MATH 217 (Fall 2021)

Honors Advanced Calculus, I

Assignment #4

1. For $0 \leq r \leq R$ and $\epsilon \in (0,1)$, determine whether or not the set

$$\left\{(x,y,z) \in \mathbb{R}^3 : r^2 \leq x^2 + z^2 \leq R^2,\ |y| \in [\epsilon, 1]\right\}$$

   is (a) open, (b) closed, (c) compact, or (d) connected.

2. A set $S \subset \mathbb{R}^N$ is called *star shaped* if there is $x_0 \in S$ such that $tx_0 + (1-t)x \in S$ for all $x \in S$ and $t \in [0,1]$. Show that every star shaped set is connected, and give an example of a star shaped set that fails to be convex.

3. Let $C \subset \mathbb{R}^N$ be connected. Show that $\overline{C}$ is also connected.

4. Let $S \subset \mathbb{R}^N$, and let $x \in \mathbb{R}^N$. Show that $x \in \overline{S}$ if and only if there is a sequence $(x_n)_{n=1}^\infty$ in S such that $x = \lim_{n\to\infty} x_n$.

5. Let $(x_n)_{n=1}^\infty$ be a convergent sequence in $\mathbb{R}^N$ with limit x. Show that $\{x_n : n \in \mathbb{N}\} \cup \{x\}$ is compact.

6*. Show that $\mathbb{R}^N \setminus \{0\}$ is disconnected if and only if $N = 1$.

Due Thursday, October 7, 2020, at 5:00 p.m.; no late assignments.