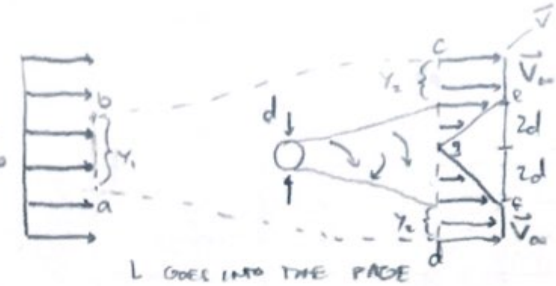
PROBLEM 1 | ME EN 512 | JOSHUA CANNAS



ASSUMPTIONS:

1. STEADY
2. INCOMPRESSIBLE
3. STREAMLINE
4. UNIFORM VELOCITY PROFILE ACROSS CS_1.

MASS:

$$0 = \cancel{\frac{\partial}{\partial t}\int_{C\forall} \rho_1 d\forall} + \int_{CS} \rho_2 (\vec{V}\cdot d\vec{A})$$

$$0 = \int_{CS_1} \rho_1 \vec{V}_1 \cdot d\vec{A}_1 + \int_{CS_2} \rho_1 \vec{V}_2 \cdot d\vec{A}_2$$

$$0 = -\int_a^b \rho_1 \vec{V}_1 \, dy \, L + \int_d^c \rho_2 \vec{V}_2 \, dy \, L$$

$$\int_a^b \rho_1 \vec{V}_1 \, dy = \int_d^c \rho_2 \vec{V}_2 \, dy \quad \underset{\Longrightarrow}{\text{MULTIPLY BOTH SIDES BY } \vec{V}_1} \quad \int_a^b \rho_1 \vec{V}_1^2 \, dy = \int_d^c \rho_2 \vec{V}_1 \vec{V}_2 \, dy$$

MOMENTUM:

D' = DRAG PER UNIT SPAN

$$D' = \frac{D}{L}$$

$$\Sigma F = \cancel{\frac{\partial}{\partial t}\int_{C\forall} \rho \vec{V} (\vec{V}\cdot dA)} + \int_{CS} \rho \vec{V} (\vec{V}\cdot dA)$$

$$-D' = -\int_a^b \rho_1 \vec{V}_1^2 \, dy + \int_d^c \rho_2 \vec{V}_2^2 \, dy$$

$$-D' = -\int_d^c \rho_2 \vec{V}_1 \vec{V}_2 \, dy + \int_d^c \rho_2 \vec{V}_2^2 \, dy$$

$$D' = \rho \int_d^c V_2 (V_1 - V_2) \, dy$$

$$V(y) = \frac{y V_\infty}{2d}$$

$$D' = \rho \int_e^c \cancel{V_\infty (V_\infty - V_\infty)} \, dy + 2\rho \int_g^e V(y)(V_\infty - V(y)) \, dy + \rho \int_d^f \cancel{V_\infty (V_\infty - V_\infty)}$$

$$D' = 2\rho \int_0^{2d} \frac{y V_\infty}{2d} \left(V_\infty - \frac{y V_\infty}{2d}\right) dy = 2\rho V_\infty^2 \int_0^{2d} \frac{y}{2d} - \frac{y^2}{4d^2} \, dy$$

$$D' = 2\rho V_\infty^2 \left[\frac{4d^2}{4d} - \frac{8d^3}{3 \cdot 12 d^2} \right] = \rho V_\infty^2 d \left(2 - \frac{4}{3}\right) = \frac{2}{3} \rho V_\infty^2 d$$

$$D = \frac{2}{3} \rho V_\infty^2 d L$$

ANSWER: $C_D = \frac{D}{\frac{1}{2}\rho V^2 d L} = \frac{\frac{2}{3}\rho V^2 d L}{\frac{1}{2}\rho V^2 d L}$

$$\boxed{C_D = 4/3}$$