

INTERNET APPENDIX

Fix the Price or Price the Fix? Resolving the Sequencing Puzzle in Corporate Acquisitions

Joshua D. Higbee* Matthew Jennejohn† Cree Jones‡ Eric Talley§

IA1 Stochastic Nash bartering

Traditional Nash bargaining assumes a convex choice set, though other work weakens this assumption (Herrero 1989; Zhou 1997; Serrano and Shimomura 1998). In our setting, we have a discrete set of possible terms that can be chosen. We wish to represent the term bartering game as an instant decision that relies on firm bargaining power in the same way as prices are set via Nash bargaining. We illustrate how the stochastic Nash bargaining solution is in expectation equal to the classic Nash bargaining solution over the convex hull of *ex ante* expected payoffs from possible choices of contract terms.

The stochastic Nash bartering solution we propose also coincides with the Nash bargaining solution on an *ex ante* convexification of the expected payoff set. We define the expected payoff set $PS_E = \{\mathbb{E}[v(m_j; \epsilon_j)] \mid m_j \in \mathcal{M}\} \cup \{\mathbb{E}[v(m_j; \epsilon_j)] \mid NP_j \geq NP_k \forall k \neq j\}$. That is, PS_E is the set of achievable expected payoffs from any individual term and the expected payoff from selecting the term with the highest realized Nash product (under whichever game is being considered). Let PS_∞ be the convex hull of the expected payoff set PS_E . Consider any randomization protocol on PS_E , so any feasible payoff in the convex hull PS_∞ can be represented by the lottery on PS_E . Then the solution of the Nash bargaining program over the convex set PS_∞ is equal to the solution of choosing

*The Ohio State University. Corresponding author. Email: higbee.25@osu.edu.

†Johns Hopkins University Applied Physics Laboratory; Brigham Young University. Email: jennejohnm@law.byu.edu.

‡Brigham Young University Law School. Email: creejones@law.byu.edu.

§Columbia Law School. Email: et2520@columbia.edu.

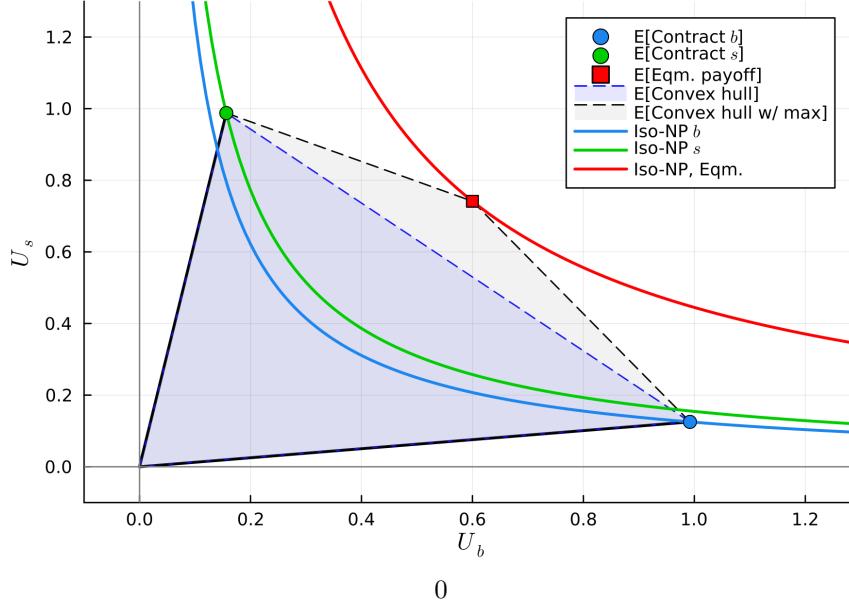
$\mathbb{E}[v(m_j; \epsilon_j) \mid NP_j \geq NP_k \forall k \neq j]$ from the restricted set PS_E .¹ The difference $\mathbb{E}[v(m_j; \epsilon_j) \mid NP_j \geq NP_k \forall k \neq j] - \max_{m_j \in \mathcal{M}} \{\mathbb{E}[v(m_j; \epsilon_j)]\}$ is strictly positive when there are at least two non-default terms in \mathcal{M} , and it represents the expected option value from having multiple potential contracts to choose from.

Figure IA1.1 illustrates the expected convex hull PS_∞ of the set of expected payoffs PS_E , represented by the grey and blue shaded areas. The blue shaded area represents the convex hull only over the proposed contracts m_0 , m_b , and m_s (i.e., $PS_E \setminus \{\mathbb{E}[v(m_j; \epsilon_j) \mid NP_j \geq NP_k \forall k \neq j]\}$). If the firms commit *ex ante* to select the term with highest Nash product once all uncertainty is resolved, the expected payoff exceeds that of any other contract selection rules in the convex hull.

1. In other words, any distinct randomization protocol over the terms in the set PS_E (that places positive probability on any term in \mathcal{M} , regardless of the realization of ϵ) will in expectation do worse (in the Nash program sense) than choosing whichever term has the highest Nash product after ϵ is drawn.

In the simple case where there are only 2 terms other than the default term (and terms cannot be added together), the expected payoff is also a weighted average of the expected payoffs from each term conditional on that term being selected; see Figure IA4.1(a) for an illustration.

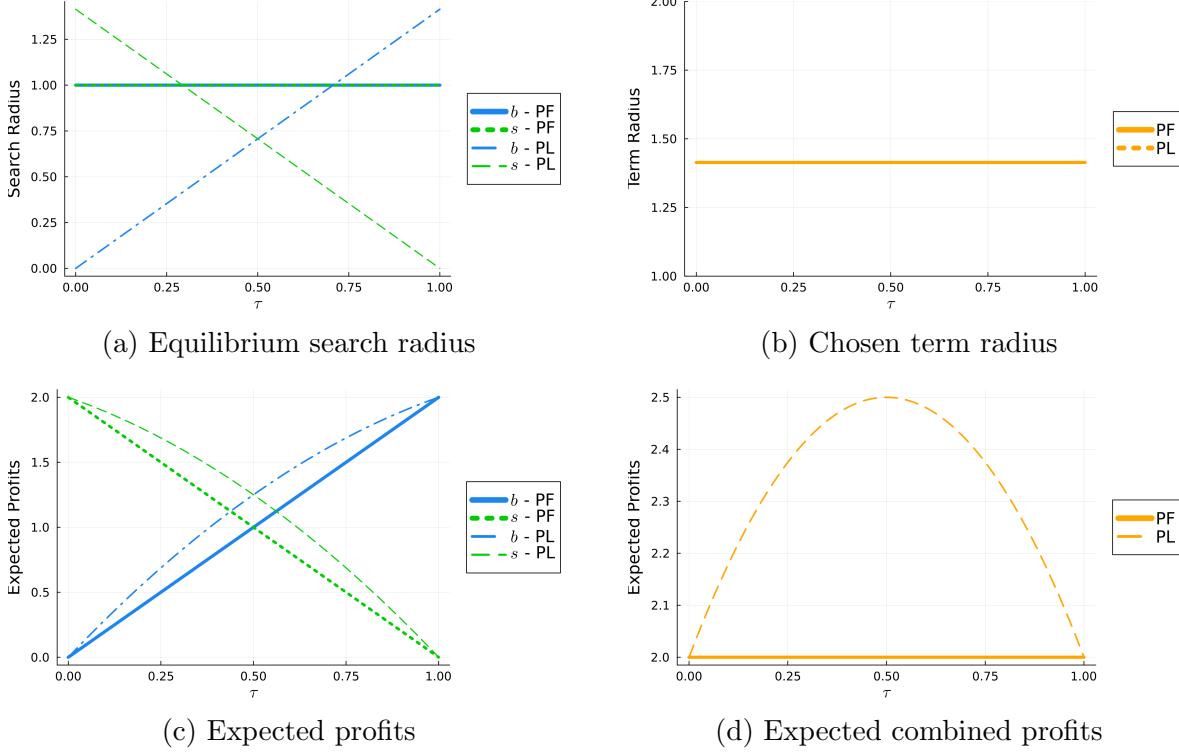
Figure IA1.1: Firm payoffs in the bartering stage of the price-first game (unrestricted model)



Notes: The plots assume $r_b = 1$, $r_s = 1.25$, $a_b = -0.19$, $a_s = 0.22$, $\alpha = 4$, and $\tau = 0.4$. None of these values necessarily represent equilibrium actions. For clarity in illustrating the expected convex hull, we present a modified case where the combined term is not considered by the firms.

IA2 Additional comparative statics

We now present additional comparative statics related to the model specifications in the main text. Figure IA2.1 presents comparative statics with respect to τ where the search angle cost parameter is $\gamma_a = 0$. In this case, it is not any more costly to search along the surplus-maximizing direction than in a purely self-serving manner, so the price-last game yields higher combined profits as well as individual profits for each party. While this case aligns with conventional intuitions about the value of leaving price as a flexible mechanism, we expect that it takes more effort to jointly maximize two objectives (the surplus of both the buyer and the seller) than to focus on one objective alone, and generally assume $\gamma_a > 0$.

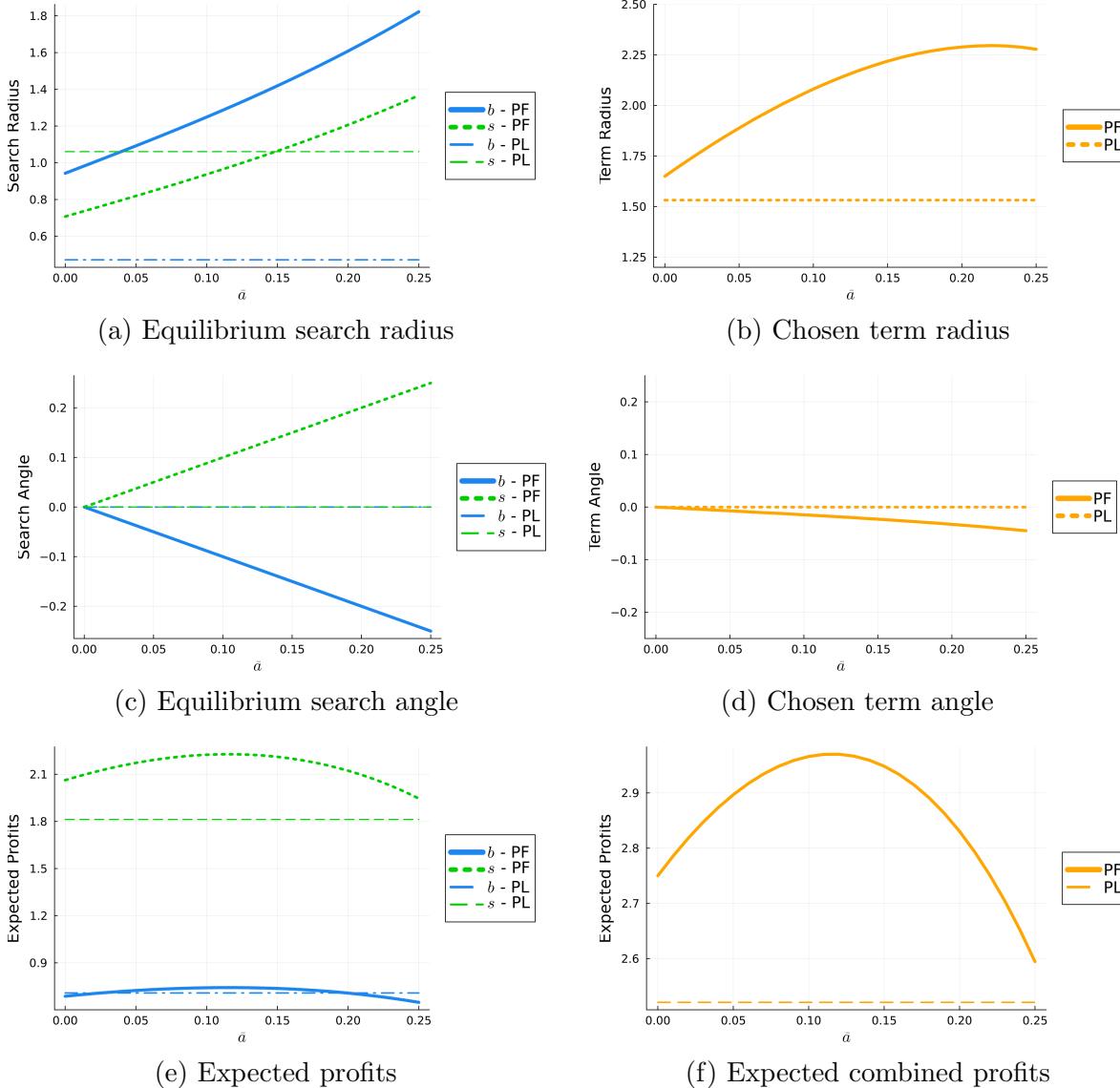
Figure IA2.1: Comparative statics with respect to τ (endogenous angle search)

Notes: The panels on the left-hand side plot the variable of interest for both firms b and s in the price-first (“PF”) and price-first (“PL”) games. The panels on the right-hand side depict the corresponding outcomes of the contract in both games, for the combined firms. The plots assume $\alpha = 2$, $\gamma_b = \gamma_s = 1$, $\gamma_a = 0$, $\pi_b = 2$, and $\pi_s = 1$. Comparative statics with respect to the search and term angles are omitted, as they are constant for all values of τ .

Figure IA2.2 next presents comparative statics with respect to the search angle constraint \bar{a} . Since γ_a is sufficiently low, the price-last game does not incentivize firms to do anything other than search along the 45-degree line. This contrasts with the price-first game, which as shown in Proposition 2(i) incentivizes firms to search at their boundary (either $-\bar{a}$ for firm b or \bar{a} for firm s). This implies that both the resulting term radius and the expected combined profits are non-monotonic in \bar{a} . Despite the two firms monotonically increasing their search efforts in panel (a), their increasingly selfish search implies that this yields less total innovation than for smaller values of \bar{a} . Interestingly, panel (f) shows that the joint profit-maximizing constraint for the price-first game is approximately $\bar{a} = 0.12$. While full

contractibility on the firms' search angles may not be possible, some partial restrictions may in fact make the price-first game generate more surplus.

Figure IA2.2: Comparative statics with respect to \bar{a} (endogenous angle search)



Notes: The panels on the left-hand side plot the variable of interest for both firms b and s in the price-first (“PF”) and price-last (“PL”) games. The panels on the right-hand side depict the corresponding outcomes of the contract in both games, for the combined firms. The plots assume $\tau = 0.25$, $\gamma_b = \gamma_s = 1$, $\gamma_a = 5$, $\pi_b = 2$, and $\pi_s = 1$.

IA3 Equilibrium contract under independent productivity shocks

We now briefly examine another specification of the model, which allows for endogenous search angles and radius under various alternative assumptions. Unlike in the previous section, firms can now search for any type of contract term—even those that may be actively harmful to their counterpart. We also allow for term-specific productivity shocks to vary across terms, implying that any individual firm’s proposed contract term may be preferred to the combined contract term in a specific setting. Thus, firms have full flexibility in using their search decisions to determine the expected payoff from their proposed term, as well as the probability it is selected. The central question is now how timing affects the contract creation process in the absence of any restrictions on firms’ search process.

In order to understand this question, we continue to make some simplifying assumptions for tractability. Instead of assuming the shocks ϵ_j are perfectly correlated, we instead assume they are independent and make a functional form assumption that yields closed-form choice probabilities. This means there is an option value to variety even if the terms have the same expected value for both parties. Thus, firms’ investment decisions are shaped by the knowledge that their own term may be chosen instead of the combined term, since the combined term may be ill-suited for a deal relative to either of the simpler individual contract terms.

Modifying these assumptions shows how firms’ incentives differ when their individual terms may be chosen. Since productivity shocks are independent, and firms have a nonzero chance of only their term being chosen in the bartering process, they offer more neutral terms (lower $|a_i|$) in the price-first game, and they increase their search radii when they have more bargaining power. Firms behave similarly in the price-last game as under the previous assumptions, generally searching in the surplus-maximizing direction because all surplus will be redistributed later and a larger “pie” is beneficial. We study this setting in detail now.

IA3.1 Alternative assumptions and implications for the expected contract

We replace assumptions **A1** and **A2** with the following two assumptions:

- B1** The direction of search may be any angle within the entire unit circle, i.e. $|a_i| \leq 1.0$.
- B2** ϵ_j is i.i.d. Frechet (inverse Weibull) with shape parameter $\alpha > 1$ and scale parameter $\sigma = \Gamma(1 - 1/\alpha)^{-1}$, implying $\mathbb{E}[\epsilon_j] = 1$ for all j .

The first assumption expands the set of possible contract terms to include those that may be value-destroying for one of the two parties.² The second assumption helps achieve tractability by yielding functional forms for conditional choice probabilities and conditional expected values, as in Eaton and Kortum (2002) and more broadly in the empirical discrete choice literature (see e.g. Berry and Haile 2021).³

Assumption **B2** implies several closed forms for important quantities in both the price first and price last game. Recall that in the bartering for terms stage of game G , firms choose whichever term of $\mathcal{M}^* = \{m_s^*, m_b^*, m_{bs}^*\}$ yields the greatest Nash product:

$$m_G^* = \operatorname{argmax}_{m_j \in \mathcal{M}^*} NP_{j,G}$$

where $NP_{j,PF} = \delta_{j,PF} \cdot \epsilon_j$ and $\delta_{j,PF} = v_b(m_j)^\tau \cdot v_s(m_j)^{1-\tau}$, while $NP_{j,PL} = \delta_{j,PL} \cdot \epsilon_j$ and $\delta_{j,PL} = v_b(m_j) + v_s(m_j)$.

Then the equilibrium choice probability for any term j is

$$\lambda_{j,G}^* \equiv \mathbb{P}[j \text{ is chosen in game } G] = \frac{(\delta_{j,G}^*)^\alpha}{\sum_k (\delta_{k,G}^*)^\alpha}$$

Thus, any term that offers strictly positive surplus to both firms in the price-first game has a strictly positive probability of being selected through Nash bartering. Using the expectation

2. These bounds are only imposed to avoid coterminal angles, i.e. those that differ by some multiple of 2π .

3. Importantly, $\epsilon_{bs} \perp \epsilon_b, \epsilon_s$. While this is a strong assumption, it will be helpful in both theoretical and empirical applications; such benefits will be highlighted below. More complex correlation structures may also be helpful; see e.g. the nested Frechet model in Lashkaripour and Lugovskyy 2023 as well as the larger literature using variations of the nested logit model for demand estimation. We use the extreme case of full independence in contrast with the other extreme, perfect correlation, which we consider above.

of the maximum of Frechet random variables,⁴ we have

$$\mathbb{E}[\epsilon_{j,G}^*] = \frac{1}{\delta_{j,G}^*} \cdot \left(\sum_k (\delta_{k,G}^*)^\alpha \right)^{1/\alpha} = (\lambda_{j,G}^*)^{-1/\alpha}$$

This expression illustrates the option value of choosing between multiple contracts - even though all ϵ_j have mean 1, conditioning on that term being chosen means the Nash product (and the associated payoffs) will be higher on average than the unconditional average payoffs. Finally, by the law of total expectation, the expected value of firm payoffs from the term stage, $U_{i,G}$, is written as

$$U_{i,G}^* = \sum_{j \in \mathcal{M}} v_i(m_{j,G}^*) \cdot (\lambda_{j,G}^*)^{1-1/\alpha}$$

That is, the expected value from each individual contract is weighted by the probability it is chosen and its expected value conditional on being chosen in game G .

IA3.2 Comparative statics

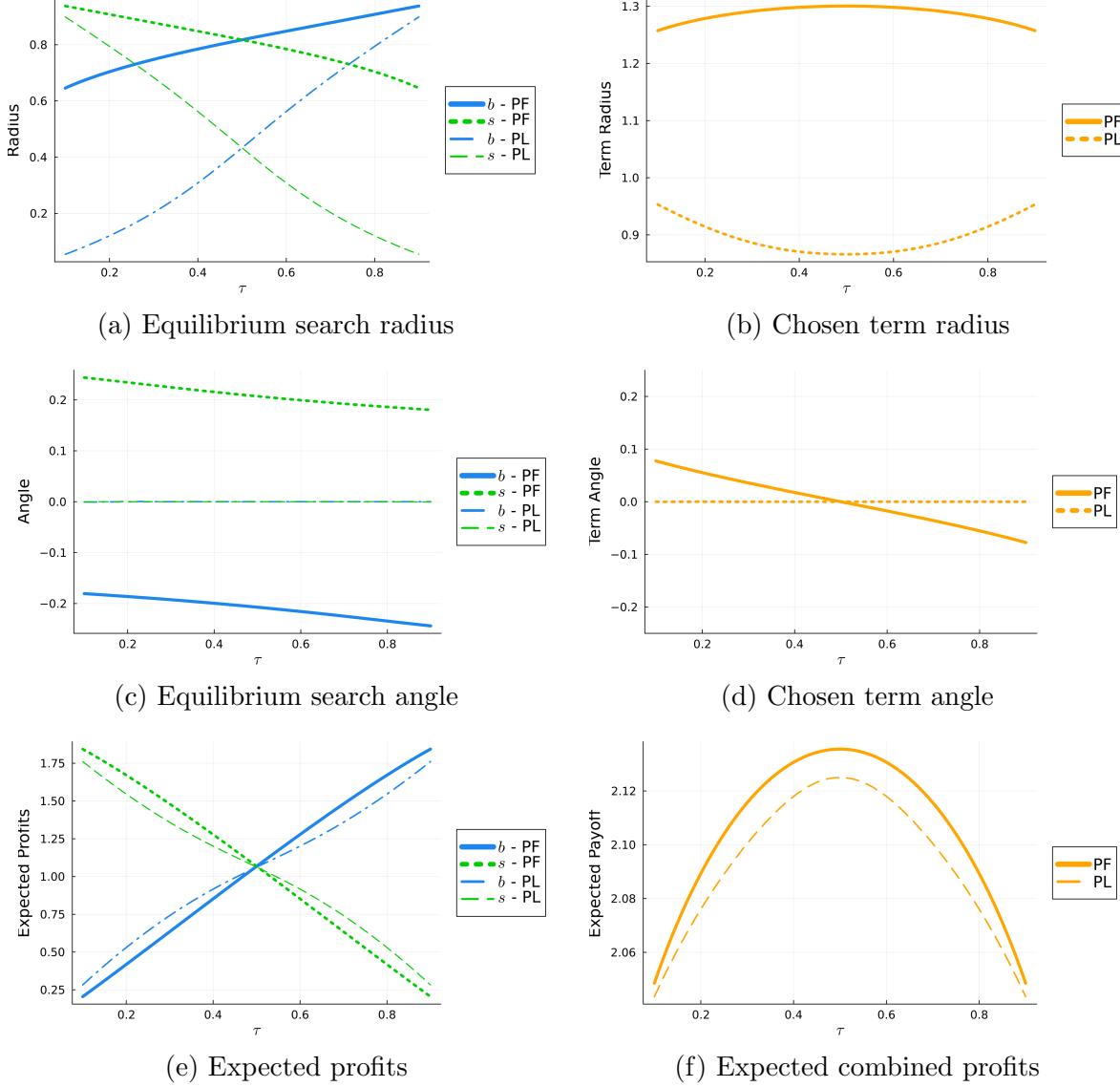
Figure IA3.1 plots the outcomes for the contracting process for different values of the bar-gaining weight τ . In this case we see that both games induce greater search efforts by the stronger bargainer, as represented by the larger search radius. Note that relative to the baseline model under endogenous angle choice, the search angles in the price-first game are shifted toward the center, particularly for the firm with less bargaining power. This is because the bartering process rewards terms that offer a higher Nash product, particularly by placing non-zero probability on choosing any individual term proposed by a single firm. However, the firms in the price-last game still search at $a_i = 0$ since it is surplus-maximizing.

As with the endogenous search angle game, the stronger bargainer generally prefers the price-last game. However, the combination of the firms' angle and radius choices in this setting imply that the expected joint profits are nearly equal across the two games (with the price-first game generating slightly higher surplus). This results in greater investment in the joint term in the price-first game than for the price-last game, particularly for intermediate

4. In particular, $\mathbb{E}[\epsilon_{j,G}^*] = \frac{1}{\delta_{j,G}^*} \mathbb{E}[NP_{j,G}] \mid j \text{ is chosen in } G]$ and under **B2** it holds that $\mathbb{E}[NP_{j,G} \mid j \text{ is chosen in } G] = (\sum_k (\delta_{k,G}^*)^\alpha)^{1/\alpha}$.

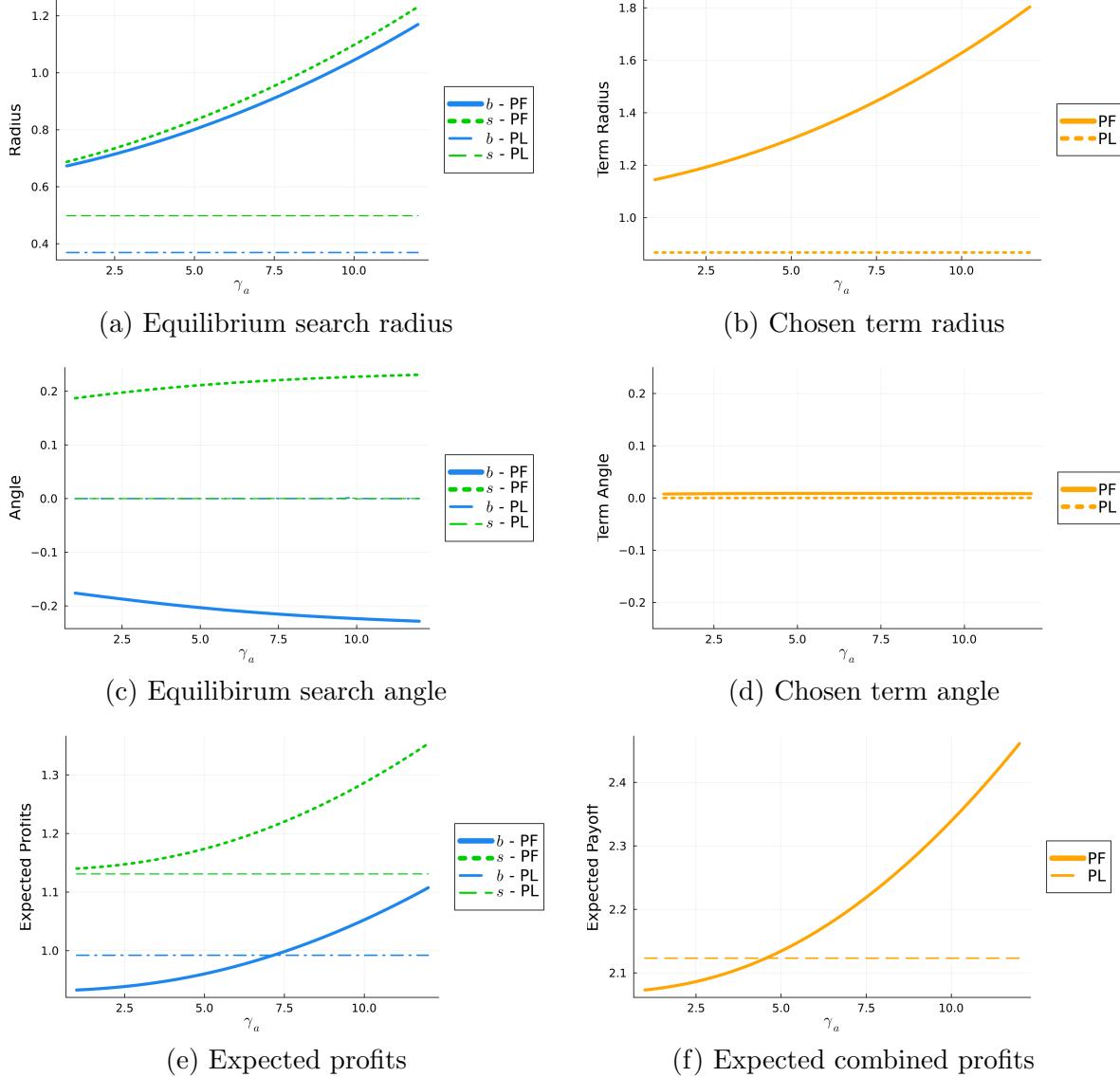
values of τ , and a term angle that is biased toward the stronger bargainer in the price-first game.

Figure IA3.1: Comparative statics with respect to τ (unrestricted term search)



Notes: The panels on the left-hand side plot the variable of interest for both firms b and s in the price-first (“PF”) and price-last (“PL”) games. The panels on the right-hand side depict the corresponding outcomes of the contract in both games, for the combined firms. The plots assume $\alpha = 2$, $\gamma_b = \gamma_s = 1$, $\gamma_a = 5$, $\pi_b = 2$, and $\pi_s = 1$.

Figure IA3.2 illustrates how the equilibrium in this model vary with the relative cost of searching for surplus-enhancing terms (i.e., along the 45-degree line). Note that the outcomes for the contracting process for different values of γ_a in the unrestricted model are almost identical to those in the partially contractible term search model in the main text. The main exception is panel (c) in which the equilibrium search angle is constant for both the buyer and the seller in the price-last game for all values of γ_a under this particular set of exogenous parameter values. This in turn implies that the firms are not incentivized to search more intensely as γ_a increases in the price-last game.

Figure IA3.2: Comparative statics with respect to γ_a (independent term shocks)

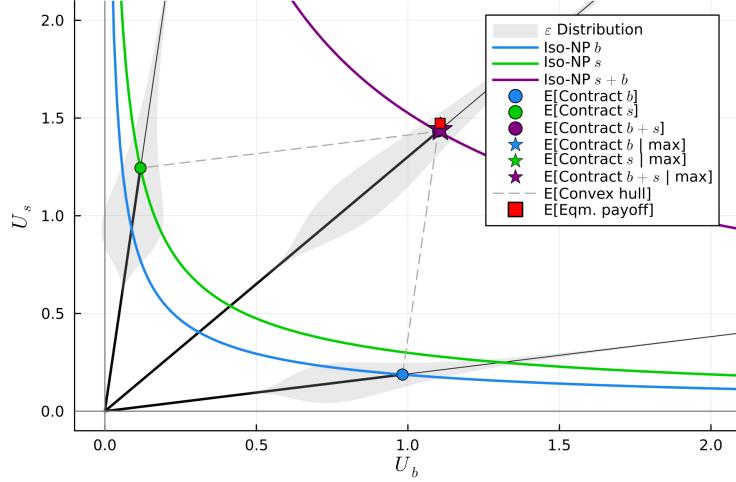
Notes: The panels on the left-hand side plot the variable of interest for both firms b and s in the price-first (“PF”) and price-last (“PL”) games. The panels on the right-hand side depict the corresponding outcomes of the contract in both games, for the combined firms. The plots assume $\tau = 0.45$, $\alpha = 2$, $\gamma_b = \gamma_s = 1$, $\pi_b = 2$, and $\pi_s = 1$.

IA4 Equilibrium contract with unrestricted term search and independent productivity shocks

Figure IA4.1 illustrates the bargaining game for the full model. In contrast to the baseline model specification, where ϵ 's components are perfectly correlated, independent realizations of term-specific shocks lead to a different “shape” for the convex hull of every bargaining set; we therefore represent the variation due to ϵ with densities graphed along the radius of each term.

Figure IA4.1 also shows the option value from variety: the red square representing the expected equilibrium payoff is closer to the top-right than any of the individual terms. This is also illustrated by the expected payoff to contract terms, represented by stars in the figure below: individual terms must have a favorable ϵ draw to be chosen, so their expected value to the firms is greater when conditioning on the terms being chosen. Note that the stars for the individual terms do not appear in the figure, since the value of the shock must be so extreme as to push the value of the joint payout to a higher iso-curve relative to the iso-curve of the combined term. For the combined term m_{bs} , this difference is only slight since it is a low-probability event that either individual term will be chosen over the combined term.

Figure IA4.1: Firm payoffs in the bartering stage of the price-first game (unrestricted model)

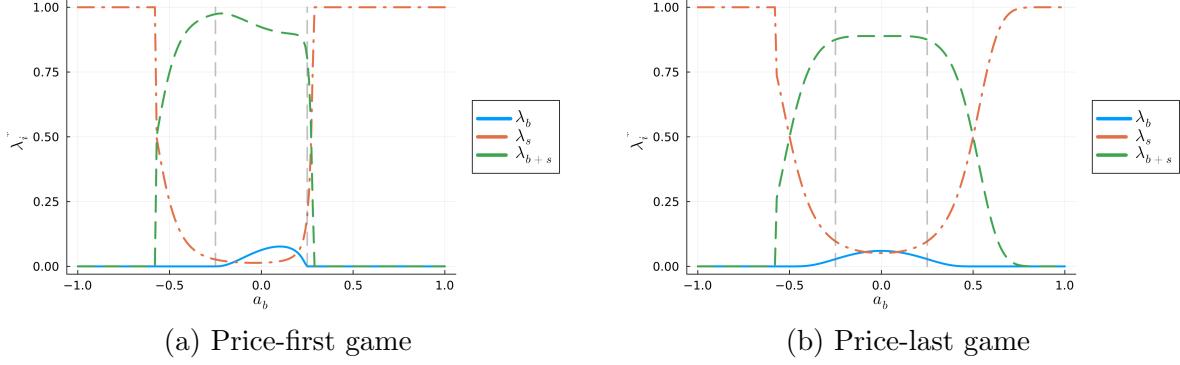


Notes: The plots assume $r_b = 1$, $r_s = 1.25$, $a_b = -0.19$, $a_s = 0.22$, $\alpha = 4$, and $\tau = 0.4$. These values are chosen for clarity of exposition and do not necessarily represent equilibrium actions. The blue and green stars are not shown on the figure, but instead lie far along their respective rays.

Figure IA4.2 similarly provides intuition for how $\lambda_{j,G}$ varies with firm search decisions. Firm b and s both have a positive probability that their term will be chosen when searching within the first quadrant ($a_b \in [-0.25, 0.25]$, marked with dashed grey lines in the figures). This highlights how Lemma 1 does not hold in this setting since all terms have a positive probability of being chosen.⁵ However, the combined term is preferred in expectation except when firm b searches in a sufficiently value-destroying direction and makes term s relatively more favorable.⁶ While the term choice probabilities look broadly similar in panels (a) and (b), the price-last game has a nonzero probability of choosing term b even when a_b is outside the first quadrant; this is due to the redistribution of term payoffs in the price-last game.

5. Note that taking the limit as $\alpha \rightarrow \infty$ (thereby decreasing the variance of ϵ) pushes $\lambda_{bs} \rightarrow 1$.

6. Searching outside the first quadrant ($a_b \notin [-0.25, 0.25]$) results in some value destruction, but as shown in panel (b), this may still result in a term that is preferred to an individual term in expectation.

Figure IA4.2: Term choice probabilities $\lambda_{j,G}$ 

Notes: The plots assume $r_b = 1$, $r_s = 1.25$, $a_s = 0.22$, $\alpha = 4$, and $\tau = 0.4$, and a_b varies to show the resulting probabilities that each term is chosen. These values are chosen for clarity of exposition and do not necessarily represent equilibrium actions.

References

- Berry, Steven T., and Philip A Haile. 2021. “Foundations of demand estimation.” In *Handbook of industrial organization*, 4:1–62. 1. Elsevier.
- Eaton, Jonathan, and Samuel Kortum. 2002. “Technology, geography, and trade.” *Econometrica* 70 (5): 1741–1779.
- Herrero, Maria Jose. 1989. “The Nash program: non-convex bargaining problems.” *Journal of Economic Theory* 49 (2): 266–277.
- Lashkaripour, Ahmad, and Volodymyr Lugovskyy. 2023. “Profits, scale economies, and the gains from trade and industrial policy.” *American Economic Review* 113 (10): 2759–2808.
- Serrano, Roberto, and Ken-Ichi Shimomura. 1998. “Beyond Nash bargaining theory: the Nash set.” *Journal of economic theory* 83 (2): 286–307.
- Zhou, Lin. 1997. “The Nash bargaining theory with non-convex problems.” *Econometrica: Journal of the Econometric Society*, 681–685.