FORMULAS FROM ELEMENTARY MATHEMATICS

A. **Algebra.**

1. *Laws of Exponents.*

$$a^m a^n = a^{m+n}, \quad (ab)^m = a^m b^m, \quad (a^m)^n = a^{mn}, \quad a^{m/n} = \sqrt[n]{a^m}.$$

If $a \neq 0$: $\dfrac{a^m}{a^n} = a^{m-n}, \quad a^0 = 1, \quad a^{-m} = \dfrac{1}{a^m}.$

2. *Zero.*

$a \cdot 0 = 0 \cdot a = 0$ for any finite number a.

If $a \neq 0$, $\quad \dfrac{0}{a} = 0, \quad 0^a = 0, \quad a^0 = 1.$

Division by zero is not defined.

3. *Fractions.*

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}, \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \quad \frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{d}{c}, \quad \frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b}.$$

$$\frac{(a/b) + (c/d)}{(e/f) + (g/h)} = \frac{(a/b) + (c/d)}{(e/f) + (g/h)} \cdot \frac{bdfh}{bdfh} = \frac{(ad + bc)fh}{(eh + fg)bd}.$$

4. *Binomial Theorem, for n = positive integer.*

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1 \cdot 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}b^3 + ... + nab^{n-1} + b^n.$$

5. *Proportionality factor.* If y is proportional to x, or y varies directly as x, then $y = kx$ for some constant k, called the *proportionality factor*.

6. *Remainder Theorem and Factor Theorem.* If the polynomial $f(x)$ is divided by $x - r$ until a remainder R independent of x is obtained, then $R = f(r)$. In particular, $x - r$ is a *factor of* $f(x)$ if and only if r is a *root* of the equation $f(x) = 0$.

7. *Quadratic formula.*   If $a \neq 0$, the roots of the equation $ax^2 + bx + c = 0$ are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

B. **Geometry.** (A = area, B = area of base, C = circumference, S = lateral area or surface area, V = volume.)

1. *Triangle.* $A = \frac{1}{2}bh.$

2. *Similar Trianles.* $\dfrac{a'}{a} = \dfrac{b'}{b} = \dfrac{c'}{c}.$

3. *Theorem of Pythagoras.* $c^2 = a^2 + b^2.$

4. *Parallelogram.* $A = bh.$

5. *Trapezoid.* $A = \frac{1}{2}(a + b)h.$

6. *Circle.* $A = \pi r^2, \quad C = 2\pi r.$

7. *Any cylinder or prism with parallel bases.* $V = Bh.$

8. *Right circular cylinder.* $V = \pi r^2 h, \quad S = \pi rs.$

9. *Any cone or pyramid.* $V = \frac{1}{3}Bh.$

10. *Right circular cone.* $V = \frac{1}{3}\pi r^2 h, \quad S = \pi rs.$

11. *Sphere.* $V = \frac{4}{3}\pi r^3, \quad A = 4\pi r^2.$

C. **Trigonometry.**

1. *Definitions and fundamental identities.*

$$sin(\theta) = \frac{y}{r} = \frac{1}{csc(\theta)}.$$

$$cos(\theta) = \frac{x}{r} = \frac{1}{sec(\theta)}.$$

$$tan(\theta) = \frac{y}{x} = \frac{1}{cot(\theta)}.$$

$$sin(-\theta) = -sin(\theta), \quad cos(-\theta) = cos(\theta).$$

$$sin^2\theta + cos^2\theta = 1.$$

$$sin(2\theta) = 2(sin\ \theta\ cos\ \theta)\ , \quad cos(2\theta) = cos^2\ \theta - sin^2\ \theta.$$

$$sin(\alpha + \beta) = sin\ \alpha\ cos\ \beta + cos\ \alpha\ sin\ \beta.$$

$$cos(\alpha + \beta) = cos\ \alpha\ cos\ \beta - sin\ \alpha\ sin\ \beta.$$

2. *Angles and sides of triangle.*

Law of cosines: $a^2 = b^2 + c^2 - 2bc\ cos\ A.$

Law of sines: $\dfrac{sin\ A}{a} = \dfrac{sin\ B}{b} = \dfrac{sin\ C}{c}.$

Area $= \dfrac{1}{2}bc\ sin\ A = \dfrac{1}{2}ac\ sin\ B = \dfrac{1}{2}ab\ sin\ C.$