MIT 6.3200 Problem Set 2

Joshua Pereira

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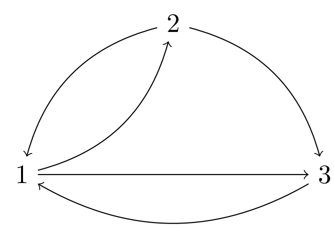
1 Problem 1

1.A Part a

Given the adjacency matrix:

$$g = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

We have the network:



1.B Part b

1.B.1 Eigenvector Centrality

To calculate eigenvector centrality, we can use the equation:

$$\lambda c = g' c$$

The transposes adjacency matrix is given by:

$$g' = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

The eigenvalues of this matrix are given by the roots of the charachteristic polynomial of g' defined by the determinant:

$$|(g' - \lambda I)|$$

So, we are given:

$$g' - \lambda I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, we are given:

$$g' - \lambda I = \begin{bmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 0 \\ 1 & 1 & -\lambda \end{bmatrix}$$

The determinant is given by:

$$-\lambda \begin{vmatrix} -\lambda & 0 \\ 1 & -\lambda \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 \\ 1 & -\lambda \end{vmatrix} + 1 \begin{vmatrix} 1 & -\lambda \\ 1 & 1 \end{vmatrix}$$

$$-\lambda^3 + 2\lambda + 1$$

There is only one positive root of this polynomial: $\frac{\sqrt{5}+1}{2}$. The associated eigenvector is therefore given as:

$$\frac{\sqrt{5}+1}{2} \cdot c = g'c$$

$$c = \begin{bmatrix} 1 \\ \frac{\sqrt{5}+1}{2} \\ 1 \end{bmatrix}$$

Normalizing by the factor: $\frac{5+\sqrt{5}}{2}$:

$$\hat{c} = \begin{bmatrix} \frac{2}{5+\sqrt{5}} \\ \frac{1+\sqrt{5}}{5+\sqrt{5}} \\ \frac{2}{5+\sqrt{5}} \end{bmatrix}$$

where $c_2 > c_1 = c_3$.

1.B.2 PageRank ($\alpha = 0.25$)

We can find the vector of all pageranks using the formula:

$$c = (I - \alpha g' D^{-1})^{-1}\mathbf{1}$$

D can be given by the diagonal matrix where each D_{ii} is given as:

$$D_{ii} = \max\{d_{i_{out}}, 1\}$$

Thus:

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore:

$$D^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Plugging into $I - \alpha g' D^{-1}$:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 0.25 \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Simplifying:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & \frac{1}{8} & \frac{1}{4} \\ \frac{1}{8} & 0 & 0 \\ \frac{1}{8} & \frac{1}{8} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{8} & -\frac{1}{4} \\ -\frac{1}{8} & 1 & 0 \\ -\frac{1}{8} & -\frac{1}{8} & 1 \end{bmatrix}$$

The Leontief inverse in g with D is given as:

$$\Lambda^{-1} = \begin{bmatrix} 1.05 & 0.165 & 0.263 \\ 0.132 & 1.02 & 0.0329 \\ 0.148 & 0.148 & 1.04 \end{bmatrix}$$

We extract the ranks as:

$$\Lambda^{-1}\mathbf{1} = \begin{bmatrix} 1.05 & 0.165 & 0.263 \\ 0.132 & 1.02 & 0.0329 \\ 0.148 & 0.148 & 1.04 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5.33 \\ 1.18 \\ 1.336 \end{bmatrix}$$

where $c_1 > c_3 > c_2$.

1.B.3 PageRank $\alpha = 0.5$

Plugging into $I - \alpha g' D^{-1}$:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 0.5 \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Simplifying:

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} - \begin{bmatrix}0 & \frac{1}{4} & \frac{1}{2}\\\frac{1}{4} & 0 & 0\\\frac{1}{4} & \frac{1}{4} & 0\end{bmatrix}$$

$$\begin{bmatrix}1 & -\frac{1}{4} & -\frac{1}{2}\\-\frac{1}{4} & 1 & 0\\-\frac{1}{4} & -\frac{1}{4} & 1\end{bmatrix}$$

The Leontief inverse in g with D is given as:

$$\Lambda^{-1} = \begin{bmatrix}\frac{32}{25} & \frac{12}{25} & \frac{16}{25}\\\frac{8}{25} & \frac{28}{25} & \frac{4}{25}\\\frac{2}{5} & \frac{2}{5} & \frac{6}{5}\end{bmatrix}$$

We extract the ranks as:

$$\Lambda^{-1}\mathbf{1} = \begin{bmatrix}\frac{32}{25} & \frac{12}{25} & \frac{16}{25}\\\frac{8}{25} & \frac{28}{25} & \frac{4}{25}\\\frac{2}{5} & \frac{2}{5} & \frac{6}{5}\end{bmatrix} \cdot \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}2.4\\1.6\\2\end{bmatrix}$$

where $c_1 > c_3 > c_2$.

1.C Part c

Yes. When the decay parameter $\alpha = 0.25$, we can see that there is increasing importance played on node 1, while the importance of nodes 2 and 3 are rather similar but distinct. This implies that node 1 has clear direct importance on the network, which makes sense due to it having the highest inward and outward degree. However, when $\alpha = 0.5$, we notice that the centrality of node 1 decreases, although it stays in the lead. The change in decay rate also led to a an increase in the centrality of nodes 2 and 3, where the difference between them grew. This implies that the algorithm was able to discern the centrality of node 2 from node 3 better, likely accounting for their indirect involvement with the highly connected node, node 1. Likewise, the indirect importance of nodes 2 and 3 likely reduced the centrality rank of node 1, signifying that there is more information reducing the relative importance of 1 as opposed to just itself: i.e. the algorithm will expand its horizon of view into less important nodes as alpha increases.

1.D Part d

We can find the vector of all Katz-Bonacich centrality scores using the formula:

$$c = (I - \alpha g')^{-1}\mathbf{1}$$

Plugging in:

$$c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 0.5 \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}$$

We can determine the Leontief inverse in g':

$$\Lambda^{-1} = \begin{bmatrix} \frac{8}{3} & 2 & \frac{4}{3} \\ \frac{4}{3} & 2 & \frac{2}{3} \\ 2 & 2 & 2 \end{bmatrix}$$

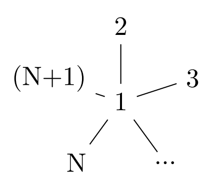
Multiplication by $\mathbf{1}$ yields c:

$$c = \begin{bmatrix} 6 \\ 4 \\ 6 \end{bmatrix}$$

2 Problem 2

2.A Part a

Consider a star-network with $N + 1$ nodes:



2.B Part b

2.B.1 Eigenvector Centrality

To calculate eigenvector centrality, we can use the equation:

$$\lambda c = g' c$$

The symmetric adjacency matrix is given by $g = g'$:

$$g = g' = \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

We can calculate the characteristic equation as:

$$\det(A - \lambda I)$$

Or:

$$\det(A-\lambda I) = \begin{vmatrix} -\lambda & 1 & 1 & \cdots & 1 \\ 1 & -\lambda & 0 & \cdots & 0 \\ 1 & 0 & -\lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -\lambda \end{vmatrix}$$

The principal eigenvalue corresponds to $\sqrt{n}$. I got this from this presentation and by performing by hand the calculations for a star graph with 3 nodes, yielding a principal eigenvalue of $\sqrt{2}$:

$$\det(A-\lambda I) = \begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = -\lambda \cdot (\lambda^2) - 1(-\lambda) + 1(0+\lambda) = -\lambda^3 + 2\lambda$$

with roots: $\lambda = \{0, \sqrt{2}, -\sqrt{2}\}$. So, I will assume a recursive pattern where the principal eigenvalue is $\sqrt{n}$.

$$\begin{bmatrix} \sqrt{n} \cdot c_1 \\ \sqrt{n} \cdot c_2 \\ \vdots \\ \sqrt{n} \cdot c_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_{n+1} \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{n} \cdot c_1 \\ \sqrt{n} \cdot c_2 \\ \vdots \\ \sqrt{n} \cdot c_{n+1} \end{bmatrix} = \begin{bmatrix} c_2 + c_3 + \ldots c_{n+1} \\ c_1 \\ c_1 \\ \vdots \\ c_1 \end{bmatrix}$$

For any $i = 2, 3, ..., n+1$:

$$c_i = \frac{c_1}{\sqrt{n}}$$

We will additionally impose the constraint that:

$$c_1 + c_2 + ... + c_{n+1} = 1$$

Or,

$$c_1 + \frac{n \cdot c_1}{\sqrt{n}} = 1$$

$$c_1 = \frac{1}{1+\sqrt{n}}$$

it follows:

$$c_i = \frac{\frac{1}{1+\sqrt{n}}}{\sqrt{n}} = \frac{1}{\sqrt{n}+n}$$

It follows:

$$c = \begin{bmatrix} \frac{1}{1+\sqrt{n}} \\ \frac{1}{\sqrt{n}+n} \\ \cdots \\ \frac{1}{\sqrt{n}+n} \end{bmatrix}$$

where we see that the hub-node, 1, has the highest centrality ranking.

2.B.2 PageRank ($\alpha = 0.25$)

We can find the vector of all pageranks using the formula:

$$c = (I - \alpha g' D^{-1})^{-1}\mathbf{1}$$

D can be given by the diagonal matrix where each D_{ii} is given as:

$$D_{ii} = \max\{d_{i_{out}}, 1\}$$

The matrix D is given as:

$$D = \begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Thus, calculating the Leontief matrix in g' with D:

$$\Lambda = I - 0.25 \cdot g' \cdot D^{-1} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} - \frac{1}{4} \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \cdot \begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} - \frac{1}{4} \cdot \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ n & 0 & 0 & \cdots & 0 \\ n & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n & 0 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{4} & -\frac{1}{4} & \cdots & -\frac{1}{4} \\ -\frac{n}{4} & 1 & 0 & \cdots & 0 \\ -\frac{n}{4} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{n}{4} & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Therefore, the vector of all pageranks with the Leontief inverse is:

$$c = \begin{bmatrix} 1 & -\frac{1}{4} & -\frac{1}{4} & \cdots & -\frac{1}{4} \\ -\frac{n}{4} & 1 & 0 & \cdots & 0 \\ -\frac{n}{4} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{n}{4} & 0 & 0 & \cdots & 1 \end{bmatrix}^{-1} \mathbf{1}$$

2.B.3 PageRank ($\alpha = 0.5$)

We can calculate the Leontif matrix in g' with the same D as previously used, switching the value of α:

$$\Lambda = I - 0.50 \cdot g' \cdot D^{-1} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} - \frac{1}{4} \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \cdot \begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} - \frac{1}{4} \cdot \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ n & 0 & 0 & \cdots & 0 \\ n & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n & 0 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \cdots & -\frac{1}{2} \\ -\frac{n}{2} & 1 & 0 & \cdots & 0 \\ -\frac{n}{2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{n}{2} & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Therefore, the vector of all pageranks using the Leontief inverse is:

$$c = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \cdots & -\frac{1}{2} \\ -\frac{n}{2} & 1 & 0 & \cdots & 0 \\ -\frac{n}{2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{n}{2} & 0 & 0 & \cdots & 1 \end{bmatrix}^{-1} \mathbf{1}$$

2.C Part c

We can see that as α increases, the denominator in the active constraints in the Leontif matrix decreases. We can use the fact that:

$$\Lambda c = \mathbf{1}$$

Implies that:

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \cdots & -\frac{1}{2} \\ -\frac{n}{2} & 1 & 0 & \cdots & 0 \\ -\frac{n}{2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{n}{2} & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ ... \\ c_{n+1} \end{bmatrix} = \mathbf{1}$$

$$\begin{bmatrix} c_1 - \frac{c_2}{2} - \frac{c_3}{2} - ... - \frac{c_{n+1}}{2} \\ -\frac{c_1 \cdot n}{2} + c_2 \\ -\frac{c_1 \cdot n}{2} + c_3 \\ ... \\ -\frac{c_1 \cdot n}{2} + c_{n+1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ ... \\ 1 \end{bmatrix}$$

Or,

$$\begin{bmatrix} c_1 - \frac{c_2}{4} - \frac{c_3}{4} - ... - \frac{c_{n+1}}{4} \\ -\frac{c_1 \cdot n}{4} + c_2 \\ -\frac{c_1 \cdot n}{4} + c_3 \\ ... \\ -\frac{c_1 \cdot n}{4} + c_{n+1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ ... \\ 1 \end{bmatrix}$$

As we can see, as α increases, in the first constraint, all c_i where $i > 1$ will reduce the score of c_1 less, raising its centrality score. Meanwhile, as c_1 increases, all c_i where $i > 2$ will need to increase to compensate for a larger negative term: $c_1 \cdot n \cdot \alpha$. Therefore, the centrality scores of these c_i where $i > 2$ will be greater as α increases. This, intuitively, corresponds allocating more centrality to "indirectly" important nodes due to their connectedness to the hub node, 1.

2.D Part d

We can find the vector of all Katz-Bonacich centrality scores using the formula:

$$c = (I - \alpha g')^{-1}\mathbf{1}$$

Plugging in:

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \cdots & -\frac{1}{2} \\ -\frac{1}{2} & 1 & 0 & \cdots & 0 \\ -\frac{1}{2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{2} & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Therefore, the vector of all Katz-Bonacich ranks is given as:

$$c = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \cdots & -\frac{1}{2} \\ -\frac{1}{2} & 1 & 0 & \cdots & 0 \\ -\frac{1}{2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{2} & 0 & 0 & \cdots & 1 \end{bmatrix}^{-1} \cdot \mathbf{1}$$

2.E Problem 3

I chose *Carvalho, Nirei, Saito, and Tahbaz-Salehi, "Supply Chain Disruptions: Evidence from the Great Japanese Earthquake"*.

2.E.1 Part a

This study quantifies the size of a node using proprietary data compiled by a major private credit reporting agency that contains information on roughly half of all private and publicly traded firms in Japan, covering almost all firms with more than five employees across all sectors of the economy. For each firm-year, the authors analyzed the set of firm-level covariates and the identities of suppliers and customers. The study coupled this information with the geographic location of firms across Japan to determine their proximity to the earthquake disaster zone. The authors quantified a node's size using a combination of the size of the trades it executes and its geographic proximity to the shock. I think this is a reasonable estimation, as it both captures the brute weight of a node in the GDP and the extent to which a firm was crippled due to the shock (i.e. size is determined by capital output and limitations imposed by a shock).

2.E.2 Part b

Using the same proprietary data compiled by a major private credit reporting agency, this study used the covariates and identities of suppliers and customers for each firm on a firm-year-basis to determine inward and outward directed edges for each node. I think this is reasonable, as it captures the transient nature of customer-supplier relations for each firm per-year and it also remains true to the purpose of the model: to describe the connectedness and emergent importance of a node.

2.E.3 Part c

"We find that in such a counterfactual economy, the disaster would have resulted in a 0.21 percentage point decline in GDP growth, thus indicating that the propagation of the shock to firms outside the disaster area over input-output linkages played a significant role in amplifying the disaster's aggregate impact."

I believe that this is the most important finding because it suggests that while the immediately disruptive effects of the 2011 Earthquake in Japan occurred at the regional level as opposed to firms "too large to fail", the network effects attributed by the interconnected Japanese economy are responsible for the significant reduction in GDP that was actually experienced in real-time. The result is concise and meaningful in the era of megaeconomies: interdependency leads to failure as opposed to the sheer effects of a negative supply-demand shock.

2.E.4 Part d

The results of this study suggest that in the event of a localized shock to our supply-chain, our company likely has enough outsourcing capabilities and connections to exhibit some degree of resilience to stay afloat until the problem can be remedied. However, in the event of a widespread shock to numerous upstream firms that we have permanent partnerships with and that we would otherwise turn to, our relatively large network of resources is greatly reduced. This is because the inherent dependency upon capacity limited firms at times when all of our competitors are seeking similar means of production means that some of our demand will not be met and we will lose revenue.

There must be some degree of balance. In other words, we must be able to retain a high degree of inter-connectivity with other firms without a high degree of inter-dependency. This means that we need to align our company's growth to meet an exploding demand with in-house capabilities that cannot go down when exogenous networks to the company go down. However, this should not be naively used to replace supply-chain partnersnhips that can otherwise save the company should these in-house means of production go down.