Fix the Price or Price the Fix? Resolving the Sequencing Puzzle in Corporate Acquisitions

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Significant corporate transactions (such as financing and acquisition agreements) are typically negotiated in stages, wherein core pricing terms are fixed early while most non-price provisions are relegated to subsequent bargaining. This ordering stands in stark (and curious) contrast with canonical theories of contract design, which overwhelmingly counsel that non-price terms should be set first, saving price negotiations for last so as to fine tune the parties' net payoffs. This longstanding disjunction between theory and practice has become a celebrated puzzle for transactional design. To reconcile this disjunction we develop an innovative framework that embeds a term search game within a discrete bartering model. In spite of (and partially because of) its discreteness, our search/bartering framework delivers a robust and tractable set of intuitions about when fixing price prior to other terms incentivizes efficient search by strategic parties. Our analysis is also amenable to making counterfactual comparisons of regimes where price is (and is not) set first, generating in the process several empirically testable implications. We illustrate our main predictions empirically through a machine-learning analysis of thousands of M&A transactions executed over two decades.

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In theory, there is no difference between theory and practice...

...while in practice, there is.[1]

1 Introduction

Social science is replete with storied examples where creative theory collides head on with institutional reality (Arteaga et al. 2024), and the law and finance literature is no exception. A notable theory/practice paradox has long vexed high-stakes corporate transactions such as acquisitions and financing agreements—deals that are undisputedly the largest, most contemplated, and most heavily negotiated in modern markets. Such transactions routinely draw significant attention from financial economists; and indeed few sub-fields of economics are as well developed as optimal contract design, garnering multiple Nobel prizes[2] and suffusing pedagogy across economics departments, business schools and law schools alike. Given the appreciable economic stakes and the sophisticated players involved, major corporate transactions would seem to be an ideal proving ground for contract design theory. If there were *any* area where participants have settled the most glaring tensions between theory and practice, this surely must be it.

And yet, the norms and practices superintending large corporate transactions have long diverged from fundamental tenets of contract design theory. In short, both camps of the theory-practice divide regard the other as operating "backwards" in assembling the price and non-price terms of a deal. According to conventional contract theory, price is a perfectly adjustable zero-sum mechanism—a numeraire that fluidly transfers payoffs between parties in a welfare-neutral manner. Non-price terms, in contrast—such as covenants, conditions, warranties, and the like—are rarely zero-sum, and their contractual allocation has real welfare consequences. Accordingly, efficiency calculus counsels that non-price provisions should be structured first (prior to setting price), with a goal of maximizing expected joint surplus. Only after those terms are fixed should pricing enter *at the very end* of the process. Such a sequence makes eminent sense (at least in theory), since the transfer-payment aspect of price makes it an ideal tool for truing up any payoff imbalances left behind after aggregating the joint welfare maximizing non-price terms, "greasing the wheels" of a mutually beneficial optimal contract. This sequential prediction is so fundamental and well-supported, in

1. This excerpt is widely (and apocryphally) attributed by turns to Yogi Berra, Albert Einstein, Richard Feynman, Nassim Taleb, and several others. The earliest invocation of it we can find appears to be substantially older. *See* Brewster (1862).

2. In the last thirty-five years alone, Nobel laureates specializing in contract theory include Milgrom and Wilson (2020), Hart and Holmstrom (2016), Tirole (2014), Roth and Shapley (2012), Hurwicz, Maskin and Meyerson (2007), Mirrlees and Vickrey (1996), Harsanyi, Nash and Selten (1994), and Coase (1991).

fact, that it permeates virtually all of contract design theory (*See* Bolton and Dewatripont 2004).

Nevertheless, and in stark contrast with contract theory, transactional practice typically proceeds in the reverse direction, fixing core financial terms at the onset, often via a succinct term sheet produced by executives and insiders. Only after pricing is locked in do outside lawyers and other transactional specialists sweep in to hammer out the non-price details. But in doing so, they work under a curious constraint: While these late-arriving actors have significant negotiating latitude, one thing they are almost *never* permitted to do is to revisit pricing: Although price re-cuts sometimes happen, they are heavily discouraged by a variety of institutional factors and are therefore extremely rare.[3] Put simply, the practice of fixing price at the onset of bargaining means that transactional professionals are left to assemble the remaining non-price components with nary a drop of the transactional grease that price adjustments can (theoretically) afford.[4]

The persistent deviation of contract theory (fix price last) from transactional practice (fix price first) has long puzzled scholars and practitioners, sparking debate and inquiry into why price—which is otherwise an ideal payoff re-leveling mechanism—remains an inflexible anchor on negotiations, especially in the realm of large corporate transactions where the monetary stakes are appreciable. Compounding this quandary further, perhaps, is a different norm in smaller transactions (such as used car sales or residential real estate) where—consistent with theory—pricing decisions typically remain more fluid throughout the negotiation process. Why would such contracts tend to conform to theoretical predictions while large corporate transactions (with billions of dollars at stake) diverge?

This paper seeks to reconcile theory and practice by introducing an innovative theoretical framework that embeds a term search game within a discrete bartering model. The key to our approach is to analogize contract design to a production process (Choi, Gulati, and Scott 2021; Choi et al. 2022), whereby the choice set of non-price terms available to the parties is endogenously revealed through the parties' efforts, expended at a private and non-contractible cost. This transforms the contract design process into a two-sided principal-agent problem where each party seeks to catalyze value-enhancing term innovation while

3. Price renegotiation in the LVMH-Tiffany transaction is a highly-publicized exception to the rule, as described in Jennejohn, Nyarko, and Talley (2022). This broader phenomenon of reference dependence in shaping economic outcomes is discussed in O'Donoghue and Sprenger (2018); other work on the role of similar benchmarks or expectations in two-stage bargaining settings includes Crawford (1982), Muthoo (1992), and Basak and Khan (2024).

4. Note that certain less frequent M&A sales processes, namely multi-bidder auctions, are more amenable to non-price terms being set first. Full-blown auctions, however, are quite rare, and some auction processes even allow bidders some degrees of freedom to adjust nonprice terms. We discuss auction structures at greater length below.

simultaneously extracting maximal rents. Within our framework, fixing the deal price at the onset can emerge as an efficient design choice, both from an incentive compatibility perspective and for joint surplus maximization. Specifically, we show that fixing price first can better incentivize parties toward efficient search for and production of welfare-enhancing non-price terms, a sort of two-sided "hostage-taking" in the spirit of Williamson (1983). Moreover, to the extent that uncovering creative non-price provisions translates into greater value in high-stakes environments (Gabaix and Landier 2008), our model predicts that "price-first" bargaining will tend to be more prevalent in large corporate transactions than in smaller-stakes deals.

The intuition behind our argument unfolds in four key steps. First, we posit (realistically) that contracting parties are heterogeneous, and consequently the optimal contract terms for a randomly chosen set of counterparties will differ from any other randomly chosen dyad. Second, we assume (again realistically) that bargaining power is an exogenous primitive, which itself cannot be bargained over. That is, if a negotiating party possesses the lion's share of the bargaining power, she cannot commit to *not* exploit that power at a later stage. Third, we argue that the universe of possible non-price terms (beyond standard-form "boilerplate" templates) is not obvious ex ante; rather, finding a bespoke non-price term that enhances payoffs requires discretionary and costly search process, undertaken by at least one (and possibly both) of the parties.

Fourth, and critically, we posit that the most skilled searcher for non-price terms need not also be the best negotiator. Thus, when search ability and bargaining power are not aligned, fixing price last (as conventional theory counsels) can disincentivize efficient search. The reason is simple: because the costs of searching for welfare-enhancing non-price terms will become sunk once those terms are unveiled, the searcher's efforts quickly become irrelevant when the parties bargain over price; instead, the bargaining outcome from that point forward hinges centrally on each party's relative bargaining power. As such, a party contemplating searching for innovative non-price terms knows that even if she succeeds, her counterparty will marshal superior bargaining power to extract the newly-created value through pricing concessions, leaving the successful searcher with little more than nonpecuniary bragging rights (and a sunk cost). The searcher's anticipatory concern over expropriation becomes especially acute, moreover, as the non-searching party's relative bargaining power increases.

When, in contrast, price is fixed from the onset, the parties' search incentives change fundamentally, typically in the direction of *more* efficient search. When (for example) the most efficient searcher has little relative bargaining power, the rigidity of an immovable price metamorphoses from a bug into a feature,[5] incentivizing her to work harder to find a value-enhancing non-price term with less fear that her counterparty will later expropriate the added value by wheedling on price. Moreover, even when the searching party also possesses significant relative bargaining power, her incentives remain roughly unchanged regardless of whether price is set first or last: in either case, she will be able to capture most of the value she brings to the table from a successful search. Aggregating across cases, our theoretical framework predicts that in a "large" set of parametric environments, fixing price *ab initio* can catalyze more efficient production of non-price terms overall, resulting in more advantageous expected outcomes for both parties.[6]

More generally, our analysis illustrates a counter-intuitive possibility for creating value by transforming a seemingly frictionless bargaining problem with modest negotiation costs into a more rigid bartering problem, where the only available currency for negotiated exchange is by "horse trading" the newly identified non-price terms. While locking in price upfront no doubt introduces certain transactional frictions, it can simultaneously promote efficient incentives, resulting actuarially in new, welfare-enhancing transactions and terms that would have been unlikely or impossible were pricing determied at the end.

In addition to developing a theoretical model capable of reconciling the longstanding disjunction between theory and practice, we also make three contributions to the contract theory literature. First, we develop a flexible contract design framework in two-dimensional payoff space—representing the buyer and seller—centered at their anticipated respective payoffs ex ante under the standard form contract. Working in polar coordinate space, our framework reduces the search strategy of each party to a decision over two variables: (1) the direction of search (measured by an angle in payoff space) and (2) the intensity of search (measured by the length of the ray in the chosen direction). By reducing the optimization problem to these two foundational dimensions, our framework delivers a tractable and powerful baseline that we believe can be deployed and extended in other contexts like trade negotiations or exclusive contracting in labor or real estate. These settings differ from smaller transactions in which the typical price-last formula is followed, highlighting the role of actively creating value-enhancing contract terms in these larger transactions.

Second, we embed a discrete "bartering" model (over non-price terms in stage 2) nested

5. *Compare* Kafka (1915).

6. This result is reminiscent of other settings, such as Weitzman (1974), where it may be optimal to fix prices and let quantities (or in our case, qualities) be chosen by the firms in question. Here, we emphasize the two-sided nature of this problem: both parties are trying to regulate each other, and the quality of proposed contract terms depends on the payoffs to both parties separately. This is particularly relevant in the price-first setting where the component of firms' payoffs that depends on non-price terms is non-transferrable. Further, the lack of price adjustment is arguably a specific example of the type of "severe limitations" predicted by Hart and Moore (1988) when some degree of renegotiation is possible.

within a Nash bargaining model (over pricing, in stage 1). This unlikely marriage mimics the reality of real world, high-stakes transactions, but at the theoretical expense of introducing discontinuities that undermine the tractability of the model. Nevertheless, we are able to find closed-form characterizations of equilibrium behavior under certain simplifying assumptions and simulate numerical solutions across a full range of associated parameter values. We do so by using a choice probability approach taken from the discrete choice literature (see e.g. McFadden 1972; Berry, Levinsohn, and Pakes 1995; Eaton and Kortum 2002), under the assumption that deal-specific heterogeneity makes the proposed contract terms vary in their suitability across negotiating dyads. This framing has the added bonus of making the analysis of more complex contracts (with myriad specific terms which are themselves the byproducts of negotiations) empirically tractable using standard tools from industrial organization. While other studies of bargaining leverage detailed data on alternating offers (Backus et al. 2020; Dunn et al. 2024), our approach is particularly suited to settings with simultaneous and/or unknown procedures for contract development.

Third, we develop a standard sister Nash bargaining model with search over non-price terms in stage 1 and subsequent price setting in stage 2. This model formalizes the more conventional intuition for setting price last but highlights its dangers in the setting where efficient contract terms are not obvious *ex ante* and must be discovered.[7] That is, the timing of this contracting game means that non-contractible efforts to develop contract terms before the price is fixed are sunk, leading to a two-sided analogue to the holdup problem in Klein, Crawford, and Alchian 1978. Moreover, the similarity between the two models enables us to compare them across different parametric settings. This comparison yields both empirical predictions and an explanation that reconciles the disjunction between practice and the currently prevailing theoretical models: setting the transaction price first–in many settings–incentivizes more efficient search for (and production of) contract terms.

Our analysis yields testable predictions about the nature of equilibrium search for non-price terms, which we explore empirically using a novel database of provisions from large corporate transactions. Specifically, our model predicts that the modest minority of “auction” deals (where prices are constrained mechanically to be set last) will manifest less innovative variation than “negotiated” deals (where prices are conventionally fixed up-front). Using data from nearly 8,000 M&A contracts in a dataset collected by Adelson et al. (2024), we compare the similarity of Material Adverse Effect (MAE) clauses within auction deals and

7. Our framework advances a deliberately flexible account of what it means to “discover” a non-price term through innovation. It could, for example, represent formulating a genuinely novel express provision (such as allocating risks due to “COVID-19” or “the identity of Elon Musk”). Or, it could represent investigating whether to deploy a provision that is already familiar but not appropriate for all deals (such as a sandbagging provision or reverse break fee). The model we develop is amenable to any of these conceptual variations.

non–auction deals. Consistent with our model's predictions, we find that MAE terms in auction deals exhibit uniformly less variety and heterogeneity than their counterparts in negotiated deals—a hallmark of the greater customization that our model shows to be induced by a fixed-price search/negotiation environment.

The account we offer here does more than resolve a longstanding conundrum using an original and tractable model, however. It also sheds light on a variety of other norms that are commonly observed in large-stakes deal negotiations as well as important legal doctrines. For example, a direct implication of our framework is the possibility of equilibrium deal failure. In our model, bargaining parties may rationally sign up a preliminary deal featuring standard "boilerplate" terms that—at least when signed—makes them jointly worse off than the *status quo ante* without a transaction. Why would they do so? Because in equilibrium they expect that the ensuing search for non-price terms may yield new payoff-enhancing structures, thereby making the risk of subsequent deal failure worth the gamble.

In a similar vein, our approach reveals a plausible rationale behind enforcing even preliminary agreements that do not have all their key terms locked in.[8] This is an area where courts have grown increasingly willing to deploy enforcement tools (such as reliance or expectation damages) against a party who fails to deploy "good faith" efforts to finalize the terms of a preliminary agreement.[9] Fixing price ex ante in our framework is important precisely in situations where it is important to incentivize parties to expend good-faith efforts to find value enhancing terms. A party's failure and/or refusal to do so can be particularly harmful in our setting, since it can increase the odds of wasteful deal failure. Consequently, courts' enhanced willingness to enforce preliminary agreements with open terms can be interpreted as consistent with catalyzing efficient search incentives within our model.

Finally, our model helps reveal the critical importance that good lawyering can play in transaction design. Highly skilled lawyers in our model face lower search costs, plausibly reflecting a combination of greater creativity and more robust firm-level experience (their own and their partners). Consequently, good lawyers are also more skilled at smoking out value-enhancing non-price terms, which expands the frontier of payoff possibilities that are available in equilibrium. In fact, when two high quality firms interact with one another, they may be in a better position to coordinate their searches, producing non-price terms that become strongly welfare enhancing when combined as part of a bartered *quid pro quo*.

8. In U.S. contract law, preliminary agreements with open terms are referred to as "Type II" agreements, differentiating them from "Type I" preliminary agreements, where all essential terms are settled and only certain formalities are lacking. *See* Teachers' Insurance v. Tribune Co., 670 F. Supp. 491 (SDNY 1987).

9. *Compare* Empro v. Ballco, 870 F.2d 423 (7th Cir. 1989) *with* SIGA v. PharmaThene, 67 A.3d 330 (Del. 2013), Pennzoil v. Texaco, 481 U.S. 1 (1987), Copeland v. Baskin Robbins, 96 Cal.App.4th 1251 (Cal. Ct. App. 2002).

Although we are not the first to observe the odd disjunction between the theoretical account of optimal contracting (where price is chosen last) and the practical reality (where price is fixed first), our framework is novel in several respects. For instance, our model provides microfoundations for observed phenomena, such as Badawi and Fontenay (2019)'s of the "first mover" advantage in the design of non-price M&A terms. It also advances earlier efforts that explored the roles of bargaining power and asymmetric information on the design of non-price terms in M&A contracts (Choi and Triantis 2012) by introducing the contractual innovation process—the non-trivial search for new terms in the context of a particular design problem (Jennejohn, Nyarko, and Talley 2022)—into its core model. Our model also uses existing tools from the literature on empirical choice models to present an empirically tractable model that can be used to analyze the value of chosen contract terms, even when there is no post-negotiation variation in price. This empirical tractability extends to other settings where firms or other agents may jointly choose among discrete options.

Our analysis unfolds as follows. In Part 2, we provide relevant context for the development of contracts in corporate transactions. In Part 3, we present the overview of our baseline model, with additional details deferred to Appendix 1. In Part 4, we describe the model's equilibria and present various comparative statics. Part 5 presents an empirical test of our model, and Part 6 discusses various implications of our analysis for both contract theorists and practitioners. Part 7 concludes.

2 Contract production in the M&A market

The modern M&A agreement is a complex piece of transactional technology, typically encompassing over 100 pages of obligations (Coates 2016; Jennejohn 2018; Hwang and Jennejohn 2018).[10] While many markets cope with similar levels of contractual complexity by standardizing terms across deals (Gulati and Scott 2012), M&A agreements are surprisingly resistant to rote use of boilerplate, and a significant amount of transaction-specific tailoring of terms often occurs in each negotiation (Coates 2016; Jennejohn 2020; Talley 2009). In short, there is space for creativity for the transaction designer, and, indeed, reputational benefits accrue to advisors who successfully innovate effective new terms.

The terms of these complex contracts can be sorted into several key categories. First, the operative terms of the agreement set forth the details of how the business combination will be accomplished, including the price for the acquisition and the nature of the consideration used (cash, the acquirer's stock, or a combination of the two). Second, the seller provides a series of representations and warranties relating to the qualities of the target company, thereby

10. Often, ancillary agreements are also attached to the main agreement (Hwang 2016). Our focus here is on the main M&A agreement that accomplishes the core transaction.

addressing potential risks that are unobservable during the acquirer's due diligence process.[11] Third, a series of covenants, which apply to the behavior of either (or both) acquirer or seller between the time the contract is executed and the time the transaction closes,[12] address pre-closing risks, such as: interim operating covenants that require the seller to operate the target in the ordinary course of business, thereby precluding extraordinary decisions that would impair the value of the target company; regulatory provisions that address the possibility of, for instance, an antitrust or national security regulator attempting to prevent or force the restructuring of the transaction; and deal protection devices, like "no-shop" provisions, that constrain the seller's ability to pursue alternative bids. Fourth, and finally, conditions to closing and termination provisions connect breaches of the aforementioned terms to the parties' duty to close the transaction, thereby incentivizing performance.

To make that complexity manageable, the advisers to a transaction—the investment bankers and deal lawyers advising both buyer and seller—tend to bifurcate the negotiation process into two steps. First, the key operative—or "business"—terms, including the price and a smattering of important terms across the four categories above, are determined and reduced to a preliminary agreement, such as a term sheet or letter of intent. The principals of both buyer and seller are heavily involved at this stage since, as one hoary treatise in the field notes, "the usual topics of discussion at the outset are generally basic business areas, on which attorneys should defer to their clients" (Freund 1975). After the core business terms are preliminarily agreed upon, the detailed "legal" terms of the agreement are then hammered out. Here, the division of labor shifts, with the deal lawyers taking the wheel.[13]

As a practical matter, the price and other key terms set in the first step of the negotiation process are typically quite sticky. Detailed accounts of such stickiness are not generally available in the public record, since secrecy in merger negotiations is jealously kept. Nevertheless, they arise from time to time, especially for imperiled deals that devolve into litigation. For instance, the M&A deal at the heart of *Frontier Oil v. Holly* provides a glimpse of how resistant the initially-set price term can be to change.[14] That transaction involved, among other assets, the acquisition of an oil rig that had been sitting on the grounds of Beverley

11. The buyer also typically provides a series of representations and warranties focused primarily upon its ability to execute the transaction, but these are usually less negotiated, especially in cash deals.

12. For most large transactions, a period of time between signing and closing is necessary in order to allow, for instance, for regulatory reviews or stockholder approvals.

13. A notable exception to this ordering can be found in deals where the target company conducts an auction rather than negotiation with an exclusive bidder. Here, it is more conventional for the non-price terms to be fixed up-front, providing a "package" against which prospective bidders formulate their competing offers. Because auction deals mechanically give significant bargaining power to the seller, our model predicts that they will be most attractive when the seller faces a low cost in searching for non-price terms, *and* when such terms are valued relatively homogenously by prospective bidders. We return to this point in Section 5 below.

14. See Frontier Oil Corp. v. Holly Corp., 2005 WL 1039027 (Del. Ch. April 29, 2005).

Hills High School for decades. When the negotiations were far advanced, news broke that environmental activist Erin Brockovich planned to bring a mass toxic tort suit against one of the target Frontier's company's subsidiaries, which operated the rig that allegedly harmed students. That potential liability risk had not been disclosed during due diligence, despite similar cases resulting in settlements worth hundreds of millions of dollars. Instead of adjusting the purchase price in light of the revelation though, the parties reworked a series of detailed terms in the agreement, resolutely remaining in the second—non-price—stage of the negotiating process rather than going back to square one and recutting the deal.

In the next section, we introduce a model that explains why this curious approach to contract design is used, shedding light on market practice and informing the legal system's approach to enforcing contractual obligations as they emerge in this negotiating process.

3 A model of two-stage contracting

In the light of general industry practices that sequentially stage price and non-price negotiations, along with the importance of deal term innovation in complex agreements, in this section we develop a novel, tractable framework that incorporates both industry practice and term innovation. We begin in Subsection 3.1 by describing the transaction in the model (the sale of a "business asset" from a seller to a buyer[15]) that involves three steps, the order of which will change depending on the model: (1) the parties set the price, (2) the parties search for new terms, and (3) the parties select the other terms of the contract based on the return of each party's search. In Subsection 3.2 we present the model in which price is set first, the search for other terms is second, and bartering over other terms occurs third. In Subsection 3.3 we present the model in which search for non-price terms occurs first, bartering over other terms occurs second, and price is set last. In Section 4 we compare closed form solutions across the two models, focusing first on three restricted cases to fix ideas, and then exploring numerical solutions to the general case.

3.1 Term innovation and alternate sequencing

Consider a potential transfer of a business asset from a representative seller s to a representative buyer b. Each respective party places a "baseline" valuation of π_i on the asset (where $i \in \{b, s\}$), and we assume these valuations to be common knowledge amongst the parties. The buyer's bargaining power is represented by an exogenous parameter $\tau \in (0, 1)$, and thus the seller enjoys complementary bargaining power $1 - \tau$.

15. The precise legal mechanism for the transaction is not critical to our inquiry, and it thus could be a stock sale, an asset sale, a statutory merger, a negotiated tender offer, or any other bargained-for means for transferring ownership of the business asset.

To transfer the asset, the parties must enter a contract consisting of a price p paid from b to s, as well as a vector of non-price terms m. The non-price terms collectively give rise to an additional expected value $v_i(m)$ to each party i, independent of (and in addition to) the parties' baseline valuations π_i. Because our key results hinge on strategic dynamics within payoff space, our analysis need not characterize the full vector space of all possible non-price terms; we instead characterize any non-price term vector m by the expected payoffs it conveys to the parties, $v(m) \equiv (v_b(m), v_s(m)) \in \mathbb{R}^2$. With one exception, the non-price terms are assumed hidden from the parties, and discovering them requires costly search (described below). The sole exception is a "default" (or "standard form" or "boilerplate") set of non-price terms m_0, which are commonly known. We normalize the coordinates of m_0 in payoff space to be at the origin, so that the expected additional payoffs delivered by the default are normalized at $v_i(m_0) = 0$ for $i \in \{b, s\}$.

For exposition purposes, it will frequently prove convenient to characterize non-price terms in payoff space using polar coordinates, with radius $r \in \mathbb{R}_+$ and angle $\theta \in [0, 2\pi]$. To further economize on notation, it will also be convenient to transform θ into $\theta(a) \equiv \pi(a+0.25)$, where $a \in [-1, 1]$.[16] Thus, the contract terms create expected valuations of:

$$v_b(m) = r\cos(\theta(a))$$
$$v_s(m) = r\sin(\theta(a))$$

The final contract terms are chosen from the subset $\mathcal{M}^*$ of terms that are known to both firms at the time of bargaining, which include by default the standard-form terms m_0, any other non-price terms discovered by the parties, and the combination of such terms.

Prior to negotiation, each party i can search for one new term m_i. The parties' search decisions are made simultaneously, and search efforts are assumed (at least for now) to be non-contractible. When each player uncovers a new term m_i, that new price vector's coordinates in payoff space are added to the choice set of possible non-price terms. We also assume that the new terms discovered by each party can be combined additively, so that the choice set expands to $\mathcal{M}^* = \{m_0, m_b, m_s, (m_b+m_s)\}$; we write the latter term as m_{bs}.[17] Thus, each set of contract terms in $\mathcal{M}^*$ is indexed by the subscript j where $j \in \{0, b, s, bs\}$.

Each player faces a cost $c_i(r_i, a_i)$ to search for non-price term innovations. The cost is assumed to be continuously differentiable, increasing and convex in search intensity r_i, but

16. Under this normalization, a search along direction $a = 0$ corresponds to a thoroughly "selfless" search, where the parties benefit symmetrically from the discovered term. In contract, the cases of $a = 1$ and $a = -1$ correspond to "selfish" zero-sum search, where the non-price term enhances value for one side in the same amount as it reduces value for the other.

17. To fix ideas, we assume that the technology for combining terms is linear in the expected payoffs, i.e.

with $\frac{\partial c_i(0,a_i)}{\partial r_i} = 0$ so search is costless on the margin near the default contract. Costs are assumed to be weakly decreasing in $|a_i|$, and thus it is costlier to search in directions that are joint-surplus improving.

Because search does not always align with success, our framework also allows for *ex ante* uncertainty in firms' ultimate success in discovering a new term given their search intensity. This assumption mimics real-world variation in the challenge of finding new terms, since both search productivity and term values may vary across deals in difficult to observe ways. (This uncertainty also may apply to attempts to combine discovered terms, as the firms' success in combining potentially-conflicting terms may also differ across deals.) To capture this uncertainty, we denote the realized search radius for term m_j as $r_j \cdot \epsilon_j$ with $\mathbb{E}[r_j \cdot \epsilon_j | r_j] = r_j$. We call ϵ_j a term-specific productivity shock that is only observed after investment decisions $\{r_i^*, a_i^*\}$ are made. This implies that realized payoffs, denoted $v_i(m_j; \epsilon_j)$, are in expectation equal to the average payoffs $v_i(m_j)$ defined above.[18]

Before proceeding, we provide some graphical intuition for the set of possible contracts in the payoff space. In Figure 1, each component is added sequentially to build a representation of the possible contract terms. Panel (a) first shows two hypothetical contracts in payoff space, characterized by the respective radii and search angles. Panel (b) then includes the compound expected payoff of the combined term, and panel (c) further illustrates the uncertainty in term production by representing the payoff as a point along a ray, with the associated density of ϵ plotted (in symmetric, "butterfly" fashion) along each ray. Though the expected contract term payoff is determined by the choice of r_i^* and a_i^*, the observed payoff will fall somewhere along the realization of its corresponding ray.

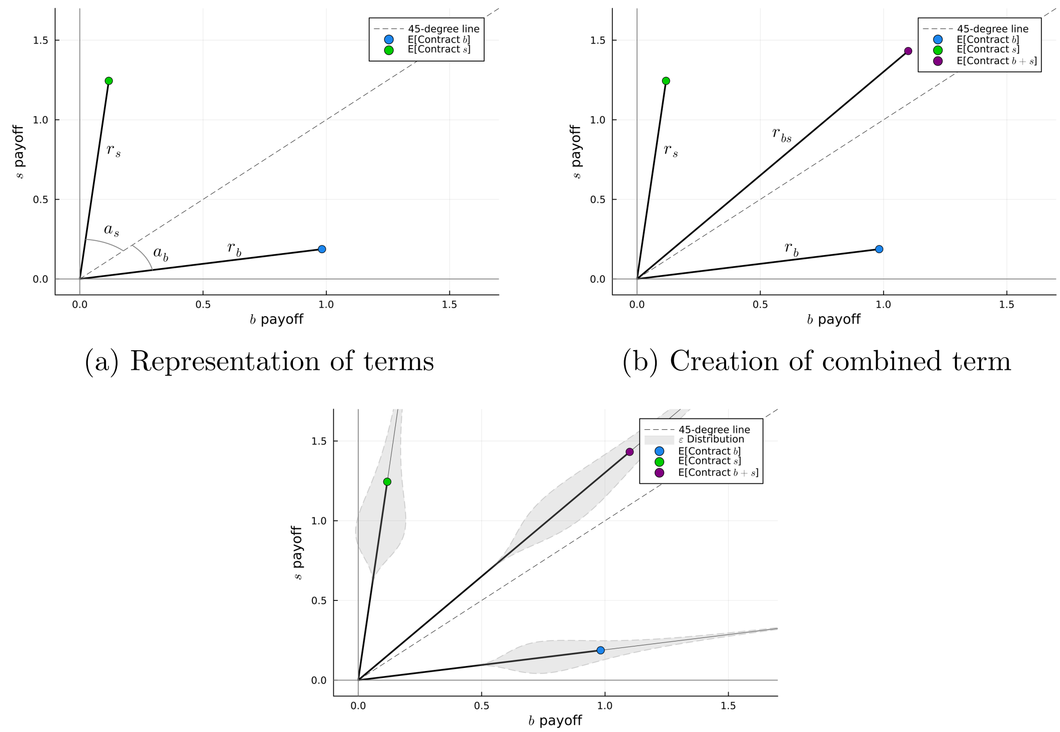
With this framework in mind, we consider two possible games that are differentiated by when the two pieces of the contract are created. In the "price-first" game (PF), firms determine the price p_{PF}^* via Nash bargaining before searching for new terms $m_{i,PF}$, and then bargain over which terms m_{PF}^* to select. In the "price-last" game (PL), firms invest in and choose the contract terms m_{PL}^* first, and then the deal price p_{PL}^*. The productivity draws associated with the chosen contract in the two games are ϵ_{PF}^* and ϵ_{PL}^*.

$v_i(m_{bs}) = v_i(m_b) + v_i(m_s)$. Consequently, the new contract bs is characterized by

$$r_{bs} = \sqrt{r_b^2 + r_s^2 + 2 r_b r_s \cos(\theta(a_s) - \theta(a_b))}$$
$$a_{bs} = \frac{1}{\pi}\left[\theta(a_b) + \tan^{-1}\left(\frac{r_s \sin(\theta(a_s) - \theta(a_b))}{r_b + r_s \cos(\theta(a_s) - \theta(a_b))}\right)\right] - 0.25$$

18. Note that while expected payoffs are linear in the component payoffs (i.e., $v_i(m_{bs}) = v_i(m_b) + v_i(m_s)$), the same does not hold for realized payoffs. We interpret ϵ as a shock to the firms' ability to implement the term in an actual contract, noting that frictions in the contract negotiation process may complicate the process of combining two distinct terms.

Figure 1: Contract term components in firm payoff space



(a) Representation of terms

(b) Creation of combined term

(c) Uncertainty in term production along angle of search

Notes: Each panel plots components of the contract term model to illustrate the choice set in payoff space. Successive panels add more features of the search game, but remove some notation to highlight the new features. Panel (a) begins with the payoffs of the proposed firm contract terms. Panel (b) adds the contract term that combines both firms' terms. Panel (c) plots densities around the search radii corresponding to the densities of each term-specific shock ϵ, where the mean contract value is at the colored dots first plotted in the preceding panels.

Table 1: Timing of the two games

t	Price-first	Price-last
0	b and s decide to transact	b and s decide to transact
1	p^*_{PF} is chosen	$\{r^*_{i,PL}, a^*_{i,PL}\}$ are chosen
2	$\{r^*_{i,PF}, a^*_{i,PF}\}$ are chosen	m^*_{PL} is chosen
3	m^*_{PF} is chosen	p^*_{PL} is chosen

We now examine in detail how the prices and contract terms are determined in the two games. The timing of the two games is summarized in Table 1. Prices in both games are determined via Nash bargaining over expected equilibrium payoffs given available information when the price is chosen. We also assume the chosen contract maximizes the weighted Nash product of firms' continuation payoffs at the contract term stage. This is similar to the standard Nash bargaining framework (Nash 1950) but with a discrete choice set $\mathcal{M}$ rather than a convex choice set; we call this Nash *bartering* to emphasize this distinction. While this modified framework does not have all the guarantees of standard Nash bargaining (in particular, the relationship of non-cooperative and cooperative bargaining), it provides a concise framing of the term bartering stage without taking a stand on the timing and rules of a sequential bargaining game.[19]

3.2 Contract creation in the price-first game

We now study the timing of the price-first contract game. We proceed by backward induction, first considering how contract terms are chosen, then firms' search choices for terms, and lastly the price bargaining game.

Bartering for terms. Since the contract price is fixed (under the default contract with standardized values $v_i(m_0) = 0$) before contract terms are chosen, the firms choose whichever term out of $\mathcal{M}^*$ yields the greatest Nash product:

$$\begin{aligned} m^*_{PF} &= \underset{m_j \in \mathcal{M}^*}{\operatorname{argmax}} \quad (v_b(m_j)\epsilon_j)^\tau \cdot (v_s(m_j)\epsilon_j)^{1-\tau} \\ &= \underset{m_j \in \mathcal{M}^*}{\operatorname{argmax}} \quad \underbrace{\left[v_b(m_j)^\tau \cdot v_s(m_j)^{1-\tau}\right]}_{\delta_{j,PF}} \cdot \epsilon_j \\ &= \underset{m_j \in \mathcal{M}^*}{\operatorname{argmax}} \quad NP_{j,PF} \\ s.t. \quad & v_i(m_j) \geq 0, \quad i \in \{b, s\} \end{aligned}$$

The non-negativity constraint holds because neither firm will accept a contract term that reduces their individual surplus.[20] We also emphasize that the set of possible terms $\mathcal{M}^*$ that is considered under bargaining is itself an equilibrium object that was previously decided by firms' investment decisions. We consider this choice now.

Search for terms. Firms choose their search angle a_i and search radius r_i to maximize their expected net payoff from the term bartering stage. Each term-specific shock ϵ_j is crucial in determining which contract term is chosen, but these are not realized until after firms' decisions are made. Thus, the expected payoffs depend both on the Nash program in the

19. See the Online Appendix for a more detailed discussion of the relationship of this model to standard Nash bargaining.

20. For the price-first game, this assumption functionally implies that neither party is made worse off than their status quo ante should a preliminary deal fail – so that neither party would become "damaged goods."

term-bartering stage and the joint distribution of ϵ, which we leave unspecified for now.

$$\{r_i^*, a_i^*\} = \underset{r_i, a_i}{\operatorname{argmax}} \quad \mathbb{E}[v_i(m^*; \epsilon^*) \mid m^* \in \mathcal{M}^* \text{ is chosen in } PF] - c_i(r_i, a_i)$$

We write the equilibrium expected term-specific payoff (conditioning on the equilibrium firm choices r_i^* and a_i^*) as $U_{i,PF}^*$. These expected payoffs, and their associated costs, are considered by the forward-looking firms when deciding on the contract price.

Bargaining for prices. Firms use their expected equilibrium net payoffs $U_{i,PF}^* - c_i(r_{i,PF}^*, a_{i,PF}^*)$ in the continuation game as a reference point when bargaining. The equilibrium price is determined by

$$\begin{aligned} p_{PF}^* &= \underset{p \in \mathbb{R}_+}{\operatorname{argmax}} \quad (\pi_b - p + U_{b,PF}^* - c_b(r_{b,PF}^*, a_{b,PF}^*))^\tau \cdot (p - \pi_s + U_{s,PF}^* - c_s(r_{s,PF}^*, a_{s,PF}^*))^{1-\tau} \\ &= \tau(\pi_s - U_{s,PF}^* + c_s(r_{s,PF}^*, a_{s,PF}^*)) + (1-\tau)(\pi_b + U_{b,PF}^* - c_b(r_{b,PF}^*, a_{b,PF}^*)) \end{aligned}$$

In other words, the firms split both the expected surplus from selling the asset and the expected net surplus from new contract terms according to their relative bargaining power.

3.3 Contract creation in the price-last game

We now study the timing of the price-last contract game. We proceed by backward induction, first considering how the price is set given contract terms, then how the contract terms are chosen, and lastly the firms' choice of term production.

Bargaining for prices. The contract price is chosen only after the contract terms are decided and the cost of finding these terms is sunk. Thus, the price for any chosen terms m_{PL}^* (with the associated shock ϵ_{PL}^*) is:

$$\begin{aligned} p_{PL}^* &= \underset{p \in \mathbb{R}_+}{\operatorname{argmax}} \quad (\pi_b - p + v_b(m_{PL}^*; \epsilon_{PL}^*))^\tau \cdot (\pi_s + p - v_s(m_{PL}^*; \epsilon_{PL}^*))^{1-\tau} \\ &= \tau(\pi_s - v_s(m_{PL}^*; \epsilon_{PL}^*)) + (1-\tau)(\pi_b + v_b(m_{PL}^*; \epsilon_{PL}^*)) \end{aligned}$$

That is, setting the price after terms are decided means that firms negotiate the price to split the newly created value from the contract.

Bartering for terms. Both firms anticipate that the price in the last stage of the game splits the surplus from the contract according to each firm's relative bargaining power (disregarding sunk costs). Thus, the firms choose the terms that solve the Nash program:

$$
\begin{aligned}
m^*_{PL} &= \underset{m_j \in \mathcal{M}}{\text{argmax}} \quad [\tau(v_b(m_j) + v_s(m_j))\epsilon_j]^\tau \cdot [(1-\tau)(v_b(m_j) + v_s(m_j))\epsilon_j]^{(1-\tau)} \\
&= \underset{m_j \in \mathcal{M}}{\text{argmax}} \quad \underbrace{[v_b(m_j) + v_s(m_j)]}_{\delta_{j,PL}} \cdot \epsilon_j \\
&= \underset{m_j \in \mathcal{M}}{\text{argmax}} \quad NP_{j,PL} \\
s.t. \quad & v_b(m_j) + v_s(m_j) \geq 0
\end{aligned}
$$

Note that this program is equivalent to maximizing the combined surplus generated by the new contract terms, regardless of who benefits from that term. This is an intuitive result: Because the last stage of the game will split the total surplus available according to exogenously given bargaining power, neither side benefits from selecting non-price terms that do not maximize the total expected "pie." In slight contrast to the price-first game, there is a non-negativity constraint implying that new terms will only be considered if they are a net *joint* improvement over the default contract m_0.

Search for terms. Each firm chooses search angle a_i and search radius r_i knowing how both the final contract terms and price will be chosen. In this case, the firms choose to maximize their share of the expected joint surplus minus the cost from finding these terms.

$$
\begin{aligned}
\{r^*_b, a^*_b\} &= \underset{r_b, a_b}{\text{argmax}} \qquad \tau \cdot \mathbb{E}\left[\sum_{i \in \{b,s\}} v_i(m^*; \epsilon^*) \,\middle|\, m^* \in \mathcal{M}^* \text{ is chosen in } PL \right] - c_b(r_b, a_b) \\
\{r^*_s, a^*_s\} &= \underset{r_s, a_s}{\text{argmax}} \quad (1-\tau) \cdot \mathbb{E}\left[\sum_{i \in \{b,s\}} v_i(m^*; \epsilon^*) \,\middle|\, m^* \in \mathcal{M}^* \text{ is chosen in } PL \right] - c_s(r_s, a_s)
\end{aligned}
$$

This stage differs materially from the firm problem in the price-first stage: instead of receiving the full benefit of their search, firms only receive a share of the combined firms' surplus from the chosen term. Equivalently, both parties are aware that their search costs will become sunk (and thus disregarded) in subsequent stages. As in the price-first game, it is helpful to denote the equilibrium expected payoff to firm i from the chosen contract term as $U^*_{i,PL}$.

4 Characterizing the equilibrium contract

Having laid out the basic structure of our bargaining/bartering model in both the price-first and price-last structures, we now explore comparisons between the competing approaches. Our framing thus far has been deliberately general, which limits our ability to solve directly for firms' equilibrium choices, since that will turn on specific functional forms related to productivity shocks (ϵ) and search costs (c_i). We now impose some additional restrictions in order to directly analyze the firms' decisions, developing core intuitions in the process.

We proceed by imposing a set of three simplifying assumptions that allow us to obtain closed-form solutions for equilibrium strategies and directly compare the price-first and price-last structures. The first two assumptions limit the directionality of and correlation between search efforts, ensuring that the equilibrium contract always incorporates terms proposed by both firms. The third assumption imposes a general functional form for search costs. These assumptions will allow us to pin down each firm's search decisions and illustrate comparative statics with respect to bargaining power and search cost parameters.

Having imposed these assumptions, we then begin by exploring two special cases of the model that tightly constrain each firm's direction of search for new terms. In the first case, each firm is restricted to searching for terms that are value-enhancing for itself but value-neutral for the other firm (i.e., $a_b = -0.25$ and $a_s = 0.25$). We call this orthogonal, or self-interested, search. In the second case, we restrict each firm to search only for terms that are symmetrically value-enhancing for both itself and the other firm (i.e., $a_b = a_s = 0$). We call this case aligned, or surplus-maximizing, search. In both restricted settings, firms are allowed their discovered terms to create a composite/joint term m_{bs} that becomes part of the choice set along with the individually discovered terms m_b and m_s. These two special cases provide helpful intuition about equilibrium behavior when firms' directionality of search is exogenous; however, neither restriction is necessary to pin down equilibrium strategies.

In both cases, the price-first model (weakly) dominates the price-last model on efficiency grounds. In the "orthogonal search" case, the price first model is Pareto optimal relative to the price last model for both firms across the full range of values for the unrestricted parameters. Notably, for all interior values of the bargaining power, this is a strict improvement for both firms. In the "aligned search" case, the price-first model is Kaldor-Hicks optimal relative to the price last model across the full range of values for the unrestricted parameters. For all cases with symmetric search costs *except* the case of equal bargaining power, the price-first model is strictly dominant on efficiency grounds.

We then consider a third case in which we assume firms' search angles are endogenously determined but bounded. These bounds may arise due to technological constraints, professional norms, good-faith bargaining obligations, or other forces that prevent searches that are "too selfish" from occurring. This case provides additional intuition for firms' incentives when both their angle and direction of search are at least partially non-contractible: the price-first structure incentivizes more investment in novel contract terms than in the price-last game. When it is sufficiently costly to search for the most welfare-enhancing terms (i.e., along the 45-degree line, or $a_b = a_s = 0$), the price-first game creates more total surplus relative to the price-last game. More broadly, the two games yield different contracts in expectation as a result of the firms' differing incentives across the two games.

Finally, we discuss a generalization of this model that allows each firms' contract development efforts to affect the probability any individual terms are chosen, which in turn affects the expected payoff from the chosen contract. This generalization draws on standard tools in the discrete choice literature and allows for closed-form choice probabilities for each of the proposed contract terms. We allow for the firms' search for new contract terms to be unrestricted, though the firms' incentives to propose valuable contract terms lead them to behave similarly to the restricted setting.

This dominance of the price-first model in the first two cases and across a large range of values in the third case and generalized case is driven by properly incentivizing each firm's search for new terms. In the price-first model each firm captures more of their realized value of the discovered terms. In the price-last model, realized value of the discovered terms is redistributed based on the relative bargaining power of the firms, which leads firms to under-invest in the search for new terms. This difference is exacerbated when one firm is a more efficient searcher and can create more expected surplus than the other firm, regardless of the firms' relative bargaining power.

4.1 Simplifying assumptions and characterization of equilibrium search

In order to achieve tractability in our comparitive statics analysis, we begin by making the following simplifying assumptions regarding the term search stage:

- **A1** The direction of search is limited to the first quadrant so that all proposed contracts have weakly positive payoffs, i.e. $|a_i| \leq \bar{a} \leq 0.25$.
- **A2** The productivity shocks are perfectly correlated, i.e. $\epsilon_{bs} = \epsilon_b = \epsilon_s$.

Assumption **A1** introduces the possibility that the directionality of each firm's search (a_i) is partially contractible and can be constrained to (weakly) Pareto improving directions. The two special cases studied below present even stronger contractibility assumptions, whereby firms can specify a precise directionality a_i instead of a range. Although this degree of contractibility may be difficult in practice due to the challenge of monitoring the other firms' contract creation efforts, it provides a useful starting point to understand each firm's incentives. Note that even in this setting, the multiplicative nature of the productivity shock ϵ_j means that firms cannot verify whether any realized contract value is due to the other firm's search intensity or dumb luck. Thus, search intensity r_i is not directly verifiable and therefore is assumed not contractible in this restricted setting.

Assumption **A2** imposes an additional restriction that firms' individual and joint term production processes have perfectly correlated shocks. This may occur because of deal-specific challenges, or perhaps due to more willingness to accept non-boilerplate contract terms (for high productivity shocks). While perfect correlation in productivity shocks is admittedly a strong assumption, it significantly simplifies the firms' expectations over term payoffs and emphasizes that the ability to innovate new terms is related to deal-specific factors. (And in any event, we explicitly relax this assumption in the subsequent section.)

Under assumptions **A1** and **A2**, we have the orderings $NP_{bs,G} > NP_{b,G}$, $NP_{s,G}$ for $G \in \{PF, PL\}$ and for all $\tau \in (0, 1)$, regardless of the choice of radii r_i. Thus implying

> **Lemma 1.** Let **A1** and **A2** hold. Then both b's and s's contract terms are always incorporated into the final contract in both games.

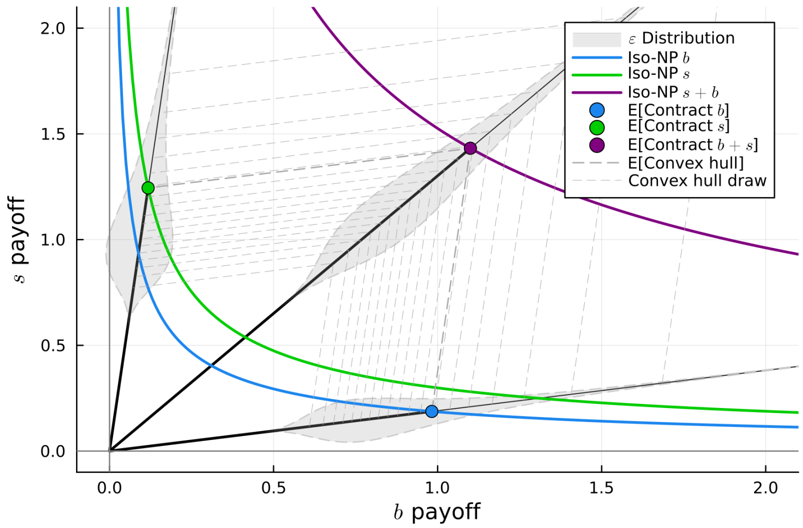
An illustration of this is presented in Figure 2 for $r_b = 1$, $r_s = 1.25$, $a_b = -0.19$, $a_s = 0.22$, and $\tau = 0.4$ for the price-first game. In Figure 2 the expected contract payoffs are plotted in the direction of search, along with the expected convex hull connecting them. Several other realizations of the convex hull are also plotted in lighter colors; these are all proportional because of assumption **A1**. The curves in Figure 2 represent the iso-Nash product lines, or the set of all contracts with equivalent Nash products. As shown in Lemma 1, the combined term yields a higher Nash product than the individual terms for any realization of ϵ.

Lemma 1 has important implications for our analysis. First, it allows us to simplify the expected term-stage payoffs from the previous section as simply the expected term-stage payoffs from the combined term m_{bs}. Second, in both of the cases we next consider, it implies that each firm's choice of r_i is not affected by the other's search costs. Since each firm's optimal search intensities are driven only by bargaining power and their own search costs, our analysis holds even when each firm's search cost is private information.

Together with general assumptions on the term cost function, these assumptions imply existence and uniqueness of each firm's investment equilibria in both the price-first and price-last games for a given set of search angles a_i. They also allow for a relative ordering of each firms' search efforts between the price-first and price-last settings.

> **Proposition 1.** Let **A1** and **A2** hold, and let a_i be fixed for $i \in \{b, s\}$. Further assume search costs $c_i(r_i, a_i)$ are increasing and strictly convex in r_i with $c_i(0, a_i) = 0$ and $\frac{\partial c_i(r_i,a_i)}{\partial r_i} \mid_{r_i=0}= 0$. Then
>
> (i) the equilibrium of the term choice stage exists and is unique for both the price-first and price-last games.
>
> (ii) the search radius for firm b is weakly higher in the price-first game than in the

Figure 2: Firm payoffs in the bartering stage of the price-first game (perfect correlation in ϵ)



Notes: None of these values necessarily represent equilibrium actions. Dashed lines represent possible convex hulls of the choice set of terms, for varying draws of ϵ with associated densities plotted around the ray corresponding to each contract as in Figure 1(c).

price-last game when $a_b \leq \frac{1}{\pi}\arctan(\frac{1-\tau}{\tau}) - 0.25$.

(iii) the search radius for firm s is weakly higher in the price-first game than in the price-last game when $a_s \geq \frac{1}{\pi}\arctan(\frac{1-\tau}{\tau}) - 0.25$.

(iv) in the price-first game, both firms search strictly less than is socially optimal except when $|a_i| = 0.25$, in which case both firms search at the socially optimal level.

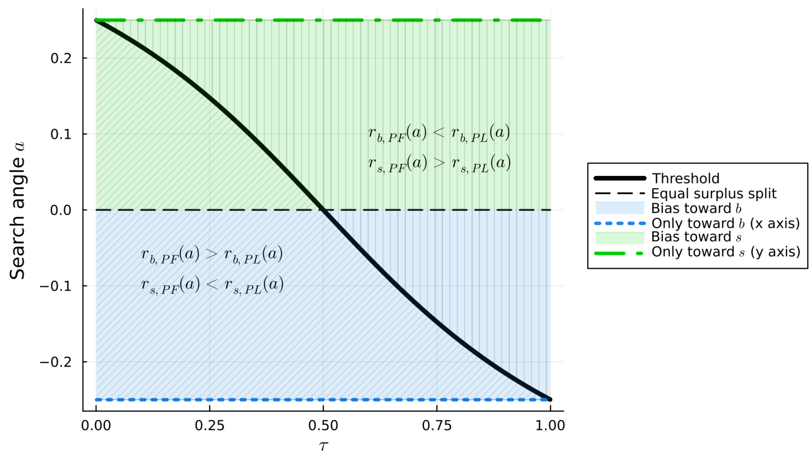
(v) in the price-last game, both firms search strictly less than is socially optimal except for when one firm has all the bargaining power ($\tau \in \{0, 1\}$), in which case only that firm searches at the socially optimal level.

(Proof in Appendix A1)

These results are straightforward: firms have a unique choice of search intensity for any given search angle. This uniqueness is important because it allows for direct comparisons between the two different versions of the contract game. In particular, firms search more intensely in price-first settings when they have relatively less bargaining power and their search angle is more biased in their own favor. Still, neither firm has incentives to search at the socially optimal level unless they can obtain all the surplus generated from their efforts; this is consistent with Hart and Moore (1988)'s finding that the first-best outcome may not be achievable when both parties make relationship-specific investments.

Figure 3 illustrates heuristically the regions in which firms search more or less in the price-first game relative to the price-last game, as a function of a given directionality a and (buyer) bargaining power τ. In the vertically hatched region, the the seller's search intensity is greatest in the price-first game while the buyer's is greatest in the price last game. In the diagonally hatched regions, these orderings reverse for both the seller and buyer.

Figure 3: Relative search intensities for firms in price-first and price-last games



Notes: This figure represents results (ii) and (iii) of Proposition 1. Each optimal search radius r_i is a posited search angle a_i. The blue and green areas represent search directions that sit on opposite sides of the 45-degree line of an angle. The vertically-hatched region corresponds to the area where b's search intensity is greatest in the price-last game, and s's search intensity is greatest in the price-first game; the opposite is depicted in the diagonally-hatched region.

To understand the intuitions behind Figure 3, consider the "northeast" quadrant where $a \in (0, 0.25)$ and $\tau \in (0.5, 1)$; that is, where the posited search angle is tilted toward the seller and the bargaining power is tilted toward the buyer. In this region, firm s searches more intensely when price is set first, since the fruits of its efforts cannot be expropriated by the powerful buyer later on. In contrast, firm b searches relatively more intensely in the price-last game, since the direction favors the seller and thus the buyer must rely on its superior bargaining power to extract price concessions. The "southwest" quadrant where $a \in (-0.25, 0)$ and $\tau \in (0, 0.5)$ is symmetric to the "northeast" quadrant, with the incentives reversed. In the remaining off-diagonal quadrants that contain the thick black curve (which represents the set of points at which each firm chooses the same search intensity in both games), bargaining power is more aligned with the posited search angle, resulting in a larger degree of near indifference (by both players) between the two approaches.

Although the qualitative characterization from Proposition 1 is interesting, additional insights (including comparative statics) are possible after assuming an explicit functional form for the firms' search cost functions. We posit a flexible and intuitive structure:

A3 The investment cost function is $c_i(r_i, a_i) = 0.5\gamma_i r_i^2 \exp(-\gamma_a a_i^2)$ where γ_i is a firm-specific cost parameter for $i \in \{b, s\}$ independent of the search angle a_i, and $\gamma_a \geq 0$ is common to both firms.

Note that under this assumption, it is cheaper to search for more biased terms, i.e. those away from the 45-degree line. While in general we may expect search for more welfare-enhancing terms to be more costly, the limiting case of $\gamma_a = 0$ (where all angles of search are equally costly) provides a useful benchmark for our analysis.

The preceding assumptions are sufficient to deliver closed form characterizations for the firms' equilibrium search decisions for any given search angles. From Lemma 1, the term-search maximization problems of firms b and s in the price-first setting are respectively

$$\max_{r_b} \quad r_b \cos(\theta(a_b)) + r_s \cos(\theta(a_s)) - 0.5\gamma_b r_b^2 \exp(-\gamma_a a_b^2)$$
$$\max_{r_s} \quad r_b \sin(\theta(a_b)) + r_s \sin(\theta(a_s)) - 0.5\gamma_s r_s^2 \exp(-\gamma_a a_s^2)$$

while for the price-last setting, the problems are

$$\max_{r_b} \quad \tau \cdot [r_b \cos(\theta(a_b)) + r_s \cos(\theta(a_s)) + r_b \sin(\theta(a_b)) + r_s \sin(\theta(a_s))] - 0.5\gamma_b r_b^2 \exp(-\gamma_a a_b^2)$$
$$\max_{r_s} \quad (1-\tau) \cdot [r_b \cos(\theta(a_b)) + r_s \cos(\theta(a_s)) + r_b \sin(\theta(a_b)) + r_s \sin(\theta(a_s))] - 0.5\gamma_s r_s^2 \exp(-\gamma_a a_s^2)$$

Evaluating the first-order conditions of the respective maximization problems yields the optimal search radii for any given search angles, as reflected in Lemma 2.

Lemma 2. Let **A1**, **A2**, and **A3** hold. Then for any a_b and a_s, the optimal search radii are as follows in each case:

(i) the price-first game

$$r^*_{b,PF}(a_b) = \frac{1}{\gamma_b} \cos(\theta(a_b)) \exp(\gamma_a a_b^2)$$
$$r^*_{s,PF}(a_s) = \frac{1}{\gamma_s} \sin(\theta(a_b)) \exp(\gamma_a a_s^2)$$

(ii) the price-last game

$$r^*_{b,PL}(a_b) = \frac{\tau}{\gamma_b} [\cos(\theta(a_b)) + \sin(\theta(a_b))] \exp(\gamma_a a_b^2)$$
$$r^*_{s,PL}(a_s) = \frac{1-\tau}{\gamma_s} [\cos(\theta(a_s)) + \sin(\theta(a_s))] \exp(\gamma_a a_s^2)$$

(iii) the socially-optimal outcome

$$r^*_{b,opt}(a_b) = \frac{1}{\gamma_b}[\cos(\theta(a_b)) + \sin(\theta(a_s))]\exp(\gamma_a a_b^2)$$
$$r^*_{s,opt}(a_s) = \frac{1}{\gamma_s}[\cos(\theta(a_s)) + \sin(\theta(a_s))]\exp(\gamma_a a_s^2)$$

Significantly, note that equilibrium investment intensity in the price-first game does not depend on the bargaining power, while in the price-last game each firm's investment intensity is increasing in its relative bargaining power.

We next turn to a comparative statics analysis under three special cases. In the first two cases, each firm's search angle is exogenously assigned, reducing the search optimization problem of each firm to choosing one variable: search intensity r_i. In the first case we explore the comparative statics in the extreme case of orthogonal (self-interested) search. In the second case we explore the comparative statics in the opposite extreme of aligned search. In the third case, we assume the firms' equilibrium search angles are endogenously chosen, but bounded in absolute value by some exogenous constant $\bar{a} \leq 0.25$.[21]

4.2 Orthogonal (self-interested) search

We first consider the special case in which both firms are constrained to search for terms that are value-enhancing for themselves but payoff-neutral for their counterparty. Subject to this constraint, we solve for each firms search intensity in both the price first and price last model. In the first comparative statics analysis we assume firm-specific search costs γ_i are the same for both firms, while in the second analysis we assume search costs differ. In both analyses we find that under orthogonal search, both firms strictly prefer the price-first game to the price-last game for all but the most extreme values of the bargaining weight τ.

In the context of our model, orthogonal search means we assume the firm search angles are $a_b = -0.25$ and $a_s = 0.25$ (that is, along the x- and y-axes in Figure 2). Since we assume search angles are exogenous for this special case, we also fix the directional cost parameter at $\gamma_a = 0$. By Lemma 1 and the assumptions above, we know that the chosen term in game G has the associated expected payoff pair $v_b(m_{bs,G}) = r_{b,G}$ and $v_s(m_{bs,G}) = r_{s,G}$. This yields

21. A fourth case, allowing unbounded endogenous search angles, is presented in the online appendix. Those results are qualitatively very similar to those of the third case.

the following equilibrium investment in the price-first and price-last games.

$$r^*_{b,PF} = \frac{1}{\gamma_b} \qquad\qquad r^*_{b,PL} = \frac{\tau}{\gamma_b}$$
$$r^*_{s,PF} = \frac{1}{\gamma_s} \qquad\qquad r^*_{s,PL} = \frac{1-\tau}{\gamma_s}$$

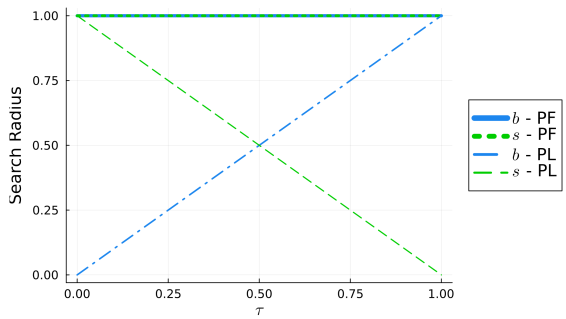
For all but extreme values of τ (i.e., $\tau = 0$ or $\tau = 1$), both firms under-invest in the price-last game relative to the price-first game. The socially optimal level of investment, given the fixed search angles, is obtained by both firms in the price-first game.

With $r^*_{i,PF}$ and $r^*_{i,PL}$ pinned down as a simple expression of exogenous parameters, we can now compare firm search intensity decisions and the corresponding firm payouts across the price first and price last models. We first compare outcomes when search costs γ_i are the same for both firms across the full range of values for the bargaining weight τ. We then compare outcomes when search costs are different for each firm by pinning down the search cost of the seller and then exploring how outcomes vary across a range of values for γ_b, the search cost of the buyer.

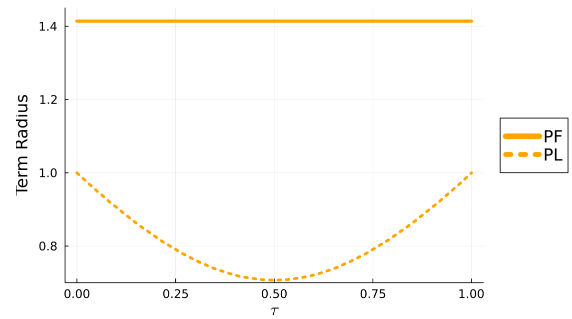
Figure 4 presents several comparative statics for both games when setting search costs equal across firms and varying τ. The default parameter values are $\gamma_b = \gamma_s = 1$, $\gamma_a = 0$, $\pi_b = 2$, and $\pi_s = 1$. Panel (a) demonstrates how, except in the extreme case of either $\tau = 0$ or $\tau = 1$, firms in the price-first setting search harder for new terms than in the price-last setting. As shown in panel (b), this yields more value-enhancing tailoring in the price-first game as well, particularly for intermediate levels of bargaining power. Regardless of the bargaining power held by each firm, panel (c) shows that both firms strictly prefer the price-first game to the price-last game for all $\tau \in (0, 1)$, yielding uniformly greater total surplus (as indicated in panel (d)).

Figure 5 examines the alternative case where firm b's bargaining power remains fixed ($\tau = 0.25$), but its search costs γ_b vary around the default seller cost parameter $\gamma_s = 1$. As before, we maintain $\pi_b = 2$ and $\pi_s = 1$. In both games, both firms ultimately benefit when the buyer faces lower search costs. As can be seen in panel (c), however, both firms gain more from a reduction in γ_b in the price-first game than in the price-last game: firm b's investment response in the price-last game is muted by its inability to recover the full fruits of the investment in the price-last game. While the relative order of the contract terms and profits is preserved between the two games (as shown in Figure 4 for $\tau = 0.25$), we note that the contract terms and price are more sensitive to changes in γ_b in the price-first game relative to the price-last game.

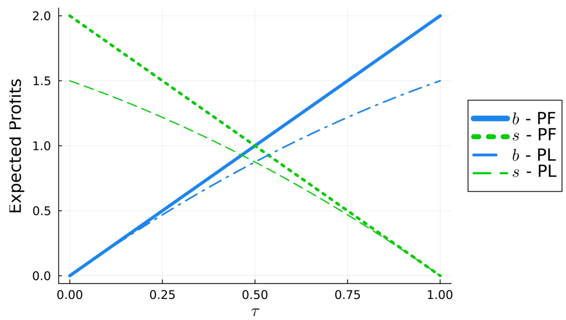
Figure 4: Comparative statics with respect to τ (orthogonal search)



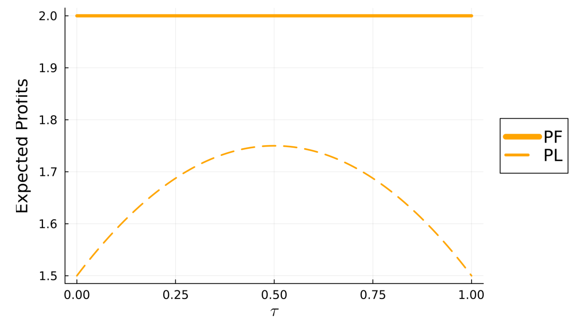
(a) Equilibrium search radius



(b) Chosen term radius



(c) Expected profits



(d) Expected combined profits

Notes: The panels on the left-hand side plot the variable of interest for both firms b and s in the price-first ("PF") and price-first ("PL") games. The panels on the right-hand side depict the corresponding outcomes of the contract in both games, for the combined firms. The plots assume $\gamma_b = \gamma_s = 1$, $\gamma_a = 0$, $\pi_b = 2$, and $\pi_s = 1$.

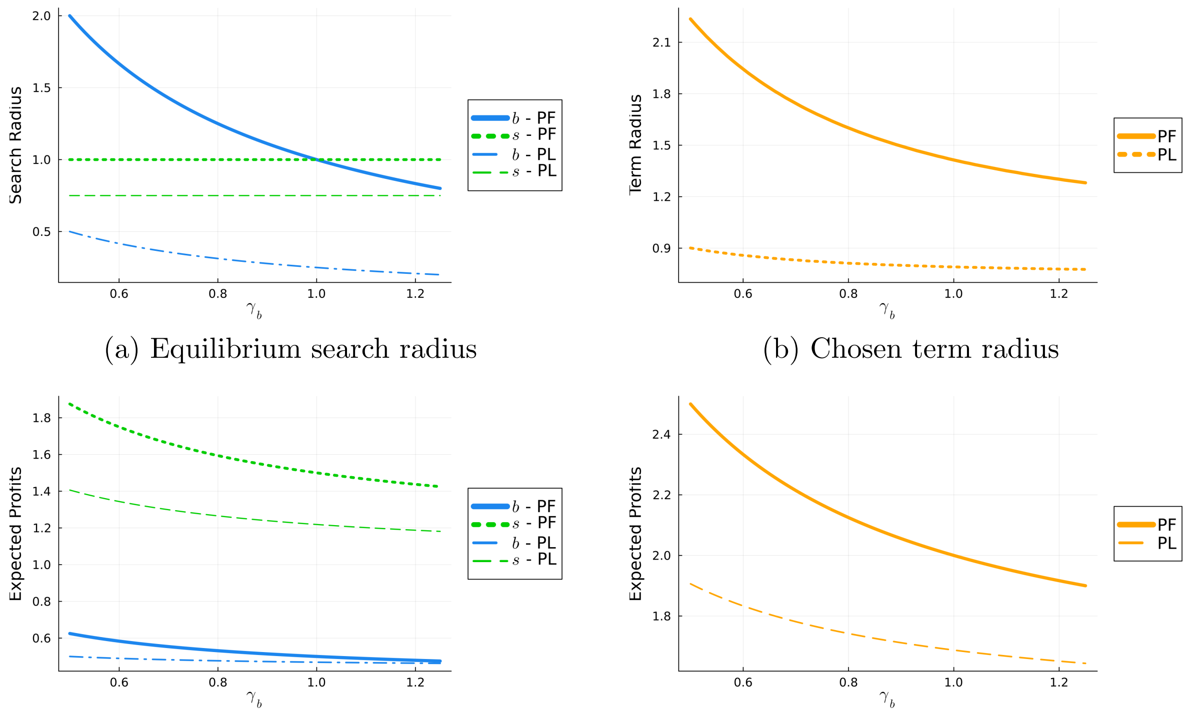
4.3 Aligned (surplus-maximizing) search

We now explore the special case where firms are constrained to search in an aligned fashion, so that their search angles are equal at $a_b = a_s = 0$ (i.e., the 45-degree line, which is the expected-surplus-maximizing angle for any fixed search radius). As before, we fix $\gamma_a = 0$ and consider the firms' search intensities and expected payoffs with both identical and heterogeneous search costs. Applying Lemma 1, we observe the expected payoffs in game G of $v_b(m_{bs,G}) = \sqrt{0.5}[r_{b,G} + r_{s,G}]$ and $v_s(m_{bs,G}) = \sqrt{0.5}[r_{b,G} + r_{s,G}]$.

From the equilibrium search intensities and the posited search angle, the firms choose the following equilibrium search radii in the price-first and price-last games.

$$r^*_{b,PF} = \frac{\sqrt{0.5}}{\gamma_b} \qquad r^*_{b,PL} = \frac{\tau\sqrt{2}}{\gamma_b}$$
$$r^*_{s,PF} = \frac{\sqrt{0.5}}{\gamma_s} \qquad r^*_{s,PL} = \frac{(1-\tau)\sqrt{2}}{\gamma_s}$$

Figure 5: Comparative statics with respect to γ_b (orthogonal search)



(a) Equilibrium search radius (b) Chosen term radius

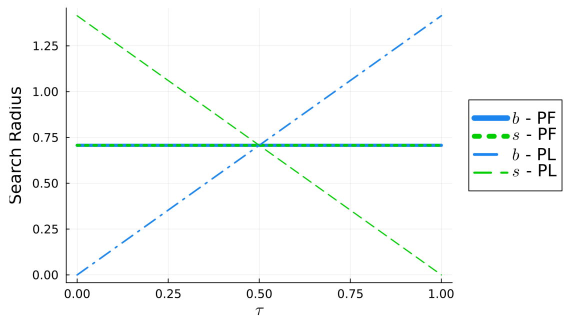
(c) Expected profits (d) Expected combined profits

Notes: The panels on the left-hand side plot the variable of interest for both firms b and s in the price-first ("PF") and price-first ("PL") games. The panels on the right-hand side depict the corresponding outcomes of the contract in both games, for the combined firms. The plots assume $\tau = 0.25$, $\gamma_s = 1$, $\gamma_a = 0$, $\pi_b = 2$, and $\pi_s = 1$.

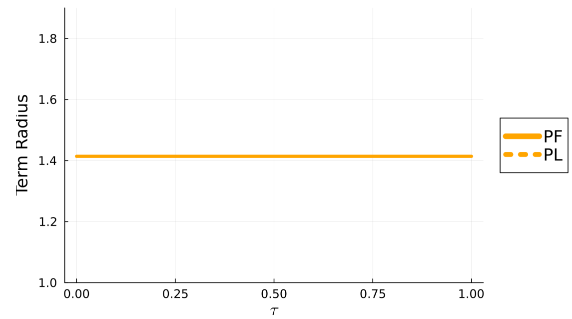
As before, the firms' respective search intensities do not turn on bargaining power in the price-first game. However, in the price-last game, the firm with more (less) bargaining power will over- (under-) invest in search relative to the price-first model; the two coincide for $\tau = 0.5$. As shown in Proposition 1, the socially optimal level of investment is only attained in the price-last game when one firm has all the bargaining power.

Figure 6 presents comparative statics in the aligned search case for varying values of τ in both games. The default values are $\gamma_b = \gamma_s = 1$, $\gamma_a = 0$, $\pi_b = 2$, and $\pi_s = 1$. Panel (a) shows that the firm with more bargaining power will choose a larger search radius in the price-last game than in the price-first game, though as indicated by panel (b) the chosen term is in expectation identical in both games. Panels (c) and (d) in turn plot the individual and combined profits for both firms in both the price-first and price-last games. In contrast to the orthogonal search setting (where the price-first game strictly dominates the price-last game), the two firms are indifferent between the two games when $\tau = 0.5$. However, their preferences diverge with unequal bargaining power. Here, the more powerful bargainer

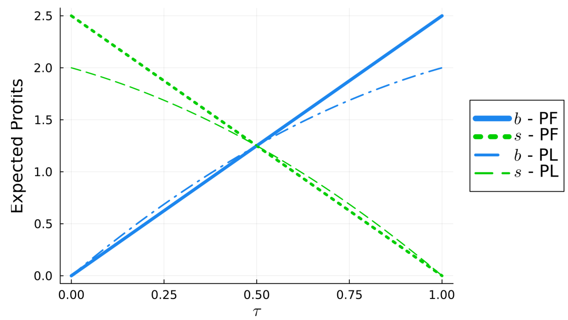
Figure 6: Comparative statics with respect to τ (aligned search)



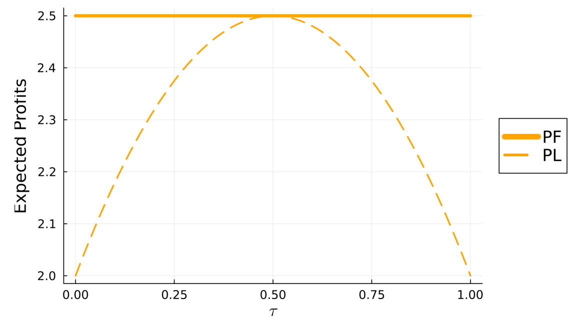
(a) Equilibrium search radius



(b) Chosen term radius



(c) Expected profits



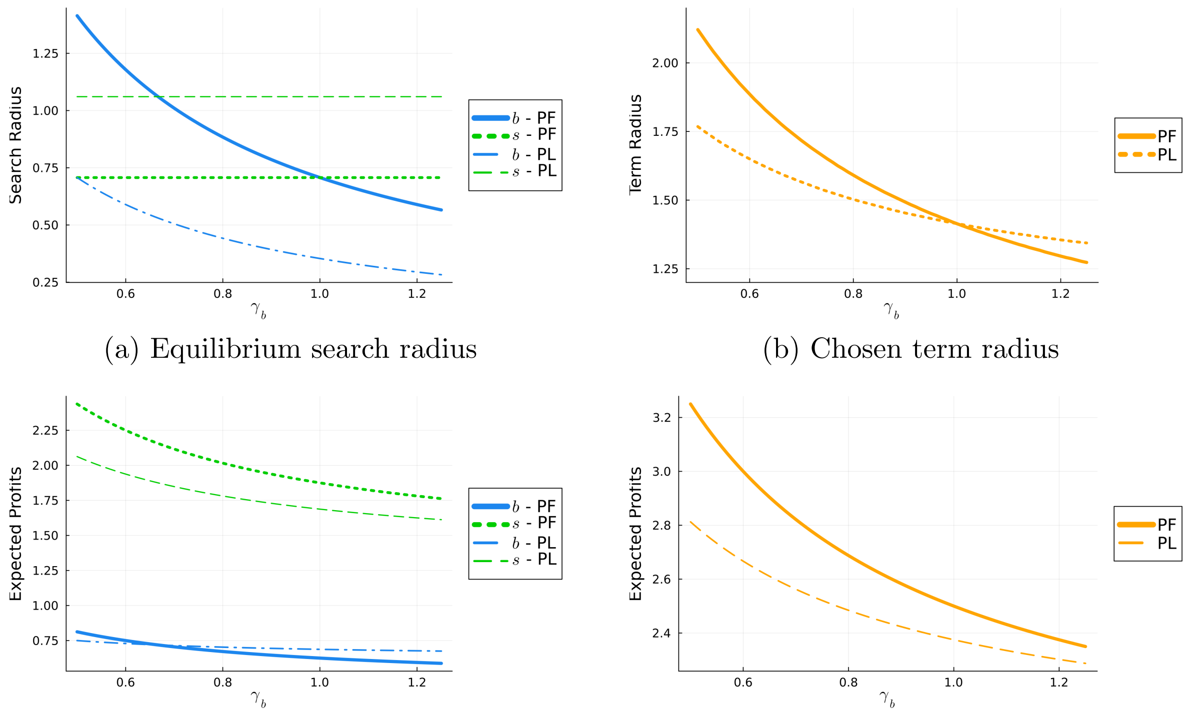
(d) Expected combined profits

Notes: The panels on the left-hand side plot the variable of interest for both firms b and s in the price-first ("PF") and price-first ("PL") games. The panels on the right-hand side depict the corresponding outcomes of the contract in both games, for the combined firms. The plots assume $\gamma_b = \gamma_s = 1$, $\gamma_a = 5$, $\pi_b = 2$, and $\pi_s = 1$.

generally prefers the price-first game, while the less powerful bargainer generally leans the other way. As panel (d) shows, however, the preferences are not zero sum, and the price-first game once again outperforms the price-last game in terms of total payoff (for all but the case of $\tau = 0.5$, where they produce equivalent payoffs).

As above, we can also illustrate comparative statics in varying heterogeneous search costs. Figure 7 again shows where firm b has low bargaining power ($\tau = 0.25$) while varying γ_b around the default seller cost parameter $\gamma_s = 1$. As before, we maintain $\pi_b = 2$ and $\pi_s = 1$. When compared to Figure 5(a), Figure 7 (a) shows that firm b's response as search costs γ_b vary are in some ways similar to the orthogonal case. At the same time, the benefit from lower search costs is more drastic for firm b in the price-first game. In fact, for low enough search costs, firm b (the weaker bargainer) no longer prefers the price-last game; this differs from the symmetric-cost bargaining setting in Figure 6(c). Panel (b) shows that the price-first game yields more investment in new terms when the weaker bargainer is the stronger searcher, while overall innovation is less sensitive to changes in firm b's cost parameter in the

Figure 7: Comparative statics with respect to γ_b (aligned search)



(a) Equilibrium search radius

(b) Chosen term radius

(c) Expected profits

(d) Expected combined profits

Notes: The panels on the left-hand side plot the variable of interest for both firms b and s in the price-first ("PF") and price-first ("PL") games. The panels on the right-hand side depict the corresponding outcomes of the contract in both games, for the combined firms. The plots assume $\tau = 0.25$, $\gamma_s = 1$, $\gamma_a = 5$, $\pi_b = 2$, and $\pi_s = 1$.

price-last game. Since both firms search along the 45-degree line in this setting, the resulting contract terms always provide equal value to both parties.

Collectively, these comparative statics analyses under both orthogonal and aligned search demonstrate that, when one accounts for the value of term innovation in the contracting process, setting price first is either weakly Pareto optimal or Kaldor Hicks optimal relative to setting price last across the full range of exogenous parameter values. This prediction, although consistent with industry practice in high stakes M&A deals, stands in stark contrast to the standard intuition in contract design that welfare is maximized when parties barter over terms first and set price last.

4.4 Partially contractible term search

We now consider the case in which firms choose both their search intensity r^* and their search angle a_i^* subject to the constraint that $|a_i^*| \leq \bar{a}$. This case weakens the assumptions of the previous two cases, in which the angles are exogenous, but still restricts the angle of search to fall within some weak subset of the first quadrant. We assume this restriction arises from some combination of professional norms or the technology by which new terms are produced, ensuring that all terms must be at least weakly value-improving for both parties.

We begin by presenting a second proposition that follows from assumptions A1, A2, and A3 when we free up the search angle to be endogenous, though still constrained to fall within the first quadrant. To build intuition, we then consider two special cases of the value of the angle cost parameter: $\gamma_a = 0$ and $\gamma_a = 5$. Building on these special cases, we then present comparative statics for the full range of values for the bargaining weight τ and a wide range of values for γ_a. These comparative statics demonstrate that the price first game induces more aggressive term innovation relative to the price last game for all values of τ and γ_a and that the price first game is Pareto and/or Kaldor-Hicks dominant relative to the price last game for a wide range of the parameter space.

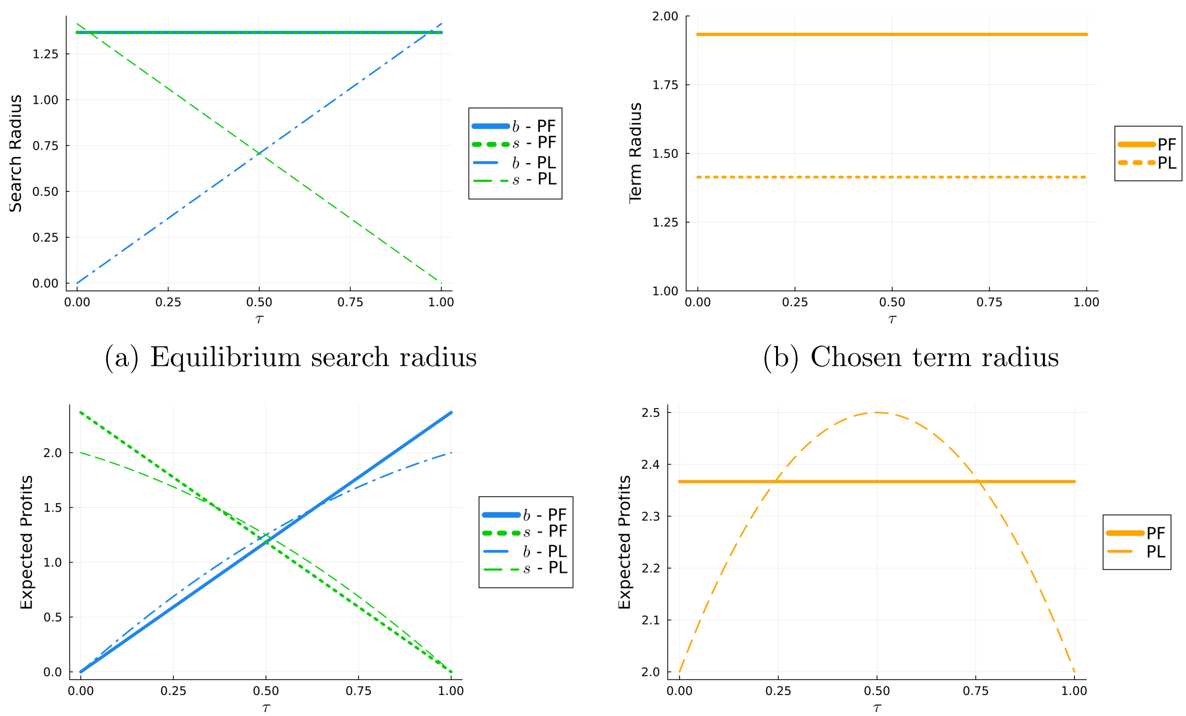
From the previous assumptions and Proposition 1, we obtain the unique optimal search radii as a function of the firms' search angles a_i. From the firms' optimal strategies for search intensity, we then solve for the optimal search angles under the constraint $|a_i| \leq \bar{a}$. We characterize this equilibrium in the following proposition.

> **Proposition 2.** Let **A1**, **A2**, and **A3** hold. An equilibrium exists for both the price-first and price-last games when firms choose both the search radius r_i and the search angle a_i. Further
>
> (i) in the price-first game, the unique optimal search angles are $a^*_{b,PF} = -\bar{a}$ and $a^*_{s,PF} = \bar{a}$.
>
> (ii) in the price-last game for $\gamma_a \in [0, \pi^2]$, there is a unique optimal search angle $a^*_{i,PL} = 0$.
>
> (iii) in the price-last game for $\gamma_a > \pi^2$, each firm has two optimal search angles that are unique up to their sign. These angles coincide with the constraint, i.e. $|a^*_{i,PL}| = \bar{a}$, for all $\gamma_a \geq \frac{\pi}{\bar{a}} \tan(\pi\bar{a})$.
>
> (Proof in Appendix A1)

The symmetric search angles exist in the price-last game because all surplus is split between both firms, so only the amount (and not the original allotments) of total surplus matters. In this case, both firms are equally compensated for their efforts when searching for terms that improve either their or their counterparts' payoffs by the same amount. To facilitate comparisons with the previous two sections, we restrict attention to equilibria where $a_b \leq 0$ and $a_s \geq 0$ (that is, each firm searches on its own side of the 45-degree line).

To fix ideas, we first examine the case where $\gamma_a = 0$ and $\bar{a} = 0.25$, i.e. when there is no penalty to searching along the 45-degree line and the entire first quadrant can be searched. In this case, the price-last game incentivizes the firms to search in the surplus-maximizing direction, since they will ultimately earn a share of the total surplus they generate. This coincides with the aligned search game considered above. In contrast, the price-first game incentivizes the firms to search in the most efficient direction to maximize their own payoff. Since the firms' search is limited only to the first quadrant, this coincides with the orthogonal search case. Evaluating firms' strategies and outcomes for various values of τ reveals that both the price-first and price-last games yield the same radius and angle for the resulting contract term, regardless of the value of τ.

Figure 8: Comparative statics with respect to τ (endogenous angle search)



(a) Equilibrium search radius

(b) Chosen term radius

(c) Expected profits

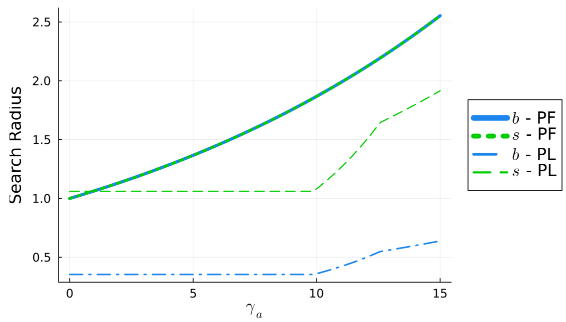
(d) Expected combined profits

Notes: The panels on the left-hand side plot the variable of interest for both firms b and s in the price-first ("PF") and price-first ("PL") games. The panels on the right-hand side depict the corresponding outcomes of the contract in both games, for the combined firms. The plots assume $\gamma_b = \gamma_s = 1$, $\gamma_a = 5$, $\bar{a} = 0.25$, $\pi_b = 2$, and $\pi_s = 1$.

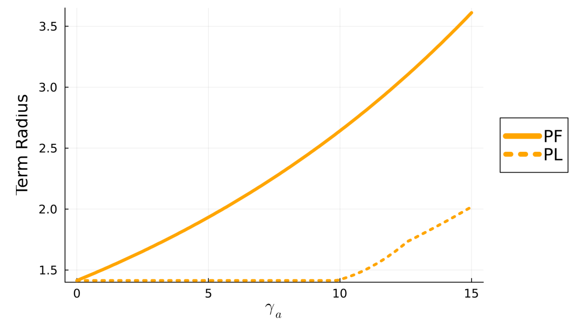
We next examine comparative statics with respect to γ_a in Figure 9. For these figures, we set $\tau = 0.25$ and $\gamma_b = \gamma_s = 1$.

To compare firms' search decisions and outcomes in this more relaxed setting, we now present several comparative statics with respect to the key parameter values in this model: the bargaining weight τ and the angle-specific cost parameter γ_a. Figure 8 illustrates the case where $\gamma_a = 5$ and $\bar{a} = 0.25$. Panel (a) shows that, with the exception of the stronger bargaining firm for extreme values of τ, the price-first game incentivizes larger firm search

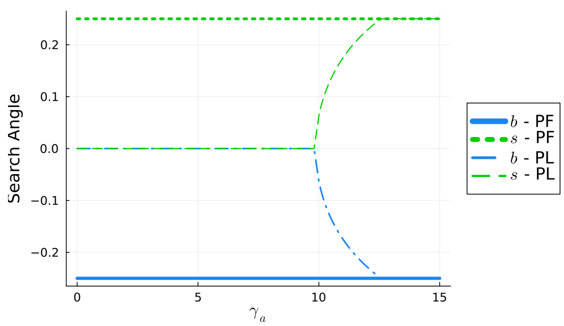
Figure 9: Comparative statics with respect to γ_a (endogenous angle search)



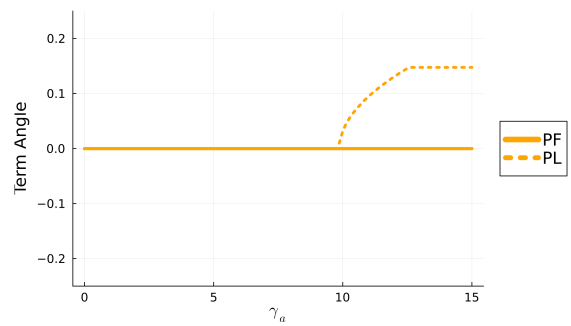
(a) Equilibrium search radius



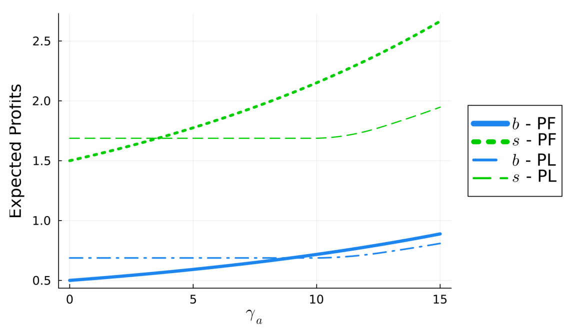
(b) Chosen term radius



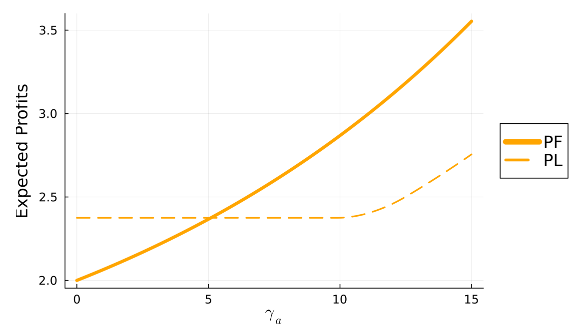
(c) Equilibrium search angle



(d) Chosen term angle



(e) Expected profits



(f) Expected combined profits

Notes: The panels on the left-hand side plot the variable of interest for both firms b and s in the price-first ("PF") and price-first ("PL") games. The panels on the right-hand side depict the corresponding outcomes of the contract in both games, for the combined firms. The plots assume $\tau = 0.25$, $\gamma_b = \gamma_s = 1$, $\pi_b = 2$, and $\pi_s = 1$.

radii than the price-last game; these imply the equilibrium radius of the chosen term is larger in the price-first game than the price-last game (see panel (b)). Panels (c) and (d) together illustrate how the higher cost to searching along the 45-degree line yields higher surplus in settings where one firm is a particularly stronger bargainer. This holds even when the cost parameter γ_a is not sufficiently large to deter firms from searching in the surplus-maximizing direction in the price-last game.

As shown in panel (a), increasing γ_a makes searching near the axes (i.e., for high $|a_i|$) cheaper, incentivizing both firms to increase their search radii whenever $a_i \neq 0$. Firms' additional efforts in increasing the search radius implies strictly more innovation in the chosen contract term in the price-first game relative to the price-last game, as shown in panel (b). Panel (c) plots the equilibrium search angles as indicated by Proposition 2, with the additional restriction that $a_b \leq 0$ and $a_s \geq 0$. When the cost of searching for surplus-maximizing (i.e., low $|a_i|$) terms is sufficiently high, the price-last game will be biased toward the firm with more bargaining power (panel (d)). Together, this implies that sufficiently high γ_a implies the price-first game generates more total surplus (see panel (f)). In fact, panel (e) indicates that the price-first game may even be strictly preferred by both firms for high enough values of γ_a.

5 Empirical validation

Although our contribution is in the first instance theoretical, it has concrete testable empirical implications as well. One such implication comes from a core prediction of our model: that transactions where prices are fixed *ab initio* should manifest greater innovation among non-price terms than comparator deals where prices are determined last. This section marshals this insight to investigate differences between typical "negotiated" deals (where price is fixed up front) and "auction" deals (where pricing is mechanically pushed to the end). Comparatively few merger transactions take place through a full-blown auction with multiple bidders; indeed, even for seller-initiated processes that invite interested prospective bidders early on, it is overwhelmingly common for a seller eventually to "go exclusive" with a preferred bidder and negotiate one on one.[22] Nevertheless, auctions do occur from time to time, and our framework offers insights to assess them empirically against negotiated deals.

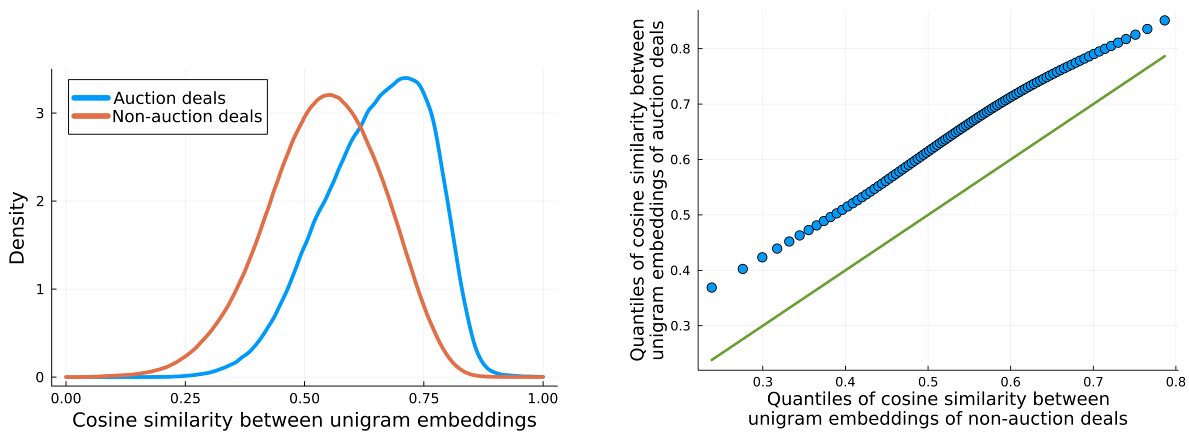
By their nature, auctions necessarily must defer pricing to the bidding process, wherein prospective buyers are invited to submit bids on a seller-prepared document that has non-financial terms locked in. While bidders are not always prohibited from submitting bid packages that "mark up" the seller's non-price terms, doing so can be risky, as noncomparability with other bidders invites a sizable discount in the bid-scoring process.[23] Thus, auction deals are one of the few circumstances in mergers and acquisitions practice where pricing is more typically pushed toward the end of the process. Our model predicts that such deals will exhibit measurably fewer markers of equilibrium search and term innovation.

22. *See* Liu, Officer, and Tu (2022).

23. See Bloomberg Law, "Auction Bid Package" (2025). Auctions *may* also skew effective bargaining power towards the seller, since buyer competition typically transfers deal surplus to the seller. This bargaining power premium, however, turns crucially on the number of bidders is small as well as the number of alternative sellers.

To assess this prediction empirically, we make use of a large publicly-available dataset of corporate transactions developed by Jennejohn, Nyarko, and Talley (2022) and Adelson et al. (2024), which contains data on definitive M&A terms for 7,931 corporate transactions valued at $100 million or more between 2000 and 2020. We restrict attention to the 7,366 deals that include a buyer-side Material Adverse Effect (MAE) clause, a critical *force majeure* (or "Act of God") provision which allows a buyer to walk from a deal if an unanticipated event has a sufficiently negative effect on the target's business, operations or financial condition.[24] To compare MAEs across deals, we extract unigram embeddings of each MAE term, using the resulting vectors to evaluate similarities across pairwise MAE dyads.[25]

Figure 10: Cosine similarity distributions of MAE clauses (auctions vs. non-auctions; unigram embeddings)



(a) Kernel density estimates (b) Empirical quantile "Q-Q" plot

Notes: Panels illustrate the distribution of pairwise cosine similarities of Material Adverse Effect (MAE) clauses in definitive merger agreements. Panel (a) plots kernel density estimates of the distances among the 6,158 non-auction deals with MAE terms and the 1,208 auction deals with MAE terms; the bandwidth is chosen using Silverman's rule. Panel (b) compares quantile realizations from these distributions against each other, parameterized at 0.01 quantile increments (with the 45-degree line superimposed in green).

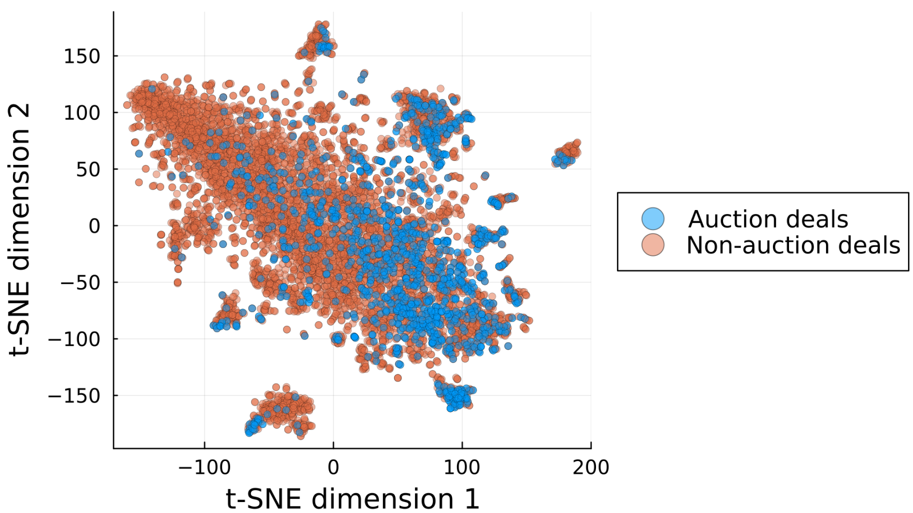
Figure 10 plots the distributions of MAE cosine similarity for all dyadic combinations of auction deals and non-auction deals (respectively). Panel (a) shows the kernel density estimates of these pairwise distances, with non-auction MAEs plotted in orange and auction

24. See generally Talley (2009).

25. We construct these measures using familiar machine learning approaches, building a lexicon of all terms that occur within each of the MAE clauses after stemming and stop-word removal. We then construct the unigram embedding of a document as the raw count of all remaining unique terms in the lexicon. (For interested readers, the Appendix provides alternative methods for embedding the MAE texts.) Finally, we compare MAE embeddings across deals using cosine similarity, which is commonly used in text comparisons because of its scale properties (see e.g. Ederer and Pellegrino 2021).

MAEs plotted in blue. While both sub-samples exhibit semantic variation, the auction MAEs are decidedly more homogeneous (i.e., "more similar"), with a distribution that skews significantly towards higher cosine similarities at every distributional quantile.[26] Panel (b) reaffirms this stochastic dominance relationship using a "Q-Q" plot, showing that at each ordered quantile of the data, auction MAEs exhibit appreciably more homogeneity than non-auction MAEs. Figure 10 presents clear evidence of measurably greater semantic variation among non-auction MAEs. We construe this variation as an artifact of greater innovation / adjustment of the MAE terms as a byproduct of more intensive search by the transacting parties when price is fixed up-front (as our model predicts).

Figure 11: Visualization of variation in MAE clauses (auctions vs. non-auctions, t-SNE of unigram embeddings)



s

Notes: This figure presents a t-distributed stochastic neighbor embedding (t-SNE) to visualize the high-dimensional MAE contracts in two-dimensional space. We first take the first 50 principal components of the normalized (zero-mean, unit-variance) unigram embeddings, and then run the t-SNE algorithm for 5,000 iterations with perplexity 50 to organize the data along two dimensions. Each contract, whether auction or non-auction, is plotted as a single point and labeled as such.

The difference in MAE clause variation across auction and non-auction structures is evident using other measurement diagnostics as well. Figure 11 presents a two-dimensional

26. The mean (standard deviation) of the non-auction deals' pairwise cosine similarities is 0.54 (0.12), compared to 0.65 (0.11) for the auction deals. The test statistics for two-sided difference-of-means and Kolmogorov-Smirnov two-sample distributional tests are 30.5 and 11.5 (respectively), which both easily satisfy any conventional measure of statistical significance. (In the absence of definitive econometric theory for dependence adjustments when studying pairwise distances along a fully-connected network—in which removing any single contract removes thousands of pairwise similarity measures—we use the number of contracts in each category rather than the number of distinct pairwise connections.)

representation of each MAE clause using t-distributed stochastic neighbor embeddings (t-SNE).[27] Once again, both sub-populations exhibit some variability, but the auction MAEs are more tightly clustered within a strict subset of the area covered by the non-auction MAEs. Similarly, while both subsets contain outliers, there appears to be discernibly more observable variation in the non-auction deals. This further supports the implications of Figure 10, that non-auction MAEs exhibit substantially more innovative customization than their auction counterparts (consistent with our model's prediction).

There are, of course, myriad more ways to analyze these data in the light of our model's predictions. While such endeavors are beyond the scope of the current paper,[28] it is striking that our model predicts—and helps explain—exactly the differential in deal term heterogeneity that we observe empirically. Under conventional contract theory, the sale process should be orthagonal to and separable from the architecture of the "pie maximizing" non-price terms (Bolton and Dewatripont 2004). Under our model, in contrast, fixing price up front (as is common in non-auction deals) invites more tailored innovation by both parties, rendering more term heterogeneity in the ensuing contract.[29]

6 Implications

The theoretical framework analyzed and explored above yields several surprising results and intuitions that relevant to both contract theory and practice. We highlight several of them below.

First, our framework demonstrates that when efficient contract structures are not obvious *a priori*, contract design protocols can play a critical role in incentivizing parties to "discover" such terms. Moreover, because bargaining power is not directly contractible, explicitly rewarding a party who discovers such terms through direct price concessions is typically infeasible, especially when that party anticipates being expropriated by a party with

27. t-SNE is a representation of higher-dimension text that maps a modest number of principal components of the original text embeddings into 2-dimensional space using a non-linear conditional probability transformation designed to accentuate data visualization and preserve proximity in higher dimensional space. See, e.g., Van der Maaten and Hinton (2008).

28. In future research, we anticipate conducting a more fulsome robustness analysis that controls for potentially unobserved attributes in observational data, such as through empirical matching techniques, analyzing "shocks" to the legal enforceability of preliminary deals, and/or running laboratory experiments.

29. It is worth observing that as the number of active bidders increases, an auction mechanism mechanically places more bargaining power in the hands of the seller. Given our model, one would predict that a rational seller who expects many bidders is more likely to embrace an auction structure when (ceteris paribus) she also tends to be the most efficient searcher for non-price terms, and can thereby enshrine such terms in the auctioned contract. Although we cannot control for this factor directly in our data, it is worth observing that this endogenous choice by the seller should *attenuate* measured differences in heterogeneity between auction and non-auction deals. And, since other factors (such as fiduciary duties and creditor arrangements) also foreordain auction structures, our empirical findings are plausibly skewed downward.

appreciable bargaining power. Rather, contract designers must fashion indirect means to encourage search for value enhancing terms. Our framework demonstrates that a seemingly inflexible protocol of cementing price first and then "bartering" non-price terms can supply an incentive compatible means for doing so across a dense space of contracting environments. Viewed in this light, the sequential inversion of canonical contract theory is not only intuitive, but it also offers a parsimonious answer to one of the long-standing puzzles from corporate transactions: why do the most sophisticated, highest-stakes business negotiators so frequently adopt a seemingly-backward "pricing first, other stuff later" approach?

In a similar vein, our framework suggests why the "price-first" approach is more typically concentrated in high-stakes contracts (such as M&A deals and large financings). The dynamics of our model operate only when the payoffs to contract innovation are sufficiently high to justify the parties' search costs. In lower-stakes contracts, by contrast, there are plausibly fewer scale economies to efficient contract design, and accordingly the benefits of incentivizing contractual tailoring are more modest.[30]

Our framework does not merely offer a solution to this longstanding puzzle, however; it also provides insights about other phenomena that observers struggle to understand. One such phenomenon is the incidence of deal failure. Although most M&A practitioners heavily prioritize the certainty of closing, between five and ten percent of publicly announced deals nonetheless fail to close.[31] The failure rate is substantially higher for preliminary deals that have signed up a term sheet but have yet to reach a definitive agreement (thought to be in the range of 20-40 percent range).[32] While deal failure no doubt has many root causes, our framework suggests an intriguing one: That a signed, price-first deal may ultimately tank because the deal was (mildly) value-destroying *from the very beginning*—and the parties had been relying on subsequent search efforts to tailor the contract language and bridge the valuation gap. In our framework, however, reliance on later search efforts is not a sure thing, even if it is a rational strategy in expectation. Accordingly, deal failure can be an equilibrium phenomenon.[33]

30. By way of comparison, Gabaix and Landier (2008) make a similar argument to predict that the highest-quality executives will sort into the largest firms, because even modest skill advantages translate into appreciable payoff differences when deployed at scale (and are reflected in higher equilibrium compensation packages as well).

31. *See* Ricks and Lin (2024) (reporting between 4 and 6 percent); Dariush Bahreini et al., "Done deal? Why many large transactions fail to cross the finish line," McKinsey & Co. (2019) (reporting 10 percent).

32. Because term sheets are not publicly disclosed, it is difficult to empirically measure deal failure before a definitive agreement is announced. The 20-40% figure, however, comports with common practitioner estimates.

33. On this note, our framework may also provide intuitions about the use of termination fees within *preliminary* (as opposed to definitive) agreements. Our model predicts that such fees can play a helpful role in incentivizing the discovery of value-enhancing non-price by the lowest cost searcher.

Relatedly, our framework helps provide insights about why courts have increasingly become attentive to the *pre-contractual* conduct of the parties. Traditionally, an aspiring contractual party enjoyed no legal rights against their counter-party unless and until a fully spelled out contract (a "Type I" agreement, in the parlance of U.S. contract law) had emerged from negotiations.[34] Until that magic moment arrived, both parties were free to walk away from (or even sabotage) the incipient deal. Over the last few decades, however, courts have warmed to the theory that, even when only a preliminary agreement is in place with price and only a few central terms (a "Type II" agreement), the parties begin bearing at least some liability exposure should they walk away.[35] In particular, a party who fails to negotiate in "good faith" may be found to have breached a preliminary agreement, and thereby subjected to damages claims. Our analysis suggests an economic rationale for this form of liability: to the extent that the parties' endogenous search efforts are at least partially contractible, their incentives to search for (and produce) payoff-enhancing terms may be efficiently augmented through some liability exposure.[36]

Our results also may bear directly on the long-simmering debate about the value transactional lawyers contribute to deals. Some commentators have suggested that deal lawyers represent little more than transactional deadwood, "churning" out contractual provisions that do little more than lard up billable hours.[37] As evidence, they note that the disclosure of the definitive merger agreement's non-price terms do not have significant effects on markets in comparison to the initial announcement of the merger some days earlier.[38] Our analysis provides an accounting not only for the value of good lawyering,[39] but also why it wouldn't be manifest in announcement-day returns: Equilibrium expectations. In our model, all parties (including the investing public) will know that a merger announcement was the product of a multi-stage equilibrium, whereby efficient terms were discovered and embraced (even if not fully disclosed alongside the bare-bones pricing terms). Since markets can price those expectations in immediately on announcement, we should not expect systematic directional returns when traders "update" their knowledge set by seeing additional granular details. To

34. *See Teachers' Insurance*, *supra* note 6.

35. *Id.*

36. *See* SIGA v. PharmaThene, 67 A.3d 330 (Del. 2013) (awarding expectation damages for breaching a Type II agreement. In a related vein, even prior to cases like *SIGA*, the emergent "promissory estoppel" doctrine may have also served to incentivize efficiency-enhancing contract design in preliminary negotiations. Hoffman v. Red Owl Stores, Inc., 26 Wis. 2d 683 (Wis. 1965).

37. *See, e.g.*, Anderson (2020) and Anderson and Manns (2013, 2017b, 2017a). Other work finds evidence of perverse bargaining outcomes arising from poorly set incentives, misaligned in ways other than the opportunistic churning of billable hours. *See, e.g.*, Clayton (2023) and Gulati and Scott (2012).

38. *See, e.g.*, Anderson and Manns (2017b).

39. In this respect, our project is consistent with the side of the debate finding evidence that transactional lawyers add value. *See, e.g.*, Gilson (1984), Coates (2016), Jennejohn, Nyarko, and Talley (2022), and Badawi, Fontenay, and Nyarko (2023)

the contrary, our model *does* predict that skilled transactional attorneys bring considerable value to a transaction, a trend that is evidently born out in market data: Evidence from recent years shows that top M&A lawyers now routinely out-earn top bankers and other financial professionals in hourly compensation rates.[40] Viewed in this light, strong legal representation is not only critical for transactional efficiency, but its importance appears to be growing.

7 Conclusion

In this article, we have presented an analytic framework that marries a bargaining model with a search game over innovative contractual provisions to reconcile a longstanding puzzle in contract design: the counterintuitive practice in complex transactions of cementing core price terms before negotiating other (non-price) terms. Our framework delivers a robust and tractable set of intuitions about when fixing price before other terms optimally incentivizes non-contractible investments by the contracting parties in contract design. We also present empirical evidence from a large corpus of M&A transactions that appears strongly consistent with our model's core results.

We are optimistic that our efforts here will serve as a metaphorical "term sheet" upon which future researchers might build to investigate how contractual design, process, and structure can efficiently interact. By modeling firms' investment decisions in the contract construction process, we allow for extensions to the case where firms can exit the negotiation process after discovering new terms. Given the empirical tractability of our model, this enables researchers to evaluate (for example) how variations in legal oversight of the negotiation process affects contract innovation. More broadly, our model can be used to more accurately estimate the value of contract terms in real-world contracts even in the absence of explicit price renegotiation.

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40. *See, e.g., Wall Street Journal*, Rock-Star Law Firms Are Billing Up to $2,500 per Hour. Clients Are Indignant (Oct. 4, 2024); *Wall Street Journal*, On Wall Street, Lawyers Make More Than Bankers Now (June 22, 2023)

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A1 Proofs and derivations

Proof of Proposition 1

The first-order conditions for b and s in PF and PL are

$$\begin{aligned}
&\{b, PF\} && \frac{\partial}{\partial r_b} c_b(r_b, a_b) = \cos(\theta(a_b)) \\
&\{s, PF\} && \frac{\partial}{\partial r_s} c_s(r_s, a_s) = \sin(\theta(a_s)) \\
&\{b, PL\} && \frac{\partial}{\partial r_b} c_b(r_b, a_b) = \tau[\cos(\theta(a_b)) + \sin(\theta(a_b))] \\
&\{s, PL\} && \frac{\partial}{\partial r_s} c_s(r_s, a_s) = (1-\tau)[\cos(\theta(a_s)) + \sin(\theta(a_s))]
\end{aligned}$$

For fixed angles a_i in the first quadrant and fixed bargaining parameter $\tau \in [0, 1]$, each of the right-hand side expressions are weakly positive constants, while the left-hand side expressions are increasing in r. Since $c_i(\cdot, a_i) = 0$ and $\frac{\partial c_i(r_i, a_i)}{\partial r_i}|_{r_i=0} = 0$, this is the unique value that satisfies the first-order conditions. Further, since the right-hand sides of the above expressions do not depend on r_i and costs are convex in r_i, the second-order conditions hold. Thus, the equilibrium exists and is unique for both the price-first and price-last games.

Further note that since c_i is increasing in the search radius r_i and search angles a_i are fixed across the two games, search intensity is higher in whichever game results in higher investment costs. For firms b and s respectively, this implies that search intensity is higher when

$$\begin{aligned}
&\{b\} && \cos(\theta(a_b)) \geq \tau[\cos(\theta(a_b)) + \sin(\theta(a_b))] \\
&\{s\} && \sin(\theta(a_s)) \geq \tau[\cos(\theta(a_s)) + \sin(\theta(a_s))]
\end{aligned}$$

or equivalently,

$$\{b\} \qquad a_b \leq \frac{1}{\pi}\arctan\left(\frac{1-\tau}{\tau}\right) - 0.25$$
$$\{s\} \qquad a_s \geq \frac{1}{\pi}\arctan\left(\frac{1-\tau}{\tau}\right) - 0.25$$

Note that the socially optimal level of investment maximizes the total surplus from producing new terms, net of search costs. That is, a social planner solves

$$\max_{r_b, r_s} \quad [r_b\cos(\theta(a_b)) + r_b\sin(\theta(a_b)) + r_s\cos(\theta(a_s)) + r_s\sin(\theta(a_s))] - c_b(r_b, a_b) - c_s(r_s, a_s)$$

Taking first-order conditions yields

$$\frac{\partial}{\partial r_b}c_b(r_b, a_b) = \cos(\theta(a_b)) + \sin(\theta(a_b))$$
$$\frac{\partial}{\partial r_s}c_s(r_s, a_s) = \sin(\theta(a_s)) + \cos(\theta(a_s))$$

The right-hand sides weakly exceed the corresponding right-hand side expressions for the first-order conditions of both firms in the both the price-first and price-last games. Since costs are convex, this implies the socially optimal level of investment is weakly higher than the investment by either firm in either game. This inequality is strict except where $\tau \in \{0, 1\}$ in the price-last game (in which case exactly one of the two firms invests at the socially optimal level) and where $|a_i| = 0.25$ in the price-first game (in which case both firms invest at the socially optimal level).

Proof of Proposition 2

Lemma 1 provides the optimal search radii in each game as functions of the search angle. Plugging in these strategies yields the following maximization problems in the price-first setting

$$\max_{a_b\colon |a_b|\leq \bar{a}} \quad \left[\frac{1}{\gamma_b}\cos(\theta(a_b))^2\exp(\gamma_a a_b^2) + \frac{1}{\gamma_s}\cos(\theta(a_s))\sin(\theta(a_s))\exp(\gamma_a a_s^2)\right] - 0.5\frac{1}{\gamma_b}\cos(\theta(a_b))^2\exp(\gamma_a a_b^2)$$

$$\max_{a_s\colon |a_s|\leq \bar{a}} \quad \left[\frac{1}{\gamma_b}\cos(\theta(a_b))\sin(\theta(a_b))\exp(\gamma_a a_b^2) + \frac{1}{\gamma_s}\sin(\theta(a_s))^2\exp(\gamma_a a_s^2)\right] - 0.5\frac{1}{\gamma_s}\sin(\theta(a_s))^2\exp(\gamma_a a_s^2)$$

and the price last setting

$$\max_{a_b\colon |a_b|\leq \bar{a}} \quad \tau \cdot \left[\frac{\tau}{\gamma_b}(1+\sin(2\theta(a_b)))\exp(\gamma_a a_b^2) + \frac{1-\tau}{\gamma_s}(1+\sin(2\theta(a_s)))\exp(\gamma_a a_s^2)\right] - 0.5\frac{\tau^2}{\gamma_b}[1+\sin(2\theta(a_b))]\exp(\gamma_a a_b^2)$$

$$\max_{a_s\colon |a_s|\leq \bar{a}} \quad (1-\tau) \cdot \left[\frac{\tau}{\gamma_b}(1+\sin(2\theta(a_b)))\exp(\gamma_a a_b^2) + \frac{1-\tau}{\gamma_s}(1+\sin(2\theta(a_s)))\exp(\gamma_a a_s^2)\right] - 0.5\frac{(1-\tau)^2}{\gamma_s}[1+\sin(2\theta(a_s))]\exp(\gamma_a a_s^2)$$

Proof of (i). We now focus on the price-first setting. First define $f_{i,PF}(a_1)$ for $i \in \{b,s\}$ as firm i's expected payoff when choosing angle a minus their expected payoff from choosing $-a$, for any fixed angle from firm i's counterpart $-i$. That is,

$$f_{b,PF}(a) = \frac{1}{2\gamma_b}\exp(\gamma_a a_1^2)\left[\cos(\theta(a))^2 - \cos(\theta(-a))^2\right]$$

$$f_{s,PF}(a) = \frac{1}{2\gamma_s}\exp(\gamma_a a_2^2)\left[\sin(\theta(a))^2 - \sin(\theta(-a))^2\right]$$

For $a > 0$, we have $f_{b,PF}(a) < 0$ ($-a$ dominates a) and $f_{s,PF}(a) > 0$ (a dominates $-a$). Thus b always chooses $a^*_{b,PF} \in [-\bar{a}, 0]$ (the "lower half" of the first quadrant) and s always chooses $a^*_{s,PF} \in [0, \bar{a}]$ (the "upper half" of the first quadrant).

Taking derivatives of the firms' profit functions with respect to their choice variables yields the following expressions

$$\{b\} \qquad \frac{1}{\gamma_b}\exp(\gamma_a a_b^2)\cdot\cos(\theta(a_b))\cdot\Big[\gamma_a a_b \cos(\theta(a_b)) - \pi\sin(\theta(a_b))\Big]$$

$$\{s\} \qquad \frac{1}{\gamma_s}\exp(\gamma_a a_s^2)\cdot\sin(\theta(a_s))\cdot\Big[\gamma_a a_s \sin(\theta(a_s)) + \pi\cos(\theta(a_s))\Big]$$

For $|a| \leq 0.25$ (i.e., a within the first quadrant), $\cos(\theta(a))$ and $\sin(\theta(a))$ at least weakly positive, which implies that the terms preceding the brackets are positive. Note that $sign(\gamma_a a_b \cos(\theta(a_b))) = sign(\gamma_a a_s \sin(\theta(a_s))) = sign(a_s)$ for a_b, a_s within the first quadrant. This implies that the derivative for firm b is weakly negative when $a_b \in [-\bar{a}, 0]$ and the derivative for firm s is weakly positive for $a_s \in [0, \bar{a}]$ (these are strict for either $\gamma_a > 0$ or $|a_i| \neq 0.25$). Therefore, for all $\gamma_a \geq 0$, it holds that the unique optimal search angles in the price-first game are $a^*_{b,PF} = -\bar{a}$ and $a^*_{s,PF} = \bar{a}$.

Proof of (ii). We now turn to the price-last setting. We have the following derivatives of the firms' maximization problems with respect to their own search angles, after applying trigonometric identities:

$$\{b\} \qquad \frac{1}{\gamma_b}\exp(\gamma_a a_b^2)\cdot\tau^2\cdot 2\pi\cos(\pi a_b)\Big[\frac{\gamma_a}{\pi}a_b\cos(\pi a_b)-\sin(\pi a_b)\Big]$$

$$\{s\} \qquad \frac{1}{\gamma_s}\exp(\gamma_a a_s^2)\cdot(1-\tau)^2\cdot 2\pi\cos(\pi a_s)\Big[\frac{\gamma_a}{\pi}a_s\cos(\pi a_s)-\sin(\pi a_s)\Big]$$

The terms preceding the brackets are weakly positive for all angles within the first quadrant (strictly so for $\tau \in (0,1)$). Therefore, the signs and zeros of these derivatives are determined solely by the signs and zeros of the bracketed terms. We now restrict attention to only the bracketed terms, which have the same functional form for both firms.

Denote $f_1(a) = \frac{a}{\pi}\cos(\pi a)$ and $f_2(a) = \sin(\pi a)$, and define $f(a,\gamma_a) \equiv \gamma_a f_1(a) + f_2(a)$. Since $f(0,\gamma_a) = 0$, the angle $a_i = 0$ always satisfies the interior first-order condition. Differentiating $f(a,\gamma_a)$ with respect to a, we have

$$\frac{1}{\pi}[\gamma_a - \pi^2]\cos(\pi a) - \gamma_a a\sin(\pi a)$$

Note that for $|a| \leq 0.25$, it holds that $\cos(\pi a) > 0$ and $a\sin(\pi a) \geq 0$ (with strict inequality for $a \neq 0$). Assume that $\gamma_a < \pi^2$, which implies that $\frac{\partial}{\partial a}f(a,\gamma_a) < 0$ for $|a| \leq \bar{a}$. Since $f(0,\gamma_a) = 0$, monotonicity of f in a implies

$$f(a,\gamma_a)\begin{cases} > 0 & \text{if } a \in [-\bar{a}, 0) \\ < 0 & \text{if } a \in (0, \bar{a}] \end{cases}$$

That is, firms' profits are increasing in a for $a < 0$ and decreasing in a for $a > 0$. This implies that the unique optimal choice of a is $a^* = 0$ for $|a| \leq 0.25$ for $\gamma_a < \pi^2$.

Proof of (iii). We now consider the case where $\gamma_a > \pi^2$. Denoting $g_{i,PL}(a,\gamma_a)$ as the derivative of firm i's maximization problem with respect to its own choice angle, note that $g_{i,PL}(a,\gamma_a) = -g_{i,PL}(-a,\gamma_a)$, implying $g'_{i,PL}(a,\gamma_a) = g'_{i,PL}(-a,\gamma_a)$. Thus, for any angle a that is optimally chosen by firm i, the angle $-a$ also satisfies both the first- and second-order conditions. We therefore restrict attention (without loss of generality) to $a \in [0, \bar{a}]$.

We first consider $a = 0$. Note that $f(0,\gamma_a) = 0$ and $\frac{\partial f(a,\gamma_a)}{a}|_{a=0} > 0$, implying by the product rule that the second derivative of firms' maximization problem is positive at $a = 0$. Thus, $a = 0$ is not optimal when $\gamma_a > \pi^2$.

We now consider $a \in (0, \bar{a}]$, and look for solutions of the first-order condition. Dividing the equation $f(a,\gamma_a) = 0$ on both sides by $\frac{a}{\pi}\cos(\pi a)$ (which is strictly positive for $0 < |a| \leq 0.25$)

yields

$$0 = \gamma_a - \frac{\pi}{a}\tan(\pi a)$$

We examine the second term, $\frac{\pi}{a}\tan(\pi a)$, to determine the behavior of this transformed first-order condition as a varies. Note that at the lower limit of this interval, we have

$$\lim_{a\downarrow 0}\frac{\pi}{a}\tan(\pi a) = \lim_{a\downarrow 0}\pi^2\sec^2(\pi a) = \pi^2$$

We examine how this function varies with a for $a > 0$. By applying trigonometric identities, we obtain

$$\frac{\partial}{\partial a}\left(\frac{\pi}{a}\tan(\pi a)\right) = \frac{\pi}{a^2}\Bigg[\pi a\sec^2(\pi a) - \tan(\pi a)\Bigg] = \frac{\pi}{a^2\cos^2(\pi a)}\Bigg[\pi a - 0.5\sin(2\pi a)\Bigg]$$

The term outside the brackets is strictly positive for $a > 0$. Defining the bracketed term in the second line as $h(a) \equiv \pi a - 0.5\sin(2\pi a)$, we have $h'(a) = \pi(1-\cos(2\pi a))$. Since $h'(a)$ is strictly positive for $a \in (0, 0.25)$ and $h(0) = 0$, we have that $h(a) > 0$ for $a \in (0, 0.25]$. Thus $\frac{\partial \frac{\pi}{a}\tan(\pi a)}{\partial a} > 0$ over the same interval. In turn, this implies that $f(\cdot, \gamma_a)$ has one zero in $(0, \bar{a}]$ if $\gamma_a \in (\pi^2, \frac{\pi}{\bar{a}}\tan(\pi\bar{a})]$.

We now prove that this zero is in fact optimal. By a similar argument as in (ii), for the angle $a^* \in (0, \bar{a}]$ such that $\gamma_a = \frac{\pi}{a^*}\tan(\pi a^*)$, the function $f(a, \gamma_a)$ is positive (and therefore the firms' profits are increasing) for any $a < a^*$ and it is negative (implying firms' profits are decreasing) for $a > a^*$. Thus a^* and $-a^*$ are optimal for the firms.

Finally, for $\gamma_a > \frac{\pi}{\bar{a}}\tan(\pi\bar{a})$, the firms' first-order condition is positive (and therefore profits are increasing in a) for all $a \in (0, 0.25]$. This implies that both the upper bound $\bar{a}$ and lower bound $-\bar{a}$ are optimal for sufficiently large γ_a.