HW #3 | CONTROLS | JOSHUA CABLAS

D.4) a) EOM: $m\ddot{z} + kz = F - b\dot{z}$

EQUILIBRIUM POINTS: $\dot{z}_e = \ddot{z}_e = 0$, $f(z_e, u_e) = 0$

$$\boxed{z_e = \text{ANYTHING}, \quad \ddot{z}_e = \dot{z}_e = 0, \quad F_e = kz_e}$$

b) $m\ddot{z} = m\ddot{z}_e + m\left.\frac{\partial \ddot{z}}{\partial \ddot{z}}\right|_e(\ddot{z} - \ddot{z}_e) = m(\ddot{z}_e + \ddot{\tilde{z}})$

$$kz = kz_e + k\left.\frac{\partial z}{\partial z}\right|_e(z - z_e) = k(z_e + \tilde{z})$$

$$F = F_e + \left.\frac{\partial F}{\partial F}\right|_e(F - F_e) = F_e + \tilde{F}$$

$$b\dot{z} = b\dot{z}_e + \left.\frac{\partial \dot{z}}{\partial \dot{z}}\right|_e(\dot{z} - \dot{z}_e) = b(\dot{z}_e + \dot{\tilde{z}})$$

$m(\ddot{z}_e + \ddot{\tilde{z}}) + k(z_e + \tilde{z}) = F_e + \tilde{F} - b(\dot{z}_e + \dot{\tilde{z}})$

SIMPLIFIES TO

$$\boxed{m\ddot{\tilde{z}} + k\tilde{z} = \tilde{F} - b\dot{\tilde{z}}}$$

c) $F = \tilde{F}$

$$\boxed{m\ddot{\tilde{z}} + k\tilde{z} = \tilde{F} - b\dot{\tilde{z}}}$$

E.4) a) EOM: $\begin{bmatrix} m_1\ddot{z} \\ 2m_1\dot{z}\dot{\theta}z + m_1z^2\ddot{\theta} + \frac{1}{3}m_2\ddot{\theta}l^2 \end{bmatrix} - \begin{bmatrix} m_1z\dot{\theta}^2 - m_1g\sin(\theta) \\ -m_1gz\cos(\theta) - m_2g\frac{l}{2}\cos(\theta) \end{bmatrix} = \begin{pmatrix} 0 \\ Fl\cos\theta \end{pmatrix}$

$\dot{z}_e = \ddot{z}_e = \dot{\theta}_e = \ddot{\theta}_e = 0$ $\quad z_e = \text{ANYTHING}$

$F_e l\cos\theta_e = +m_1gz_e + m_2g\frac{l}{2}$

$$\boxed{\begin{array}{l} m_1g\sin(\theta_e) = 0, \quad \theta_e = 0, \pm\pi, \pm 2\pi \\ F_e = \frac{+m_1g}{l}z_e + \frac{m_2g}{2} \end{array}}$$

b) $z\dot{\theta}^2 = z_e\dot{\theta}_e^2 + \left.\frac{\partial z\dot{\theta}^2}{\partial z}\right|_e(z - z_e) + \left.\frac{\partial z\dot{\theta}^2}{\partial \dot{\theta}}\right|_e(\dot{\theta} - \dot{\theta}_e)$

$$= 0 + \cancel{\dot{\theta}_{eq}^2}\tilde{z} + 2\cancel{\dot{\theta}_{eq}}\dot{\tilde{\theta}} = 0$$

$$\sin\theta = \sin\theta_e + \left.\frac{\partial \sin\theta}{\partial\theta}\right|_e(\theta - \theta_e) = \cos\theta_e\tilde{\theta} = \tilde{\theta}$$

① $m_1\ddot{\tilde{z}} + m_1g\tilde{\theta} = 0$

$\dot{z}\dot{\theta}z = \dot{z}_e\dot{\theta}_e z_e + \left.\frac{\partial \dot{z}_e\dot{\theta}_e z_e}{\partial \dot{z}_e}\right|_e(\dot{z} - \dot{z}_e) + \left.\frac{\partial \dot{z}_e\dot{\theta}_e z_e}{\partial \dot{\theta}_e}\right|_e(\dot{\theta} - \dot{\theta}_e) + \left.\frac{\partial \dot{z}_e\dot{\theta}_e z_e}{\partial z_e}\right|_e(z - z_e) = 0$

$z^2\ddot{\theta} = z_e^2\ddot{\theta}_e + \left.\frac{\partial z^2\ddot{\theta}}{\partial z}\right|_e(z - z_e) + \left.\frac{\partial \ddot{\theta}z^2}{\partial \ddot{\theta}}\right|_e(\ddot{\theta} - \ddot{\theta}_e) = 0 + 2\cancel{z_e\ddot{\theta}_e}\tilde{z} + z_e^2\ddot{\tilde{\theta}} = z_e^2\ddot{\tilde{\theta}}$

$z\cos\theta = z_e\cos(\theta_e) + \left.\frac{\partial z\cos\theta}{\partial z}\right|_e(z - z_e) + \left.\frac{\partial z\cos\theta}{\partial\theta}\right|_e(\theta - \theta_e) = z_e + \cos\theta_e\tilde{z} - \cancel{\sin(\theta_e)}\tilde{\theta} = z_e + \tilde{z}$

$\cos\theta = \cos\theta_e + \left.\frac{\partial\cos\theta}{\partial\theta}\right|_e(\theta - \theta_e) = 1 + \sin(\theta_e)\tilde{\theta} = 1$

$$m_1\ddot{\tilde{z}} + m_1 g\tilde{\theta} = 0$$

$$m_1 z_e^2\ddot{\tilde{\theta}} + \frac{1}{3}m_2\ddot{\tilde{\theta}}l^2 + m_1 g\tilde{z} + \cancel{m_1 g z_e} + \cancel{m_2 g\frac{l}{2}} = \cancel{F_e l} + \tilde{F}l$$

$$\boxed{\begin{aligned} &m_1\ddot{\tilde{z}} + m_1 g\tilde{\theta} = 0 \\ &\left(m_1 z_e^2 + \frac{1}{3}m_2 l^2\right)\ddot{\tilde{\theta}} + m_1 g\tilde{z} = \tilde{F}l \end{aligned}}$$

c) CAN'T USE FEEDBACK LINEARIZATION.

F.4) a) EOM:

$$\begin{pmatrix} m_c\ddot{z} + 2m_r\ddot{z} \\ m_c\ddot{h} + 2m_r\ddot{h} \\ 2d^2m_r\ddot{\theta} + J_c\ddot{\theta} \end{pmatrix} + \begin{pmatrix} 0 \\ (m_c+2m_r)g \\ 0 \end{pmatrix} = \begin{pmatrix} -F\sin\theta \\ F\cos\theta \\ \tau \end{pmatrix} - \begin{pmatrix} \mu\dot{z} \\ 0 \\ 0 \end{pmatrix}$$

EQUILIBRIUM POINTS:

$$F_e\sin\theta_e = 0 \ , \quad \theta_e = 0, \pm\pi, \pm 2\pi, \ldots$$

$$F_e\cos\theta_e = (m_c+2m_r)g \Rightarrow F_e = (m_c+2m_r)g$$

$$\tau_e = 0$$

b) $F\sin\theta = F_e\sin\theta_e + \left.\frac{\partial F\sin\theta}{\partial\theta}\right|_e(\theta-\theta_e) + \left.\frac{\partial F\sin\theta}{\partial F}\right|_e(F-F_e) = 0 + F_e\cos\theta_e\tilde{\theta} + \sin\theta_e\tilde{F} = F_e\tilde{\theta}$

① $m_c\ddot{\tilde{z}} + 2m_r\ddot{\tilde{z}} = -F_e\tilde{\theta} - \mu\dot{\tilde{z}} \Rightarrow (m_c+2m_r)\ddot{\tilde{z}} = -(m_c+2m_r)g\tilde{\theta} - \mu\dot{\tilde{z}}$

$F\cos\theta = F_e\cos\theta_e + \left.\frac{\partial F\cos\theta}{\partial\theta}\right|_e(\theta-\theta_e) + \left.\frac{\partial F\cos\theta}{\partial F}\right|_e(F-F_e) = F_e - \cancel{F_e\sin\theta_e\tilde{\theta}} + \cos\theta_e\tilde{F} = F_e + \tilde{F}$

② $m_c\ddot{\tilde{h}} + 2m_r\ddot{\tilde{h}} + (m_c+2m_r)g = F_e + \tilde{F} \Rightarrow (m_c+2m_r)\ddot{\tilde{h}} = \tilde{F}$

③ $2d^2m_r\ddot{\tilde{\theta}} + J_c\ddot{\tilde{\theta}} = \tilde{\tau} \Rightarrow (2d^2m_r + J_c)\ddot{\tilde{\theta}} = \tilde{\tau}$

$$\boxed{\begin{aligned} &(m_c+2m_r)\ddot{\tilde{z}} = -(m_c+2m_r)g\tilde{\theta} - \mu\dot{\tilde{z}} \\ &(m_c+2m_r)\ddot{\tilde{h}} = \tilde{F} \\ &(2d^2m_r + J_c)\ddot{\tilde{\theta}} = \tilde{\tau} \end{aligned}}$$

c) CAN'T USE FEEDBACK LINEARIZATION

D.5) a) LINEARIZED EOM: $m\ddot{\tilde{z}} + k\tilde{z} = \tilde{F} - b\dot{\tilde{z}}$

$$\mathcal{L}\{m\ddot{\tilde{z}} + k\tilde{z} = \tilde{F} - b\dot{\tilde{z}}\} \Rightarrow \boxed{ms^2\tilde{Z}(s) + k\tilde{Z}(s) = \tilde{F}(s) - bs\tilde{Z}(s)}$$

b) $$\tilde{Z}(s)\left[ms^2 + k + bs\right] = \tilde{F}(s)$$

$$\tilde{Z}(s) = \underbrace{\left(\frac{1}{ms^2+k+bs}\right)}_{\text{TRANSFER FUNCTION}}\tilde{F}(s)$$

c) $$\tilde{F}(s) \longrightarrow \boxed{\frac{1}{ms^2+bs+k}} \longrightarrow \tilde{Z}(s)$$

E.5) a) LINEARIZED EOM:

$$m_1\ddot{\tilde{z}} + m_1 g\tilde{\theta} = 0$$

$$(m_1 z_e^2 + \tfrac{1}{3}m_2 l^2)\ddot{\tilde{\theta}} + m_1 g\tilde{z} = \tilde{F}l$$

$$\mathcal{L}\{m_1\ddot{\tilde{z}} + m_1 g\tilde{\theta} = 0\} \Rightarrow \boxed{m_1 s^2\tilde{Z}(s) + m_1 g\tilde{\Theta}(s) = 0}$$

$$\boxed{(m_1 z_e^2 + \tfrac{1}{3}m_2 l^2)s^2\tilde{\Theta}(s) + m_1 g\tilde{Z}(s) = l\tilde{F}(s)}$$

b) $$\begin{pmatrix} m_1 s^2 & m_1 g \\ m_1 g & (m_1 z_e^2 + \tfrac{1}{3}m_2 l^2)s^2 \end{pmatrix}\begin{pmatrix} Z(s) \\ \Theta(s) \end{pmatrix} = \begin{pmatrix} 0 \\ l \end{pmatrix}\tilde{F}(s)$$

$$\begin{pmatrix} Z(s) \\ \Theta(s) \end{pmatrix} = \frac{\begin{pmatrix} * & -m_1 g \\ * & m_1 s^2 \end{pmatrix}}{(m_1 s^2)(m_1 z_e^2 + \tfrac{1}{3}m_2 l^2)s^2 + (m_1 g)^2}\begin{pmatrix} 0 \\ l \end{pmatrix}\tilde{F}(s)$$

$$\tilde{Z}(s) = \frac{-m_1 g l}{(m_1 s^2)(m_1 z_e^2 + \tfrac{1}{3}m_2 l^2)s^2 - (m_1 g)^2}\tilde{F}(s)$$

$$\tilde{\Theta}(s) = \frac{m_1 s^2 l}{(m_1 s^2)(m_1 z_e^2 + \tfrac{1}{3}m_2 l^2)s^2 - (m_1 g)^2}\tilde{F}(s)$$

$$\frac{\tilde{Z}(s)}{\tilde{\Theta}(s)} = \frac{\tilde{Z}(s)}{\tilde{F}(s)}\,\frac{\tilde{F}(s)}{\tilde{\Theta}(s)} = \frac{-\cancel{m_1}g\cancel{l}}{\cancel{m_1}s^2\cancel{l}} = \frac{-g}{s^2} \Rightarrow \tilde{Z}(s) = \frac{-g}{s^2}\tilde{\Theta}(s)$$

c) IT WOULD SIMPLIFY THE DENOMINATOR FOR THE TRANSFER FUNCTION FROM F TO Θ.

$$\tilde{\Theta}(s) = \frac{m_1 s^2 \ell}{(m_1 s^4)(m_1 z_e^2 + \frac{1}{3} m_2 \ell^2)} \Rightarrow \tilde{\Theta}(s) = \frac{\ell}{(m_1 z_e^2 + \frac{1}{3} m_2 \ell^2)s^2}$$

d)

$$\tilde{F}(s) \longrightarrow \boxed{\frac{\ell}{(m_1 z_e^2 + \frac{1}{3} m_2 \ell^2)s^2}} \xrightarrow{\tilde{\Theta}(s)} \boxed{-\frac{g}{s^2}} \xrightarrow{\tilde{z}(s)}$$

F.5) LINEARIZED EOM:

$$(m_c + 2m_r)\ddot{\tilde{z}} = -(m_c + 2m_r)g\tilde{\theta} - \mu\dot{\tilde{z}}$$

$$(m_c + 2m_r)\ddot{\tilde{h}} = \tilde{F}$$

$$(2d^2 m_r + J_c)\ddot{\tilde{\theta}} = \tilde{\tau}$$

a) LAPLACE:

$$(m_c + 2m_r)s^2 \tilde{Z}(s) = -(m_c + 2m_r)g\tilde{\Theta}(s) - \mu s\tilde{Z}(s)$$

$$(m_c + 2m_r)s^2 \tilde{H}(s) = \tilde{F}(s)$$

$$(2d^2 m_r + J_c)s^2 \tilde{\Theta}(s) = \tilde{\tau}(s)$$

b)

$$\tilde{H}(s) = \frac{1}{(m_c + 2m_r)s^2}\tilde{F}(s)$$

c)

$$\begin{pmatrix} (m_c+2m_r)s^2 + \mu s \ , & (m_c+2m_r)g \\ 0 & , (2d^2 m_r + J_c)s^2 \end{pmatrix} \begin{pmatrix} \tilde{Z}(s) \\ \tilde{\Theta}(s) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tilde{\tau}(s)$$

$$\begin{pmatrix} \tilde{Z}(s) \\ \tilde{\Theta}(s) \end{pmatrix} = \frac{\begin{pmatrix} * & -(m_c+2m_r)g \\ * & (m_c+2m_r)s^2 + \mu s \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\left((m_c+2m_r)s^2 + \mu s\right)\left((2d^2 m_r + J_c)s^2\right)} \tilde{\tau}(s)$$

$$\tilde{Z}(s) = \frac{-(m_c + 2m_r)g}{\left((m_c+2m_r)s^2 + \mu s\right)\left((2d^2 m_r + J_c)s^2\right)} \tilde{\tau}(s) \qquad \tilde{Z}(s) = \frac{-(m_c + 2m_r)g}{(m_c+2m_r)s^2 + \mu s} \tilde{\Theta}(s)$$

$$\tilde{\Theta}(s) = \frac{\cancel{\left((m_c+2m_r)s^2 + \mu s\right)}}{\cancel{\left((m_c+2m_r)s^2 + \mu s\right)}\left((2d^2 m_r + J_c)s^2\right)} \tilde{\tau}(s) \Rightarrow \tilde{\Theta}(s) = \frac{1}{(2d^2 m_r + J_c)s^2}$$

d)

$$\tilde{F}(s) \longrightarrow \boxed{\frac{1}{(m_c+2m_r)s^2}} \xrightarrow{\tilde{H}(s)} \ , \quad \tilde{\tau}(s) \longrightarrow \boxed{\frac{1}{(2d^2 m_r + J_c)s^2}} \xrightarrow{\tilde{\Theta}(s)} \boxed{\frac{-(m_c+2m_r)g}{(m_c+2m_r)s^2 + \mu s}} \xrightarrow{\tilde{Z}(s)}$$