Simulation Analysis.

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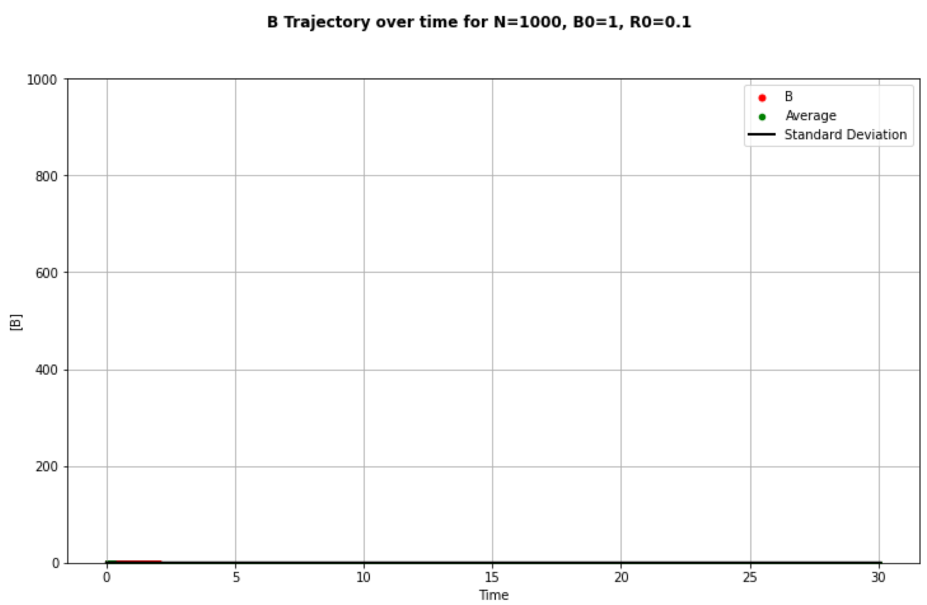
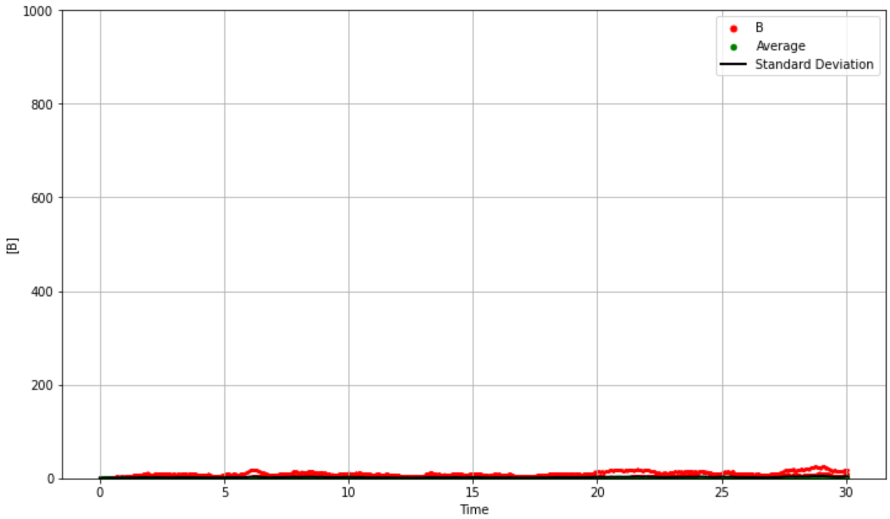
In response to the the $nota\ bene$: Gillespie algorithm uses exponentially distributed times which results in varying lengths for every run of the same parameters. Furthermore, the amount of time signatures will vary and creates a problem with calculating the average and error for B across the realisations. Therefore, I have employed a nearest neighbour technique to produce an approximation of the stochastic realisations. Within this implementation a linear time scale is used for comparison with the stochastic timings. For the closest stochastic time to the time in the linear time array, its respective $[B]$ value is used. Additionally, all the initial graphs have had the average and error imposed upon them to reduce the need for more graphs and ease of comparison.

Analysis.

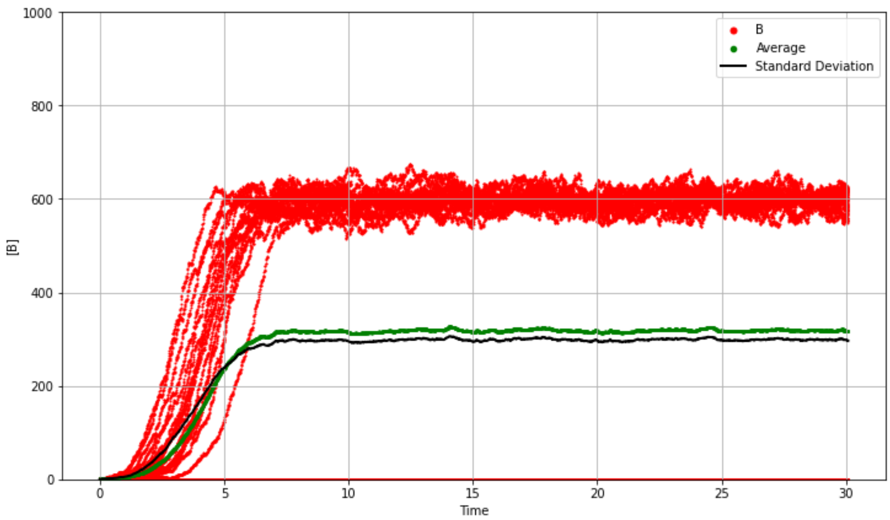
Gillespie for small B_0, in this case, $B_0 = 1$ and 30 with varying R_0 (0.1, 1, 2.5 , 8). In each graph's heading it can be seen the initial conditions. γ is set to 1 throughout these realisations.



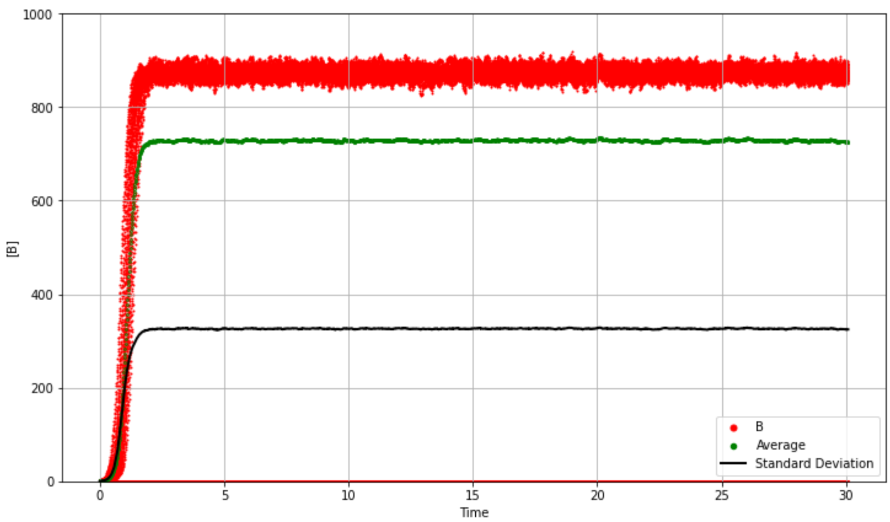
B Trajectory over time for N=1000, B0=1, R0=1.0



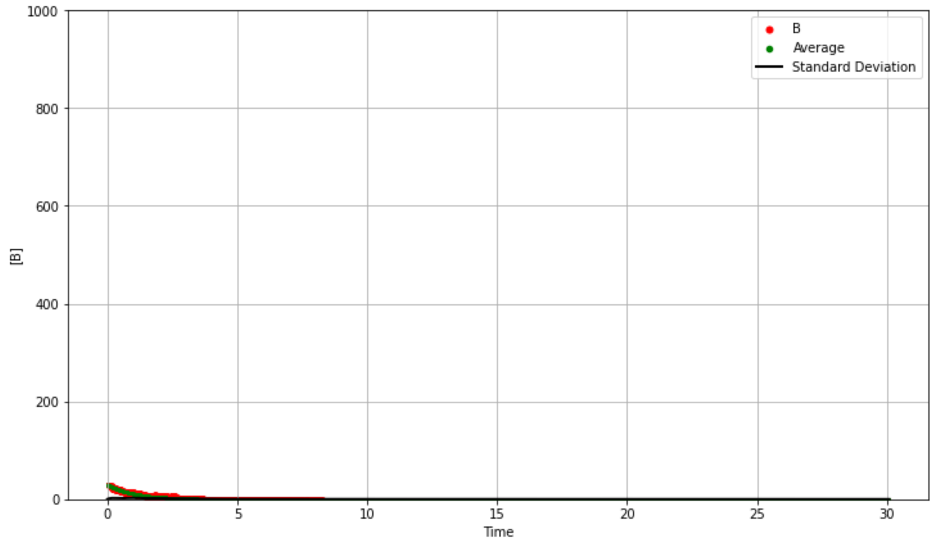
B Trajectory over time for N=1000, B0=1, R0=2.5



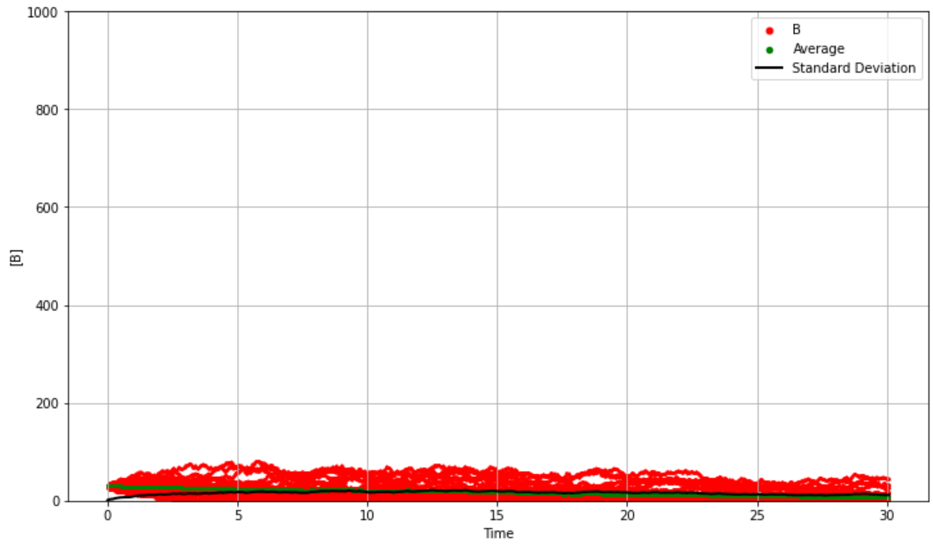
B Trajectory over time for N=1000, B0=1, R0=8.0



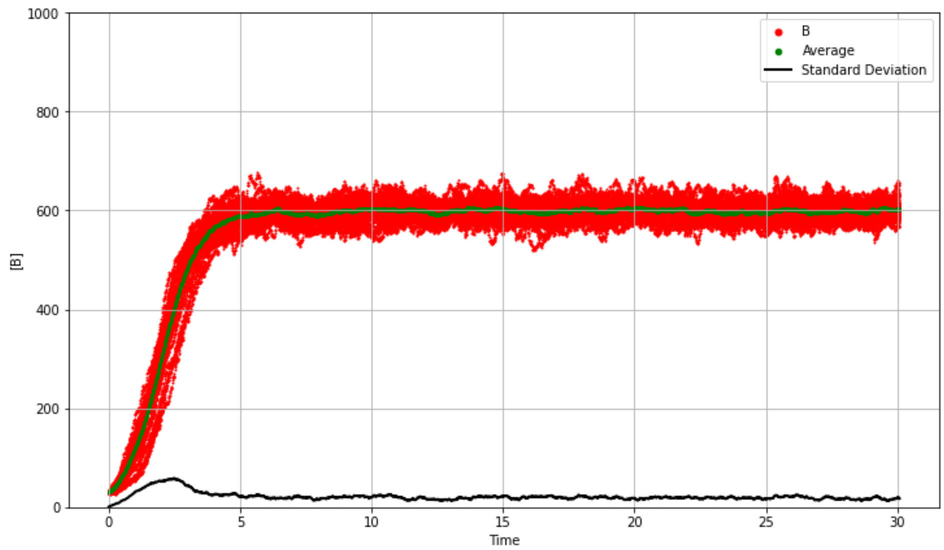
B Trajectory over time for N=1000, B0=30, R0=0.1

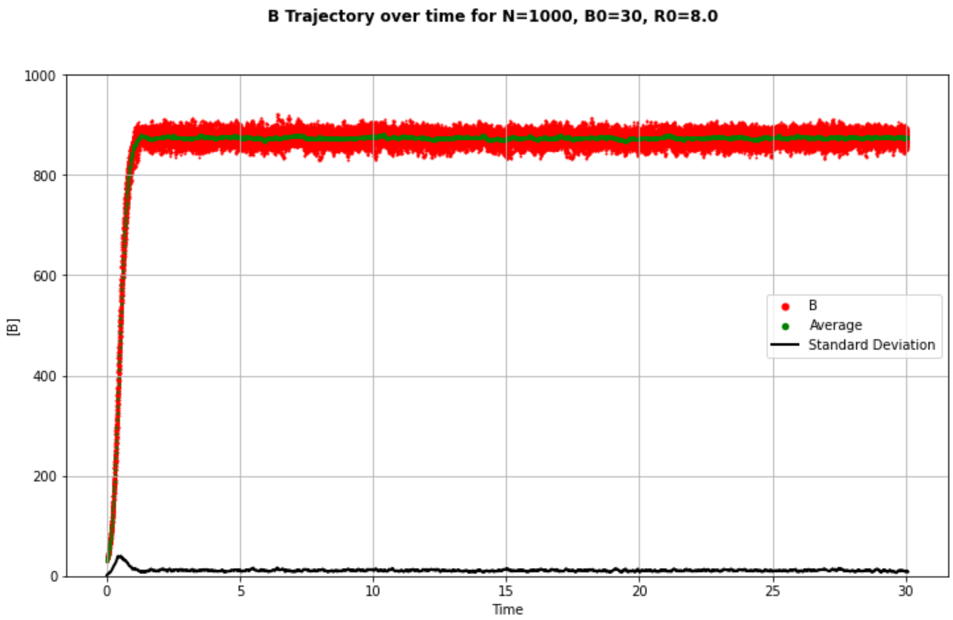


B Trajectory over time for N=1000, B0=30, R0=1.0



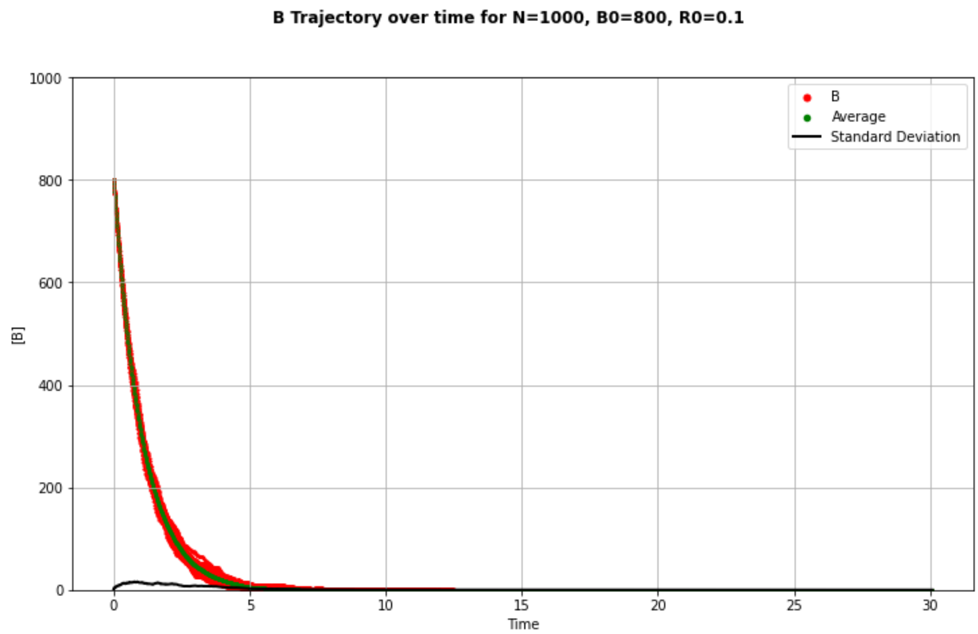
B Trajectory over time for N=1000, B0=30, R0=2.5

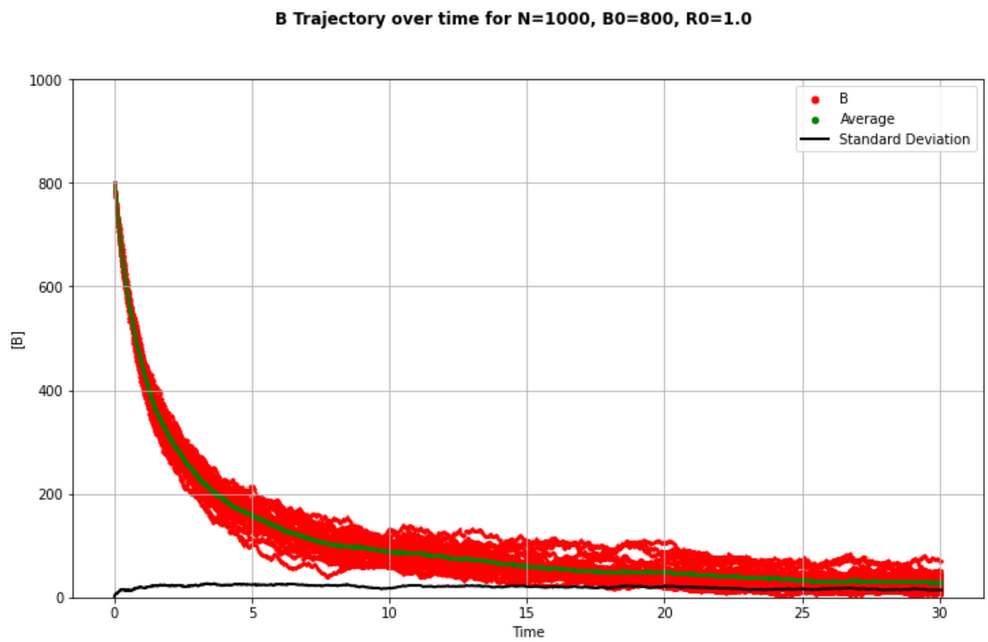




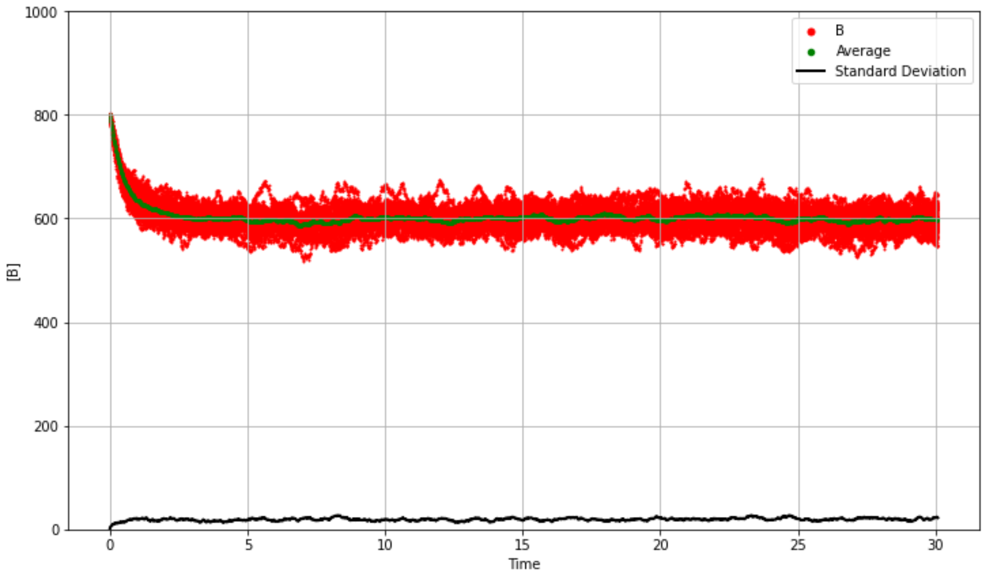
The interesting cases are when B_0 = 1 with $R_0 = 8$ (large), and when the parameter $R_0 = 1$. Firstly, when $R_0 = 1$, this can be regarded as an inflection point because the second derivative is oscillating its sign around zero. In other words, the Jacobian stated that when $\beta > \gamma$ the equilibria for $[B]$ is stable. But if the Gillespie changes the rate and γ becomes greater than β, the equilibria will become unstable. Thus, realisations close to 0 may converge. It is a critical point between the system being unstable or stable. In terms of a large R_0, the realisations stay close to the positive equilibria, but it is shown that a handful of realisations converge to zero. This is due to the nature of the Gillespie algorithm, where the stochasticity of the first iterations can result in the value of $[B]$ transitioning to 0. If this occurs then the rate vector turns to zero and these realisations become trapped.

$B_0 = 800, 999$ (Large), identical R_0s.

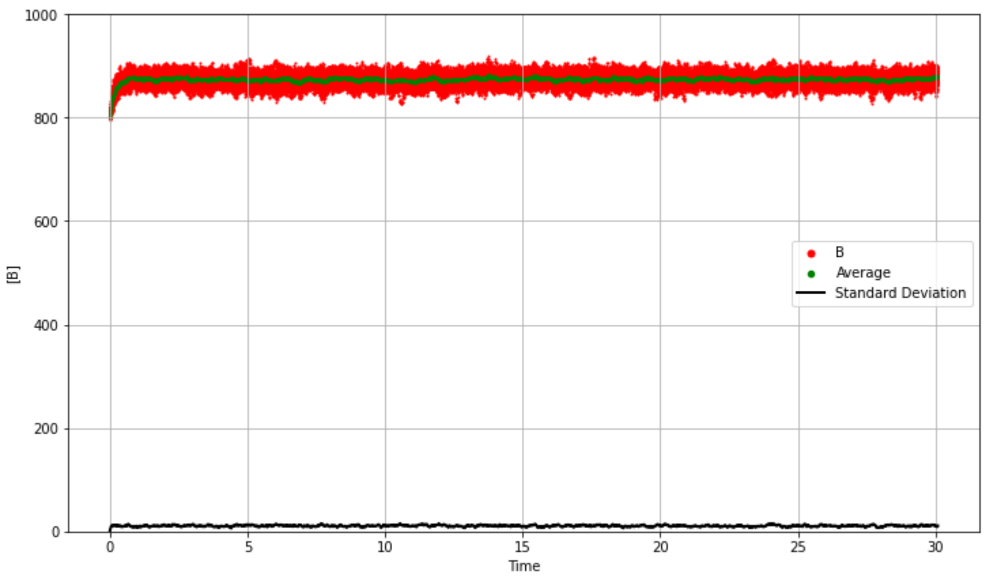




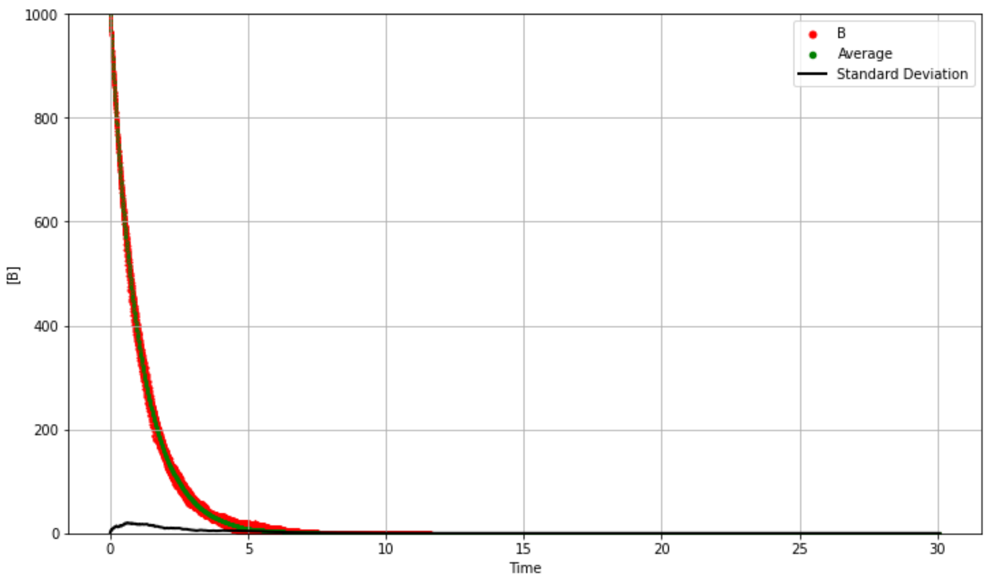
B Trajectory over time for N=1000, B0=800, R0=2.5



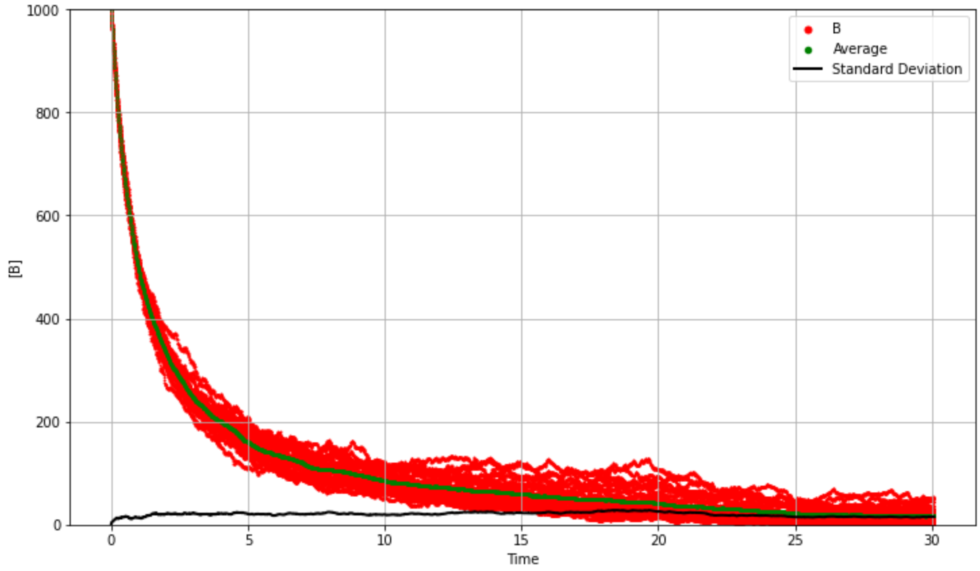
B Trajectory over time for N=1000, B0=800, R0=8.0



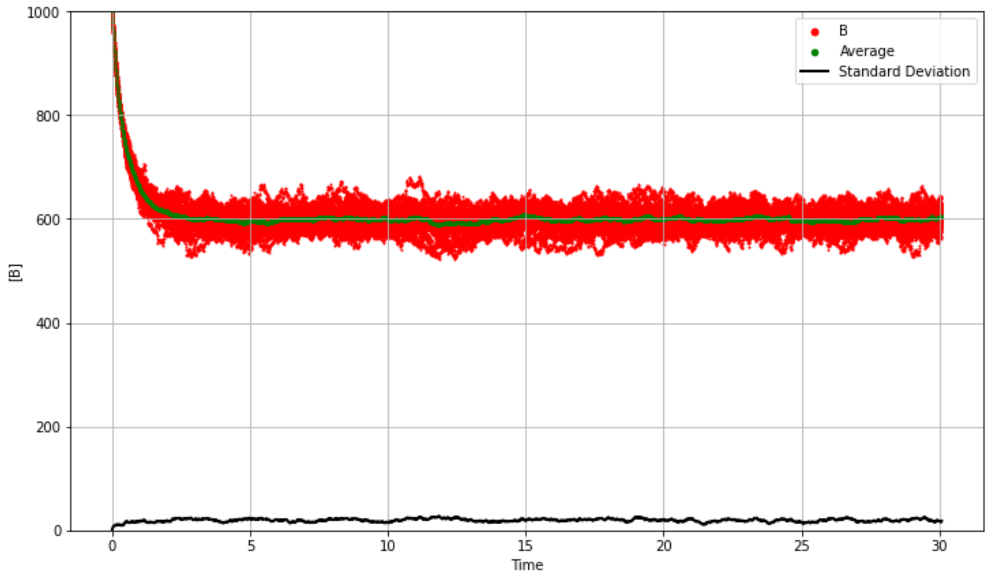
B Trajectory over time for N=1000, B0=999, R0=0.1



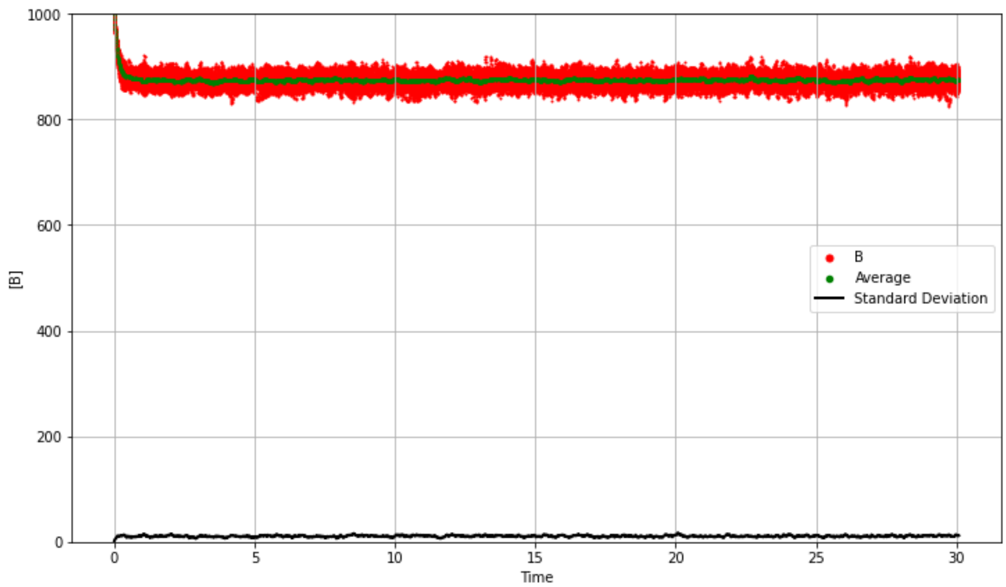
B Trajectory over time for N=1000, B0=999, R0=1.0



B Trajectory over time for N=1000, B0=999, R0=2.5



B Trajectory over time for N=1000, B0=999, R0=8.0

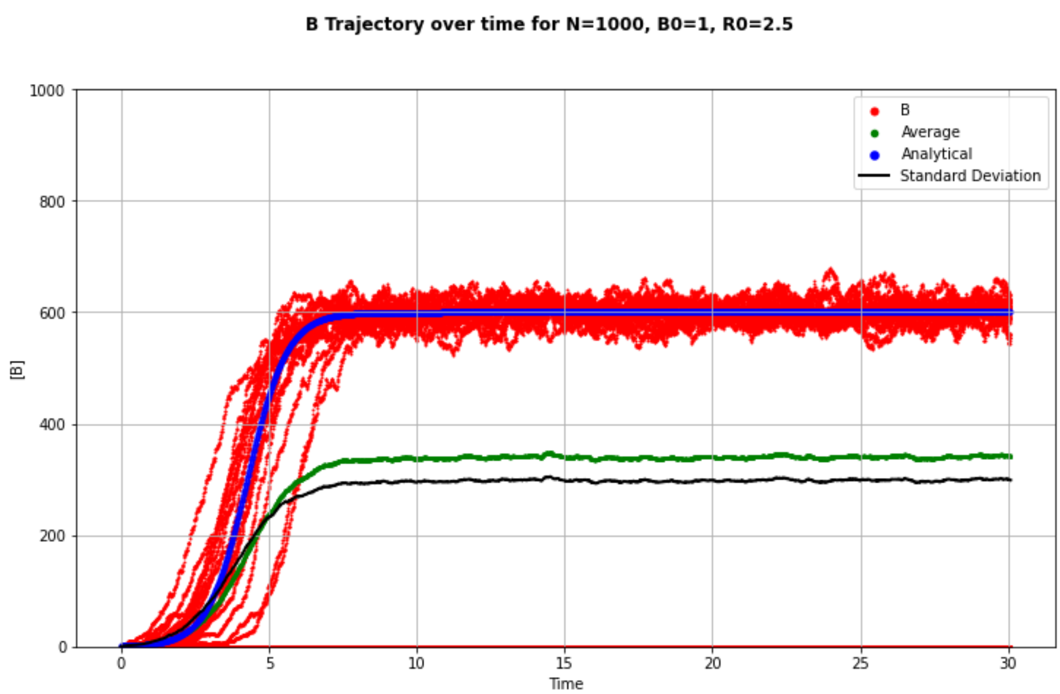


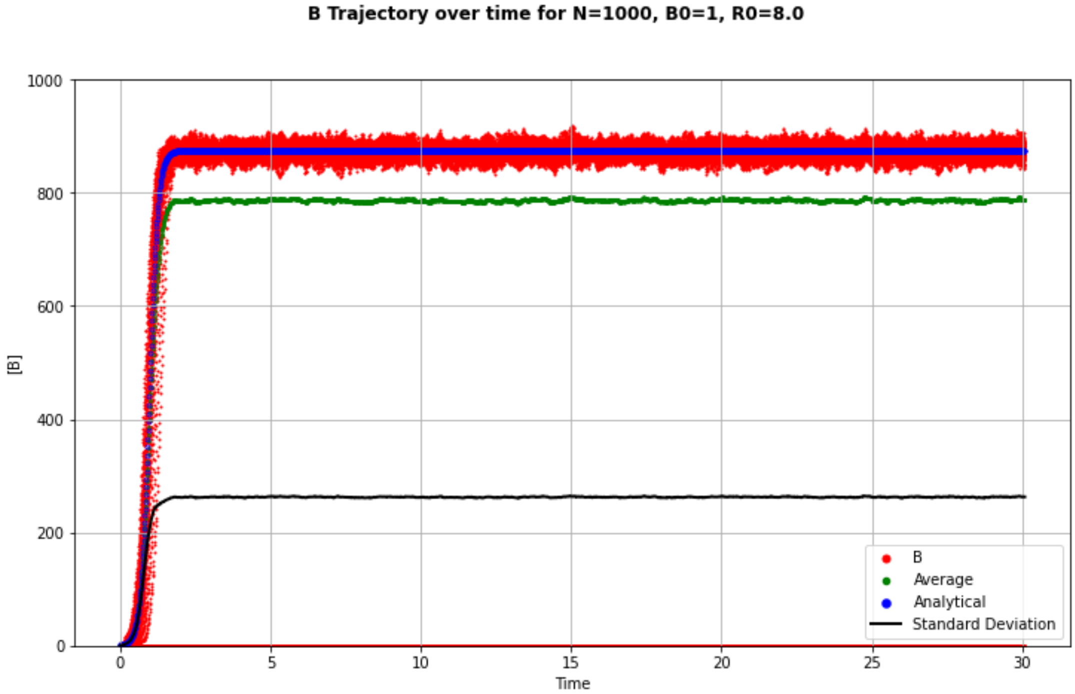
When $R_0 < 1$, the trajectories of the realisations are close to the mean. The same goes for large values of R_0. The comparison between $R_0 = 2.5$ and $R_0 = 8$ showcases this with the difference in realisations pulling further away from the equilibria and resulting in a larger spread. This is because when R_0 approaches 1 the system is approaching a critical point.

Interpreting the results as a collective. When $[B]$ reaches stability faster a lower error is produced than when there is a smoother trajectory to stability. This steepness is determined by the initial parameters, specifically B_0 and β. But as observed in the analytical section, a larger γ in its relation to a fixed R_0, makes reaching asymptotic equilibria faster. These parameters are represented in Gillespie by the rate vector, and are the reason for the stochasticity in these areas. The transitions that take higher values of t to stabilise allow for realisations to deviate away from the mean field.

In regards to larger N, test were undertaken but no significant differences were observed except for longer execution times.

Interpreting the agreement between the average and mean field.

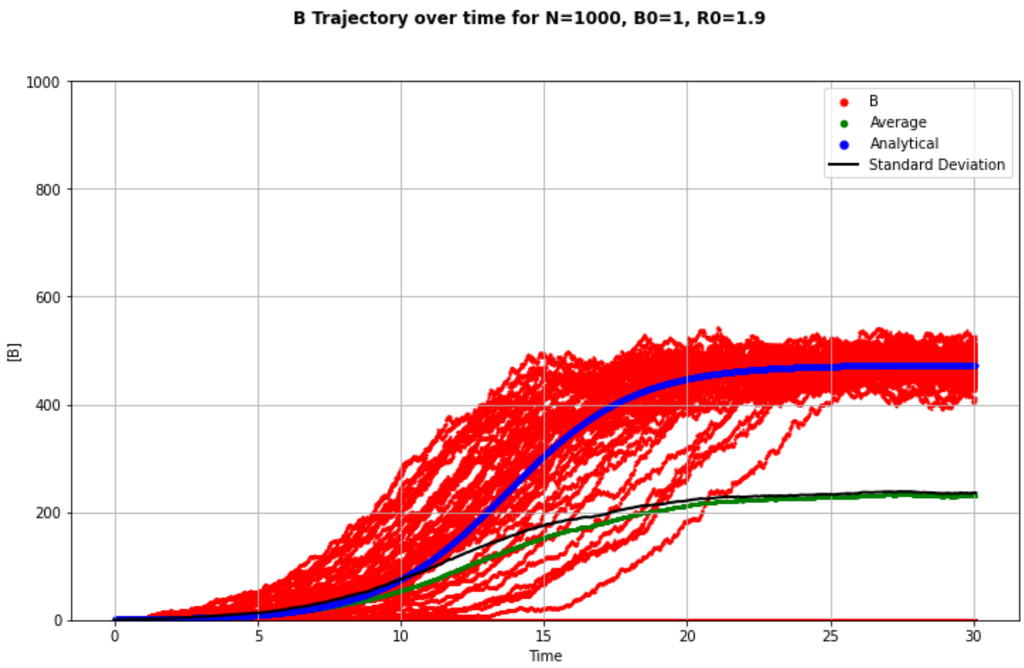


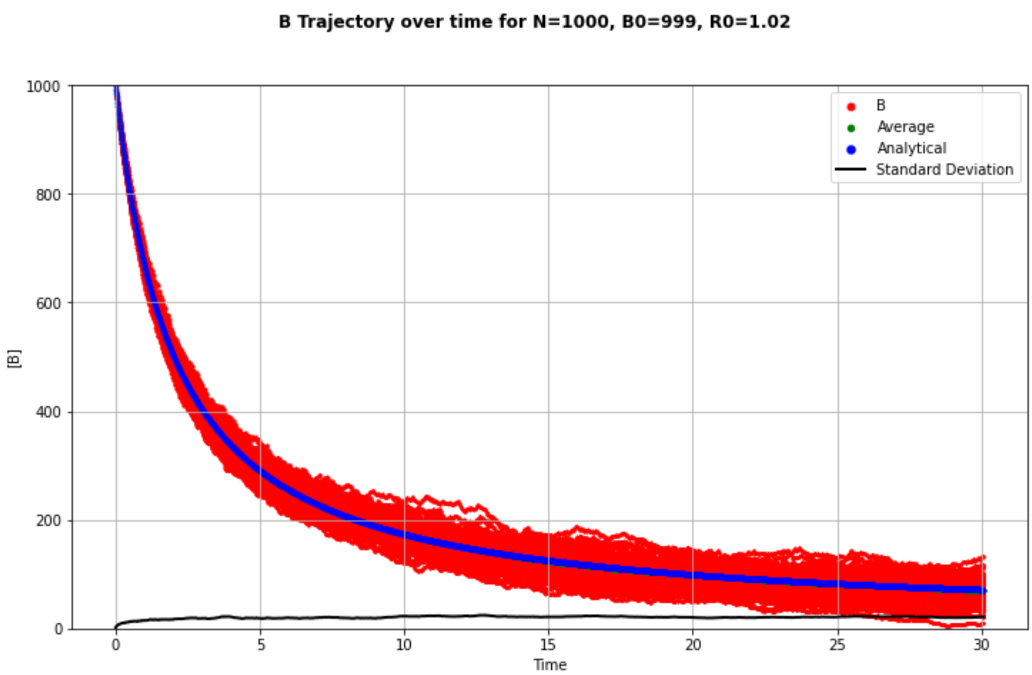


When B_0 is large the average of the stochastic trajectories match with the mean field solution. The poor agreement between the mean field and average begin to occur when B_0 is close to 1. As shown in the bifurcation plot, no population converges to N when $R_0 > 1$. Which is contrary to what is occurring with a sample of the realisations. I believe this agreement is poor because of the previous observation. Where realisations converge to zero and never leave due to the nature of the Gillespie method. This is inevitably going to bring the average of the realisations down, thus increase the standard deviation.

Improving the agreement between the mean field and stochastic average.

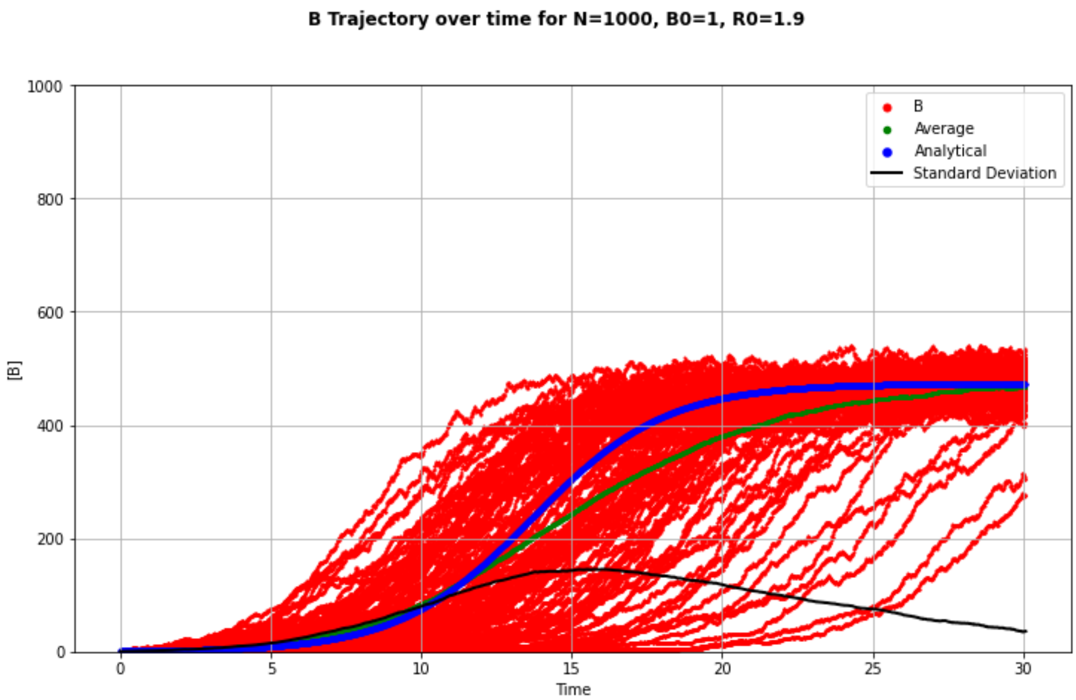
The graphs below consider $\beta = 0.51, \gamma = 0.5, (R_0 = 1.02)$ and $\beta = 0.95, \gamma = 0.5, (R_0 = 1.9)$.

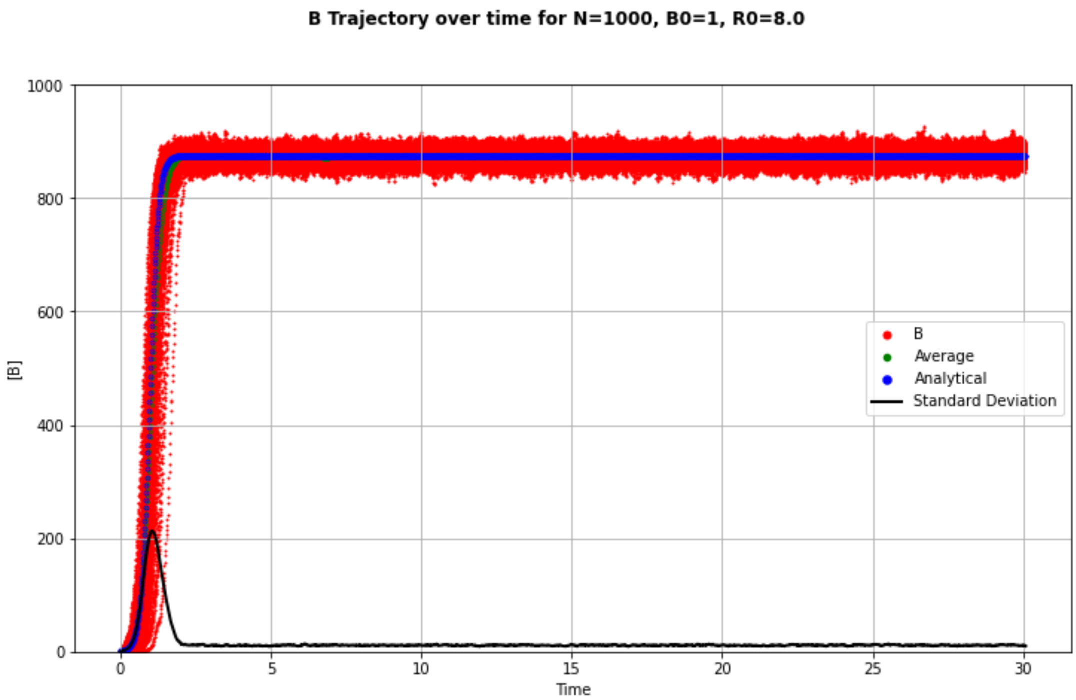




What can be observed specially from the graph above, again, the average falls below the predict mean field when $[B]$ is 1 or close to. The disparity between them is the result of realisations of $[B]$ becoming trapped at zero. In order to improve the agreement for these scenarios, a check for whether $[B]$ is greater than 1 to initiate the change of an individual in state $[B]$ to $[A]$.

Results of implementation





The improvement is noticeable, the above graph having matched the average with mean field. In terms of the conditions put forward for consideration. As time increases, the realisations are drawn to the mean field as demonstrated by the average nearly matching when t approaches 25. An additional condition was implemented to allow the transition of an individual from $[B]$ to $[A]$ when $R <= 1$. Convergence to zero is within the scope of those parameters. A simple check to see if $\beta < \gamma$ allows for the system to behave as previous. For ease of comparison, the implementation of this method is located can be found in gillespie_ABA_stable.

Critical Thinking

Applications:

Disease outbreak/Pandemic.

The states $[A]$ and $[B]$ could be denoted respectively as non-infected or infected. In this example, β would represent how contagious the disease being monitored is and γ the recoverer rate. The equilibria for this model would represent the impact of the infection on the population overtime. In other words, what percentage of the population is going to become infected given the severity of the disease. A few assumptions have to be made in order for this system to be implemented. Being, there is no lasting immunity once an individual has recovered, all individuals are in contact and the non infected are all susceptible to catching the disease no matter how healthy their individual lifestyles are. The latter assumption is partially tricky because the definition of an optimum healthy lifestyle will vary. A lot of factors that could potentially be hard to model. For example, the mental health of an individual, recent studies have shown that stress is a huge indicator of how well the immune system is functioning.

Improving the model.

A connection matrix would allow for the system to take into account the probability of a person being in contact with an infectious person or a healthy one. This connection matrix could potentially involve further parameters to effect the probability and rate of change of a population into either state. We will consider a weight that introduces the each member of the population and their respective locations to every other member of the population. A range from 0 to 1 for example, with 1 representing being in contact and 0 being non. The weighted matrix in terms of the mean field equation, it could be represented by the constant ω, which could be placed next to β to effect the parameters determining the change of one state to another. Demonstrated below:

$$[\dot{B}] = [B](\omega\beta - \frac{\beta[B]}{N} - \gamma)$$