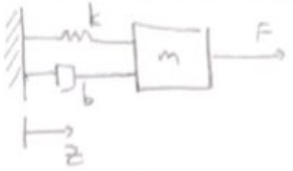
HW #1 | EC EN 483 | JOSHUA CANLAS

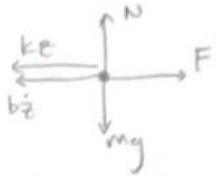
D.2)



PARAMETERS:
$m = 5$ kg $b = 0.5$ N-sec/m
$k = 3$ N/m

* NO FRICTION

$\dot{z} = \begin{pmatrix} \dot{z} \\ 0 \\ 0 \end{pmatrix}$



$$K = \frac{1}{2} m \dot{z}^T \dot{z} = \frac{1}{2} m \begin{pmatrix} \dot{z} \\ 0 \\ 0 \end{pmatrix}^T \begin{pmatrix} \dot{z} \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2} m \dot{z}^2$$

* SPRING PE

$$m\ddot{z} = F - kz - b\dot{z}$$

D.a) SEE SIMULATION

E.2)

$$P = \begin{pmatrix} z\cos\theta \\ z\sin\theta \\ 0 \end{pmatrix} \qquad V = \begin{pmatrix} \dot{z}\cos\theta - z\dot{\theta}\sin\theta \\ \dot{z}\sin\theta + z\dot{\theta}\cos\theta \\ 0 \end{pmatrix}$$

$$K_{BALL} = \frac{1}{2} m_1 \dot{z}^T \dot{z} = \frac{1}{2} m_1 \left[(\dot{z}\cos\theta - z\dot{\theta}\sin\theta)^2 + (\dot{z}\sin\theta + z\dot{\theta}\cos\theta)^2 \right]$$

$$= \frac{1}{2} m_1 \left[\left(\dot{z}^2\cos^2\theta - 2\dot{z}z\dot{\theta}\cos\theta\sin\theta + z^2\dot{\theta}^2\sin^2\theta \right) + \left(\dot{z}^2\sin^2\theta + 2\dot{z}z\dot{\theta}\sin\theta\cos\theta + z^2\dot{\theta}^2\cos^2\theta \right) \right]$$

$$K_{BALL} = \frac{1}{2} m_1 \left(\dot{z}^2 + z^2\dot{\theta}^2 \right)$$

14/48

BEAM:

$$P_B = \begin{pmatrix} \frac{l}{2}\cos\theta \\ \frac{l}{2}\sin\theta \\ 0 \end{pmatrix} \qquad V_B = \begin{pmatrix} -\frac{l}{2}\dot{\theta}\sin\theta \\ \frac{l}{2}\dot{\theta}\cos\theta \\ 0 \end{pmatrix} \qquad \omega = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}$$

$$K = \frac{1}{2} m_2 V^T V + \frac{1}{2} J_z \omega^T \omega$$

$$= \frac{1}{2} m_2 \left[\frac{l^2}{4}\dot{\theta}^2\sin^2\theta + \frac{l^2}{4}\dot{\theta}^2\cos^2\theta \right] + \frac{1}{2}\dot{\theta}^2 \frac{m_2 l^2}{12}$$

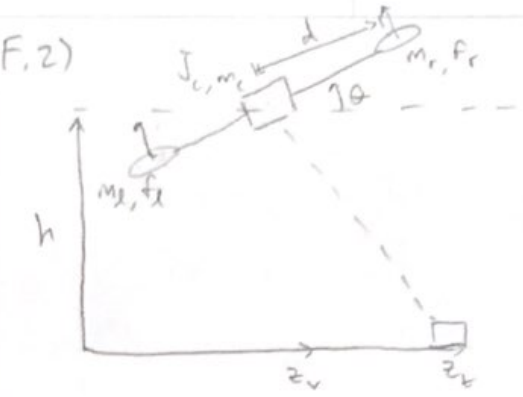
$$= \frac{1}{4} m_2 \dot{\theta}^2 l^2 + \frac{1}{24} m_2 \dot{\theta}^2 l^2$$

$$K_{BEAM} = \frac{7}{24} m_2 \dot{\theta}^2 l^2$$

$$K_{TOTAL} = \frac{1}{2} m_1 \left(\dot{z}^2 + z^2\dot{\theta}^2 \right) + \frac{7}{24} m_2 \dot{\theta}^2 l^2$$

E.a) SEE SIMULATION

F.2)



$$P_l = \begin{pmatrix} z_v - d\cos\theta \\ h - d\sin\theta \\ 0 \end{pmatrix}, \quad V_l = \begin{pmatrix} \dot{z}_v + d\dot{\theta}\sin\theta \\ \dot{h} - d\dot{\theta}\cos\theta \\ 0 \end{pmatrix}$$

$$P_r = \begin{pmatrix} z_v + d\cos\theta \\ h + d\sin\theta \\ 0 \end{pmatrix}, \quad V_r = \begin{pmatrix} \dot{z}_v - d\dot{\theta}\sin\theta \\ \dot{h} + d\dot{\theta}\cos\theta \\ 0 \end{pmatrix}$$

$$P_c = \begin{pmatrix} z_v \\ h \\ 0 \end{pmatrix}, \quad V_c = \begin{pmatrix} \dot{z}_v \\ 0 \\ 0 \end{pmatrix}$$

$$\omega_c = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}$$

$$K_{TOTAL} = \frac{1}{2} m_l V_l^T V_l + \frac{1}{2} m_c V_c^T V_c + \frac{1}{2} J_c \omega_c^T \omega_c + \frac{1}{2} m_r V_r^T V_r$$

$$= \frac{1}{2} m_l \left[\left(\dot{z}_v + d\dot{\theta}\sin\theta\right)^2 + \left(\dot{h} - d\dot{\theta}\cos\theta\right)^2 \right] + \frac{1}{2} m_c \dot{z}_v^2 + \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2} m_r \left[\left(\dot{z}_v - d\dot{\theta}\sin\theta\right)^2 + \left(\dot{h} + d\dot{\theta}\cos\theta\right)^2 \right]$$

$$= \frac{1}{2} m_l \left[\dot{z}_v^2 + 2\dot{z}_v d\dot{\theta}\sin\theta + d^2\dot{\theta}^2 \sin^2\theta + d^2\dot{\theta}^2\cos^2\theta + \dot{h}^2 - 2\dot{h}d\dot{\theta}\cos\theta \right] + \frac{1}{2} m_c \dot{z}_v^2 + \frac{1}{2} J_c \dot{\theta}^2$$

$$+ \frac{1}{2} m_r \left[\dot{z}_v^2 - 2\dot{z}_v d\dot{\theta}\sin\theta + d^2\dot{\theta}^2\sin^2\theta + d^2\dot{\theta}^2\cos^2\theta + \dot{h}^2 + 2\dot{h}d\dot{\theta}\cos\theta \right]$$

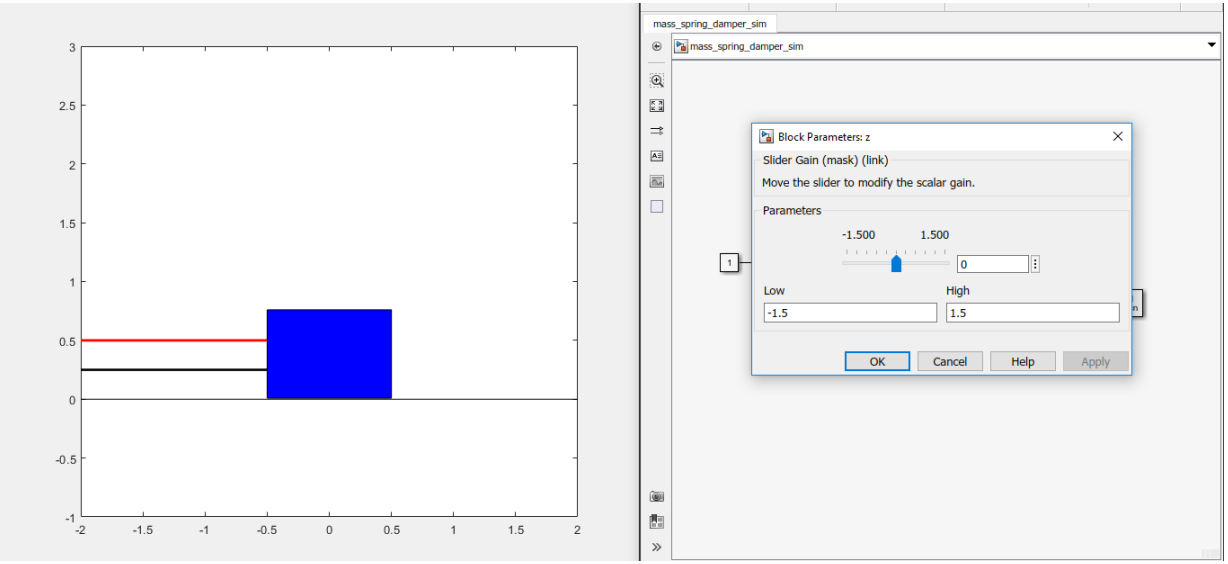
$$= \frac{1}{2} m_l \dot{z}_v^2 + m_l \dot{z}_v d\dot{\theta}\sin\theta + \frac{1}{2} m_l d^2\dot{\theta}^2 + \frac{1}{2} m_c \dot{z}_v^2 + \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2} m_r \dot{z}_v^2 - m_r \dot{z}_v d\dot{\theta}\sin\theta$$

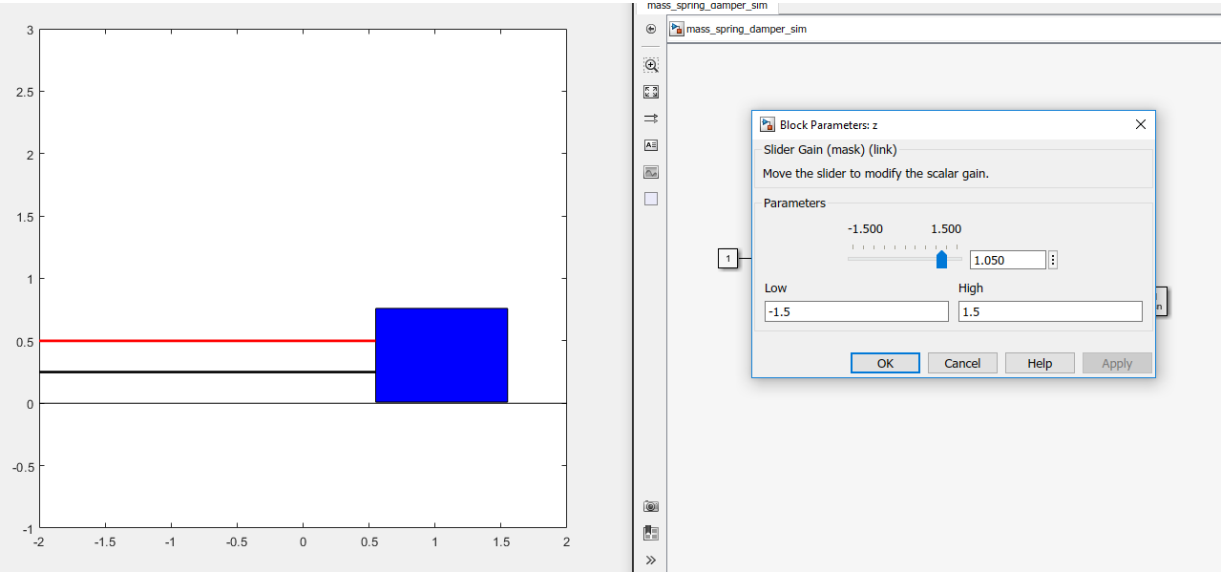
$$+ \frac{1}{2} m_r d^2\dot{\theta}^2 + \frac{1}{2} m_l \dot{h}^2 + \frac{1}{2} m_r \dot{h}^2$$

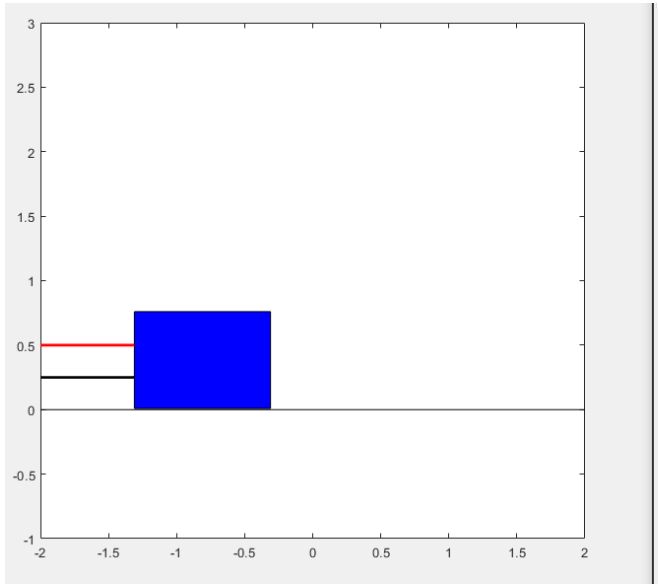
$$\boxed{K = \frac{1}{2}\dot{z}_v^2 \left(m_l + m_c + m_r\right) + \dot{z}_v d\dot{\theta}\sin\theta \left(m_l - m_r\right) + \frac{1}{2} d^2\dot{\theta}^2 \left(m_l + m_r\right) + \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2}\dot{h}^2 \left(m_l + m_r\right) + \dot{h}\dot{\theta}d\cos\theta \left(m_r - m_l\right)}$$

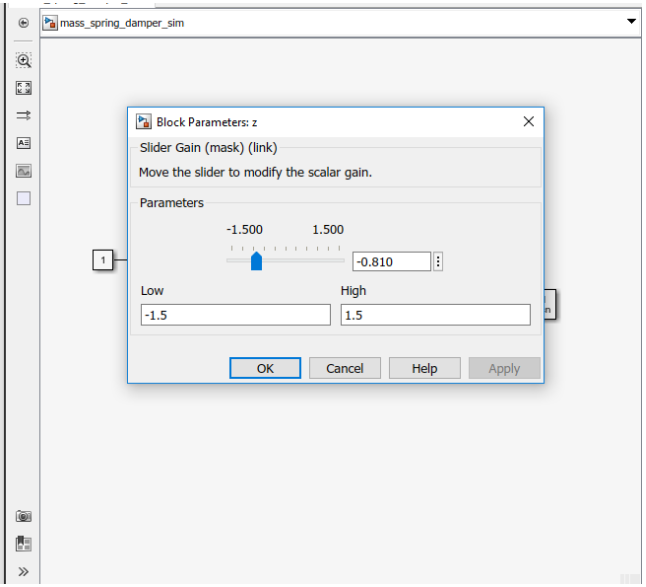
F.3) SEE SIMULATION

D.a) Mass-spring-damper

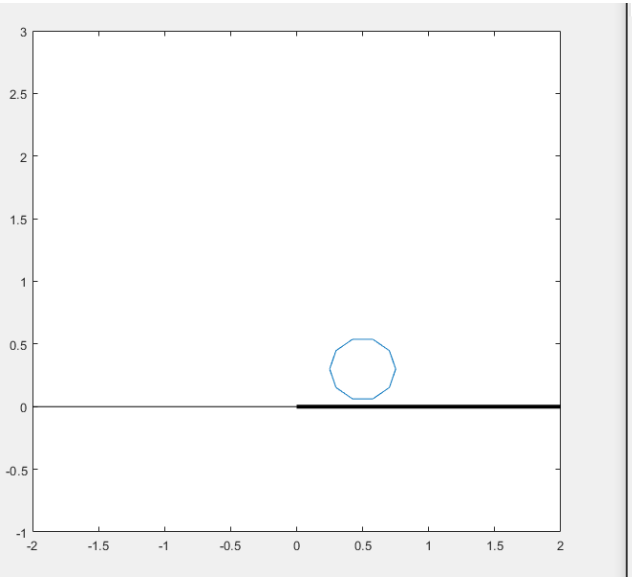


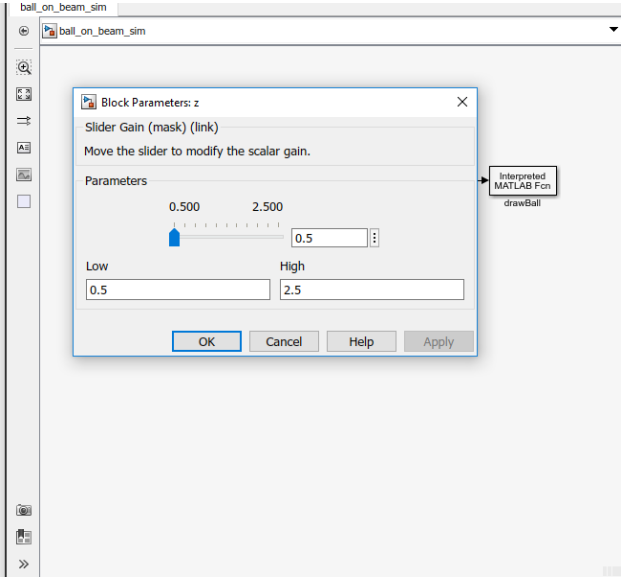


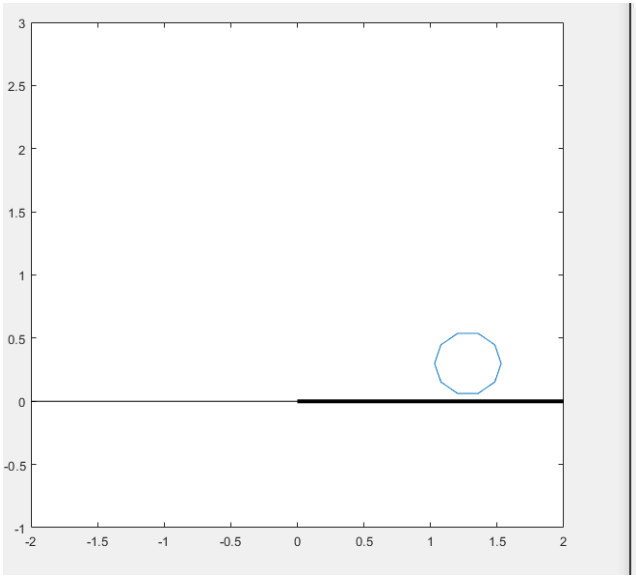

3
2.5
2
1.5
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

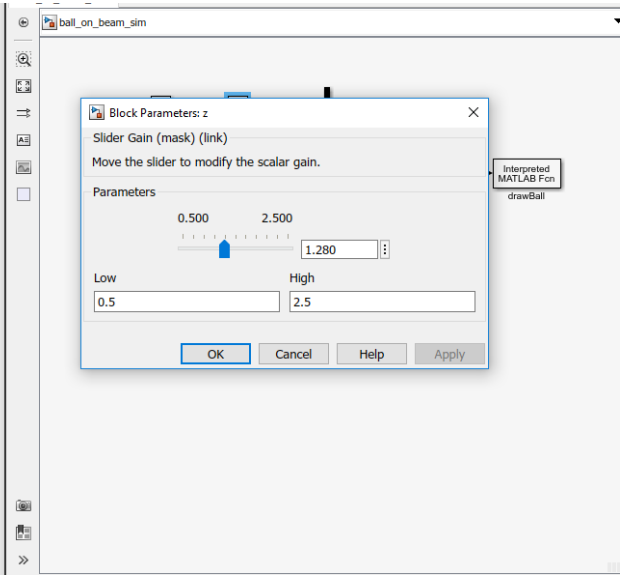

mass_spring_damper_sim
Block Parameters: z
Slider Gain (mask) (link)
Move the slider to modify the scalar gain.
Parameters
-1.500
1.500
-0.810
1
Low
High
-1.5
1.5
OK
Cancel
Help
Apply

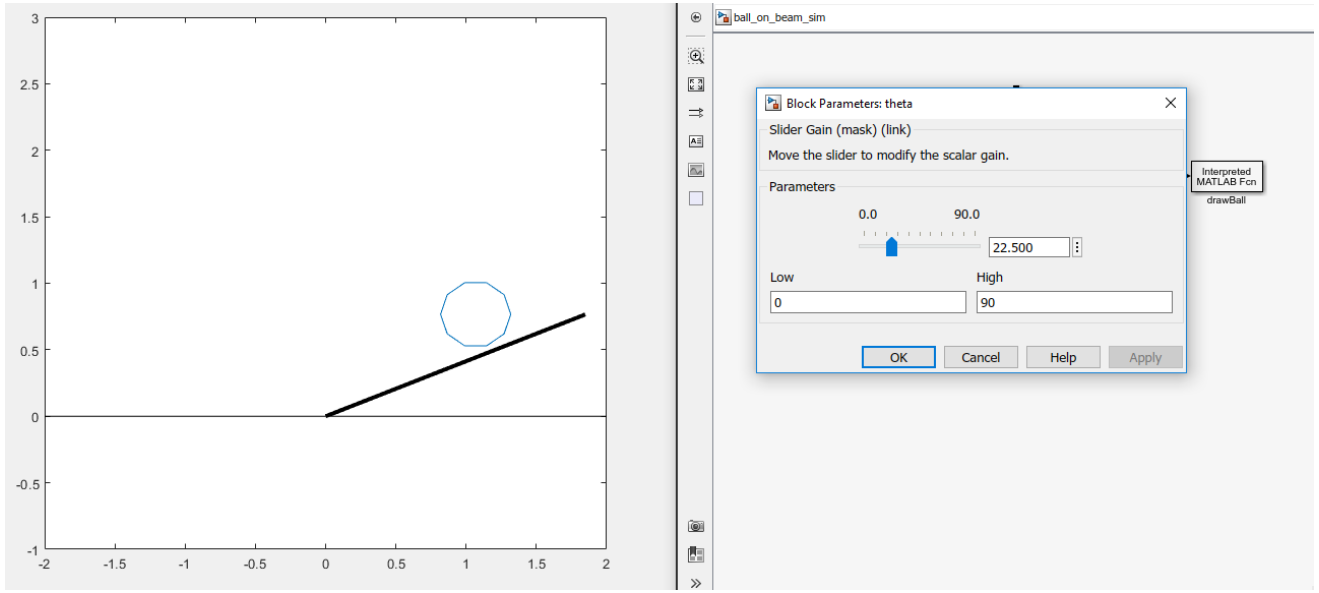
E.a) Ball on beam

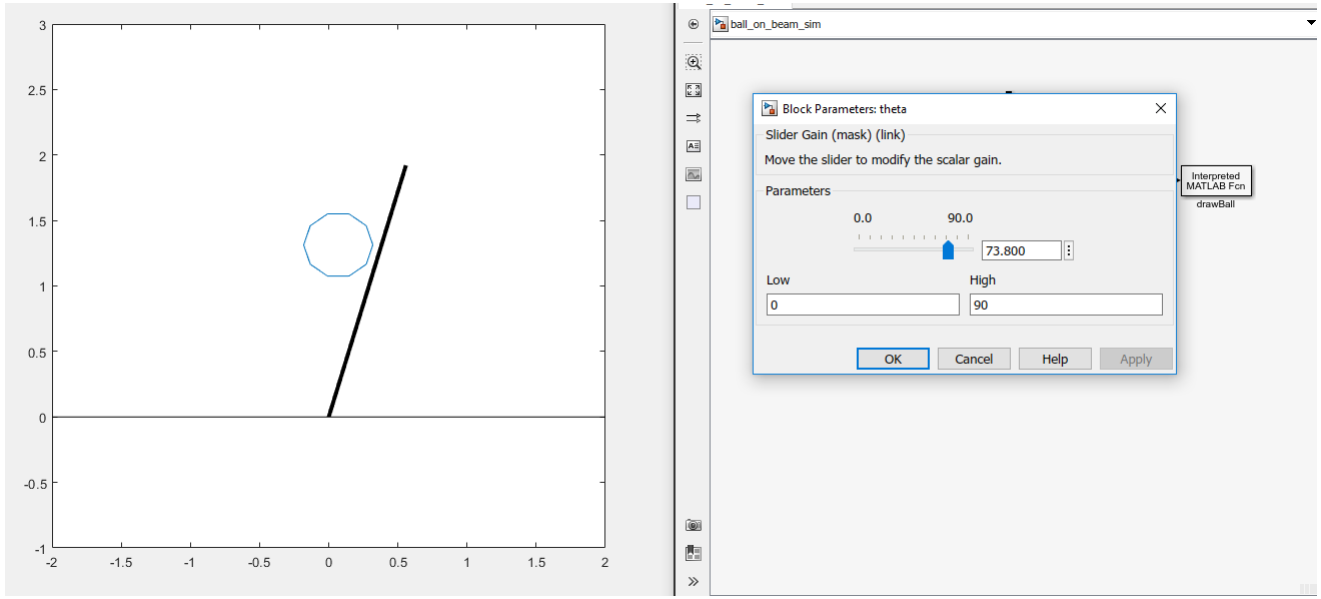




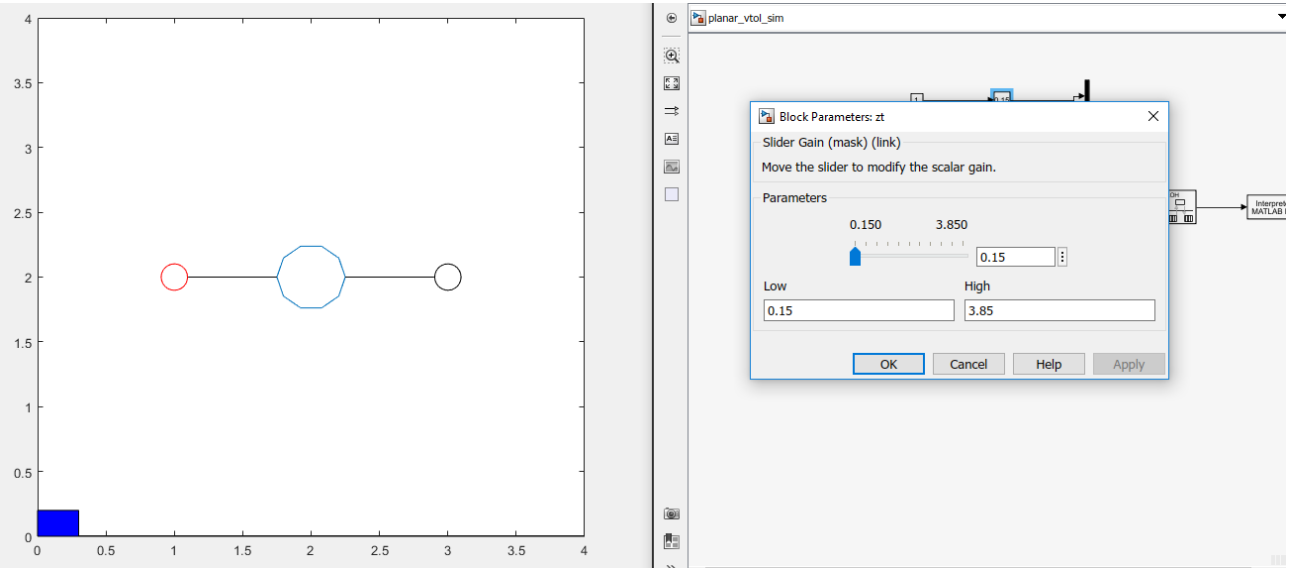


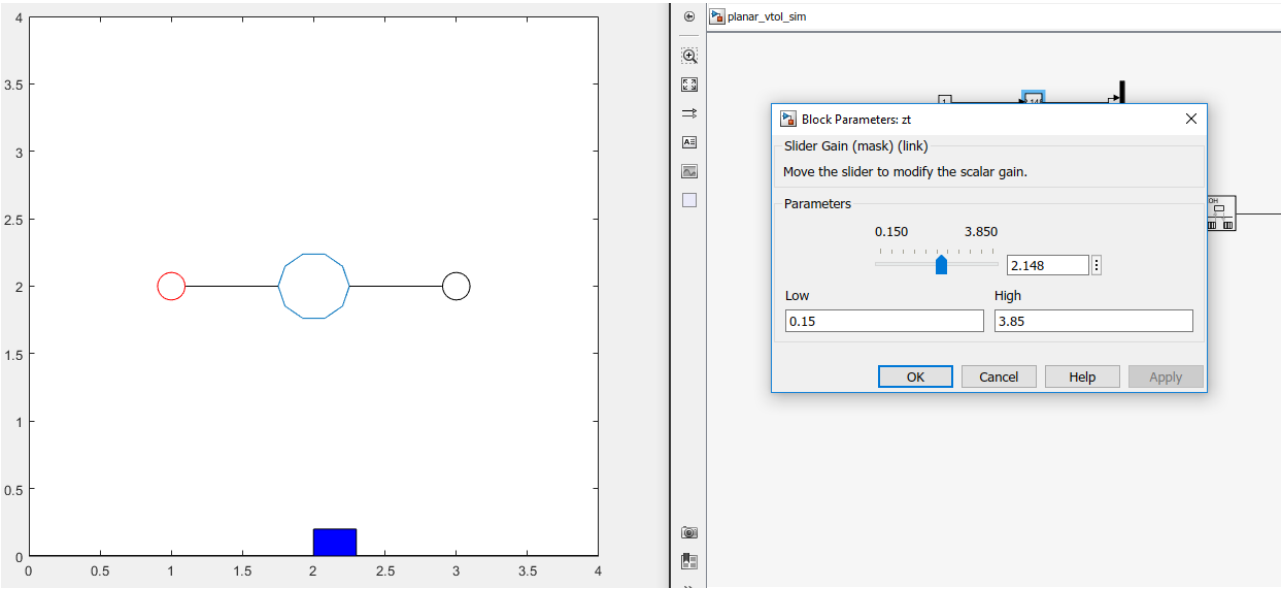


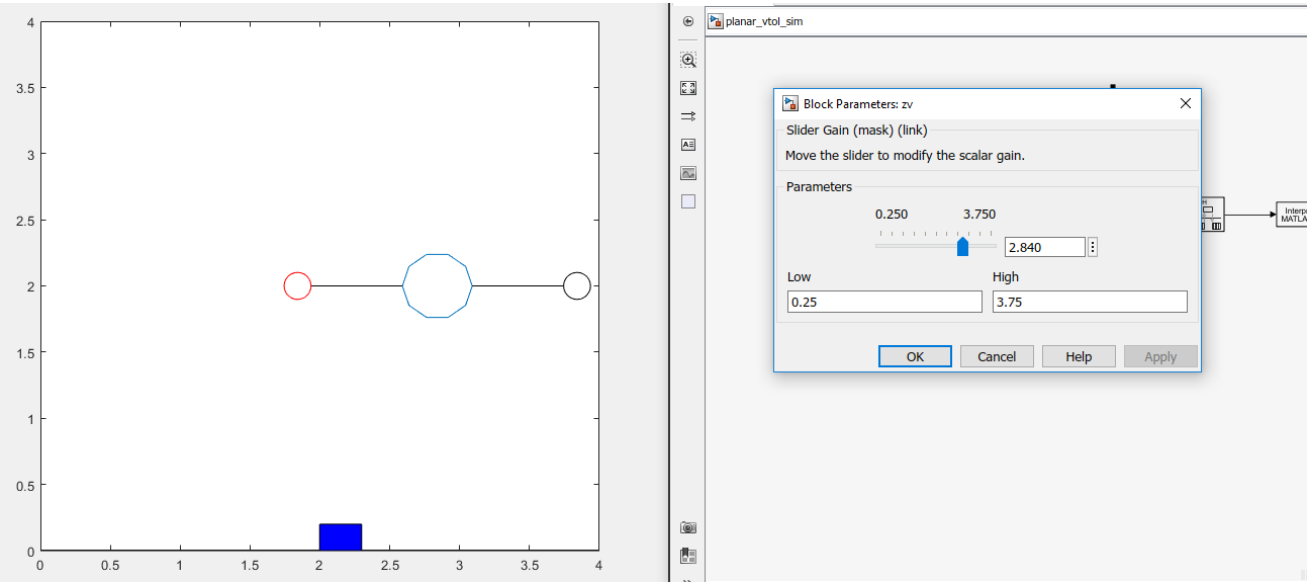

ball_on_beam_sim
Block Parameters: theta
Slider Gain (mask) (link)
Move the slider to modify the scalar gain.
Parameters
0.0
90.0
22.500
Low
High
0
90
OK
Cancel
Help
Apply
Interpreted MATLAB Fcn
drawBall

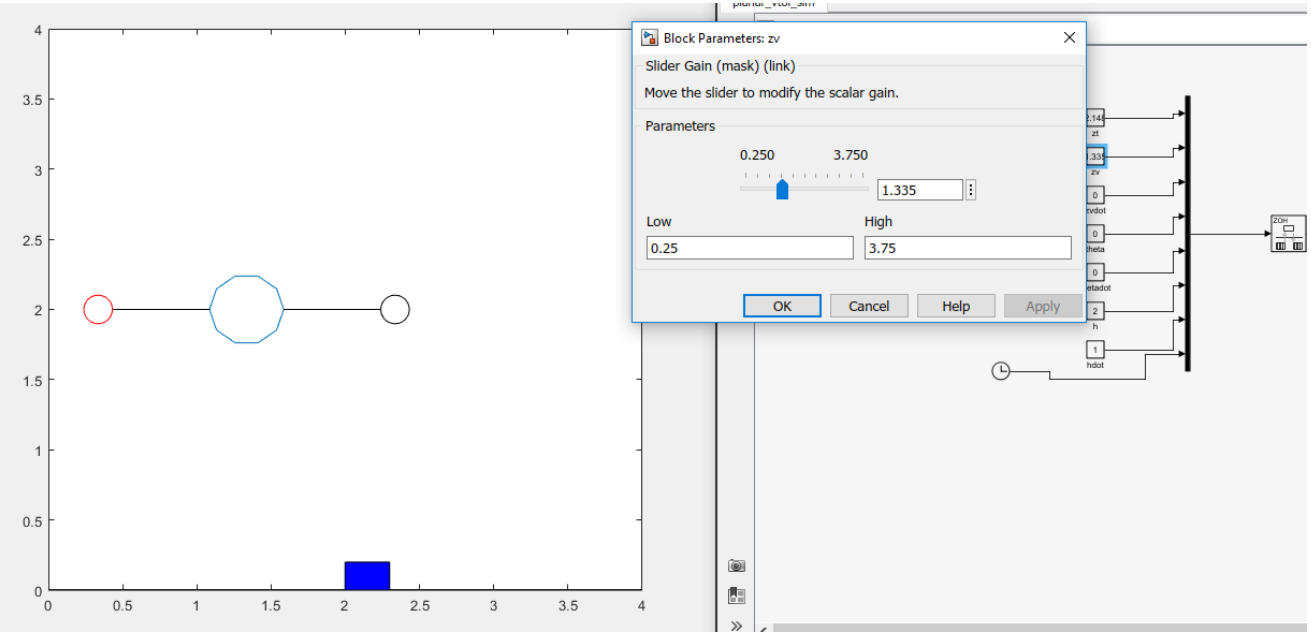

ball_on_beam_sim
Block Parameters: theta
Slider Gain (mask) (link)
Move the slider to modify the scalar gain.
Parameters
0.0
90.0
73.800
Low
High
0
90
OK
Cancel
Help
Apply
Interpreted MATLAB Fcn
drawBall

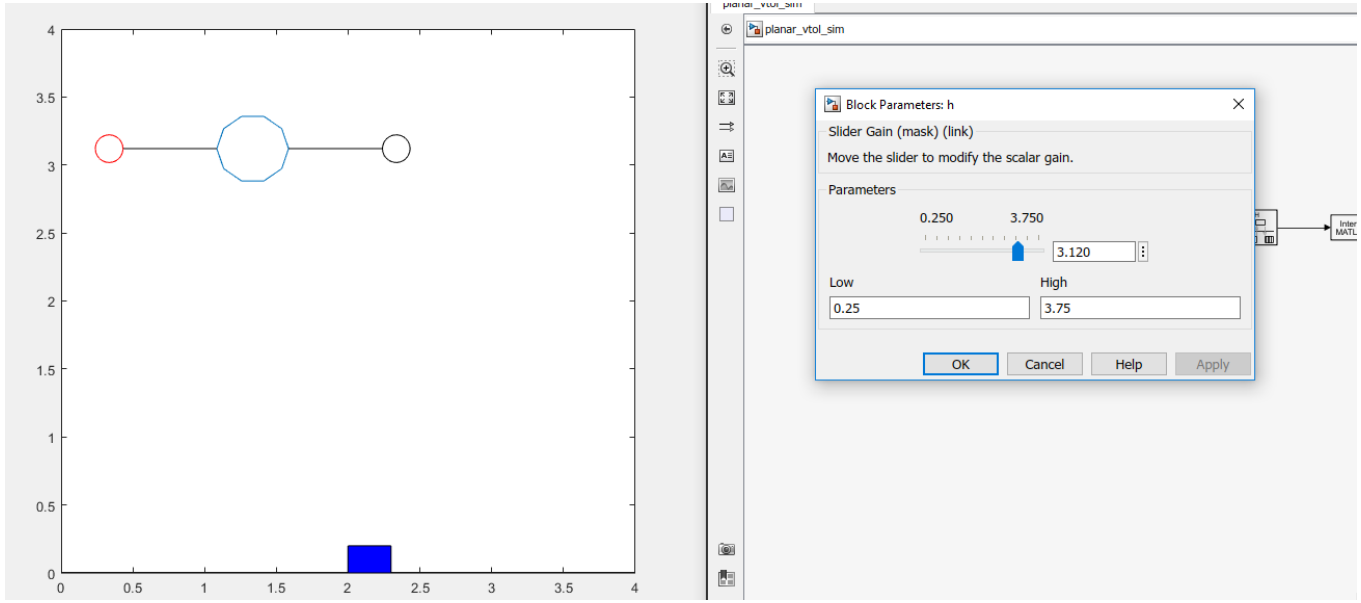
F.a) Planar VTOL

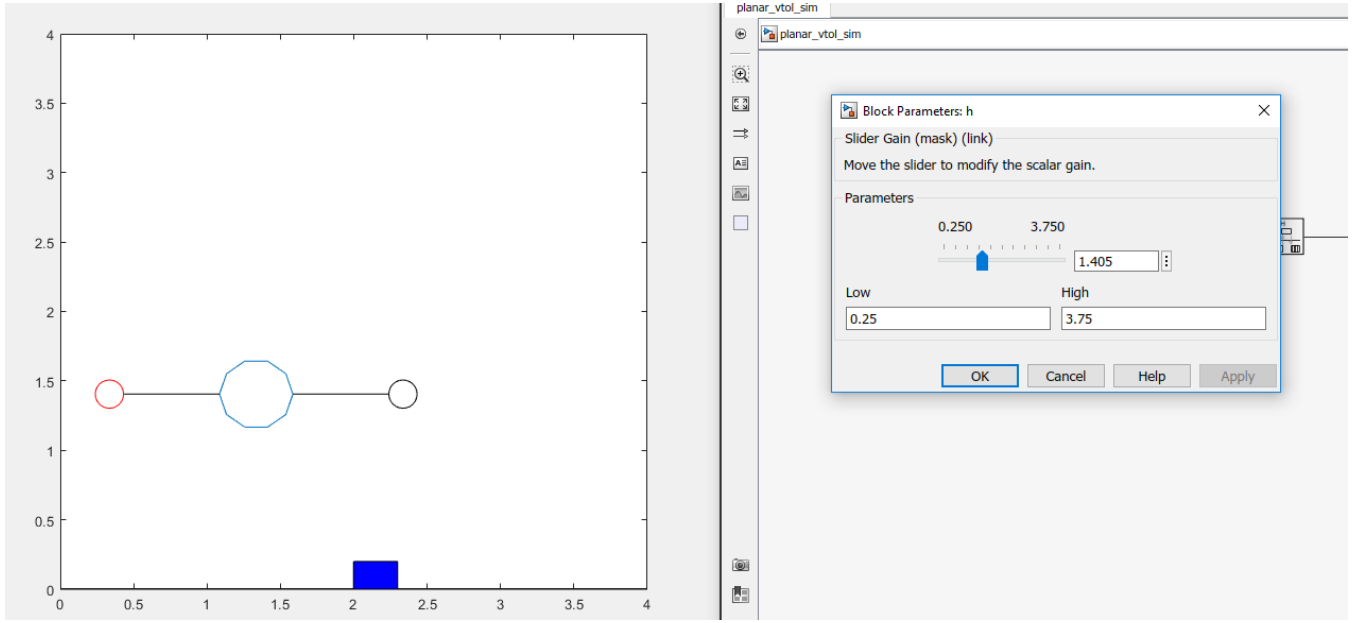


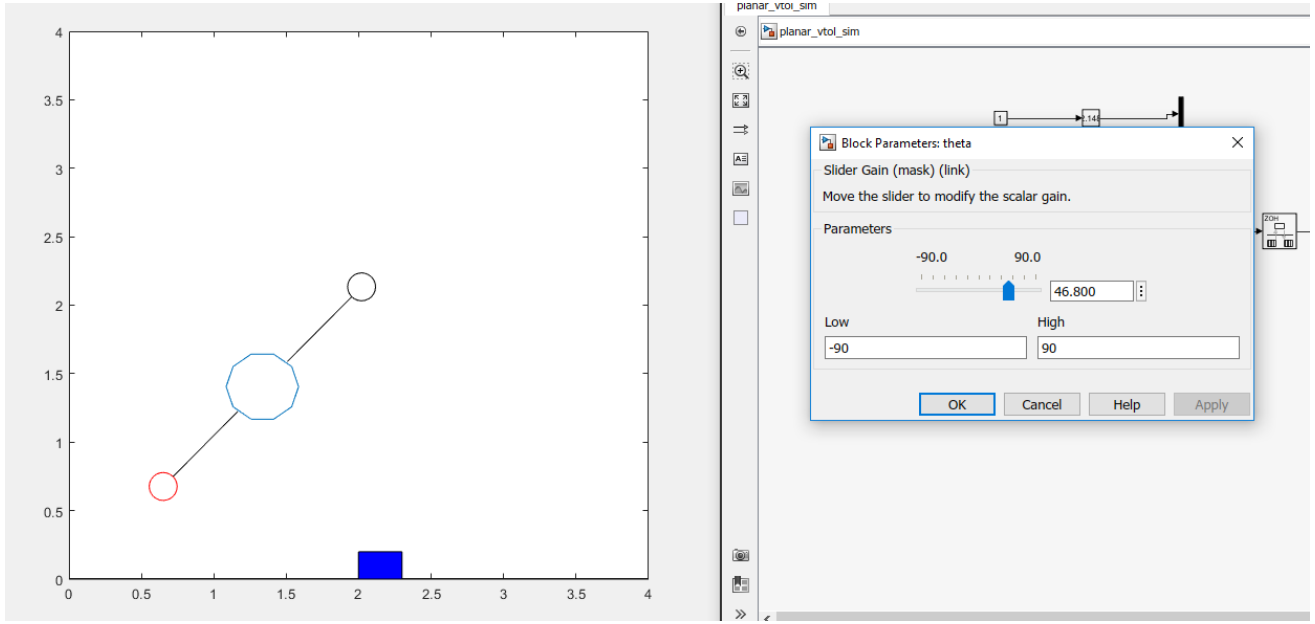


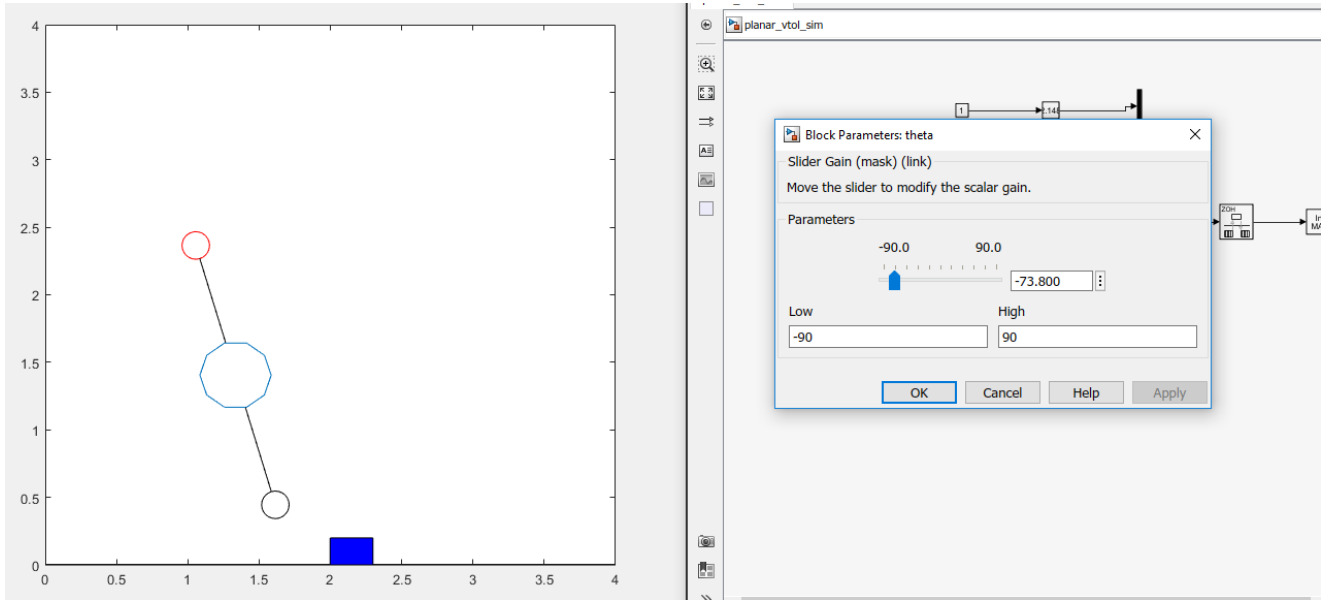

planar_vtol_sim
Block Parameters: zv
Slider Gain (mask) (link)
Move the slider to modify the scalar gain.
Parameters
0.250
3.750
2.840
Low
High
0.25
3.75
OK
Cancel
Help
Apply


Block Parameters: zv
Slider Gain (mask) (link)
Move the slider to modify the scalar gain.
Parameters
0.250
3.750
1.335
Low
High
0.25
3.75
OK
Cancel
Help
Apply


planar_vtol_sim
Block Parameters: h
Slider Gain (mask) (link)
Move the slider to modify the scalar gain.
Parameters
0.250
3.750
3.120
Low
High
0.25
3.75
OK
Cancel
Help
Apply


planar_vtol_sim
planar_vtol_sim
Block Parameters: h
Slider Gain (mask) (link)
Move the slider to modify the scalar gain.
Parameters
0.250
3.750
1.405
Low
High
0.25
3.75
OK
Cancel
Help
Apply


planar_vtol_sim
Block Parameters: theta
Slider Gain (mask) (link)
Move the slider to modify the scalar gain.
Parameters
-90.0
90.0
46.800
Low
High
-90
90
OK
Cancel
Help
Apply


planar_vtol_sim
Block Parameters: theta
Slider Gain (mask) (link)
Move the slider to modify the scalar gain.
Parameters
-90.0
90.0
-73.800
Low
High
-90
90
OK
Cancel
Help
Apply