

# The Diffusion of Sticky Particles in One Dimension

Driven Stochastic Transport in Low-Dimensional Systems

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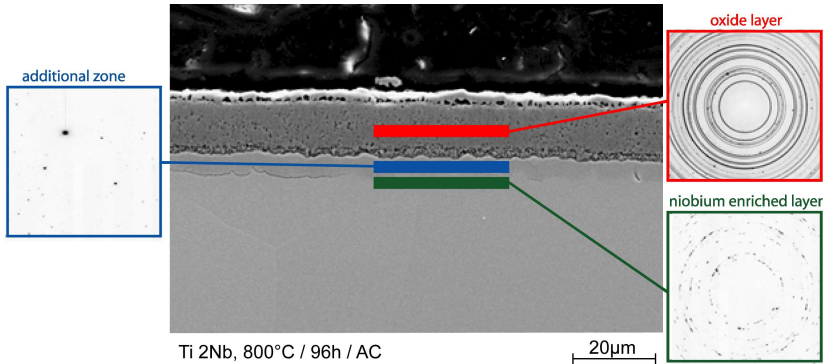
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# Model Motivation

## Formation of an Oxide Layer in Titanium



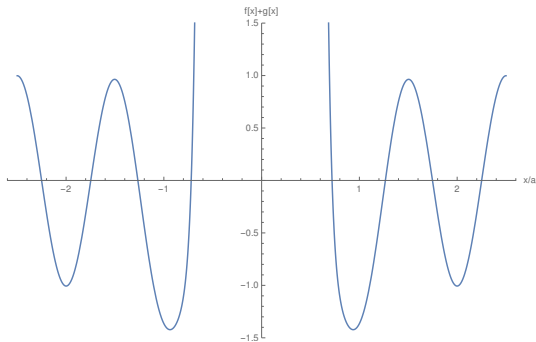
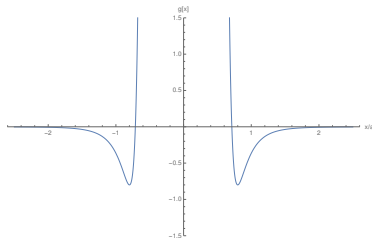
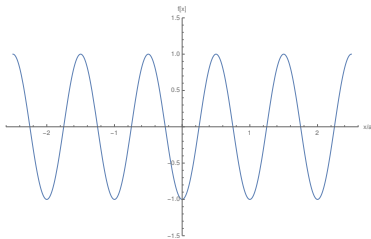
# Model Motivation

## The Diffusion of Oxygen Atoms in Titanium



# Model Motivation

## Interactions Between Particles in a Periodic Potential



# Model Motivation

## My 1D Sticky Lattice Model

$\dots VOV \dots \longrightarrow \dots VVO \dots$  with rate 1  
 $\dots VOV \dots \longrightarrow \dots OVV \dots$  with rate 1  
 $\dots OOV \dots \longrightarrow \dots OVO \dots$  with rate  $\lambda$   
 $\dots VOO \dots \longrightarrow \dots OVO \dots$  with rate  $\lambda$

# Model Phenomenology

## MFT Master Equations

- Let  $\zeta = 1 - \lambda$ , and  $\rho_i$  be the ensemble-averaged occupation probability of the  $i^{\text{th}}$  site in the mean-field approximation.
- Then

$$\begin{aligned} \frac{\partial \rho_i}{\partial t} = & (1 - \rho_i) [(1 - \zeta \rho_{i-2}) \rho_{i-1} + (1 - \zeta \rho_{i+2}) \rho_{i+1}] \\ & - \rho_i [2\zeta \rho_{i-1} \rho_{i+1} - (3 - \zeta) (\rho_{i-1} + \rho_{i+1}) + 2]. \end{aligned}$$

- Just set the LHS to zero for the steady state.

# Model Phenomenology

## Continuum-Limit MFT

- Let the lattice spacing be  $a$ , and let us take the long-wavelength limit of our MFT, with  $x$  as our spatial coordinate, promoting  $\rho_i(t)$  to the field  $\rho(x, t)$ .
- Then we find that

$$\frac{\partial \rho}{\partial t} = \frac{1}{2} a^2 \left[ (2 - 2\zeta\rho(4 - 3\rho)) \frac{\partial^2 \rho}{\partial x^2} - \zeta(2 - 3\rho) \left( \frac{\partial \rho}{\partial x} \right)^2 \right] + \mathcal{O}(a^4).$$

- We can rewrite this as the continuity equation  $\frac{\partial \rho}{\partial t} + \frac{\partial J}{\partial x} = 0$ , where

$$J = -a^2 [1 - \zeta(4 - 3\rho)\rho] \frac{\partial \rho}{\partial x}$$





# Model Phenomenology

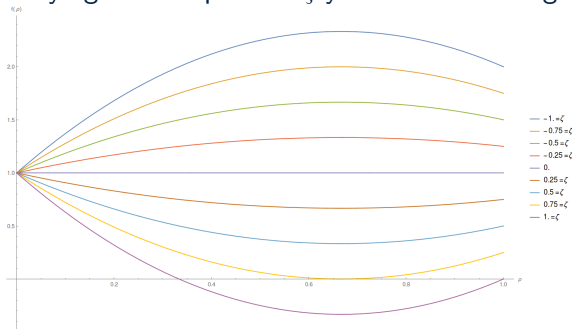
## Continuum-Limit MFT Current Flows

- So we can write the current as

$$J = -a^2 f(\rho) \frac{\partial \rho}{\partial x},$$

where  $f$  varies with  $\zeta$ .

- Varying with respect to  $\zeta$  yields the following:



# Model Phenomenology

## Steady State PDE Solutions

- We can actually solve the steady state MFT PDE analytically in the bulk, by recalling that the current flow must be constant and absorbing  $a$ :

$$J_0 = [1 - \zeta(4 - 3\rho)\rho] \frac{\partial \rho}{\partial x}$$
$$\int dx J_0 = \int d\rho [1 - \zeta(4 - 3\rho)\rho]$$
$$J_0(x - x_0) = \rho(2\zeta\rho - \zeta\rho^2 - 1),$$

- All we have to do now is solve for  $\rho$ !