

Python Codes

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1 Exact SFT Prediction for $R_o(t)/R_{UCZ}$

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import numpy as np import matplotlib.pyplot as plt
— Parameters —  $H_0$  in  $km/s/Mpc$  = 67.77  $Hubbleconstant[km/s/Mpc]$   $H_0 =$ 
 $H_0$  in  $km/s/Mpc * 1000.0/3.0857e22$  convert to  $1/s$   $\Omega_m = 0.317$   $\Omega_L = 0.683$ 
 $R_{today} = 4.4e26$  Present observable radius  $[m]$   $R_{ucz} = 1.4466e27$  UCZ radius  $[m]$   $ratio_{R_{today}/R_{ucz}} =$ 
 $R_{today}/R_{ucz}$ 
 $sec_{perGyr} = 3.15576e16$ 
— Integration grid —  $a_{min}, a_{max}, N = 1e-8, 50.0, 20000$   $a = np.logspace(np.log10(a_{min}), np.log10(a_{max}),$ 
 $E = np.sqrt(\Omega_m * a^{*-3} + \Omega_L)$   $integrand = 1.0/(a * E)$ 
 $cum = np.cumsum(0.5 * (integrand[1:] + integrand[:-1]) * (a[1:] - a[:-$ 
 $1]))$   $t_{sec} = np.empty_like(a)$   $t_{sec}[0] = 0.0$   $t_{sec}[1:] = (1.0/H_0) * cum$   $t_{Gyr} =$ 
 $t_{sec}/sec_{perGyr}$ 
Invert to get  $a(t)$   $t_{plot} = np.linspace(0.01, 40.0, 800)$   $a_{of_t} = np.interp(t_{plot}, t_{Gyr}, a, left =$ 
 $a[0], right = a[-1])$ 
 $Ro_{norm} = ratio_{R_{today}/R_{ucz}} * a_{of_t}$ 
Present age and crossing  $t_0 = float(np.interp(1.0, a, t_{Gyr}))$   $idx_{star} = np.argmax(Ro_{norm} >=$ 
 $1.0)$   $t_{star} = t_{plot}[idx_{star}]$  if  $np.any(Ro_{norm} >= 1.0)$  else  $None$ 
— Plot —  $plt.figure(figsize=(10,6))$   $plt.plot(t_{plot}, Ro_{norm}, color='orange', lw =$ 
 $2, label = r'R_o(t)/R_{UCZ}$  (SFT exact)')  $plt.axhline(1.0, color='red', ls='-', lw=1.5,$ 
 $label='UCZ boundary')$  if  $t_{star} :$   $plt.axvline(t_{star}, color='gray', ls=':', label =$ 
 $fr' t_* = t_{star} : .2f$  Gyr')  $plt.scatter([t_0], [ratio_{R_{today}/R_{ucz}}], color='black', marker =$ 
 $x', s = 80, label = fr' t_0 = t_0 : .2f$  Gyr')
 $plt.xlabel("Cosmic time t (Gyr)")$   $plt.ylabel("Normalized radius  $R_o(t)/R_{UCZ}$ ")$   $plt.title("Exact SFT prediction$ 
 $R_o(t)/R_{UCZ}$  (Friedmann integral)")  $plt.legend()$   $plt.grid(True)$   $plt.tight_layout()$   $plt.savefig("SFT_{Ro_{today}/R_{ucz}}$ 
 $300)$   $plt.show()$ 
```