MATH 217 (Fall 2021)
Honors Advanced Calculus, I

Assignment #10

1. Let D in spherical coordinates be given as the solid lying between the spheres given by $r = 2$ and $r = 4$, above the xy-plane and below the cone given by the angle $\theta = \frac{\pi}{3}$. Evaluate the integral $\int_D xyz$.

2. Let K be the triangle with vertices $(1, 8)$, $(2, 7)$, and $(9, 3)$. Evaluate the line integral

$$\int_{\partial K} \sin y \, dx + x \cos y \, dy$$

where ∂K is positively oriented.

3. Let $P, Q \colon \mathbb{R}^2 \to \mathbb{R}$ be given by

$$P(x, y) = e^x + y^3 \qquad \text{and} \qquad Q(x, y) = 4xy^2.$$

Suppose that the force field (P, Q) moves a particle once along the boundary of the ellipse $\left\{(x, y) \in \mathbb{R}^2 : x^2 + \frac{y^2}{4} \le 1\right\}$ in counterclockwise direction. Compute the work done.

4. Let $a, b > 0$. Use Green's Theorem to compute the area of the ellipse

$$E := \left\{(x, y) \in \mathbb{R}^2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\}.$$

5. Let $\varnothing \neq U \subset \mathbb{R}^3$ be open, and let $f, g : U \to \mathbb{R}$ be twice continuously partially differentiable. Show that $\operatorname{div}(\nabla f \times \nabla g) = 0$ on U, where $\times$ denotes the cross product in $\mathbb{R}^3$.

6*. Let $D \subset \mathbb{R}^2$ be the trapeze with vertices $(1, 0)$, $(2, 0)$, $(0, -2)$, and $(0, -1)$. Evaluate $\int_D \exp\left(\frac{x+y}{x-y}\right)$. (*Hint*: Consider

$$\phi \colon \mathbb{R}^2 \to \mathbb{R}^2, \quad (u, v) \mapsto \left(\frac{1}{2}(u + v), \frac{1}{2}(u - v)\right)$$

and apply Change of Variables.)

Due Thursday, December 2, 2020, at 5:00 p.m.; no late assignments.