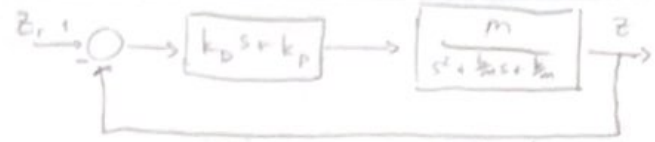
HW #5 | EC EN 483 | JOSHUA CANNAS

0.7) a)



$E(s) = \frac{1}{1+PC} R$

OPEN-LOOP SYSTEM. $P(s)C(s) = (k_D s + k_P)\left(\frac{1/m}{s^2 + \frac{b}{m}s + \frac{k}{m}}\right)$

THIS SYSTEM IS **TYPE 0** B/C IT HAS NO FREE INTEGRATORS

S.S. ERROR FOR:

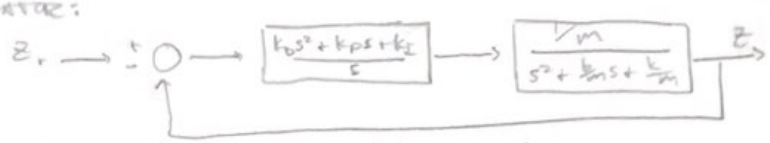
STEP: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \frac{1}{1+P(s)C(s)} \frac{1}{s} = \lim_{s\to 0} \frac{1}{1 + \frac{k_D s + k_P/m}{s^2 + \frac{b}{m}s + \frac{k}{m}}} = \frac{1}{1 + \frac{k_P/m}{k/m}}$

$= \boxed{\frac{k}{k + k_P}}$

RAMP: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \frac{1}{1+P(s)C(s)} \frac{1}{s^2} = \lim_{s\to 0} \frac{1}{s + sP(s)C(s)} = \boxed{\infty}$

PARABOLA: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \frac{1}{1+P(s)C(s)} \frac{1}{s^3} = \boxed{\infty}$

c) AN INTEGRATOR:



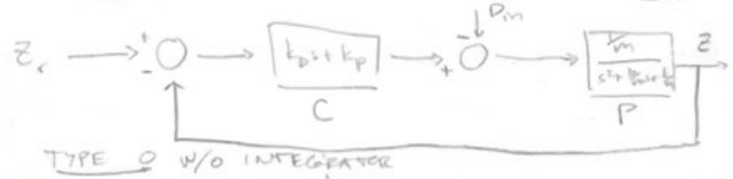
$P(s)C(s) = \left(\frac{k_D s^2 + k_P s + k_I}{s}\right)\left(\frac{1/m}{s^2 + \frac{b}{m}s + \frac{k}{m}}\right)$

STEP: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \frac{1}{1+P(s)C(s)} \frac{1}{s} = \lim_{s\to 0} \frac{1}{1 + \left(\frac{k_D s^2 + k_P s + k_I}{s}\right)\left(\frac{1/m}{s^2 + b/m s + k/m}\right)} = \boxed{0}$

RAMP: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \frac{1}{(1+P(s)C(s))} \frac{1}{s^2} = \frac{1}{s + \frac{k_I/m}{b/m}} = \boxed{\frac{k}{k_I}}$

PARABOLA: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \frac{1}{1+P(s)C(s)} \frac{1}{s^3} = \boxed{\infty}$

b)



$E(s) = \frac{P}{1+PC} P_{in}$

TYPE 0 W/O INTEGRATOR

STEP: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \frac{\frac{1}{m}}{1 + (k_D s + k_P)\left(\frac{1/m}{s^2 + \frac{b}{m}s + \frac{k}{m}}\right)} \frac{1}{s} = \frac{\frac{1}{k}}{1 + \frac{k_P}{k}} = \boxed{\frac{1}{k + k_P}}$

RAMP: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \frac{P(s)}{1+P(s)C(s)} \frac{1}{s^2} = \boxed{\infty}$

PARABOLA: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \frac{P(s)}{1+P(s)C(s)} \frac{1}{s^3} = \boxed{\infty}$

W/ AN INTEGRATOR:

STEP: $q=0$ $\lim_{t\to\infty} e(t) = \lim_{s\to 0}$ $\dfrac{\frac{1/m}{s^2+\frac{k_D}{m}s+\frac{k_P}{m}}}{1+\left(\frac{1/m}{s^2+\frac{k_D}{m}s+\frac{k_P}{m}}\right)\left(\frac{k_D s^2+k_P s+k_I}{s}\right)}\ \frac{1}{s^2} = \boxed{0}$

RAMP: $q=1$ S.S. ERROR: $\lim_{s\to 0}$ $\dfrac{P(s)}{1+P(s)C(s)}\ \frac{1}{s^2} = \boxed{\frac{1}{k_I}}$

PARABOLA: $q=2$ S.S. ERROR $= \lim_{s\to 0}$ $\dfrac{P(s)}{1+P(s)C(s)}\ \frac{1}{s^3} = \boxed{\infty}$

E.9) $\tilde{\Theta}_r \to \Sigma \to \boxed{k_{D_\theta}s + k_{P_\theta}} \to \boxed{\frac{1}{\left(m_1\frac{l^2}{4}+m_2\frac{l^2}{3}\right)s^2}} \to \tilde{\Theta}$

a)

THIS SYSTEM IS $\boxed{\text{TYPE 2}}$

S.S. ERROR FOR:

STEP: $q=0$ $\lim_{s\to 0}$ $\dfrac{1}{1+(k_{D_\theta}s+k_{P_\theta})\left(\frac{1}{\left(m_1\frac{l^2}{4}+m_2\frac{l^2}{3}\right)s^2}\right)}\ \frac{1}{s^2} = \boxed{0}$

RAMP: $q=1$ $\lim_{s\to 0}$ $\dfrac{1}{1+P(s)C(s)}\ \frac{1}{s^2} = \boxed{0}$

PARABOLA: $q=2$ $\lim_{s\to 0}$ $\dfrac{1}{1+P(s)C(s)}\ \frac{1}{s^2} = \boxed{\frac{1}{k_{P_\theta}b_0}}$ $\qquad b_0 = m_1 z_2^2 + m_2\frac{l^2}{3}$

THIS SYSTEM IS $\boxed{\text{TYPE 0}}$ W/ RESPECT TO AN INPUT DISTURBANCE.

b) $k_{PC_\theta} = 1$ $\quad z_r \to \Sigma \to \boxed{k_{P_z}+k_{D_z}s} \to \boxed{\frac{g}{s^2}} \to z$

THIS SYSTEM IS $\boxed{\text{TYPE 2}}$

STEP: $q=0$ $\lim_{s\to 0}$ $\dfrac{1}{1+\left(k_{D_\theta}s+k_{P_\theta}\right)\left(-\frac{g}{s^2}\right)}\ \frac{1}{s^2} = \boxed{0}$

RAMP: $q=1$ $\lim_{s\to 0}$ $\dfrac{1}{1+PC}\ \frac{1}{s^2} = \boxed{0}$

PARABOLA: $q=2$ $\lim_{s\to 0}$ $\dfrac{1}{\left[1+\left(k_{D_\theta}s+k_{P_\theta}\right)\left(-\frac{g}{s^2}\right)\right]}\ \frac{1}{s^2} = \boxed{\frac{-1}{k_{P_\theta}g}}$

$PC = \dfrac{k_D s^2+k_P s+k_I}{s}\left(-\frac{g}{s^2}\right)$

W/ INTEGRATOR: TYPE 3

STEP: S.S. ERROR = $\boxed{0}$

RAMP: S.S. ERROR = $\boxed{0}$

PARABOLA: S.S. ERROR = $\boxed{0}$

FOR DISTURBANCE INPUT:

PD: $\boxed{\text{TYPE 0}}$

PID: $\boxed{\text{TYPE 1}}$

F.9) a] $\tilde{H}_i \rightarrow \pm \bigcirc \rightarrow \boxed{k_D s + k_P} \rightarrow \boxed{\frac{1}{(m_c + 2m_r)s^2}} \rightarrow \tilde{H}$ $\quad b_0 = \frac{1}{m_c + 2m_r}$

THIS SYSTEM IS TYPE 2.

S.S. ERROR:

STEP: $\lim_{s \to 0} \frac{1}{1 + (k_D s + k_P)\left(\frac{b_0}{s^2}\right)} \frac{1}{s^2} = 0$

RAMP: S.S. ERROR = 0

PARABOLA: $\lim_{s \to 0} \frac{1}{1 + (k_D s + k_P)\left(\frac{b_0}{s^2}\right)} \frac{1}{s^2} = \frac{1}{k_P b_0}$

W/ AN INTEGRATOR:

S.S. ERROR FOR STEP = 0

S.S. ERROR FOR RAMP = 0

S.S. ERROR FOR PARABOLA = 0

SYSTEM TYPE W/ RESPECT TO AN INPUT DISTURBANCE:

PD CONTROL: TYPE 0

PID CONTROL: TYPE 1

b] $\tilde{\theta}_d \rightarrow \pm \bigcirc \rightarrow \boxed{k_{P_\theta} s + k_{P_\theta}} \rightarrow \boxed{\frac{1}{(2d^2 m_r + J_c)s^2}} \rightarrow \tilde{\theta}$ $\quad c_0 = \frac{1}{2d^2 m_r + I_c}$

THIS SYSTEM IS TYPE 2.

S.S. ERROR:

S.S. ERROR FOR STEP = 0

S.S. ERROR FOR RAMP = 0

PARABOLA: $\lim_{s \to 0} \frac{1}{1 + \frac{c_0}{s^2}(k_{P_\theta} s + k_{P_\theta})} \frac{1}{s^2} = \frac{1}{c_0 k_{P_\theta}}$

SYSTEM TYPE W/ RESPECT TO INPUT DISTURBANCE: TYPE 0

c] $\tilde{z}_d \rightarrow \pm \bigcirc \rightarrow \boxed{k_{P_z} s + k_{P_z}} \rightarrow \frac{-F_e/(m_c + 2m_r)}{s\left(s + \frac{\mu}{m_c + 2m_r}\right)}$ $\quad a_0 = m_c + 2m_r$

THIS SYSTEM IS TYPE 1

S.S. ERROR:

S.S. ERROR FOR STEP = 0

RAMP: $\lim_{s \to 0} \frac{1}{1 + \frac{-F_e/a_0}{s\left(s + \frac{\mu}{a_0}\right)}(k_{P_z} s + k_{P_z})} \frac{1}{s} = \frac{-\mu}{F_e k_{P_z}}$

S.S. ERROR FOR PARABOLA = ∞

W/ AN INTEGRATOR: $P(s)C(s) = \frac{k_{D_0}s^2 + k_p s + k_I}{s}\left(\frac{-F_\varepsilon/a_0}{s(s + a_1/a_0)}\right)$

S.S. ERROR FOR STEP: 0

S.S. ERROR FOR RAMP: 0

PARABOLA: $\lim_{s\to 0} \frac{1}{1 + \frac{-F_\varepsilon/a_0}{s(s+a_1/a_0)}\left(\frac{k_{D_0}s^2 + k_p s + k_I}{s}\right)} \frac{1}{s^2} = \frac{-a_1}{F_\varepsilon k_I}$

SYSTEM TYPE W/ RESPECT TO A DISTURBANCE:

PD: TYPE 0

PID: TYPE 1