CHAPTER 12

Finned Tube Heat Exchangers

Contents

12.1 WHY FINNED TUBE HEAT EXCHANGERS?

Finned tubes are used if the heat transfer coefficient on the outside of the tubes is very much lower than the heat transfer coefficient inside the tubes. The key point for the heat transfer coefficient is the heat conduction through the boundary layer δ on the tube.

$$\alpha = \frac{\lambda}{\delta} \quad \left(\mathrm{W/m^2\,K}\right)$$

In media having poor heat conductivity λ, for instance, gases or for high viscous materials with larger boundary film thickness δ, the heat load Q can be improved by an outer area enlargement.

$$Q = \alpha_o \times A_o \times \Delta t_o = \alpha_i \times A_i \times \Delta t_i$$

The required area ratio considering the fouling factors is calculated as follows:

$$\left(\frac{A_o}{A_i}\right)_{req} \geq \frac{\frac{1}{\alpha_o} + r_o}{\frac{1}{\alpha_i} + r_i}$$

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A_o = outer surface area of the tube (m^2/m)
A_i = inner surface area of the tube (m^2/m)
α_o = outer heat transfer coefficient (W/m^2 K)
α_i = inner heat transfer coefficient (W/m^2 K)
r_o = outer fouling factor
r_i = inner fouling factor

Example 1: Calculation of the required area ratio

$$\alpha_0 = 100\,\mathrm{W/m^2\,K} \quad \alpha_i = 1000\,\mathrm{W/m^2\,K} \quad r_0 = 0.0001 \quad r_i = 0.0002$$

$$\left(\frac{A_o}{A_i}\right)_{req} = \frac{\frac{1}{100} + 0.0001}{\frac{1}{1000} + 0.0002} = 8.5$$

Due to the poor outer heat transfer coefficient, the outer surface area should be larger than the inner surface tube area by 8.5!

Advantages of finned tubes

- higher heat load per m tube or per construction volume
- smaller equipment dimensions/less tubes
- smaller flow cross-section in the tubes/better heat transfer
- less pressure loss

A distinction is made between the following types of finned tubes:

- *high-finned cross-finned tubes* with finned heights of 10—16 mm
- *high-finned longitudinal-finned tubes* with finned heights of 12.7—25 mm
- *low-finned cross-finned tubes* with finned heights of 1.5—3 mm

The high-finned cross-finned tubes are used in air coolers and gas heat exchangers.

The longitudinal-finned tubes are used as vessel heater or in double pipe heat exchangers for high viscous media, for instance oil or bitumen, because—as opposed to the cross-finned tubes—the distances between the fins are much greater than the laminar boundary layer on the tube so that a flow between the fins is possible.

For a face flow length of 10 mm, the following laminar boundary layer thickness results depending on the Reynolds number Re at plates:

$$\mathrm{Re} = 100 \quad | \quad \delta = 4.64\,\mathrm{mm}$$
$$\mathrm{Re} = 200 \quad | \quad \delta = 3.28\,\mathrm{mm}$$
$$\mathrm{Re} = 500 \quad | \quad \delta = 2.07\,\mathrm{mm}$$

The fin spacing must be larger than the boundary layer thickness! Low-finned tubes are used to increase the efficiency in existing heat exchangers or to reduce the size of the equipment, for instance, for the refrigerant evaporation and condensation or for the decrease of the construction heights of heating bundles.

At convective heat transfer on the shell side, the Reynolds number should be >500.

12.2 WHAT PARAMETERS INFLUENCE THE EFFECTIVENESS OF FINNED TUBES?

In Figure 12.1, it can be seen how strong the outer surface area of a tube with 20 mm inner diameter increases depending on the fin height.

With increasing fin height, however, the fin efficiency η_F, which includes the temperature drop from the core tube up to the fin tip falls.

This is shown in Figure 12.2. It can also be seen how strong the heat conductivity of the fin material has an influence on the fin efficiency.

The different curves for various α_o-values in Figure 12.3 show the influence of the outer heat transfer coefficient α_o on the fin efficiency η_F.

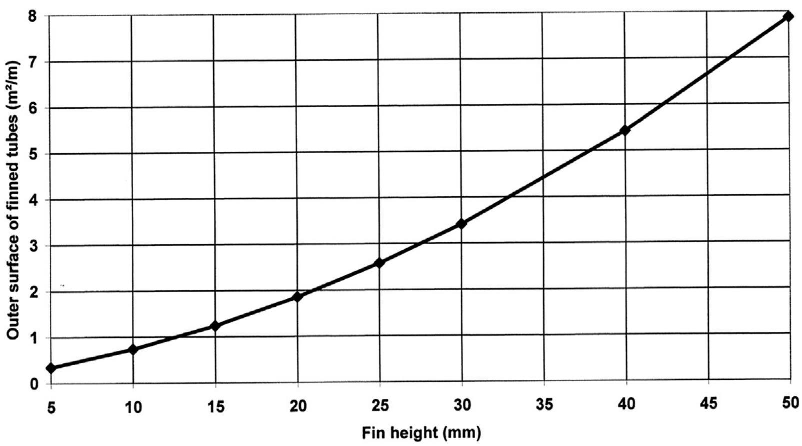

Figure 12.1 Outer tube surface area as a function of the fin height for a core tube diameter of 20 mm.

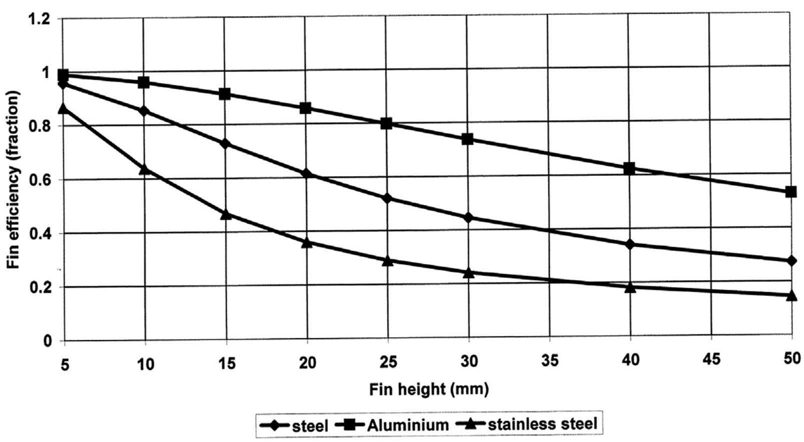

Figure 12.2 Fin efficiency η_F as a function of the fin height and the fin material.

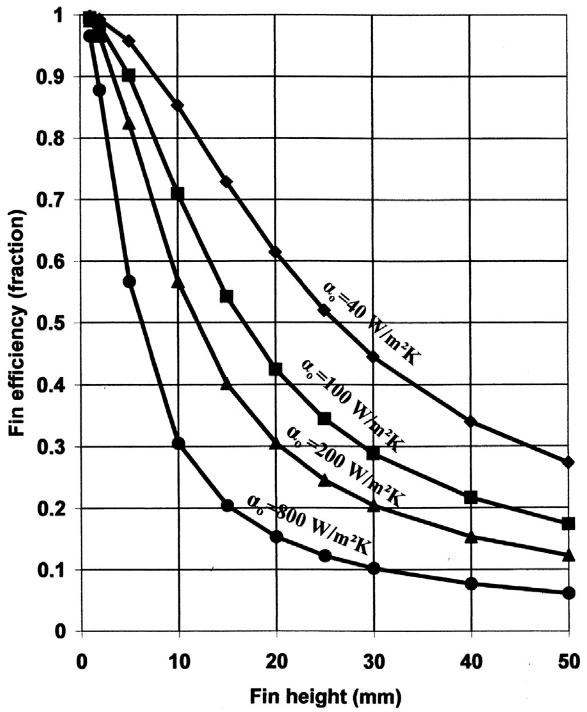

Figure 12.3 Fin efficiency η_F as a function of the fin height for different outer heat transfer coefficients of $\alpha_o = 40\text{–}800\ \text{W/m}^2\ \text{K}$.

With increasing fin height and a rising α_o-value, the fin efficiency falls.

Larger fin heights are only of interest with small heat transfer coefficients on the outer side of the tubes, for instance, with gases.

- Large fin heights are economic for small outer heat transfer coefficients up to $\alpha_o = 50\ \text{W/m}^2\ \text{K}$.
- Low-finned tubes have an advantage at higher α_o-values up to $1000\ \text{W/m}^2\ \text{K}$.

The deterioration by the fin efficiency η_F of the effective Δt for the heat transfer is only valid for the fin surface area and not for the core tube outer surface area.

This is considered in the weighted fin efficiency η_W, which is lightly better than the fin efficiency η_F. From economic view, the weighted fin efficiency should be >80%.

The following listing shows the use of different steel-finned tubes with an example:

Heat duty $Q = 100\ \text{kW}$ $\quad\Delta t = 50\ \text{K}$ $\quad\alpha_i = 3000\ \text{W/m}^2\ \text{K}$

The required tube length L for the heat duty $Q = 100$ kW is determined for different α_o-values and finned tube types.

			$\alpha_o = 50$ W/m^2 K	$\alpha_o = 500$ W/m^2 K
Type 1:	$h_F = 16$ mm	$d_C = 38$ mm	$L = 340$ m	$L = 145$ m
Type 2:	$h_F = 10$ mm	$d_C = 20$ mm	$L = 905$ m	$L = 313$ m
Type 3:	$h_F = 1.5$ mm	$d_C = 22.2$ mm	$L = 1695$ m	$L = 317$ m
Plain tube 25 × 2			$L = 4348$ m	$L = 623$ m

h_F = fin height
d_C = core tube diameter

12.3 FINNED TUBE CALCULATIONS [1–3]

12.3.1 Calculation of the fin efficiency η_F

$$\eta_F = \frac{\tanh X}{X}$$

$$\tanh X = \frac{e^X - e^{-X}}{e^X + e^{-X}}$$

$$X = h_F \times \sqrt{\frac{2 \times \alpha_o}{b_F \times \lambda_F}}$$

α_o = outer heat transfer coefficient (W/m^2 K)
λ_F = heat conductivity of the fin material (W/m K)
b_F = fin width (m)
h_F = fin height (m)

Correction for disk-finned tubes in which X_{DF} is inserted for the calculation of the fin efficiency η_F instead of X:

$$X_{DF} = X \times \left(1 + 0.35 \times \ln \frac{d_F}{d_C}\right)$$

d_F = fin diameter (mm)
d_C = core tube diameter (mm)

12.3.2 Calculation of the weighted fin efficiency η_W

$$\eta_F = \frac{\eta_F \times A_F + A_C}{A_o}$$

A_o = total outer surface area = $A_F + A_C$ (m^2)
A_C = core tube surface area (m^2)
A_F = fin surface area (m^2)

Example 2: Calculation of the fin efficiency for $\alpha_o = 40\ W/m^2\ K$

$d_C = 20$ mm	$d_F = 40$ mm	$A_o = 0.55\ m^2/m$
$b_F = 0.3$ mm	$\lambda_F = 50$ W/m K	$d_i = 16$ mm
$h_F = 10$ mm	$A_C = 0.07\ m^2/m$	$A_F = 0.48\ m^2/m$

$$X = \sqrt{\frac{2 \times 40}{0.0003 \times 50}} \times 0.01 = 0.7303$$

$$X_{DF} = 0.7303 \times \left(1 + 0.35 \times \ln\frac{40}{20}\right) = 0.9075 \qquad \tanh X_{DF} = 0.7199$$

$$\eta_F = \frac{\tanh X}{X} = \frac{0.7199}{0.9075} = 0.7933$$

$$\eta_W = \frac{0.7933 \times 0.48 + 0.07}{0.55} = 0.82$$

Example 3: The same data as in Example 2, but $\alpha_o = 300\ W/m^2\ K$

$$X = 0.01 \times \sqrt{\frac{2 \times 300}{0.0003 \times 50}} = 2$$

$$X_{DF} = 2.48$$

$$\eta_F = 0.3968$$

$$\eta_W = \frac{0.3968 \times 0.48 + 0.07}{0.55} = 0.4736$$

Example 4: The same data as in Example 2, but $\alpha_o = 1000\ W/m^2\ K$

$$X = 0.01 \times \sqrt{\frac{2 \times 1000}{0.0003 \times 50}} = 3.65$$

$$X_{DF} = 4.5373$$

$$\eta_F = 0.22 \quad \eta_W = 0.32$$

12.3.3 Calculation of the overall heat transfer coefficient U_i for the inner tube surface area A_i without fouling

$$\frac{1}{U_i} = \frac{1}{\alpha_{oi}} + \frac{1}{\alpha_i} + \frac{s}{\lambda_F}$$

$$\alpha_{oi} = \alpha_o \times \eta_F \times \frac{A_o}{A_i} = \frac{\alpha_o}{A_i} \times (\eta_F \times A_F + A_C)$$

$$Q_i = U_i \times A_i \times \Delta t \ (\mathrm{W})$$

α_o = outer heat transfer coefficient (W/m^2 K)
α_{oi} = outer heat transfer coefficient based on the core tube surface area
α_i = inner heat transfer coefficient (W/m^2 K)
λ_t = heat conductivity of the core tube (W/m K)
s = tube wall thickness of the core tube (m)
Δt = driving temperature gradient (K)
Q_i = heat duty in relation to the inner tube surface area (W)

Example 5: Calculation of the overall heat transfer coefficient U_i based on the tube *inner surface area* A_i *per m tube without fouling consideration*

$\alpha_i = 3000$ W/m^2 K	$s = 2$ mm	$\lambda_t = 50$ W/m K	$\Delta t = 30$ K
$A_i = 0.05$ m^2/m	$A_o = 0.55$ m^2/m	$A_o/A_i = 11$	

1. $\alpha_o = 40$ W/m^2 K $\eta_F = 0.7933$ $\eta_W = 0.82$

$$\alpha_{oi} = \alpha_o \times \eta_W \times \frac{A_o}{A_i} = 40 \times 0.82 \times 11 = 360 \ \mathrm{W/m^2\,K}$$

$$\alpha_{oi} = \frac{\alpha_o}{A_i} \times (\eta_F \times A_F + A_C) = \frac{40}{0.05} \times (0.7933 \times 0.48 + 0.07) = 360 \ \mathrm{W/m^2\,K}$$

$$\frac{1}{U_i} = \frac{1}{360} + \frac{1}{3000} + \frac{0.002}{50} = 0.0032$$

$$U_i = 317 \ \mathrm{W/m^2\,K}$$

$$Q_i = U_i \times A_i \times \Delta t = 317 \times 0.05 \times 30 = 475.5 \ \mathrm{W/m\ tube}$$

2. $\alpha_o = 300$ W/m^2 K $\eta_W = 0.4736$

$$\alpha_{oi} = 300 \times 0.4736 \times 11 = 1563 \ \mathrm{W/m^2\,K}$$

$$\frac{1}{U_i} = \frac{1}{1563} + \frac{1}{3000} + \frac{0.002}{50} = 0.001 \quad U_i = 987 \ \mathrm{W/m^2\,K}$$

$$Q_i = 987 \times 30 \times 0.05 = 1480.5 \ \mathrm{W/m\ tube}$$

3. $\alpha_o = 1000\ \mathrm{W/m^2\,K}$ $\eta_W = 0.32$

$$\alpha_{oi} = 1000 \times 0.32 \times 11 = 3520\ \mathrm{W/m^2\,K}$$

$$\frac{1}{U_i} = \frac{1}{3520} + \frac{1}{3000} + \frac{0.002}{50} = 0.0007 \quad U_i = 1521\ \mathrm{W/m^2\,K}$$

$$Q_i = 1521 \times 30 \times 0.05 = 2281.5\ \mathrm{W/m\ tube}$$

12.3.4 Calculation of the overall heat transfer coefficient U_o based on the outer tube area A_o without fouling

$$\frac{1}{U_o} = \frac{1}{\alpha_{oW}} + \frac{A_o}{A_i} \times \left(\frac{s}{\lambda} + \frac{1}{\alpha_i}\right)$$

$$\alpha_{oW} = \alpha_o \times \left(1 - (1 - \eta_F) \times \frac{A_F}{A_o}\right) = \frac{\alpha_o}{A_o} \times (\eta_F \times A_F + A_C) = \alpha_o \times \eta_W$$

$$Q_o = U_o \times A_o \times \Delta t \ \ (\mathrm{W})$$

α_o = outer heat transfer coefficient ($\mathrm{W/m^2\,K}$)
α_{oW} = weighted outer heat transfer coefficient under consideration of the fin efficiency for the fin surface area
Q_a = heat duty based on the outer tube surface area (W)

$$U_o = U_i \times \frac{A_i}{A_o} \qquad U_i = U_o \times \frac{A_o}{A_i}$$

Example 6: Calculation of the overall heat transfer coefficient U_a based on the outer tube surface area per *m* tube without fouling consideration

Data as in Example 5

1. $\alpha_o = 40\ \mathrm{W/m^2\,K}$ $\eta_F = 0.7933$

$$\alpha_{oW} = 40 \times \left(1 - \left(1 - 0.7933 \times \frac{0.48}{0.55}\right)\right) = 32.8\ \mathrm{W/m^2\,K}$$

$$\frac{1}{U_o} = \frac{1}{32.8} + \frac{0.55}{0.05} \times \left(\frac{0.002}{50} + \frac{1}{3000}\right) = 0.0346$$

$$U_a = 28.82\ \mathrm{W/m^2\,K} \qquad F_o = 0.55\ \mathrm{m^2/m\ tube}$$

$$Q_o = 28.82 \times 30 \times 0.55 = 475.5\ \mathrm{W/m\ tube}$$

Conversion:

$$U_i = U_o \times A_o/A_i = 28.82 \times 11 = 317\ \mathrm{W/m^2\,K}$$

$$Q_i = 317 \times 30 \times 0.05 = 475.5\ \mathrm{W/m\ tube}$$

2. $\alpha_A = 300\ \mathrm{W/m^2\,K}$ $\eta_R = 0.3963$

$$\alpha_{oW} = 300 \times \left(1 - (1 - 0.3963) \times \frac{0.48}{0.55}\right) = 141.9\ \mathrm{W/m^2\,K}$$

$$\frac{1}{U_o} = \frac{1}{141.9} + \frac{0.55}{0.05} \times \left(\frac{0.002}{50} + \frac{1}{3000}\right) = 0.0112$$

$$U_o = 89.72\ \mathrm{W/m^2\,K}$$

$$Q_a = 89.72 \times 30 \times 0.55 = 1480.4\ \mathrm{W/m\ tube}$$

Conversion : $U_i = 89.72 \times 11 = 986.9\ \mathrm{W/m^2\,K}$

$Q_i = 986.9 \times 30 \times 0.05 = 1480.35\ \mathrm{W/m\ tube}$

12.3.5 Calculation of the overall heat transfer coefficient U_o based on the finned outer surface area A_o considering the fouling factors r_o and r_i

$$\frac{1}{U_o} = \frac{\frac{1}{\alpha_o} + r_o}{\eta_F} + \frac{A_o}{A_i} \times \left(\frac{1}{\alpha_i} + r_i \times \frac{s_W}{\lambda_W}\right)$$

$$\frac{1}{U_o} = \frac{1}{\alpha_{oW}} + \frac{r_o}{\eta_W} + \frac{A_o}{A_i} \times \left(\frac{1}{\alpha_i} + \frac{s_W}{\lambda_W} + r_i\right)$$

$$Q_a = U_a \times A_A \times \Delta t$$

Example 7: Calculation of the overall heat transfer coefficient U_a considering fouling

$\alpha_o = 800\ \mathrm{W/m^2\,K}$	$\alpha_i = 6000\ \mathrm{W/m^2\,K}$	
$A_o = 0.207\ \mathrm{m^2/m}$	$A_o/A_i = 3.27$	$\eta_W = 0.9368$
$r_o = r_i = 0.00015$	$s_W = 1\ \mathrm{mm}$	$\lambda_W = 50\ \mathrm{W/m\,K}$
$\Delta t = 25\ \mathrm{K}$		

$$\alpha_{oW} = \alpha_o \times \eta_W = 800 \times 0.9368 = 749.4\ \mathrm{W/m^2\,K}$$

$$\frac{1}{U_o} = \frac{1}{749.4} + \frac{0.00015}{0.9368} + 3.27 \times \left(\frac{1}{6000} + \frac{0.001}{50} + 0.00015\right) = 0.0026$$

$$U_o = 385.3\ \mathrm{W/m^2\,K}$$

$$Q_o = U_o \times A_o \times \Delta t = 385.3 \times 0.207 \times 25 = 1994\ \mathrm{W/m\ \ tube}$$

12.3.6 Calculation of the overall heat transfer coefficient U_i for the inner tube surface area A_i considering the fouling factors r_o and r_i

$$\frac{1}{U_i} = \frac{1}{\alpha_{oi}} + \left(\frac{r_o}{\eta_W} \times \frac{A_i}{A_o}\right) + \frac{1}{\alpha_i} + r_i + \frac{s_W}{\lambda_W}$$

$$Q_i = U_i \times A_i \times \Delta t$$

Example 8: Calculation of U_i considering fouling data as in Example 7

$$\alpha_{oi} = \alpha_o \times \eta_W \times \frac{A_o}{A_i} = 800 \times 0.9368 \times 3.27 = 2450.6\ \mathrm{W/m^2\ K}$$

$$\frac{1}{U_i} = \frac{1}{2450.6} + \left(\frac{0.00015}{0.9368} \times \frac{1}{3.27}\right) + \frac{1}{6000} + 0.00015 + \frac{0.001}{50} = 0.0008$$

$$U_i = 1259.9\ \mathrm{W/m^2\ K}$$

$$Q_i = U_i \times A_i \times \Delta t = 1259.9 \times 0.0633 \times 25 = 1994\ \mathrm{W/m\ \ tube}$$

Conversion:

$$U_o = U_i \times \frac{A_i}{A_o} = 1259.9 \times \frac{1}{3.27} = 385.3\ \mathrm{W/m^2\ K}$$

$$Q_o = U_o \times A_o \times \Delta t = 385.3 \times 0.207 \times 25 = 1994\ \mathrm{W/m\ \ tube}$$

12.3.7 Fouling and Temperature Gradient

A further advantage of fin tubes is the smaller temperature drop by foulings on the enlarged outer tube surface area F_A. The fouling does not have such a strong effect as with plain tubes.

$$\Delta t_{ro} = r_o \times \frac{q}{A_o}\ (K) \quad \Delta t_{ri} = r_i \times \frac{q}{A_i}\ (\mathrm{K})$$

Δt_{ro} = temperature drop by fouling on A_o
Δt_{ri} = temperature drop by fouling on A_i
q = heat flux density (W/m tube)

Since the surface area of finned tubes is much larger than the surface area of the inner tube the fouling effect on the outer side is considerably less.

Example 9: Calculation of the temperature drop by fouling

$A_o = 0.207\ \mathrm{m^2/m}$ $\quad A_i = 0.0638\ \mathrm{m^2/m}$
$r_o = r_i = 0.0002$ $\quad U_o = 531\ \mathrm{W/m^2\ K}$ $\quad \Delta t = 25\ \mathrm{K}$

$$Q = U_o \times A_o \times \Delta t = 531 \times 0.207 \times 25 = 2748 \text{ W/m tube}$$

$$\Delta t_{ro} = 0.0002 \times \frac{2748}{0.207} = 2.6 \text{ K}$$

$$\Delta t_{ri} = 0.0002 \times \frac{2748}{0.0638} = 8.6 \text{ K}$$

The temperature drop on the large outer surface area is much smaller!

The temperature gradient at the outer side is also reduced due to the larger outer surface area.

The temperature difference between the wall and the medium is smaller so that a more careful heating with less Δt is possible, or at high temperatures on the tube outer side the material is treated carefully.

Example 10: Calculation of the temperature gradients for plain tube and finned tube

$Q = 500$ kW	$\alpha_o = 800$ W/m^2 K	$\alpha_i = 6000$ W/m^2 K	
$r_o = r_i = 0.00015$	$s_W = 1$ mm	$\lambda_W = 50$ W/m K	$\Delta t = 25$ K

1. Plain tube: 25 × 1 $\quad F_o = 0.07854$ m^2/m $\quad A_o/A_i = 1.087$

$$\frac{1}{U_o} = \frac{1}{800} + 0.00015 + \frac{0.001}{50} + 1.087 \times \left(\frac{1}{6000} + 0.00015\right)$$

$$U_o = 566.8 \text{ W/m}^2 \text{ K}$$

$$q = U_o \times \Delta t = 566.8 \times 25 = 14{,}170 \text{ W/m}^2$$

$$\Delta t_{\alpha o} = \frac{14170}{800} = 17.71\ ^\circ\text{K}$$

$$\Delta t_{ro} = 0.00015 \times 14170 = 2.13\ ^\circ\text{K}$$

$$\Delta t_{\alpha i} = \frac{14170}{6000} \times 1.087 = 2.57\ ^\circ\text{K}$$

$$\Delta t_{ri} = 0.00015 \times 14170 \times 1.087 = 2.31\ ^\circ\text{K}$$

$$\Delta t_W = \frac{0.001}{50} \times 14170 = 0.28\ ^\circ\text{K}$$

2. Finned tube Trufin S/T

$A_o = 0.207$ m^2/m $\quad A_o/A_i = 3.27 \quad \eta_W = 0.9368$

$$\frac{1}{U_o} = \frac{\frac{1}{800} + 0.00015}{0.9368} + 3.27 \times \left(\frac{1}{6000} + 0.00015 + \frac{0.001}{50}\right)$$

$$U = 385.3 \text{ W/m}^2 \text{ K}$$

$$q = U_o \times \Delta t = 385.3 \times 25 = 9632.5 \text{ W/m}^2$$

$$\Delta t_{\alpha o} = \frac{9632.5}{800 \times 0.9368} = 12.85\ ^{\circ}\mathrm{K}$$

$$\Delta t_{ro} = \frac{0.00015 \times 0632.5}{0.9368} = 1.55\ ^{\circ}\mathrm{K}$$

$$\Delta t_{\alpha i} = \frac{9632.5}{6000} \times 3.27 = 5.25\ ^{\circ}\mathrm{K}$$

$$\Delta t_{ri} = 0.00015 \times 9632.5 \times 3.27 = 4.72\ ^{\circ}\mathrm{K}$$

$$\Delta t_{W} = \frac{0.001}{50} \times 9632.5 \times 3.27 = 0.63\ ^{\circ}\mathrm{K}$$

$\Delta t_{\alpha o}$ = temperature gradient for α_o
$\Delta t_{\alpha i}$ = temperature gradient for α_i
Δt_{ro} = temperature gradient for the outer fouling r_o
Δt_{ri} = temperature gradient for the inner fouling r_i
Δt_{W} = temperature gradient for the heat conductivity through the wall

12.3.8 Comparison of the specific heat duties $U_i \times A_i$ (W/m K) of different tubes

In Figure 12.4, the specific heat duties per m tube of different tubes as a function of the outer heat transfer coefficient are shown.

The curves are valid for an inner heat transfer coefficient of $\alpha_i = 3000\ \mathrm{W/m^2\,K}$ in the tube.

- Plain steel tubes 25 × 2 made, $A_o/A_i = 1.19$
- Low-finned Trufin tubes 22.2 × 1.65 with 1.5 mm steel fin height, $A_o/A_i = 3.6$
- High-finned Applifin tubes 20 × 2 with 10 mm fin height, $A_o/A_i = 11$

It can be clearly seen that by the finning, the heat duty can be increased.

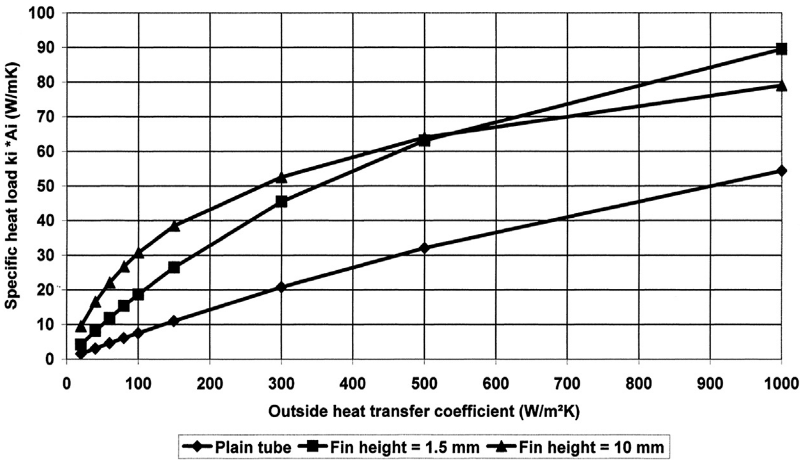

Figure 12.4 Heat load (W/m K) as a function of the outer heat transfer coefficients.

The advantage is seen with the high-finned tubes in the area of lower outer heat transfer coefficients and also the possibilities for an increase of the efficiency by the use of low-finned tubes in the area of higher outer heat transfer coefficients.

12.4 APPLICATION EXAMPLES

From the discussions so far, it follows that finned tubes are preferably applied if a low heat transfer coefficient should be compensated by a large surface area. From the following calculated examples, the preferred application of different fin tube types for specific tasks results:

- From Example 11, the advantages of low-finned tubes for heating bundles for the evaporation if low construction heights and low bundle widths are desired result.
- Example 12 emphasizes the advantages of longitudinal fin tubes for heating coils for heating storage tanks, because the heat transfer by natural convection is very poor.
- From Example 13, it is clear that the high-finned cross-finned tubes are suitable for the gas cooling or heating.
- In Example 14, it is shown that double-tube heat exchangers with finned tubes and multitube heat exchangers are very suitable for high viscous and gaseous media, because in small room large duties are possible [4].

Example 11: Evaporator heating bundle for a distillation still without consideration of fouling and the heat conduction resistance of the tube wall

$Q = 500$ kW $\quad \alpha_i = 6000$ W/m^2 K $\quad \alpha_o = 800$ W/m^2 K $\quad \Delta t = 25$ K

1. With plain tubes 25 × 1 $\quad L = 4$ m $\quad A_o = 0.0785$ m^2/m

$$\frac{1}{U_o} = \frac{1}{6000} \times \frac{25}{23} + \frac{1}{800} \quad U_o = 699 \text{ W/m}^2 \text{ K}$$

$$A_{req} = \frac{Q}{U \times \Delta t} = \frac{500{,}000}{699 \times 25} = 28.6 \text{ m}^2 = 364 \text{ m tube DN 25}$$

Arrangement: 91 Rohre, 4 m long, pitch 32 mm
Bundle width $B = 90 \times 32 + 25 = 2905$ mm

2. With low-finned Trufin tubes S/T 60-197042

$d_o = 25.4$ mm	$h_F = 1.5$ mm	$b_F = 0.3$ mm	$\lambda_F = 50$ W/m K
$A_o = 0.207$ m^2/m	$A_o/A_i = 3.27$	$d_F/d_C = 1.135$	
$X = 0.4898$	$X_{DF} = 0.5116$	$\eta_F = 0.921$	$\eta_W = 0.9368$

$$\alpha_{oW} = \eta_W \times \alpha_o = 0.9368 \times 800 = 749.4 \text{ W/m}^2\text{K}$$

$$\frac{1}{U_o} = \frac{1}{749.4} + \frac{1}{6000} \times 3.27 \quad U_o = 532 \text{ W/m}^2\text{K}$$

$$A_{req} = \frac{500{,}000}{532 \times 25} = 37.6 \text{ m}^2 = 182 \text{ m tube DN 25}$$

Arrangement: 46 tubes, 4 m long, pitch 32 mm
Bundle width $B = 45 \times 32 + 25.4 = 1465.4$ mm

3. Longitudinal-finned tubes, $d_C = 25.4$ mm, with 20 longitudinal fins 12.7 mm high,

$b_F = 0.81$ mm $A_o = 0.5869\ \mathrm{m^2/m}$ $A_o/A_i = 8.838$
$X = 2.52$ $\eta_F = 0.391$ $\eta_W = 0.4559$ $\alpha_{oW} = 364.7\ \mathrm{W/m^2\,K}$

$$\frac{1}{U_o} = \frac{1}{364.7} + \frac{1}{6000} \times 8.838 \quad U_o = 237\ \mathrm{W/m^2\,K}$$

$$A_{req} = \frac{500{,}000}{237 \times 25} = 84.4\ \mathrm{m^2} = 144\ \mathrm{m\ \ tube}$$

Arrangement: 36 tubes, 4 m long, pitch 66 mm
Bundle width $B = 35 \times 66 + 50.8 = 2360.8$ mm
Corollary: The smallest bundle results by applying the low-finned tubes!

Example 12: Heating bundle for the heating of a product in a storage tank with 10 m diameter without considering the fouling and the heat conduction resistance of the tube wall

$Q = 200$ kW $\alpha_i = 6000\ \mathrm{W/m^2\,K}$ $\alpha_o = 50\ \mathrm{W/m^2\,K}$ $\Delta t = 50$ K

Tube data as in Example 11.

1. With plain tubes 25 × 1 $A_o = 0.0785\ \mathrm{m^2}$

$$\frac{1}{U_o} = \frac{1}{6000} \times \frac{25}{23} + \frac{1}{50} \quad U_o = 49.5\ \mathrm{W/m^2\,K}$$

$$A_{req} = \frac{200{,}000}{49.5 \times 50} = 80.8\ \mathrm{m^2} = 1029\ \mathrm{m\ pipe\ \ DN\ 25}$$

Arrangement: 128 tubes, 8 m long, single pass

2. With Trufin-finned tubes $A_o = 0.207\ \mathrm{m^2/m}$ $A_o/A_i = 3.27$

$X = 0.122$ $X_{DF} = 0.1279$ $\eta_F = 0.9945$ $\eta_W = 0.9957$
$\alpha_{AW} = 0.9957 \times 50 = 49.8\ \mathrm{W/m^2\,K}$

$$\frac{1}{U_o} = \frac{1}{49.8} + \frac{1}{6000} \times 3.27 \quad U_o = 48.5\ \mathrm{W/m^2\,K}$$

$$A_{req} = \frac{200{,}000}{48.5 \times 50} = 82.5\ \mathrm{m^2} = 398\ \mathrm{m\ pipe}$$

Arrangement: 50 tubes, 8 m long, single pass

3. With longitudinal-finned tubes

$d_C = 25.4$ mm $h_F = 12.7$ mm $A_o = 0.5869\ \mathrm{m^2/m}$ $A_o/A_i = 8.838$
$\eta_F = 0.885$ $\eta_W = 0.897$ $\alpha_{oW} = 0.897 \times 50 = 44.85\ \mathrm{W/m^2\,K}$

$$\frac{1}{U_o} = \frac{1}{44.85} + 8.838 \times \frac{1}{6000} \quad U_o = 42\ \mathrm{W/m^2\,K}$$

$$A_{req} = \frac{200{,}000}{42 \times 50} = 95.2\ \mathrm{m^2} = 162\ \mathrm{m\ pipe}$$

Arrangement: 20 tubes, 8 m long, single pass
Recommendation: Heating bundle consisting of longitudinal-finned tubes

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Example 13: Gas cooling in a cross flow bundle with cooling water in the tubes

$\alpha_i = 5000$ W/m^2 K in the tubes
Gas flow rate $V_{Shell} = 904{,}000$ m^3/h $Q = 8$ Mio W $\Delta t = 40$ K
Allowable pressure loss = 3.8 mbar
Allowable bundle width = 6 m
Properties of the gases: $\varrho = 0.885$ kg/m^3 $\lambda = 0.0332$ W/m K
$\nu = 25$ mm^2/s   Pr = 0.68

1. With plain tubes 38 × 3.6 without fouling

Estimation of the required area A with $U = 90$ W/m^2 K:

$$A = \frac{Q}{U \times \Delta t} = \frac{8 \times 10^6}{90 \times 40} = 2222 \text{ m}^2$$
$$L_{req} = 19{,}000 \text{ m} = 3166 \text{ tubes with } L = 6 \text{ m}$$

Due to the low allowable pressure losses, an aligning arrangement with $P_C = P_L = 2 \times da = 76$ mm is chosen.

P_C = cross pitch of the tubes
P_L = longitudinal pitch of the tubes
Arrangement: 100 tubes one over the other, 6 m long, 36 tube rows one behind the other, aligned arrangement

Total tube length = 100 × 36 × 6 = 21,600 m   Bundle length $L = 6$ m
Total surface area = 2578 m^2 Bundle height $H = 7.6$ m
Bundle cross-sectional area $A_{bundle} = 6 \times 7.6 = 45.6$ m^2 (width × height)

Free flow cross-sectional area A_{free} for the gas through the tube bundle:

$$A_{free} = \frac{P_C - d_o}{P_C} \times H \times L = \frac{76 - 38}{76} \times 7.6 \times 6 = 22.8 \text{ m}^2$$
$$w_{gas} = \frac{904{,}000}{22.8 \times 3600} = 11 \text{ m/s}$$
$$\text{Re} = \frac{w_{gas} \times d_o}{\nu} = \frac{11 \times 0.038}{25 \times 10^{-6}} = 16{,}720$$

Pressure loss calculation according to Jakob (Section 11.3): $\zeta = 0.189$

$$\Delta P = \frac{w^2 \times \rho}{2} \times n_R \times \zeta = \frac{11 \times 0.88}{2} \times 36 \times 0.189 = 362 \text{ Pa}$$

Heat transfer calculation according to Grimison (Section 11.3):

$$\text{Nu} = 0.229 \times 16{,}720^{0.632} \times 1 = 106.8$$
$$\alpha = \frac{106.8 \times 0.0332}{0.038} = 93.3 \text{ W/m}^2 \text{ K}$$

Calculation of the overall heat transfer coefficient for steam heating with $\alpha_i = 5000$ W/m^2 K:

$$\alpha_{io} = 5000 \times \frac{30.8}{38} = 4428 \text{ W/m}^2 \text{ K}$$
$$\frac{1}{U} = \frac{1}{4428} + \frac{1}{93.3} + \frac{0.0036}{50} = 0.0011$$

$U = 90.7\ \text{W/m}^2\ \text{K}$

Heat duty Q: $Q = k \times A \times \Delta t = 90.7 \times 2578 \times 40 = 9.35$ W

Required heat duty $Q_{req} = 8$ Mio W   Fouling reserve: 16.9%
Bundle dimensions: Height = 7.6 m   Width = 6 m   Depth = 2.7 m

2. With finned tubes Applifin 95,725 without fouling

Outer surface area $A_o = 1.5\ \text{m}^2/\text{m}$ $d_C = 25.4$ mm $d_F = 57.2$ mm
Cross pitch $P_C = 57.2$ mm   Longitudinal pitch $P_L = 49.5$ mm, staggered

Arrangement: 153 tubes one over the other, 6 m long, five rows one behind the other

Total tube length $L_{tot} = 153 \times 6 \times 5 = 4590$ m tube   Total surface area $A = 6885\ \text{m}^2$
$A_{bundle} = 7.6 \times 6 = 45.6\ \text{m}^2$ $f_{proj} = 0.03\ \text{m}^2/\text{m}$ tube $A_{free} = 45.6 \times 6 \times 0.03 = 18\ \text{m}^2$

$$w = \frac{904{,}000}{3600 \times 18} = 13.95\ \text{m/s} \quad \text{Re} = \frac{13.95 \times 0.0254}{25 \times 10^{-6}} = 14{,}174$$

Pressure loss calculation with $\zeta = 0.8$ from manufacturer data

$$\Delta P = \frac{13.95^2 \times 0.88}{2} \times 5 \times 0.8 = 343\ \text{Pa}$$

Calculation of the heat transfer coefficients according to manufacturer data:

$$\frac{\text{Nu}}{\text{Pr}^{1/3}} = 0.37 \times \text{Re}^{0.553} \times f_R = 0.37 \times 14{,}174^{0.553} \times 1 = 73.1$$

$$\text{Nu} = 73.1 \times 0.68^{0.33} = 64.4$$

$$\alpha_o = \frac{64.4 \times 0.0332}{0.0254} = 84.2\ \text{W/m}^2\ \text{K}$$

Calculation of the fin efficiency:

$$X = 0.0159 \times \sqrt{\frac{2 \times 84.2}{50 \times 0.0004}} = 1.459 \quad X_{DF} = 1.8735 \quad \eta_F = 0.5092$$

$$\eta_W = \frac{0.5092 \times 1.46 + 0.04}{1.5} = 0.5222$$

$$\alpha_{oW} = 0.5222 \times 84.2 = 44\ \text{W/m}^2\ \text{K}$$

Calculation of the overall heat transfer coefficient:

$\alpha_i = 4000\ \text{W/m}^2\ \text{K}$ $\alpha_{oW} = 44\ \text{W/m}^2\ \text{K}$ = effective α-value shell side

$$\frac{1}{U} = \frac{1}{44} + 23.5 \times \left(\frac{0.0025}{50} + \frac{1}{4000}\right) = 0.0298 \quad U = 33.6\ \text{W/m}^2\text{K}$$

Heat duty $Q = 33.6 \times 6885 \times 40 = 9.25$ Mio W

Required heat load: $Q_{req} = 8$ Mio W   Reserve: 15.7%
Bundle dimensions: Height: 7.6 m   Width: 6 m   Depth: 255.2 mm

Comparison between plain tube and finned tube:

	Plain tube	Finned tube
Height (m)	7.6	7.6
Width (m)	6,-	6
Depth (m)	2.7	0.255
Tube length (m)	21,600	4590
Surface area (m^2)	2578	6885

Corollary: By the use of finned tubes, the tube length can be reduced from 21,600 to 4590 m and the bundle depth from 2.7 to 0.26 m.

Example 14: Double pipe oil cooler with an inner longitudinal-finned tube

Pipe shell with $D_i = 77.9$ mm $A_o/A_i = 5.5$

Inner tube with longitudinal fins: $d_o = 48.2$ mm, fin height 12.7 mm, surface area 0.76 m^2/m

Tube-side product: cooling water

Required heat load $Q_{req} = 7$ kW   Effective $\Delta t = 23$ °C

$s_W = 2$ mm $\lambda_W = 59$ W/m K $r_a = r_i = 0.0001$

Shell side product rate $V_{Shell} = 3.5$ m^3/h Flow cross-section $A_{free} = 0.0026$ m^2

Flow velocity $w_{Shell} = 0.374$ m/s.

Oil data: $\rho = 846$ kg/m^3 $c = 0.58$ Wh/kg K $\lambda = 0.131$ W/m K $\nu = 19$ mm^2/s

$$\mathrm{Pr} = \frac{3600 \times 19 \times 10^{-6} \times 0.58 \times 846}{0.131} = 256$$

$$\mathrm{Re} = \frac{0.374 \times 0.0104}{19 \times 10^{-6}} = 204.7 \rightarrow \text{Laminar Flow}$$

Calculation of the heat transfer coefficient:

$$\mathrm{Nu} = 1.86 \times \left(204.7 \times 256 \times \frac{0.0104}{6}\right)^{1/3} = 8.36$$

$$\alpha_o = \frac{8.36 \times 0.131}{0.0104} = 105\ \mathrm{W/m^2\ K}$$

$$X = 0.0127 \times \sqrt{\frac{2 \times 105}{0.001 \times 50}} = 0.823 \quad \eta_F = 0.822$$

$$\eta_W = \frac{0.822 \times 0.61 + 0.15}{0.76} = 0.857$$

$$\alpha_{oW} = 0.857 \times 105 = 90 \quad \mathrm{W/m^2\ K}$$

Tube side: 3 m^3/h cooling water.

$d_i = 44.6$ mm $w_T = 0.54$ m/s   Pr = 6.94 $\lambda = 0.604$ W/m K

$$\mathrm{Re} = \frac{0.54 \times 0.0442}{1 \times 10^{-6}} = 23{,}868$$

$$\mathrm{Nu} = 0.023 \times 23{,}868^{0.8} \times 6.94^{1/3} = 139.45$$

$$\alpha_T = \frac{139.45 \times 0.604}{0.0442} = 1905\ \mathrm{W/m^2\ K}$$

Calculation of the overall heat transfer coefficient U_o:

$$\frac{1}{U_o} = \frac{1}{\alpha_{oW}} + \frac{r_o}{\eta_W} + \frac{A_o}{A_i} \times \left(\frac{1}{\alpha_i} + \frac{s_W}{\lambda_W} + r_i\right)$$

$$\frac{1}{U_o} = \frac{1}{90} + \frac{0.0001}{0.857} + 5.5 \times \left(\frac{0.002}{59} + \frac{1}{1905} + 0.0001\right) \quad U_o = 68.1\ \mathrm{W/m^2\,K}$$

$$\text{Required area } A_{req} = \frac{7000}{23 \times 68.1} = 4.47\ \mathrm{m^2}$$

$$\text{Required tube length } L_{req} = \frac{4.47}{0.76} = 5.88\ \text{m pipe}$$

Alternative calculation for a multipipe heat exchanger with plain tubes:
Shell diameter $D_i = 84$ mm with seven inner tubes 20 × 2
Shell side: 3.5 m^3/h oil

Flow cross-section $f_{Shell} = 0.0033\ m^2$ De = 0.0304 m Surface area = 0.4398 m^2/m
$w_{Shell} = 0.295$ m/s Re = 471

$$\mathrm{Nu} = 1.86 \times \left(471 \times 256 \times \frac{0.0304}{6}\right) = 15.78$$

$$\alpha_o = \frac{15.78 \times 0.131}{0.0304} = 68\ \mathrm{W/m^2\,K}$$

Tube side: 3 m^3/h cooling water $w_T = 0.59$ m/s Re = 18,009
Nu = 111.3 $\alpha_i = 4202$ W/m^2 K $\alpha_{io} = 3362$ W/m^2 K

Calculation of the overall heat transfer coefficient U:

$$\frac{1}{U} = \frac{1}{3362} + \frac{1}{68} + \frac{0.002}{50} + 0.0002$$

$$U = 65.6\ \mathrm{W/m^2\,K}$$

$$\text{Required area } A_{req} = \frac{7000}{65.6 \times 23} = 4.64\ \mathrm{m^2}$$

$$\text{Required length } L_{req} = \frac{4.64}{0.4398} = 10.6\ \mathrm{m}$$

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