Kevin Hoy, Joshua Kingsbury, Avidaan Srivastava, Logan Steele **Spring '22**

Measuring Sodium and Iron Abundance in the Sun

Introduction

As of the writing of this work 5,009 confirmed exoplanets have been detected [1]. The information we've gleaned from these planets so far paints a diverse picture of the formation of star systems, most appearing quite different from our own. With limitations for each type of detection method, our catalogue of exoplanets is incredibly incomplete, but there is a wealth of information we can glean about these exoplanets and star systems to inform models of exoplanet formation and the tendencies we would expect to see.

Since planets and their star form from the same primordial dust cloud and should have had access to the same elemental abundances, we would expect to see this correlated with their compositions. This information is much easier to obtain in the form of stellar spectra. By analyzing the stellar composition, and applying models to planet formation based on the materials that would have been available, we can learn more about the processes that may have lead to the planets we observe, their potential properties, and about the planets we can't yet detect.

In this exercise we use spectra measured from the Sun and attempt to compute the stellar abundances of Na and Fe from one of their absorption lines. After calculating the total number densities, we check how well our values match to the accepted values for the Sun, and then display the calculated abundances in formats commonly used to multiple disciplines.

Methods

By zooming in on the spectrum we can focus on distinct absorption lines and begin by examining the elemental abundance causing the dip. Our goal is to methodically extrapolate and tally the total counts of each element starting from an initial estimate of atoms in their ground state. The first step is calculating the Equivalent Width (EW) of one of the absorption lines. The EW is defined to be the width of a box of the same height as the continuum emission and containing the same area in the spectral line. Figure 1 below shows the spectra highlighting the Sodium 5890 (bounded) and 5896 doublet. Using a range of [5887.5, 5892.5] Å for Na and [5167.3, 5167.95] Å for Fe and taking Riemann sums we arrive at EW of 0.83 Å and 0.15 Å, respectively.

[1] `https://exoplanetarchive.ipac.caltech.edu/`

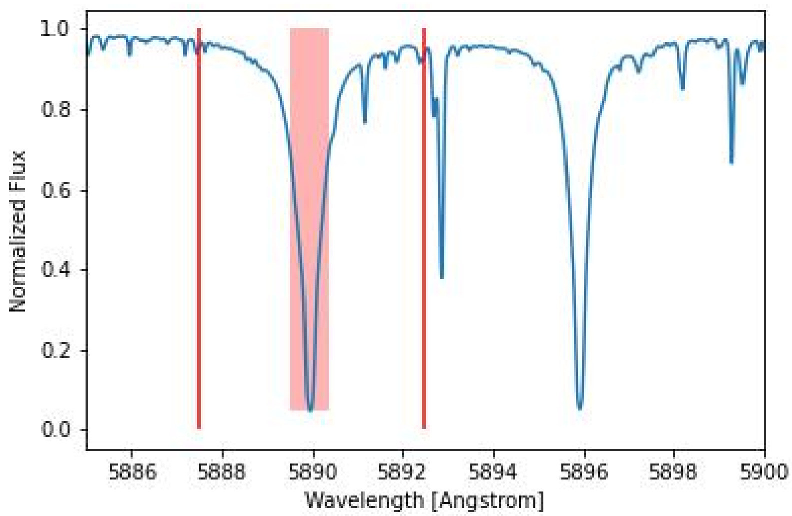

Figure 1: Spectrum featuring the Sodium doublet. Vertical red lines denote the range around the absorption dip used to calculate the equivalent width; the red box features the same area of the absorption line within the range, at the same depth and equivalent width.

Using a curve of growth plot (Appendix A; Figure 3) we can take the log of the EW and relative wavelength on the y-axis to obtain a value on the x-axis we can use to calculate the number of atoms in an absorbing, or ground, state. The parameter f is the oscillator strength obtained from the BASS2000 Solar Survey Archive [2] and unique to each line.

$$\log\left(\frac{EW}{\lambda}\right) \sim \log\left(N_1 f \frac{\lambda}{5000\mathring{A}}\right)$$

Ground state density for Na:

$$\log\left(\frac{0.83\text{Å}}{5890.0\text{Å}}\right) \approx -3.85 \sim \log\left(N_1\ 0.65\ \frac{5890.0\text{Å}}{5000\text{Å}}\right) \approx 14.8$$

$$\log\left(N_1\ 0.65\ \frac{5890.0\text{Å}}{5000\text{Å}}\right) \approx 14.8 \Rightarrow N_1 \approx 8.24\text{E}14 \text{ atoms cm}^2$$

Ground state density for Fe:

$$-4.53 \sim \log\left(N_1\ 7.09\text{E}-6\ \frac{5166.3\text{Å}}{5000\text{Å}}\right) \approx 13.4 \Rightarrow N_1 \approx 3.43\text{E}18 \text{ atoms cm}^2$$

Once we've obtained values for ground state quantities, our next task is to find the number of atoms in an excited state. The Boltzmann equation below allows us to calculate

[2] https://bass2000.obspm.fr/solar_spect.php

the ratio between atoms in the excited state (N_2) to those in the ground state (N_1). In this equation: g_i is the number of degenerate states ($g_i = 2i^2$), the Boltzmann constant $k = 1.38E{-}23$ m^2 kg s^{-2} K^{-1}, the temperature $T = 5770$ K, and E_i is the energy of the respective state. The difference in energies was calculated as $E_2 - E_1 = \frac{hc}{\lambda}$ with Planck's constant $h = 6.63\text{E} - 34$ m^2 kg s^{-1} and the speed of light $c = 3.00\text{E}8$ m s^{-1}.

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(-\frac{E_2 - E_1}{kT}\right)$$

The total number of ground state and excited atoms provides a value for the number density of neutral atoms (Na_I). We still need to account for the ionized atoms (Na_{II}), and the Saha equation below is used to find the ratio between ionized and neutral atoms. Parameters below include: electron pressure $P_e = n_e kT = 1$N m^{-2}, electron mass $m_e =$ 9.11E$-$31 kg, partition functions Z_I & Z_{II}, and the ionization energy χ.

$$\frac{Na_{II}}{Na_I} = \frac{2kT}{P_e}\frac{Z_{II}}{Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi}{kT}\right)$$

Table 1 below shows the values used for each element, and the calculated values will be presented in the Results section.

	Boltzmann			Saha		
Elements	g_1	g_2	$E_2 - E_1$ (eV)E$-$19	Z_I	Z_{II}	χ (eV)
Na	3p	3s	3.37	2.4	1.0	5.1
Fe	4p	4s	3.85	2.4	1.0	7.9

Table 1

Finally, once we have all of our constituent abundances (ground/excited/neutral/ionized), we can calculate a total column density as below, and convert it to the preferred abundance measures by discipline.

$$N_{tot} = N_1 \times \left(1 + \frac{N_2}{N_1}\right) \times \left(1 + \frac{Na_{II}}{Na_I}\right)$$

Physicist's Molar Ratio (assuming column density towards Sun $N_H = 6.6\text{E}23$):

$$N_H/N_x$$

Galactic Astronomer's Relative Abundance (using log abundance of Hydrogen = 12):

$$12 + \log(N_H/N_x)$$

Stellar Astronomer's Solar Abundance Ratio (using log abundance of Hydrogen = 12):

$$\log\left(\frac{N_H/N_x}{10^{\text{Accepted Solar Relative Abundance}-12}}\right)$$

Results

Our calculated number densities and abundances for both iron and sodium are below in Table 2. In calculating the solar abundance ratios, we used the accepted values of 7.5 for iron and 6.3 for sodium [1]. Since we are using solar spectra we would expect our calculated solar abundance to be zero, instead of super-solar as we found. We found a large ratio of ionized sodium to neutral as we would expect at such temperatures as the Sun, and number densities of elements less than that of Hydrogen, which also makes sense.

	Number Densities				Abundances		
Elements	Ground (atoms cm^{-2})	Excited/Ground	Ionized/Neutral	Total (atoms cm^{-2})	Molar Ratio N_x/N_H	Relative Abundance $12 + \log(N_x/N_H)$	Solar Abundance [x/H]
Na	8.24E14	0.043	2482.694	2.14E18	3.2E-6	6.51	0.21
Fe	3.43E18	0.024	8.899	3.48E19	5.27E-5	7.72	0.22

Table 2

Conclusion

We were able to reasonably estimate the elemental abundances for iron and sodium from solar spectra, demonstrating a capability to apply the procedure to stellar spectra in general. Moving forward we would pay closer attention to error analysis to try and determine the discrepancy between our values and the accepted values for solar abundances. One factor might be more careful and analytical selection of the spectral range used to calculate the EW. Additionally, more rigorous sourcing and vetting of our input parameters may have contributed to a smaller error.

Contributers

Avidaan - Sodium Research
Logan - Iron Research
Kevin - Presentation
Joshua - Report

Appendix A

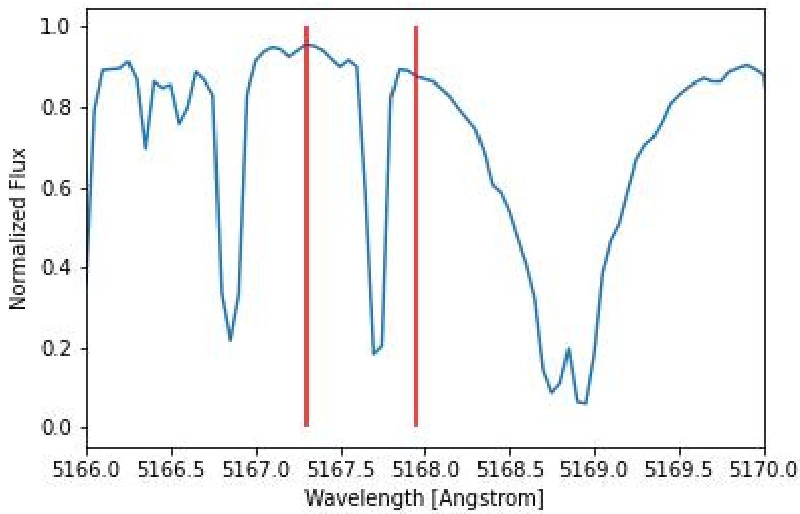

Figure 2: Spectrum featuring an Iron line. Vertical red lines denote the range around the absorption dip used to calculate the effective width.

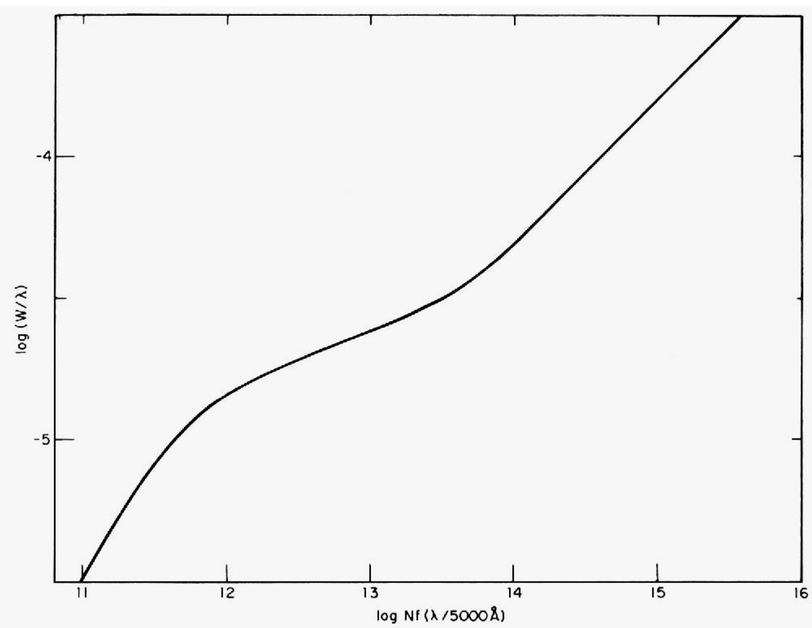

Figure 9.22 A general curve of growth for the Sun. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)

Figure 3

References

[1] Palme, Lodders, Jones. Solar System Abundances of the Elements. *Elsevier*. Treatise on Geochemistry, Vol. 2 (15:36). 2014.