89.9301	y_4	0.0002	9.5554
x_5	287.2824	202.5691	y_5	0.0000	4.9434
x_6	218.6001	194.7662	y_6	0.0000	199.5982
x_7	293.0581	272.1678			
x_8	387.2795	293.0828			



(a) CEALM (b) PSO

Figure 7: Parameter X of problem 4 with lowest cost



(a) CEALM (b) PSO

Figure 8: Parameter Y of problem 4 with lowest cost

Appendix

Listing 1: Main code

```
%
%
%%%  Main Program of CEALM & PSO

  clear; clc; close all;

%------ Problem to be solved --------------------
%  fprintf('\n prob_m01 \n' ) ;
  prob_m01;                                  % defines the problem to be solved1

%---------- Monte-Carlo computation -----------------------------------------------
   nrun           = 10  ;                    % no. of MC computation
   MaxGen         = 6000;                 % Maximum Generation

   nPrint = MaxGen * 10;      % print every nPrint generations

%------ Output control ------------------------------------------
  outfile = fopen ('out_v20.dat','w');


%---------- Strategy ------------------------------------------------------

   method = 'pso';
%  pso    = Particle swarm optimization
%  cealm = Co-Evolution Augmented Lagrangian Method

   istrategy = 0;
%  ONLY FOR CEALM. PSO ALWAYS USE SECURITY STRATEGY
%  0 = security strategy for both players (recommended)
%  1 = man-to-man strategy for the parameter, which is the follower
%

%---------- Populations -----------------------------------------------------

   NumOffspringX   = 40;                 % CEALM Offspring & PSO Population
   NumParentX      = 8;                  % CEALM Parent Population

   NumOffspringY   = 40;                 % CEALM Offspring & PSO Population
   NumParentY      = 8;                  % CEALM Parent Population

%----------------------------------------------------------------------

  done = false;
  for irun=1:nrun
    fprintf(outfile,'\n Run=%3d ', irun) ;
    fprintf(outfile,'\n') ;
    if strcmp(method, 'pso')
        pso_v1
    else
        cealm_v20
    end

  end
  done = true;
  graph_data;

  fclose(outfile);

%---------- end ------------
```

Listing 2: PSO method code

```
%--------------------------ParamMaxX------------------------------------------------------------
% Particle Swarm Optimization
%--------------------------------------------------------------------------------
%
%% Version 1.0
% June 8, 2020
%
% Algorithm from https://en.wikipedia.org/wiki/Particle_swarm_optimization
% Modified from cealm_v20.m
%
% Compiled by:
% Joshua Julian Damanik (20194701)
% Aerospace Enginnering
% Korea Advanced Institute of Science and Technology (KAIST)

%----------- initial bounds of Lagrange multipliers

   NumParamY = NumIneq + NumEq;

   if(NumIneq ~= 0)
       for k=1:NumIneq
          ParamMaxY(k) = 0.01;
          ParamMinY(k) = 0;
       end
   end
   if(NumEq ~= 0)
       for k=1:NumEq
          ParamMaxY(NumIneq+k) = 0.01;
          ParamMinY(NumIneq+k) = -0.01;
       end
   end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

   Gencount     = 1 ;                          % Generation counter

   InitFlag      = 0 ;

   IntervalX      = ParamMaxX - ParamMinX;
   IntervalY      = ParamMaxY - ParamMinY;

   VelocityMinX   = -IntervalX;
   VelocityMaxX   = IntervalX;

   VelocityMinY   = -IntervalY;
   VelocityMaxY   = IntervalY;

   OffspringPopX     = zeros(NumOffspringX,NumParamX);
   OffspringPopY     = zeros(NumOffspringY,NumParamY);

   VelocityPopX   = zeros(NumOffspringX,NumParamX);
   VelocityPopY   = zeros(NumOffspringY,NumParamY);

   BestPopX       = zeros(NumOffspringX,NumParamX);
   BestPopY       = zeros(NumOffspringY,NumParamY);

   Tol            = Tolerance*ones(1,NumParamY);

   PrevF           = 1000000000000000.;
   PrevV           = 1000000000000000.;

%---------------------------------------------------------------
```

```
% Initial Population:
%    Uniform distrubution within the search space
%------------------------------------------------------

%   randn('seed',sum(100*clock));                  % Used 'seed' instead of 'state' for
%   randn('seed',sum(100*clock));                 % compatability of Matcom3 to Matlab4

  for i=1:NumOffspringX
     for j=1:NumParamX
         OffspringPopX(i,j) = rand*(ParamMaxX(j)-ParamMinX(j)) + ParamMinX(j);
         BestPopX(i,j) = OffspringPopX(i,j);
         VelocityPopX(i,j) = (rand*(VelocityMaxX(j)-VelocityMinX(j)) + VelocityMinX(j));
%          OffspringSigmaX(i,j)= VarIntervalX(j);
     end
  end

  for i=1:NumOffspringY
     for j=1:NumParamY
        OffspringPopY(i,j) = rand*(ParamMaxY(j)-ParamMinY(j)) + ParamMinY(j);
        BestPopY(i,j) = OffspringPopY(i,j);
        VelocityPopY(i,j) = (rand*(VelocityMaxY(j)-VelocityMinY(j)) + VelocityMinY(j));
%         OffspringSigmaY(i,j)= VarIntervalY(j);
     end
  end

SX = ceil(rand*NumOffspringX);
SY = ceil(rand*NumOffspringY);

BestParamX = BestPopX(SX,:);
BestParamY = BestPopY(SY,:);

SigInitX      = 0.000001*ones(1,NumParamX);
SigInitY      = 0.000001*ones(1,NumParamY);

%  SigInitX = VarIntervalX;
%  SigInitY = VarIntervalY;

SigMinX       = 0.000001*ones(1,NumParamX);
SigMinY       = 0.000001*ones(1,NumParamY);

%---------- mutation lower bounds ---------------------------------------------

   alpha   = 10^(-1/DeciGen);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% >>======= Iteration Starts Here =======<< %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

iPrint = 1;
while (Gencount <= MaxGen)

%------------------------------------------------------
% Recombination Process   (evolutionary strategy)
%------------------------------------------------------

for i=1:NumOffspringX
    RP = rand();
    RG = rand();
    for j=1:NumParamX
        VelocityPopX(i,j) = omega * VelocityPopX(i,j)...
            + phiP * RP * (BestPopX(i,j) - OffspringPopX(i,j))...
            + phiG * RG * (BestParamX(j) - OffspringPopX(i,j));
```

```
%          VelocityPopX(i,j) = min(max(VelocityPopX(i,j),VelocityMinX(j)),VelocityMaxX(j));
        OffspringPopX(i,j) = OffspringPopX(i,j) + VelocityPopX(i,j);
    end
end

for i=1:NumOffspringY
    RP = rand();
    RG = rand();
    for j=1:NumParamY
        VelocityPopY(i,j) = omega * VelocityPopY(i,j)...
            + phiP * RP * (BestPopY(i,j) - OffspringPopY(i,j))...
            + phiG * RG * (BestParamY(j) - OffspringPopY(i,j));
%          VelocityPopY(i,j) = min(max(VelocityPopY(i,j),VelocityMinY(j)),VelocityMaxY(j));
        OffspringPopY(i,j) = OffspringPopY(i,j) + VelocityPopY(i,j);
    end
end

%---------------------------------------------
%  search space restriction (softwalls)
%---------------------------------------------

for i=1:NumOffspringX
   for j=1:NumParamX
      violate1 = OffspringPopX(i,j)-ParamMaxX(j);
      violate2 = ParamMinX(j)-OffspringPopX(i,j);
      if violate1 >0
         OffspringPopX(i,j) = OffspringPopX(j) - rand*min(violate1,IntervalX(j)) ;
      end
      if violate2 >0
         OffspringPopX(i,j) = OffspringPopX(j) + rand*min(violate2,IntervalX(j)) ;
      end
   end
end

%-------------- lower bound check only for inequality constraints --------------

for i=1:NumOffspringY
   for j=1:NumIneq
      violate1 = OffspringPopY(i,j)-ParamMaxY(j);
      violate2 = ParamMinY(j)-OffspringPopY(i,j);
%        if violate1 >0
%           OffspringPopY(i,j) = OffspringPopY(j) - rand*min(violate1,IntervalY(j)) ;
%        end
      if violate2 >0
         OffspringPopY(i,j) = OffspringPopY(j)+rand*min(violate2,IntervalY(j)) ;
      end
   end
end



%---------------------------------------------
% Matching Process (full match)
%---------------------------------------------


for i=1:NumOffspringX

  [costf,cnstr] = feval(CostDef,OffspringPopX(i,:));

  for j=1:NumOffspringY
    AL1 = 0;
    AL2 = 0;
```

```
    AL3 = 0;
    if(NumIneq ~= 0)
        for k=1:NumIneq
            AL1 = AL1 + (max(cnstr(k)+0.5*OffspringPopY(j,k)/rho, 0))^2;
            AL2 = AL2 + OffspringPopY(j,k)^2;
        end
        AL1 = rho*AL1;
        AL2 = - 0.25*AL2/rho;
    end
    if(NumEq ~= 0)
        for k=1:NumEq
            AL3 = AL3 + OffspringPopY(j,NumIneq+k)*cnstr(NumIneq+k) + rho*cnstr(NumIneq+k)^2
        end
    end
    ALV = costf + AL1 + AL2 + AL3;
    CostOffspringX(i,j) = ALV;
    CostOffspringY(j,i) = ALV;
  end
end



%-----------------------------------------------------------------
%  Fitness evaluation and ordering
%-----------------------------------------------------------------



if(istrategy==0)     % Y & X = security
    [maxcost,mxcind] = max(CostOffspringX');
    [mincost,mncind] = min(CostOffspringY');
end

if exist('CostBestX','var') == 0
    BestPopX = OffspringPopX;
    BestPopY = OffspringPopY;

    BestCostPopX = maxcost;
    BestCostPopY = mincost;

    [BestCostX,bestindexx] = min(BestCostPopX);
    [BestCostY,bestindexy] = max(BestCostPopY);

    BestParamX = BestPopX(bestindexx,:);
    BestParamY = BestPopY(bestindexy,:);
end

for i=NumOffspringX
    if (maxcost(i) < BestCostPopX(i))
        BestPopX(i,:) = OffspringPopX(i,:);
        BestCostPopX(i) = maxcost(i);
    end
    if BestCostPopX(i) < BestCostX
        BestParamX = BestPopX(i,:);
        BestCostX = BestCostPopX(i);
    end
end

% [BestCostY, bestindexy] = max(CostOffspringY(:,bestindexx));
% BestParamY = OffspringPopY(bestindexy,:);

for i=NumOffspringY
```

```
    if (mincost(i) > BestCostPopY(i))
        BestPopY(i,:) = OffspringPopY(i,:);
        BestCostPopY(i) = mincost(i);
    end
    if BestCostPopY(i) > BestCostY
        BestParamY = BestPopY(i,:);
        BestCostY = BestCostPopY(i);
    end
end

%---------------------------------------------------
% Output to Display & File
%---------------------------------------------------


  [CostF,Cnstr] = feval(CostDef,BestParamX);

  if(NumEq ~= 0)
     for k=1:NumEq
        Cnstr(NumIneq+k)=abs(Cnstr(NumIneq+k));
     end
  end
  Vx   = max(Cnstr,0);
  Vsum = sum(Vx);

  C1 = Cnstr - Tol;
  V1    = max(C1,0);
  CV   = sum(V1);


  if CV <= 0
    CnstrFlag = 1;
  else
    CnstrFlag = 0;
  end

  if InitFlag == 0  & CnstrFlag ==1              % set BestF if constraint is satisfied
    InitFlag = 1;                                % for the first time
    PrevF = CostF;
    PrevV = Vsum;
    PrevPX = BestParamX;
    PrevPY = BestParamY;
  end

  if InitFlag ==1 & CnstrFlag ==1  & CostF < PrevF      % check if cost improved
    ImprvFlag = 1;
  else
    ImprvFlag=0;
  end

  if ImprvFlag == 1
     PrevF = CostF;
     PrevV = Vsum;
     PrevPX = BestParamX;
     PrevPY = BestParamY;
  end

%------  Write to the output file if improved ------------------

  if ImprvFlag == 1 || (InitFlag == 0  & CnstrFlag ==1 )
    fprintf(outfile, '\n Run=%3d   G=%3d', irun, Gencount) ;
    fprintf(outfile, '\n CostF =%10.7f', CostF) ;
    fprintf(outfile,'\n');
```

```
    if NumIneq ~= 0
        for i=1:NumIneq
            fprintf(outfile,' G%d=%10.7f',i, Cnstr(i));
        end
        fprintf(outfile,'\n');
    end
    if NumEq ~= 0
       for i=1:NumEq
           fprintf(outfile,' H%d=%10.7f',i, Cnstr(i+NumIneq));
       end
       fprintf(outfile,'\n');
    end
    for i=1:NumParamX
      fprintf(outfile,' X%d=%10.7f',i, BestParamX(i));
    end
    fprintf(outfile,'\n');
    for i=1:NumParamY
      fprintf(outfile,' Y%d=%10.7f',i, BestParamY(i));
    end
    fprintf(outfile,'\n');
  end

%--------- Print at every nPrint generation ------------------------------------------

   if iPrint ==  nPrint
     fprintf('\n Run=%3d G=%3d ', irun, Gencount);
     fprintf('\n CV     =%10.7f',CV);
     for i=1:NumParamY
       fprintf('   G%d=%10.7f',i, Cnstr(i));
     end
     fprintf('\n CostX =%10.7f', BestCostX) ;
     for i=1:NumParamX
       fprintf('   X%d=%10.7f',i, BestParamX(i));
     end
     fprintf('\n CostY =%10.7f', BestCostY) ;
     for i=1:NumParamY
       fprintf('   Y%d=%10.7f',i, BestParamY(i));
     end
     fprintf('\n');
     iPrint = 0;
   end
   iPrint = iPrint + 1;


%---------Screen output at the end of evolution --------------

  if(Gencount >= MaxGen)
     fprintf('\n Evolution Summary: Run=%3d G=%3d ', irun, Gencount);

    if(InitFlag == 1)
     else
        fprintf('   Unable to find a feasible solution!');
     end

     fprintf('\n CostF=%10.7f Vsum=%10.7f AL_X=%10.7f AL_Y=%10.7f', CostF, Vsum, BestCostX,
     fprintf('\n');
     for i=1:NumParamX
       fprintf('   X%d=%10.7f',i, BestParamX(i));
     end
     fprintf('\n');
     for i=1:NumParamY
       fprintf('   Y%d=%10.7f',i, BestParamY(i));
     end
```

```
      fprintf('\n');

   end
%  check_transient;
 graph_data;
%  if done == true, break, end
 Gencount = Gencount + 1 ;



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

end          %%%%%   End of 'While' Loop   %%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

References

[1] M.-J. Tahk and B.-C. Sun, "Coevolutionary augmented lagrangian methods for constrained optimization," *IEEE Transactions on Evolutionary Computation*, vol. 4, no. 2, p. 114–124, 2000.

[2] M. Zambrano-Bigiarini, M. Clerc, and R. Rojas, "Standard particle swarm optimisation 2011 at cec-2013: A baseline for future pso improvements," *2013 IEEE Congress on Evolutionary Computation*, 2013.