Homework 1: Solutions
AE 552: Advanced Linear Stability and Control

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1 NLPD Model

By using the missile parameters provided, please write simulation code for the NLPD model (you can utilize MATLAB Simulink or m file coding)

Answer:

The solution is given in MATLAB Simulink file `model.slx`

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2 Trim Point and Linearized Model

Please write m-file codes for determining trim state/input/output and obtaining the linearized model at a trim point.

Answer:

The solution is given in MATLAB m-file `trim.m` and `linearization.m`

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3 Shifted IMU

Please determine the transfer function from the control input to the pitch rate and the acceleration when IMU is not located at the center of gravity. Please discuss how pole/zero values are altered depending on the locations of IMU.

Answer:

When we shift the IMU by X_{IMU}, the acceleration in z-axis becomes

$$\begin{aligned} a_z &= \frac{QS}{m}\left[C_{z,0} + C_{z,\delta}\delta\right] - \dot{q}X_{IMU} \\ &= \frac{QS}{m}\left[C_{z,0} + C_{z,\delta}\delta\right] - \frac{QSdX_{IMU}}{I_{yy}}\left[C_{M,0} + C_{M,q}\left(\frac{d}{2V}\right)q + C_{M,\delta}\delta\right] \end{aligned} \tag{1}$$

While the transfer function of $G_q(s) = \frac{q}{d}$ is identical with the one on lecture note (pg. 50), the transfer function of $G_{a_z}(s) = \frac{a_z}{\delta}$ is changed into

$$G_{a_z}(s) = \frac{a_z}{\delta} = \frac{(A_\delta - M\delta X_{IMU})s^2 + \left[A_\alpha Z_\delta - (Z_\alpha + M_q)A_\delta + (M_\delta Z_\alpha - M_\alpha Z_\delta)X_{IMU}\right]s + A\alpha M_\delta - A_\delta M_\alpha + (A_\delta Z_\alpha - A_\alpha Z_\delta)M_q}{s^2 - (Z_\alpha + M_q)s + Z_\alpha M_q - M_\alpha} \tag{2}$$

The pole and zero plot of transfer function $G_q(s) = \frac{q}{d}$ at center of gravity and shifted position is identical. It is shown in figure 1. The pole and zero plot of transfer function $G_{a_z}(s) = \frac{a_z}{\delta}$ at center of gravity is shown in figure. The plot when the IMU is shifted to the positive and negative x-axis is shown in figures 3 and 4 respectively.

As we can see from the figures, shifting the IMU to positive x-axis up until 0.4 m, the zeros are getting larger, both on the positive and negative plane. However, if we increase the shift larger than 0.6, the zeros moved to the negative imaginary plane. And if we keep increasing the shift, the zeros move closer to the y-axis of the plane.

In contrary, if we shift the IMU to negative x-axis, the zeros are keep on the real plane, but as we shift further, the zeros move closer to the y-axis by a little bit.

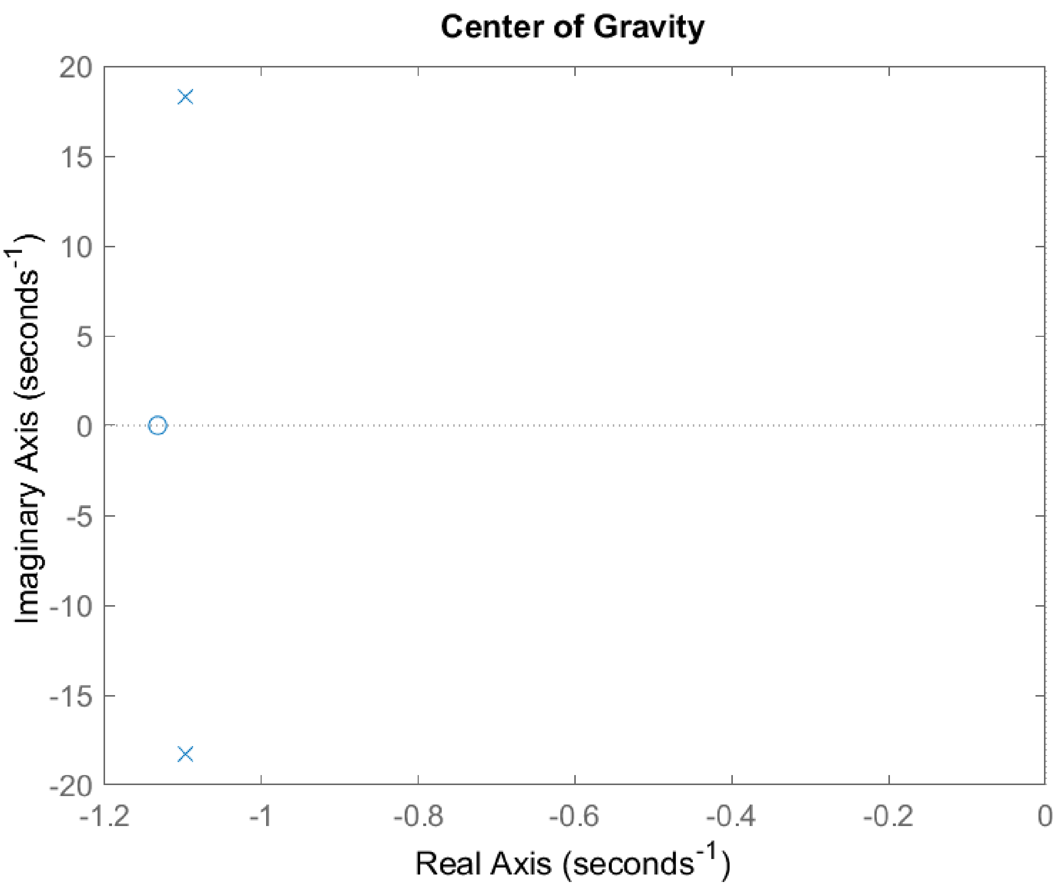


Figure 1: *Zero and pole plot of transfer function* $G_q(s) = \frac{q}{d}$

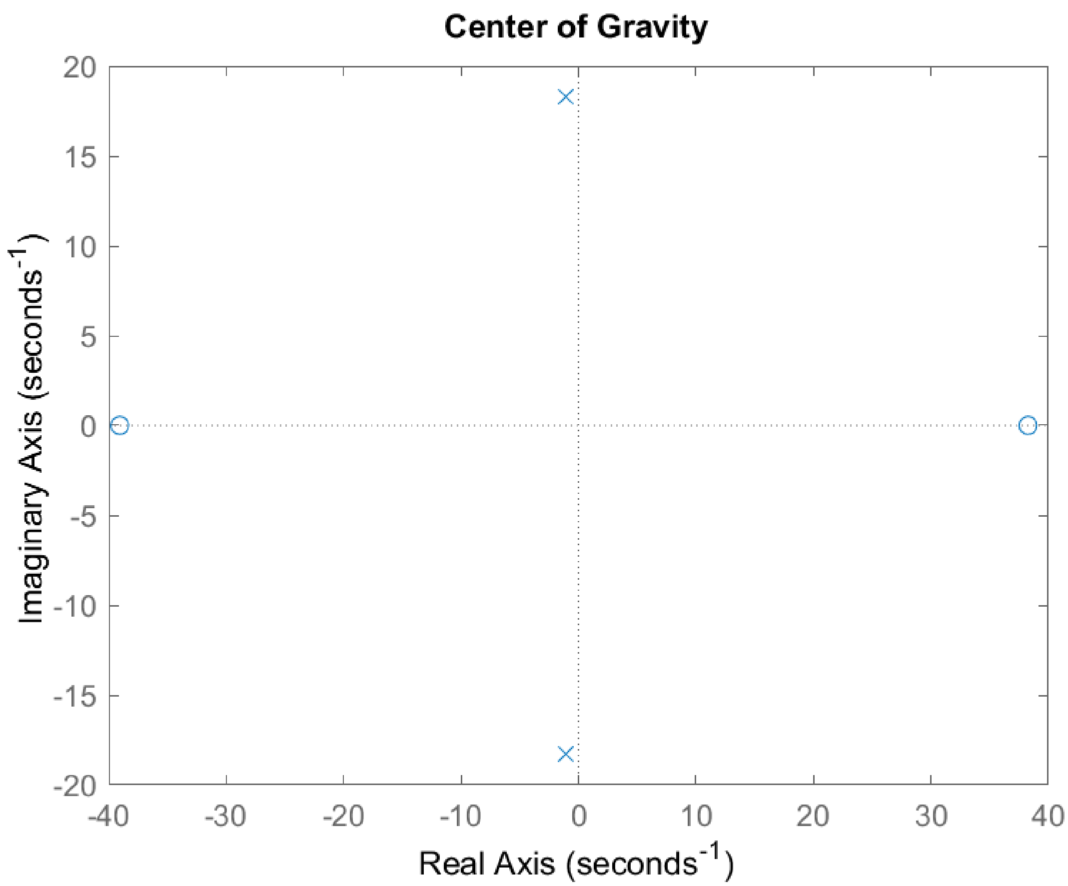
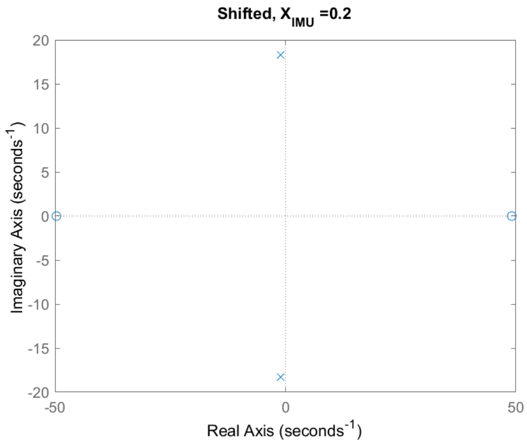
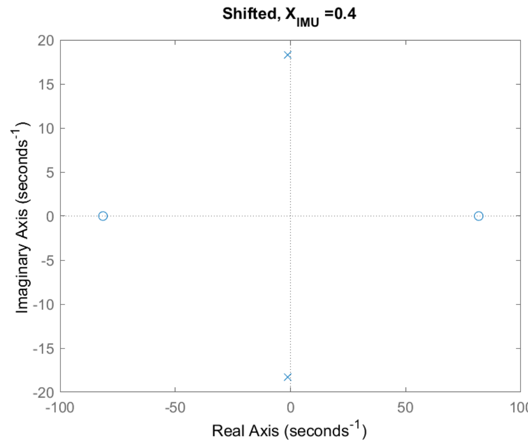


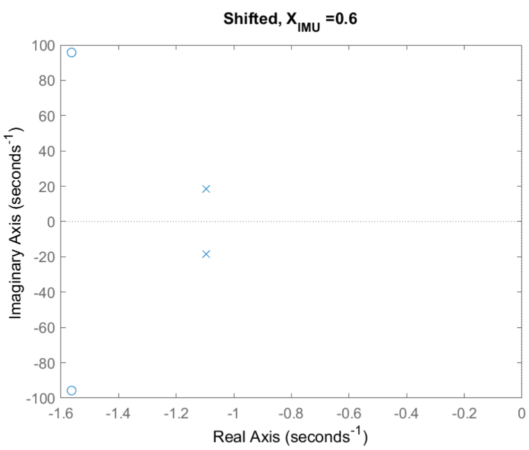
Figure 2: *Zero and pole plot of transfer function* $G_{a_z}(s) = \frac{a_z}{\delta}$ *at center of gravity*



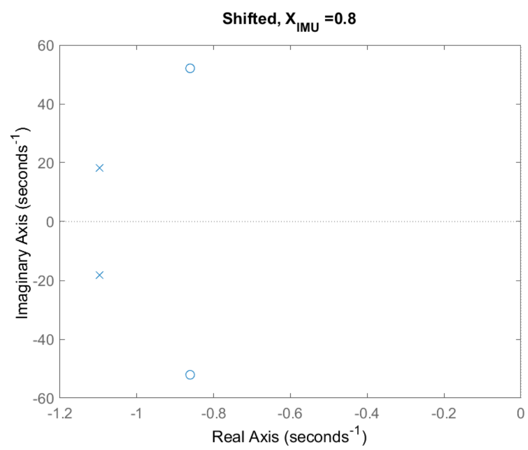
(a) *Shifted by* +0.2



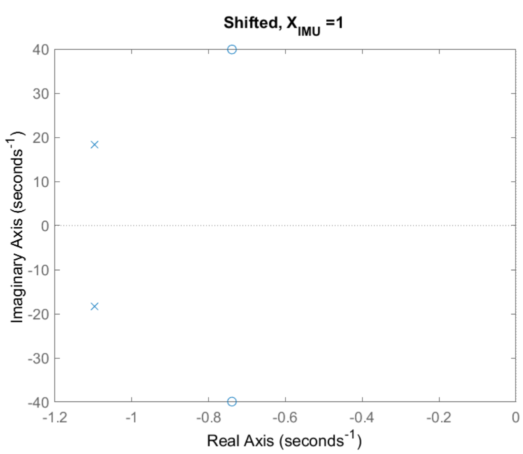
(b) *Shifted by* +0.4



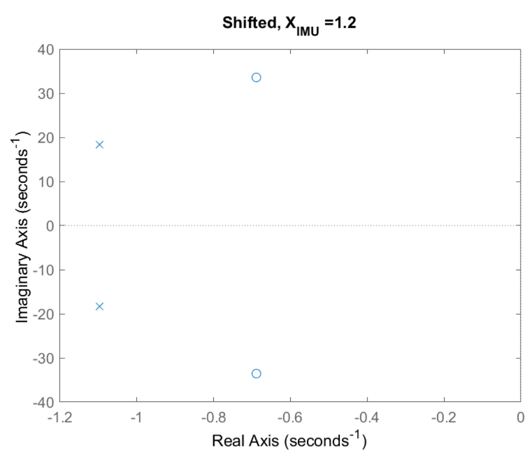
(c) *Shifted by* +0.6



(d) *Shifted by* +0.8

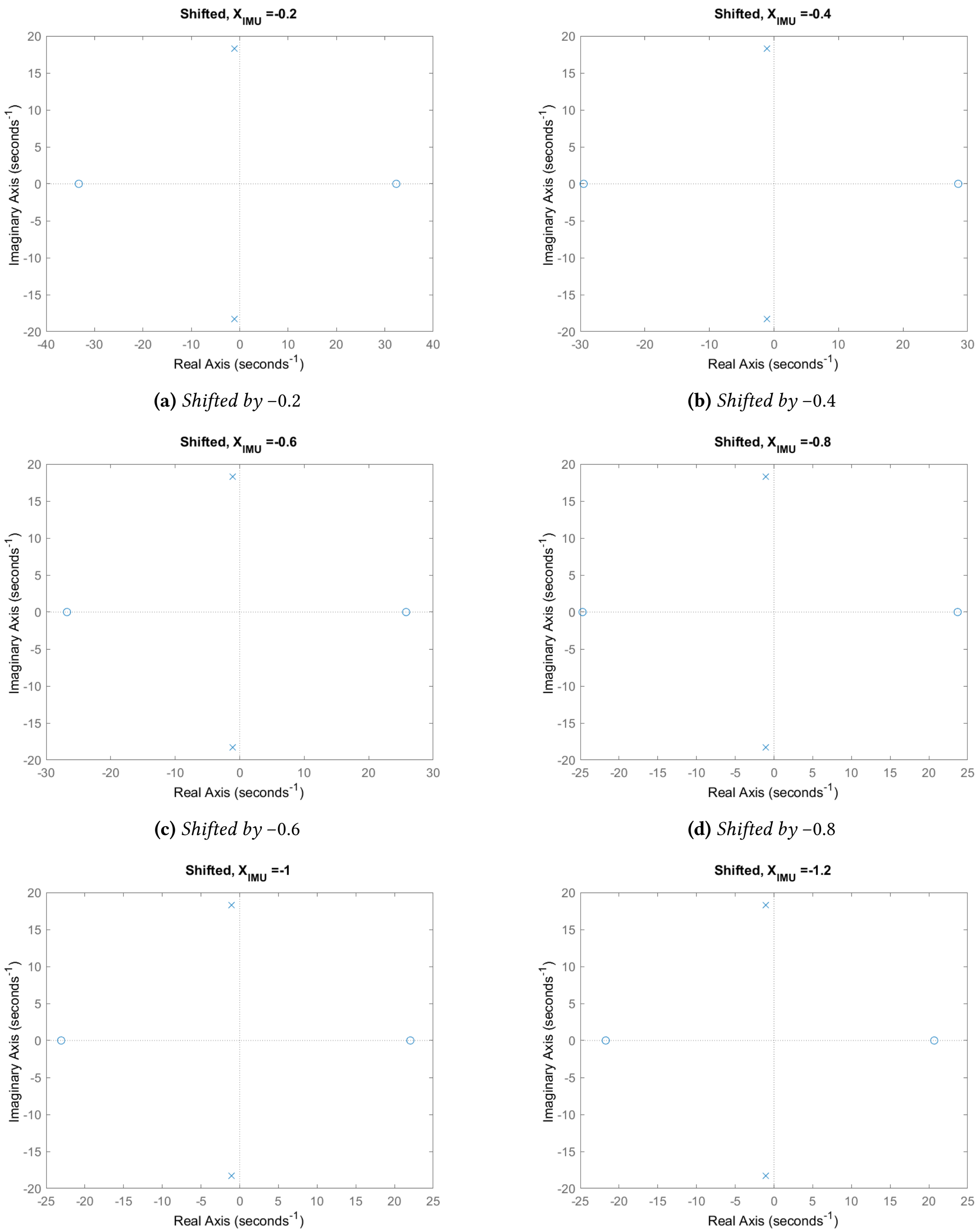


(e) *Shifted by* +1.0



(f) *Shifted by* +1.2

Figure 3: *Zero and pole plot of transfer function* $G_{a_z}(s) = \frac{a_z}{\delta}$ *when IMU is shifted to the positive x-axis*



(a) *Shifted by* -0.2

(b) *Shifted by* -0.4

(c) *Shifted by* -0.6

(d) *Shifted by* -0.8

(e) *Shifted by* -1.0

(f) *Shifted by* -1.2

Figure 4: *Zero and pole plot of transfer function* $G_{a_z}(s) = \frac{a_z}{\delta}$ *when IMU is shifted to the negative x-axis*