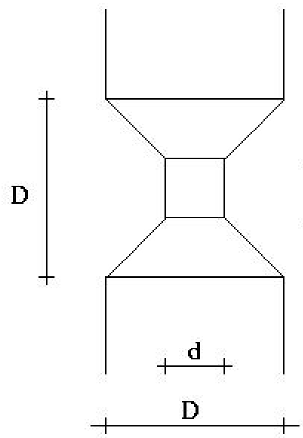
Beavergnaw

Joshua T. Guerin, Ph.D.

September 30, 2017

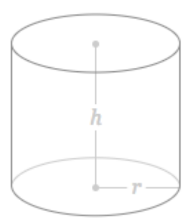
Abstract

open.kattis.com problem beavergnaw



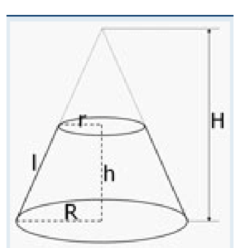
Relevant Formulas

Volume of a cylinder:



$$V = \pi r^2 h$$

Volume of a tapered cylinder:



$$V = \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

Problem Setup

To simplify the arithmetic assume:

$$R = \frac{D}{2}, r = \frac{d}{2}$$

These will represent the radiuses of the figures.
Volume of the outer cylinder:

$$\begin{aligned} V_0 &= \pi R^2 2R \\ &= 2\pi R^3 \end{aligned}$$

Same for the inner:

$$V_i = 2\pi r^3$$

And the 2 tapered cyliners:

$$\begin{aligned} 2V_t &= \frac{2}{3}\pi(R-r)(R^2 + Rr + r^2) \\ &= \frac{2}{3}\pi\big(R^3 + R^2 r + Rr^2 - R^2 r - Rr^2 - r^3\big) \\ &= \frac{2}{3}\pi\big(R^3 - r^3\big) \end{aligned}$$

Solution

Given V and D, we have that:

$$\begin{aligned} V &= V_o - V_i - 2V_t \\ &= 2\pi R^3 - 2\pi r^3 - \frac{2}{3}\pi\big(R^3 - r^3\big) \\ &= 2\pi\left(R^3 - r^3 - \frac{1}{3}\big(R^3 - r^3\big)\right) \\ &= 2\pi\left(R^3 - r^3 - \frac{1}{3}R^3 + \frac{1}{3}r^3\right) \\ &= 2\pi\left(\frac{2}{3}R^3 - \frac{2}{3}r^3\right) \\ &= 2\pi\frac{2}{3}\left(R^3 - r^3\right) \\ &= 2\pi\frac{2}{3}\left(\big(\frac{D}{2}\big)^3 - \big(\frac{d}{2}\big)^3\right) \qquad \text{because } R = \frac{D}{2},\ r = \frac{d}{2} \\ &= \frac{2\pi}{3}\left(\frac{D^3}{4} - \frac{d^3}{4}\right) \end{aligned}$$

and so:

$$V = \frac{2\pi}{3}\left(\frac{D^3}{4} - \frac{d^3}{4}\right)$$

$$\frac{3V}{2\pi} = \frac{D^3}{4} - \frac{d^3}{4}$$

$$4 \cdot \frac{3V}{2\pi} = D^3 - d^3$$

$$\frac{6V}{\pi} = D^3 - d^3$$

so:

$$d = \sqrt[3]{\frac{-6V}{\pi} + D^3}$$

The solution is:

```
print(((-6*V)/pi + D**3)**(1/3))
```