MATH 217 (Fall 2021)

Honors Advanced Calculus, I

Assignment #7

1. Determine and classify the stationary points of

$$f\colon \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}, \quad (x,y) \mapsto \frac{1}{y} - \frac{1}{x} - 4x + y.$$

   If f attains a local minimum or maximum at a stationary point, evaluate the function there.

2. Determine and classify the stationary points of

$$f\colon \mathbb{R}^2 \to \mathbb{R}, \quad (x,y) \mapsto (x^2 + 2y^2)e^{-(x^2+y^2)}.$$

   If f has a local extremum at a stationary point, determine the nature of this extremum and evaluate f there.

3. Determine the minimum and the maximum of

$$f\colon D \to \mathbb{R}, \quad (x,y) \mapsto \sin x + \sin y + \sin(x+y),$$

   where $D := \left\{(x,y) \in \mathbb{R}^2 : 0 \leq x, y \leq \frac{\pi}{2}\right\}$, and all points of D where they are attained.

4. Let $(x_n)_{n=1}^{\infty}$ be a convergent sequence in $\mathbb{R}^N$ with limit x. Show that $\{x_n : n \in \mathbb{N}\} \cup \{x\}$ has content zero.

5. Let $I \subset \mathbb{R}^N$ be a compact interval. Show that ∂I has content zero.

6*. Let $I_1, \ldots, I_n \subset \mathbb{R}$ be compact intervals such that $\mathbb{Q} \cap [0,1] \subset I_1 \cup \cdots \cup I_n$. Show that $\sum_{j=1}^{n} \mu(I_j) \geq 1$.

Due Thursday, November 4, 2020, at 5:00 p.m.; no late assignments.