Untitled8

January 31, 2022

[2]:
```
import matplotlib.pyplot as plt
import numpy as np
import scipy as sp
import sympy as sy
from IPython.display import Math,display
from IPython.display import Latex

%matplotlib inline
```

[3]:
```
# Problem 8.2.1
```

[4]:
```
x, a = sy.symbols('x, a')
expr = a*x*sy.exp(-x)-x
sy.solve(expr,x)
```

[4]: `[0, log(a)]`

[5]:
```
Math('f(x) =x =  a.x.e^{-x}')
```

[5]: $f(x) = x = a.x.e^{-x}$

[6]:
```
# (a)

print("The stability of x* changes when")
Math('a = 1')
```

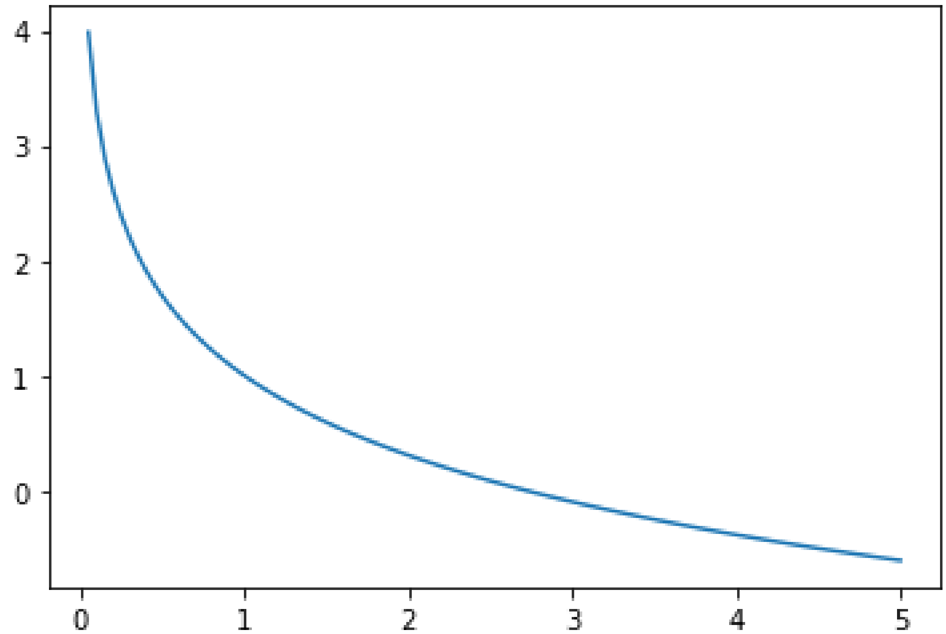
```
The stability of x* changes when
```

[6]: $a = 1$

[7]:
```
# (b)
a = np.linspace(0,5,100)
y = 1-np.log(a)
plt.plot(a,y)
```

```
<ipython-input-7-ddd2b933eab3>:3: RuntimeWarning: divide by zero encountered in
log
  y = 1-np.log(a)
```

[7]: [<matplotlib.lines.Line2D at 0x7fb1e30fc4c0>]



[8]:
```
Math(' |g(a)| < 1  = |1-\ln(a)|<1 \Leftrightarrow 1<a<e^2')
```

[8]: $|g(a)| < 1 = |1 - \ln(a)| < 1 \Leftrightarrow 1 < a < e^2$

[9]:
```
Math('|g(a)|>1 = |1-\ln(a)|>1 \Leftrightarrow \{a\in \mathbb{R} : 0<a<1 \cup␣
↪e^2 < a\}')
```

[9]: $|g(a)| > 1 = |1 - \ln(a)| > 1 \Leftrightarrow \{a \in \mathbb{R} : 0 < a < 1 \cup e^2 < a\}$

[10]:
```
# Problem 8.2.3
```

$$\operatorname{sgn}(x) := \begin{cases} -1, \text{ if } x < 0 \\ 0, \text{ if } x = 0 \\ 1, \text{ if } x > 0 \end{cases}$$

[11]:
```
x = int(input("Enter your number:"))
np.sign(x)
```

Enter your number:10

[11]: 1

[48]:
```python
# 2.4.6 (d)
x, r = sy.symbols('x, r')
expr = r*r*x*(1-x)*(1-r*x*(1-x))-x
y = sy.solve(expr,x)
y
```

[48]:
```
[0,
 (r - 1)/r,
 (r - sqrt(r**2 - 2*r - 3) + 1)/(2*r),
 (r + sqrt(r**2 - 2*r - 3) + 1)/(2*r)]
```

[56]:
```python
expr_2 = r*r - 2*r*r*r*x - 2*r*r*x-4*x*x*x*r*r*r + 6*r*r*r*x*x
m = expr_2.subs(x,y[3])
m
```

[56]:

$$-r^2\left(r+\sqrt{r^2-2r-3}+1\right)+r^2+\frac{3r\left(r+\sqrt{r^2-2r-3}+1\right)^2}{2}-r\left(r+\sqrt{r^2-2r-3}+1\right)-\frac{\left(r+\sqrt{r^2-2r-3}+1\right)^3}{2}$$

[61]:
```python
# x = np.linspace(0,2,100)
# plt.plot(abs(m),x)
abs(m) <= 1
sy.solve(m,r)
```

[61]:
```
[1 - sqrt(5), 1 + sqrt(5)]
```

Since r > 3 from (c) and we get

$$|m| \leq 1 \Leftrightarrow |f'(x=y[1,2])| <= 1 \implies r \in [1+\sqrt(5), 1+\sqrt(5)]$$

we get its stable from

$$r \in (3, 1+\sqrt(6))$$

and unstable from

$$r > 1$$

[ ]:
```python
```