Fall 2021, Math 328, Homework 2

Due: End of day on 2021-09-24

1 (10 points)

Compute the order of each of the elements in the following groups: $D_6,\ D_8,\ D_{10}$.

2 (10 points)

For $n = 3, 4$, write out all elements of S_n in cycle notation, and compute the order of each element.

3 (10 points)

Given a group G, an element $g \in G$ is called *central* provided that for all $h \in G$, one has $g \cdot h = h \cdot g$. The identity element is clearly central.

1. Prove that the product of two central elements is central.
2. Prove that the inverse of a central element is central.
3. Find all central elements in the following groups: S_4, Q_8, D_{2n} (arbitrary $n \geq 3$).

4 (10 points)

Suppose that σ is an element of S_n which has the form

$$\sigma = (a_1, a_2, \ldots, a_m).$$

Let i be any integer. Prove that $\sigma^i(a_k) = a_r$ where $r \equiv k + i \bmod m$. Determine the order of the element σ.

5 (10 points)

Let $\varphi : G \to H$ be a homomorphism of groups and let $g \in G$ be given. Prove that $\varphi(g^a) = \varphi(g)^a$ for all $a \in \mathbb{Z}$.

6 (10 points)

1. Prove that S_3 and D_6 are isomorphic.
2. Prove that S_4 and D_{24} are not isomorphic.
3. For a pair of groups G and H, prove that $G \times H$ and $H \times G$ are isomorphic.
4. Let G be a group, and let $\mathrm{Aut}(G)$ be the set of automorphisms of G. Prove that $\mathrm{Aut}(G)$ is a group under composition of automorphisms.
5. Suppose that G and H are isomorphic. Prove that $\mathrm{Aut}(G)$ is isomorphic to $\mathrm{Aut}(H)$.