HOMEWORK #1 | ME EN 534 | JOSHUA CANAS

1)
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [R_z(\phi)] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = [R_{x'}(\theta)] \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\begin{bmatrix} x''' \\ y''' \\ z''' \end{bmatrix} = [R_{z''}(\psi)] \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{[R_{z''}(\psi)][R_{x'}(\theta)][R_z(\phi)]}_{R} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

a)
$$\begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\psi & \sin\psi\cos\theta & \sin\psi\sin\theta \\ -\sin\psi & \cos\psi\cos\theta & \cos\psi\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[R] = \begin{bmatrix} (c\psi c\phi - c\psi c\theta s\phi), & (c\psi s\phi + s\psi c\theta c\phi), & (s\psi s\theta) \\ (-s\psi c\phi - c\psi c\theta s\phi), & (-s\psi s\phi + c\psi c\theta c\phi), & (c\psi s\theta) \\ (s\theta s\phi) & , (-s\theta c\phi) & , c\theta \end{bmatrix} \leftarrow \quad c = \cos,\ s = \sin$$

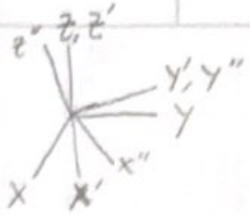
b) $[R]$ FOR $\phi = 30°$, $\theta = 45°$, $\psi = -60°$

USING MATLAB SCRIPT,

$$[R] = \begin{bmatrix} 0.7392 & -0.2803 & -0.6124 \\ 0.5732 & 0.7392 & 0.3536 \\ 0.3536 & -0.6124 & 0.7071 \end{bmatrix} \leftarrow$$

c)
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = [R]^T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [R]^T \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -1.9763 \\ 3.6192 \\ 4.1225 \end{bmatrix} \leftarrow$$

2) PROBLEM 3.3



X = N-S
Y = E-W
Z = V

① $R_z(40°)$

② $R_{y'}(-20°)$

③ $R_{x''}(-10°)$

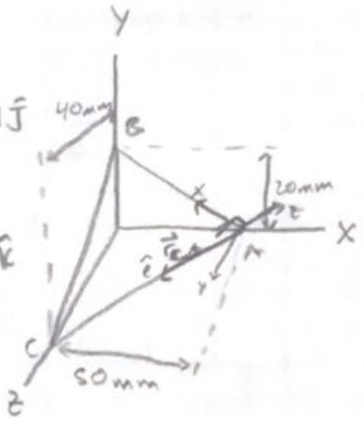
$$[R] = [R_{x''}(-10)][R_{y'}(-20)][R_z(40)]$$

$$NWU = [R]^T \begin{bmatrix} 0.5 \\ 0 \\ -2 \end{bmatrix} \text{ft} \quad \xrightarrow{\text{USING MATLAB FUNCTIONS}} \quad NWU = \begin{bmatrix} 0.6527 \\ 1.0011 \\ -1.6798 \end{bmatrix} \text{ft} \longleftarrow$$

3) PROBLEM 3.4

$$\hat{i} = \frac{-50\hat{I} + 20\hat{J}}{\sqrt{50^2 + 20^2}} = -0.9285\hat{I} + 0.3714\hat{J}$$

$$\hat{e} = \frac{-50\hat{I} + 40\hat{K}}{\sqrt{(50)^2 + 40^2}} = -0.7809\hat{I} + 0.6247\hat{K}$$



$$\vec{r}_{B/A} = -50\hat{I} + 20\hat{J}$$

$$\vec{r}_{C/A} = -50\hat{I} + 40\hat{K}$$

$$\hat{k} = \hat{i} \times \hat{e}$$

$$= (-0.9285\hat{I} + 0.3714\hat{J}) \times (-0.7809\hat{I} + 0.6247\hat{K})$$

$$\hat{k} = 0.2320\hat{I} + 0.5800\hat{J} + 0.2900\hat{K}$$

$$\hat{j} = \hat{k} \times \hat{i}$$

$$= (0.2320\hat{I} + 0.5800\hat{J} + 0.2900\hat{K}) \times (-0.9285\hat{I} + 0.3714\hat{J})$$

$$\hat{j} = -0.1077\hat{I} - 0.2693\hat{J} + 0.6247\hat{K}$$

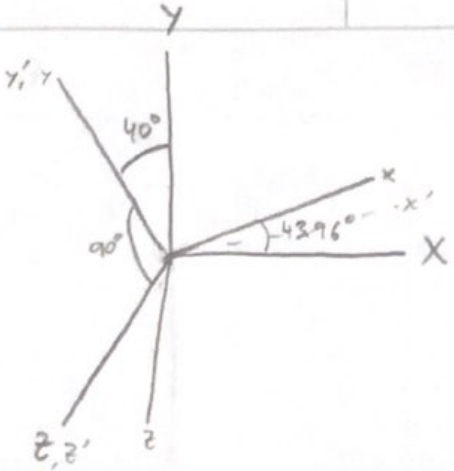
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} -0.9285 & 0.3714 & 0 \\ -0.1564 & -0.3910 & 0.9070 \\ 0.3369 & 0.8422 & 0.4211 \end{bmatrix}}_{[R]} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \longleftarrow$$

$$\vec{r}_{O/A} = -50\hat{I}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [R] \begin{bmatrix} -50 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 46.4238 \\ 7.8192 \\ -16.8430 \end{bmatrix} \text{mm} \longleftarrow$$

4)
FIND [R] WHERE

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [R] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



$$[R] = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

$l_{11} = \cos(43.96)$
$l_{22} = \cos(40)$
$l_{23} = \cos(90)$

$$[R] = [R_{y'}(\theta)][R_z(40)]$$

$$\begin{bmatrix} \cos(43.96) & l_{12} & l_{13} \\ l_{21} & \cos(40) & \cos(90) \\ l_{31} & l_{32} & l_{33} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos 40 & \sin 40 & 0 \\ -\sin 40 & \cos 40 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

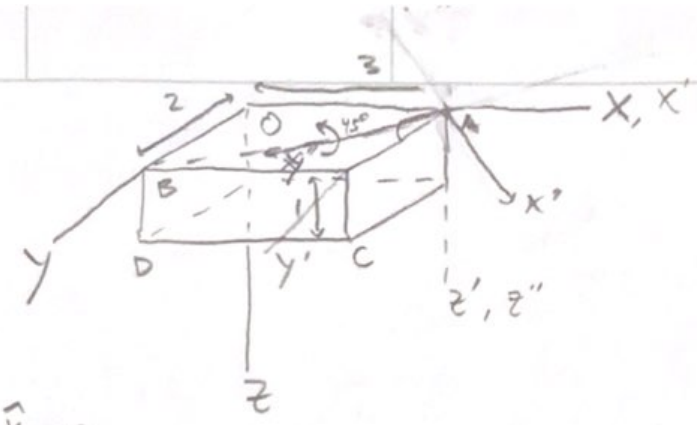
$$= \begin{bmatrix} \cos\theta\cos 40 & \cos\theta \sin 40 & -\sin\theta \\ -\sin 40 & \cos 40 & 0 \\ \sin\theta\cos 40 & \sin\theta\sin 40 & \cos\theta \end{bmatrix}$$

$$\cos\theta \cos 40 = \cos 43.96$$

$$\theta = \cos^{-1}\left(\frac{\cos 43.96}{\cos 40}\right) = 20.00°$$

$$[R] = \begin{bmatrix} 0.7198 & 0.6040 & -0.3421 \\ -0.6428 & 0.7660 & 0 \\ 0.2621 & 0.2199 & 0.9397 \end{bmatrix} \longleftarrow$$

5)



$\vec{CO} = 3\hat{I} + 2\hat{J} + 1\hat{K}$

$\vec{AO} = 3\hat{I}$

$\vec{AC} = \vec{CO} - \vec{AO} = 2\hat{J} + 1\hat{K}$ C_0

① TRANSLATE TO A C_1

② $R_{z'}(a\tan d(3/2))$ C_2

③ $R_{x''}(45)$ - ROTATE WHOLE BODY $\therefore$ $C_2 = C_3^*$

$$[R] = [R_{x''}(45)][R_{z'}(a\tan d(3/2))][\text{TRANSLATE TO A}]\begin{bmatrix}3\\2\\1\end{bmatrix}$$

USING MATLAB SCRIPTS:

$\vec{C}^* = 3.1219\,\hat{I} + 2.1828\,\hat{J} - 0.4696\,\hat{K}$ ⟵