AE 551: Introduction to Optimal Control (Due: 2020/06/12)

Homework #9 Submission

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Solve the four problems treated in the paper "Coevolutionary Augmented Lagrangian Methods for Constrained Optimization" by using CEALM and PSO. The paper is included in the ZIP file of cealm_v20. For each problem, run the codes of CEALM and PSO for at least 10 times to find the best and worst results of each method.

- CEALM: Use the CEALM code posted on KLMS.
- PSO:
  - You can use any PSO code but you need to explicitly state the source of the code (for example, URL or the reference papers)
  - Describe in detail the PSO algorithm you use in the homework report. (Do not use any code you don't understand the details.)
  - Attach the code to your report.

(Solution) Code for PSO method is attached at the Appendix section. However, the rest of the code can be accessed from `https://github.com/joshuadamanik/Homework-9`.

For the CEALM method [1], the code used in this homework is the one provided at cealm_v20_ae551.zip. However, the code for the PSO method is created based on proposed algorithm [2] by Zambrano-Bigiarini, et al.

Algorithm 1: Standard Particle Swarm Optimization

for *each particle:* $i \leftarrow 1, \dots, N$ **do**
  initialize random particle's parameter: $X_i \in U(X_{min}, X_{max})$;
  initialize random particle's velocity: $V_i \in U(-(X_{max} - X_{min}), (X_{max} - X_{min}))$;
  initialize particle's best parameter: $\bar{X}_i \leftarrow x_i$;
  initialize swarm's best parameter: $X^* \leftarrow \arg\min f(\bar{X}_i)$;
end
while *termination criteria is not satisfied* **do**
  for *each particle:* $i \leftarrow 1, \dots, N$ **do**
    for *each dimension:* $j \leftarrow 1, \dots, M$ **do**
      initialize random numbers: $r_p, r_g \in U(0, 1)$;
      update particle's velocity: $V_{i,j} \leftarrow \omega V_{i,j} + c_1 r_p (\bar{X}_{i,j} - X_{i,j}) + c_2 r_g (X^*_j - X_{i,j})$;
    end
    update particle's parameter: $X_i \leftarrow X_i + v_i$;
    if $f(X_i) < f(\bar{X}_i)$ **then**
      update particle's best parameter: $\bar{X}_i \leftarrow X_i$;
      if $f(\bar{X}_i) < f(X^*)$ **then**
        update swarm's best parameter: $X^* \leftarrow \bar{X}_i$;
      end
    end
  end
end

However, the algorithm 1 is an unconstrained optimization method. For constrained optimization, the objective function $f(X)$ is replaced by augmented Lagrangian function $L_A(X, Y)$, defined as

$$L(X_i, \lambda_i, \mu_i) = f(X_i) + \rho \sum_{j=1}^{N_{ineq}} \max^2 \left\{ g_j(X_i) + \frac{\mu_{i,j}}{2\rho} \right\} + \sum_{j=1}^{N_{ineq}} \frac{\mu_{i,j}^2}{4\rho} + \lambda_i^T h(X_i) + \rho h(X_i)^T h(X_i) \tag{1}$$

where $g(X_i) \leq 0$ is set of inequality constraints and $h(X_i) = 0$ is set of equality constraints with their Lagrange multiplier μ_i and λ_i respectively. The multiplier μ_i and λ_i is then combined into a particle object Y_i with dimension $N_{ineq} + N_{eq}$.

By using augmented Lagrangian function (eq. 1), the optimization is done in an unconstrained fashion by solving both the X and Y particles. But, instead of searching for minimum value, we need to find the maximum value for particle Y. Then, the swarm's best parameter both for X and Y are selected by using security strategy [1].

In addition, because all of the problems defined in this homework have restricted search space, the softwall algorithm [1] is added into the Standard PSO (algorithm 1). While the search space of particle X is defined in each problem, the search space for particle Y is defined as

$$Y_{min} = \begin{cases} 0, & \text{inequality constraint} \\ -10, & \text{equality constraint} \end{cases} \tag{2}$$

$$Y_{max} = 10; \tag{3}$$

Problem 1

$$\min f(X) = 5x_1 + 5x_2 + 5x_3 + 5x_4 - 5\sum_{i=1}^{4} x_i^2 - \sum_{i=5}^{1} 3x_i \tag{4}$$

$$\begin{aligned} \text{subject to} \quad 2x_1 + 2x_2 + x_{10} + x_{11} &\le 10 \\ 2x_1 + 2x_3 + x_{10} + x_{12} &\le 10 \\ 2x_1 + 2x_3 + x_{10} + x_{12} &\le 10 \\ -8x_1 + x_{10} &\le 0 \\ -8x_2 + x_{11} &\le 0 \\ -8x_3 + x_{12} &\le 0 \\ -2x_4 - x_5 + x_{10} &\le 0 \\ -2x_6 - x_7 + x_{11} &\le 0 \\ -2x_8 - x_9 + x_{12} &\le 0 \end{aligned}$$

search space: $0 \le x_i \le 1, i = 1, \ldots, 9; \quad 0 \le x_i \le 10, i = 10, 11, 12; \quad 0 \le x_{13} \le 1$

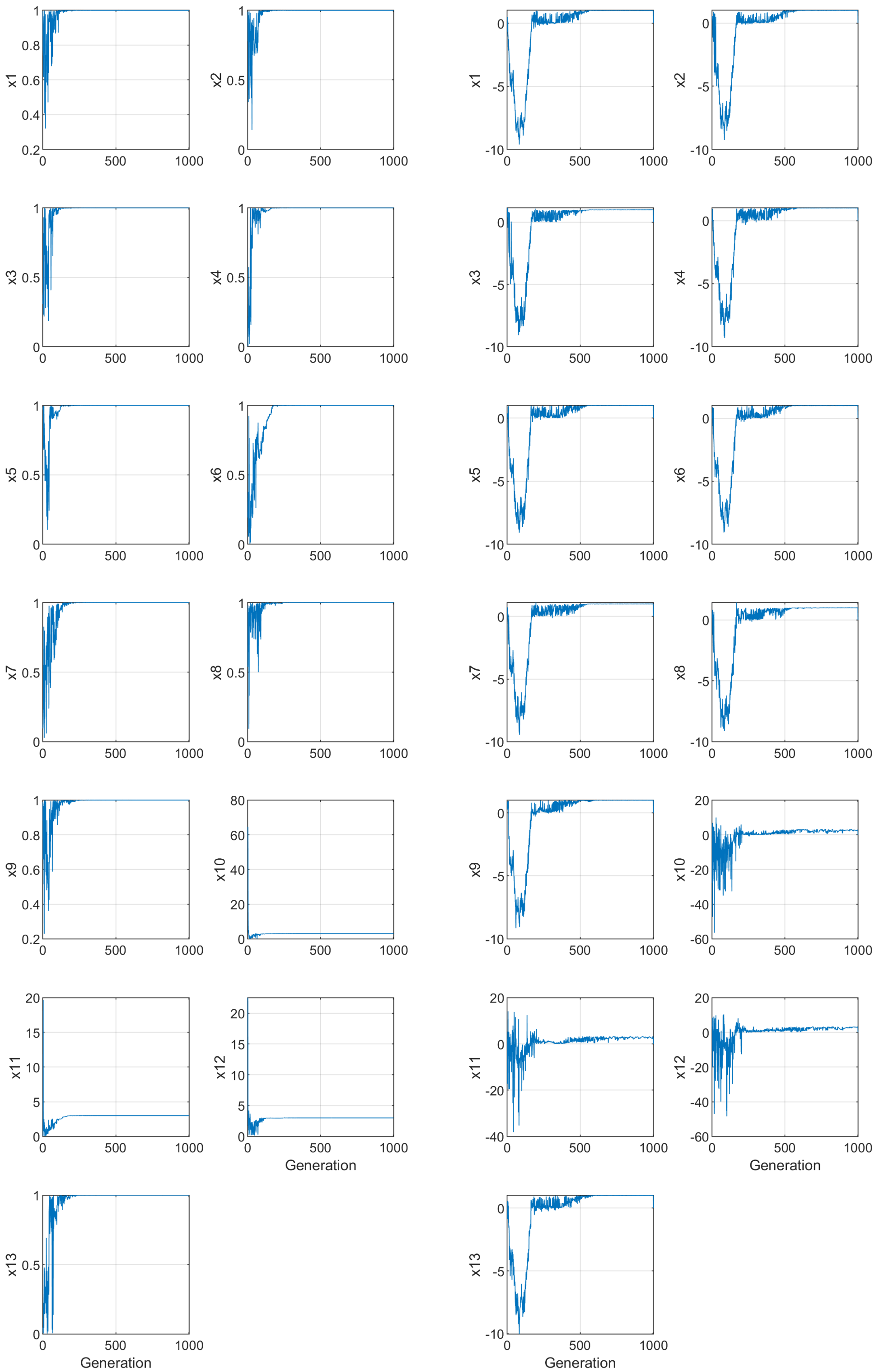
(Solution)

Table 1: Cost comparison of CEALM and PSO. Lowest cost is labelled with bold.

Run	CEALM	PSO	Analytical
1	-15.0000	-13.6296	-15.0000
2	-15.0000	**-14.5906**	
3	-15.0000	-4.7454	
4	-13.8281	-13.0326	
5	-15.0000	-14.1221	
6	-15.0000	-4.5065	
7	-13.8281	-13.1253	
8	-15.0000	-14.4504	
9	-15.0000	-3.8945	
10	**-15.0000**	-14.2518	

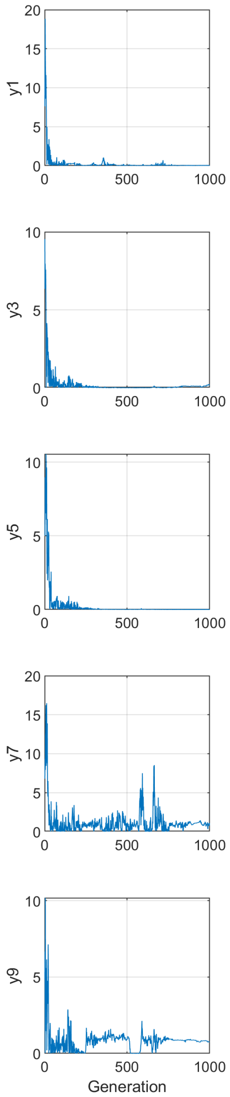
Table 2: Optimal parameter value of problem 1 with lowest cost

	CEALM	PSO		CEALM	PSO
x_1	1.0000	0.9993	y_1	0.0289	-0.0002
x_2	1.0000	0.9982	y_2	0.0754	-0.0004
x_3	1.0000	0.9995	y_3	0.1624	1.5536
x_4	1.0000	0.9999	y_4	0.0000	1.4803
x_5	1.0000	0.9991	y_5	0.0002	0.0763
x_6	1.0000	0.9999	y_6	0.0000	-0.0002
x_7	1.0000	0.9999	y_7	0.8247	2.6092
x_8	1.0000	0.9995	y_8	0.8140	-0.0005
x_9	1.0000	0.9990	y_9	0.7531	0.0017
x_{10}	3.0000	2.8896			
x_{11}	3.0000	2.7007			
x_{12}	3.0000	3.0186			
x_{13}	1.0000	0.9997			

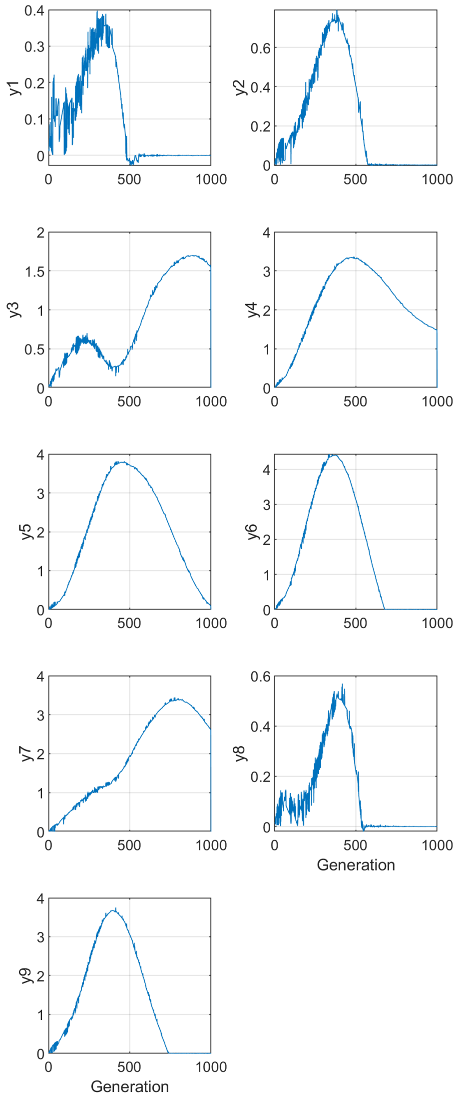


(a) CEALM (b) PSO

Figure 1: Parameter X of problem 1 with lowest cost



(a) CEALM



(b) PSO

Figure 2: Parameter Y of problem 1 with lowest cost

Problem 2

$$\begin{aligned}\min f(X) = {} & x_1^2 + x_2^2 + x_1x_2 - 14x_1 - 16x_2 + (x_3-10)^2 + 4(x_4-5)^2 + (x_5-3)^2 + 2(x_6-1)^2 \\ & + 5x_7^2 + 7(x_8-11)^2 + 2(x_9-10)^2 + (x_10-7)^2 + 45\end{aligned} \tag{5}$$

$$\begin{aligned}\text{subject to} \quad 105 - 4x_1 - 5x_2 + 3x_7 - 9x_8 &\geq 0 \\ -3(x_1-2)^2 - 4(x_2-3)^2 - 2x_3^2 + 7x_4 + 120 &\geq 0 \\ -10x_1 + 8x_2 + 17x_7 - 2x_8 &\geq 0 \\ -x_1^2 - 2(x_2-2)^2 + 2x_1x_2 - 14x_5 + 6x_6 &\geq 0 \\ 8x_1 - 2x_2 - 5x_9 + 2x_{10} + 12 &\geq 0 \\ -5x_1^2 - 8x_2 - (x_3-6)^2 + 2x_4 + 40 &\geq 0 \\ 3x_1 - 6x_2 - 12(x_9-8)^2 + 7x_{10} &\geq 0 \\ -0.5(x_1-8)^2 - 2(x_2-4) - 3x_5^2 + x_6 + 30 &\geq 0\end{aligned}$$

$$\text{search space:} \quad -10 \leq x_i \leq 10, i = 1, \ldots, 10$$

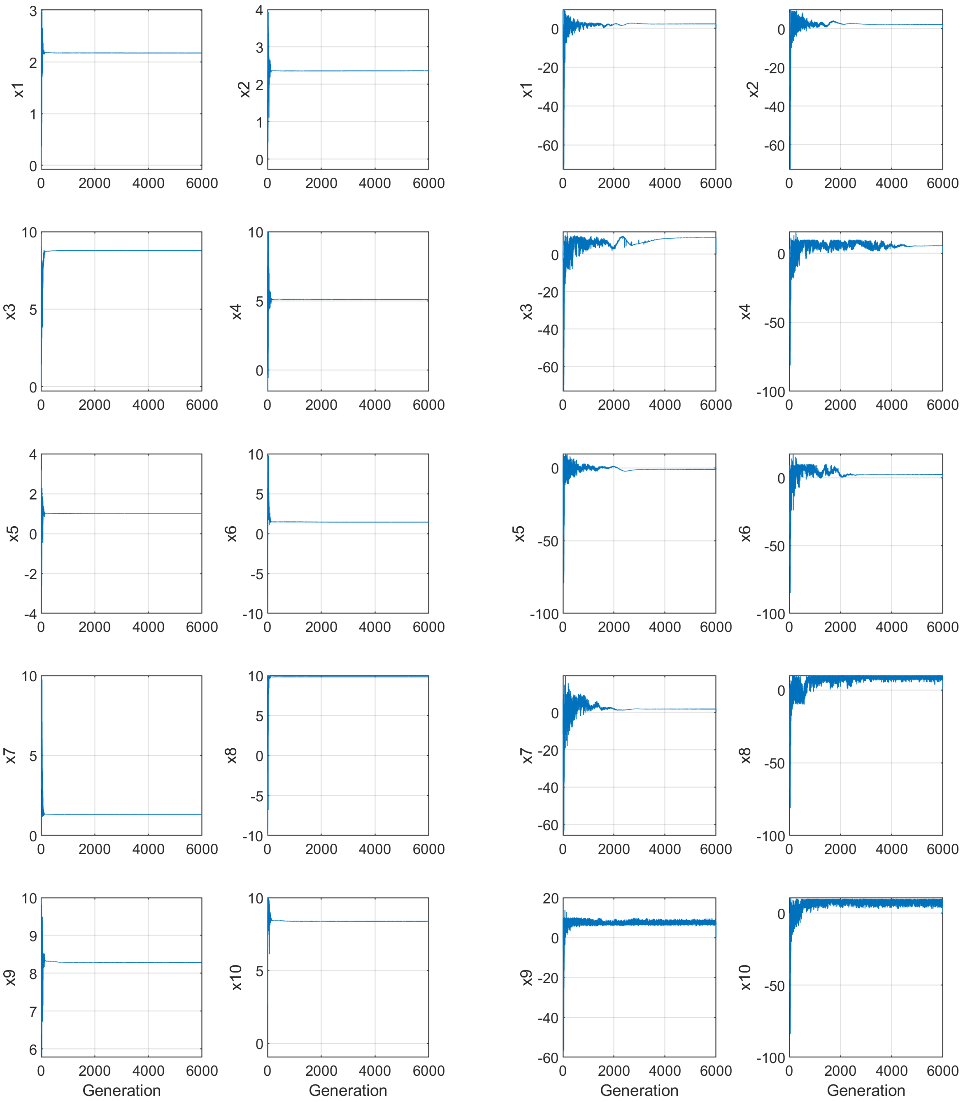
(Solution)

Table 3: Cost comparison of CEALM and PSO. Lowest cost is labelled with bold.

Run	CEALM	PSO	Analytical
1	24.4014	356.8283	24.3060
2	24.3097	107.9832	
3	24.4034	105.9014	
4	24.3591	270.3384	
5	24.3325	63.6385	
6	24.3077	203.3939	
7	24.3547	170.0315	
8	24.3079	93.1403	
9	24.3078	**61.1770**	
10	**24.3065**	70.9870	

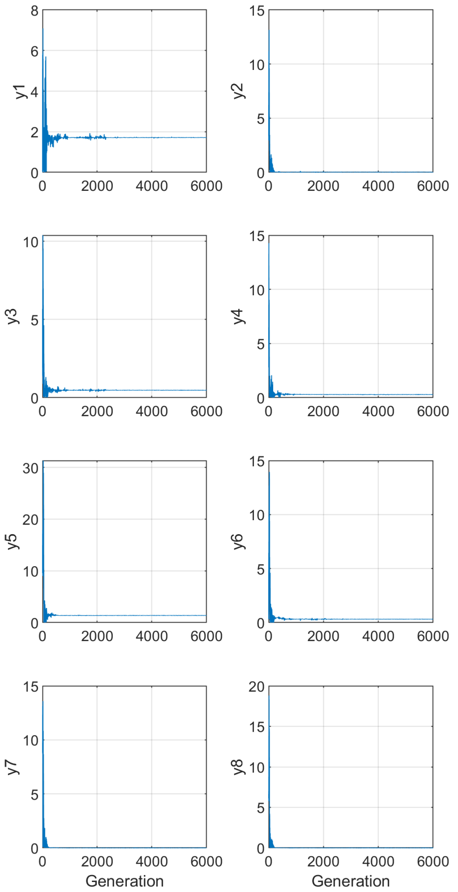
Table 4: Optimal parameter value of problem 2 with lowest cost

	CEALM	PSO		CEALM	PSO
x_1	2.1736	2.2521	y_1	1.7130	0.0000
x_2	2.3598	2.1415	y_2	0.0205	4.3195
x_3	8.7734	8.7517	y_3	0.4761	1.7603
x_4	5.0963	5.5805	y_4	0.2856	0.7250
x_5	0.9904	-0.9687	y_5	1.3778	5.3716
x_6	1.4318	2.3595	y_6	0.3054	13.0650
x_7	1.3247	1.7573	y_7	0.0001	7.4777
x_8	9.8312	9.4168	y_8	0.0004	13.7182
x_9	8.2833	8.0274			
x_{10}	8.3736	9.0898			

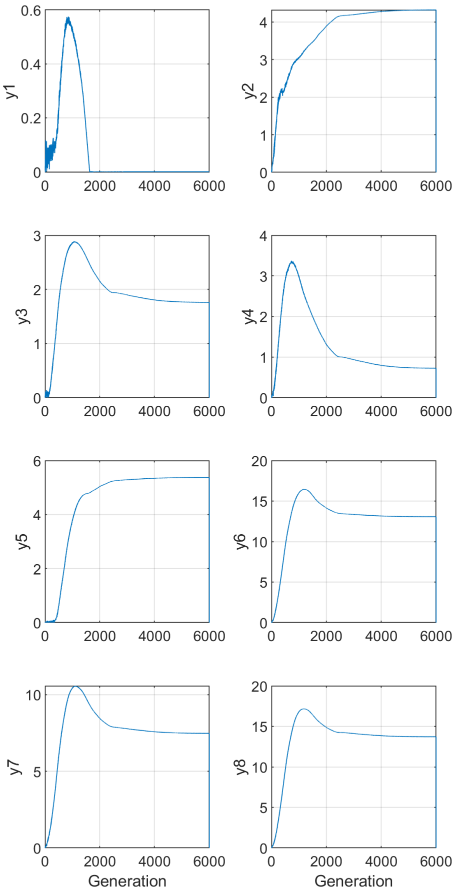


(a) CEALM (b) PSO

Figure 3: Parameter X of problem 2 with lowest cost



(a) CEALM



(b) PSO

Figure 4: Parameter Y of problem 2 with lowest cost

Problem 3

$$\min f(X) = (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 + 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7 \tag{6}$$

$$\begin{aligned} \text{subject to} \quad 127 - 2x_1^2 - 3x_2^4 - x_3 - 4x_4^2 - 5x_5 &\geq 0 \\ 282 - 7x_1 - 3x_2 - 10x_3^2 - x_4 + x_5 &\geq 0 \\ 196 - 23x_1 - x_2^2 - 6x_6^2 + 8x_7 &\geq 0 \\ -4x_1^2 - x_2^2 + 3x_1x_2 - 2x_3^2 - 5x_6 + 11x_7 &\geq 0 \end{aligned}$$

$$\text{search space:} \quad -10 \leq x_i \leq 10, i = 1, \ldots, 7$$

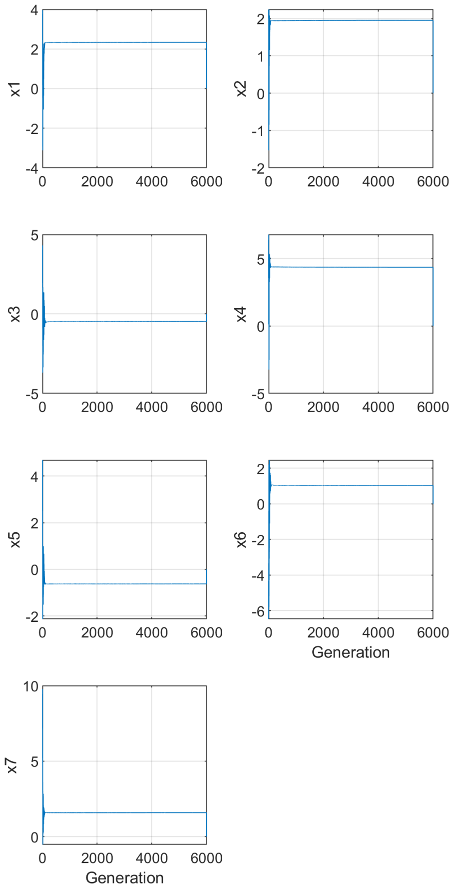
(Solution)

Table 5: Cost comparison of CEALM and PSO. Lowest cost is labelled with bold.

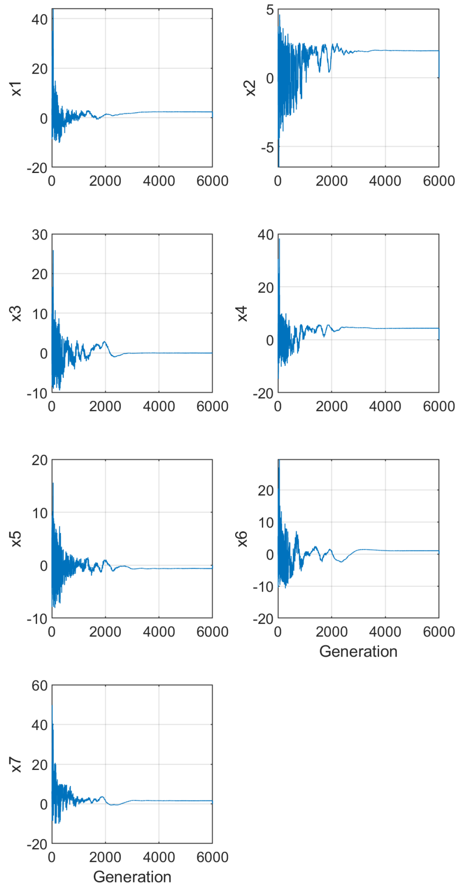
Run	CEALM	PSO	Analytical
1	680.6305	686.4466	680.6300
2	680.6300	693.6093	
3	680.6301	10007452.1252	
4	680.6301	838.4517	
5	680.6303	39618.8533	
6	**680.6300**	**698.7194**	
7	680.6314	681.0175	
8	680.6305	782.3090	
9	680.6300	803.8013	
10	680.6301	1307.3733	

Table 6: Optimal parameter value of problem 3 with lowest cost

	CEALM	PSO		CEALM	PSO
x_1	2.3303	2.3486	y_1	1.1404	0.6142
x_2	1.9511	1.9554	y_2	0.0001	0.0531
x_3	-0.4777	-0.0253	y_3	0.0002	3.3404
x_4	4.3665	4.3385	y_4	0.3685	0.1580
x_5	-0.6245	-0.6265			
x_6	1.0382	1.0181			
x_7	1.5941	1.5908			

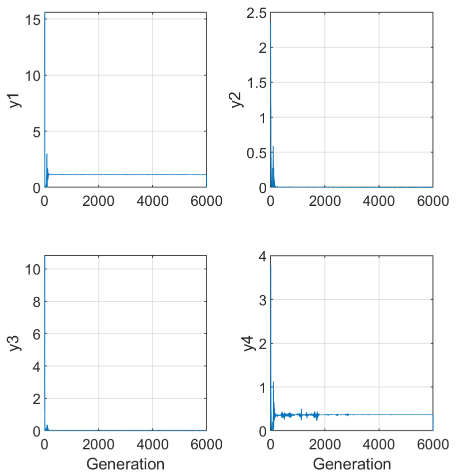


(a) CEALM

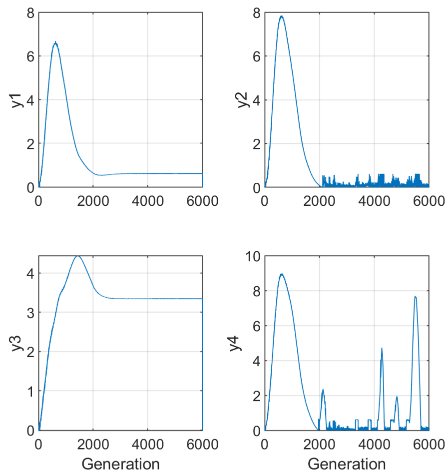


(b) PSO

Figure 5: Parameter X of problem 3 with lowest cost



(a) CEALM



(b) PSO

Figure 6: Parameter Y of problem 3 with lowest cost

Problem 4

$$\min f(X) = x_1 + x_2 + x_3 \tag{7}$$

$$\begin{aligned} \text{subject to} \quad 1 - 0.0025(x_4 + x_6) &\geq 0 \\ 1 - 0.0025(x_5 + x_7 - x_4) &\geq 0 \\ 1 - 0.01(x_8 - x_5) &\geq 0 \\ x_1x_6 - 833.33252x_4 - 100x_1 + 83333.333 &\geq 0 \\ x_2x_7 - 1250x_5 - x_2x_4 + 1250x_4 &\geq 0 \\ x_3x_8 - 1250000 - x_3x_5 + 2500x_5 &\geq 0 \end{aligned}$$

search space: $100 \leq x_1 \leq 10000$; $1000 \leq x_i \leq 10000, i = 2, 3$; $10 \leq x_i \leq 1000, i = 4, \ldots, 8$

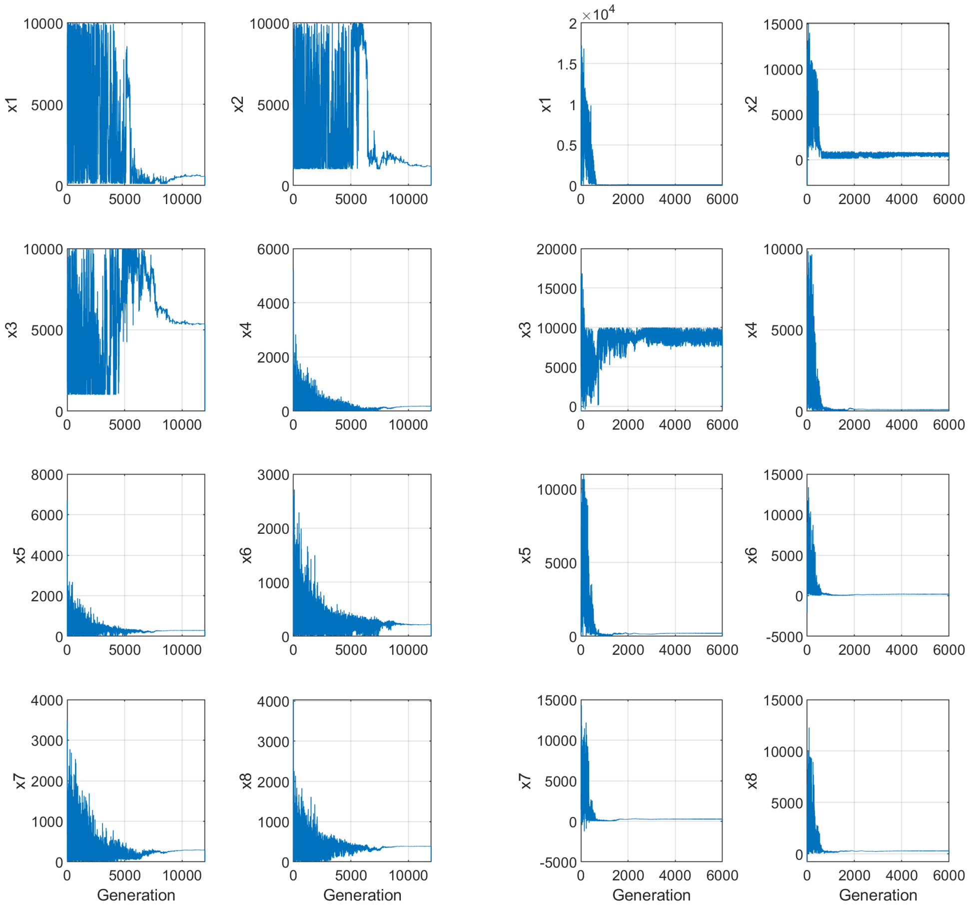
(Solution)

Table 7: Cost comparison of CEALM and PSO. Lowest cost is labelled with bold.

Run	CEALM	PSO	Analytical
1	**7107.6943**	Infeasible	7049.3310
2	7205.8859	Infeasible	
3	7122.9906	Infeasible	
4	7157.6723	Infeasible	
5	7140.7222	Infeasible	
6	7122.7595	Infeasible	
7	7145.1340	Infeasible	
8	7149.9819	Infeasible	
9	7156.5398	**10016.9597**	
10	7216.4679	Infeasible	

Table 8: Optimal parameter value of problem 4 with lowest cost

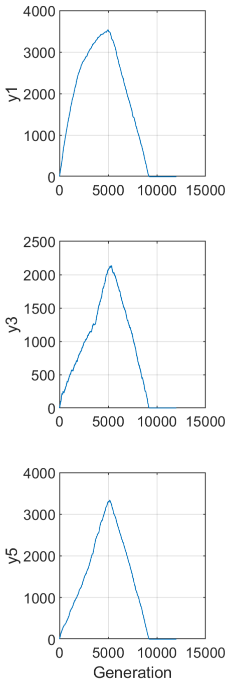
	CEALM	PSO		CEALM	PSO
x_1	572.9815	100.0000	y_1	0.0000	11.9362
x_2	1190.8736	836.9181	y_2	0.0002	9.7719
x_3	5343.8391	9080.0416	y_3	0.0000	20.1803
x_4	180.6481	89.9301	y_4	0.0002	9.5554
x_5	287.2824	202.5691	y_5	0.0000	4.9434
x_6	218.6001	194.7662	y_6	0.0000	199.5982
x_7	293.0581	272.1678			
x_8	387.2795	293.0828			



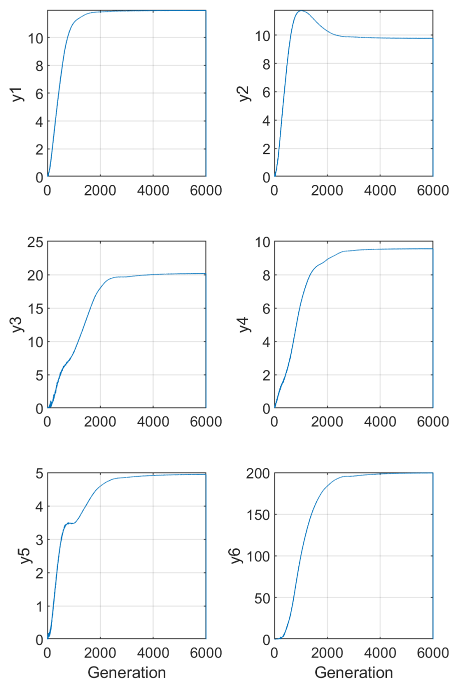
(a) CEALM

(b) PSO

Figure 7: Parameter X of problem 4 with lowest cost



(a) CEALM



(b) PSO

Figure 8: Parameter Y of problem 4 with lowest cost

Appendix

References

[1] M.-J. Tahk and B.-C. Sun, "Coevolutionary augmented lagrangian methods for constrained optimization," *IEEE Transactions on Evolutionary Computation*, vol. 4, no. 2, p. 114–124, 2000.

[2] M. Zambrano-Bigiarini, M. Clerc, and R. Rojas, "Standard particle swarm optimisation 2011 at cec-2013: A baseline for future pso improvements," *2013 IEEE Congress on Evolutionary Computation*, 2013.