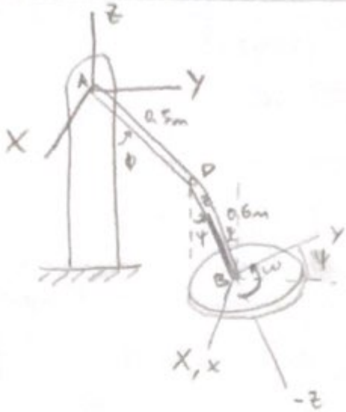
HOMEWORK #2 | ME 534 | JOSHUA CANALAS

1)

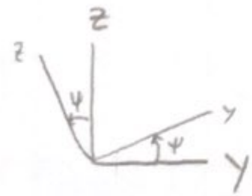


$\omega = 1500$ rpm

$\phi = 90^\circ$ $\quad \dot{\phi} = 0.2$ rad/s

$\psi = 60^\circ$ $\quad \dot{\psi} = -0.3$ rad/s

FIND ANG. VEL & ANG. ACCEL. IN TERMS OF INERTIAL COORDINATES (XYZ).



$\vec{\omega} = \vec{\omega}_{B/A} = \vec{\omega}_B + \vec{\omega}_D$

$\hat{k} = -\sin\psi\,\hat{J} + \cos\psi\,\hat{K}$

$\vec{\omega}_D = \dot{\psi}\,\hat{I}$

$\vec{\omega}_B = \omega\hat{k} = -\omega\sin\psi\,\hat{J} + \omega\cos\psi\,\hat{K}$

$\vec{\omega} = (\dot{\psi})\hat{I} - \omega\sin\psi\,\hat{J} + \omega\cos\psi\,\hat{K}$

$\vec{\omega} = -0.3\,\hat{I} - 136.035\,\hat{J} + 78.5398\,\hat{K}$ rad/s $\longleftarrow$

$\vec{\alpha} = \dot{\vec{\omega}} = (\dot{\vec{\omega}})_{rel} + \vec{\omega}_{FRAME} \times \vec{\omega}$ $\qquad \vec{\omega}_{FRAME} = 0$

$= \cancel{\ddot{\psi}}^{\,0} - (\cancel{\dot{\omega}\sin\psi} + \omega\dot{\psi}\cos\psi)\hat{J} + (\cancel{\dot{\omega}\cos\psi} - \omega\dot{\psi}\sin\psi)\hat{K}$

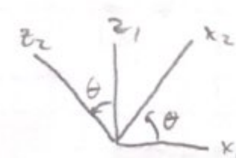
$\vec{\alpha} = 23.5619\,\hat{J} + 40.8105\,\hat{K}$ rad/s² $\longleftarrow$

2) $\omega_3 = 5000$ rpm $\qquad \theta = 75^\circ$

$\omega_1 = 3$ rad/s $\qquad \dot{\theta} = 0$ rad/s

$\dot{\omega}_1 = -1.8$ rad/s² $\qquad \ddot{\theta} = 3$ rad/s²

FIND ANG. ACCEL IN TERMS OF x_1, y_1, z_1



$\vec{\omega} = \vec{\omega}_{OG} + \vec{\omega}_{IG} + \vec{\omega}_{FW}$

$\vec{\omega}_{OG} = \omega_1\,\hat{i}_1$

$\vec{\omega}_{IG} = -\dot{\theta}\,\hat{j}_1$

$\hat{i}_2 = \cos\theta\,\hat{i}_1 + \sin\theta\,\hat{k}_1$

$\vec{\omega}_{FW} = \omega_3\,\hat{i}_2 = \omega_3\cos\theta\,\hat{i}_1 + \omega_3\sin\theta\,\hat{k}_1$

$\vec{\omega} = (\omega_1 + \omega_3\cos\theta)\hat{i}_1 - \dot{\theta}\,\hat{j}_1 + \omega_3\sin\theta\,\hat{k}_1$

$\vec{\omega} = 138.5173\,\hat{i}_1 + 505.7576\,\hat{k}_1$

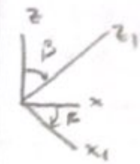
$\vec{\alpha} = (\dot{\vec{\omega}})_{rel} + \vec{\omega}_{FRAME} \times \vec{\omega}$ $\qquad \dot{\omega}_2 = 0 \quad \dot{\theta} = 0$

$(\dot{\vec{\omega}})_{rel} = \left[\dot{\omega}_1 + \left(\cancel{\dot{\omega}_2 \cos\theta} - \cancel{\omega_2 \dot{\theta} \sin\theta}\right)\right]\hat{i}_1 - \ddot{\theta}\,\hat{j}_1 + \left(\cancel{\dot{\omega}_2 \sin\theta} + \cancel{\omega_2 \dot{\theta} \cos\theta}\right)\hat{k}_1$

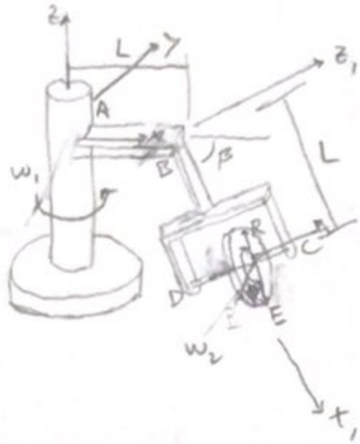
$(\dot{\vec{\omega}})_{rel} = \dot{\omega}_1\, \hat{i}_1 - \ddot{\theta}\, \hat{j}_1$

$\vec{\omega}_{FRAME} = \omega_1\, \hat{i}_1$

$\vec{\alpha} = -1.8\, \hat{i}_1 - 1520.3\, \hat{j}_1 \ \text{rad/s}^2 \longleftarrow$



3)



$\vec{r}_{E/A} = \vec{r}_{B/A} + \vec{r}_{E/B}$

$\vec{r}_{B/A} = L\hat{i} = L\cos\beta\,\hat{i}_1 + L\sin\beta\,\hat{k}_1$

$\hat{i} = \cos\beta\,\hat{i}_1 + \sin\beta\,\hat{k}_1$

$\vec{r}_{E/B} = (L + R\cos\phi)\hat{i}_1 - R\sin\phi\,\hat{j}_1$

$\vec{r}_{E/A} = (L + L\cos\beta + R\cos\phi)\hat{i}_1 - R\sin\phi\,\hat{j}_1 + L\sin\beta\,\hat{k}_1$

ASSUME SMALL ANGLE: $\sin\phi = \phi \quad \cos\phi = 1$

$\vec{r}_{E/A} = (L + L\cos\beta + R)\hat{i}_1 - R\phi\,\hat{j}_1 + L\sin\beta\,\hat{k}_1$

$\vec{V} = \dot{\vec{r}} = (\dot{\vec{r}}_{rel}) + \omega_{FRAME} \times \vec{r}$

$\omega_{FRAME} = \omega_1 \hat{k} = -\omega_1 \sin(\beta)\,\hat{i}_1 + \omega_1 \cos(\beta)\,\hat{k}_1$ $\qquad$ CONSTANTS: L, β, R

$\hat{k} = -\sin\beta\,\hat{i}_1 + \cos\beta\,\hat{k}_1$ $\qquad$ @ END $\phi = 0$

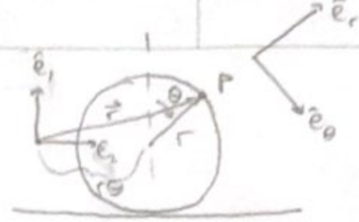
$(\dot{\vec{r}})_{rel} = -R\dot{\phi}\,\hat{j}_1 = -R\omega_2\,\hat{j}_1$ $\qquad \dot{\phi} = \omega_2$

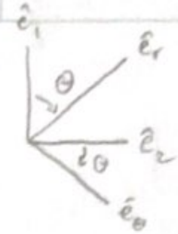
$$\vec{V} = \dot{\vec{r}} = \begin{bmatrix} R\phi\omega_1 \cos\beta \\ L\omega_1 - R\omega_2 + L\omega_1\cos\beta + R\omega_1\cos\beta \\ R\phi\,\omega_1 \sin\beta \end{bmatrix}$$ (THIS DESCRIBES VELOCITY AT ANY POINT)

VELOCITY AT $\phi = 0$ IS:

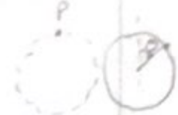
$\vec{V} = L\omega_1 - R\omega_2 + L\omega_1\cos\beta + R\omega_1\cos\beta \longleftarrow$

4)





$$\begin{bmatrix} r \\ \theta \\ z \end{bmatrix}$$



$\vec r = r\cos\theta\,\hat e_1 + (r\theta + r\sin\theta)\hat e_2$

$\vec\omega_{FRAME} = \omega\,\hat e_z$

$\hat e_1 = -\sin\theta\,\hat e_\theta + \cos\theta\,\hat e_r$

$\dot\theta = \omega = \alpha t$

$\ddot\theta = \dot\omega = \alpha$

$\hat e_2 = \cos\theta\,\hat e_\theta + \sin\theta\,\hat e_r$

$\vec r = -r\cos\theta\sin\theta\,\hat e_\theta + \underline{r\cos^2\theta\,\hat e_r} + (r\theta + r\sin\theta)\cos\theta\,\hat e_\theta + \underline{(r\theta + r\sin\theta)\sin\theta}\,\hat e_r$

$\vec r = (r\theta\sin\theta + r)\,\hat e_r + r\theta\cos\theta\,\hat e_\theta$

r IS CONSTANT

$(\dot{\vec r})_{rel} = (r\omega\sin\theta + r\omega\theta\cos\theta)\hat e_r + (r\omega\cos\theta - r\omega\theta\sin\theta)\hat e_\theta$

$\dot{\vec r} = (\dot{\vec r})_{rel} + \vec\omega_{FRAME} \times \vec r$

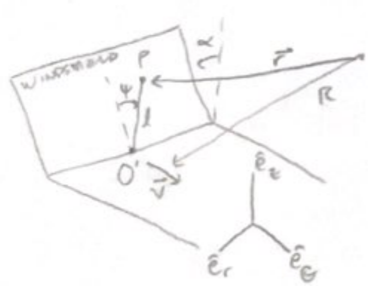
$\dot{\vec r} = r\omega\sin\theta\,\hat e_r + (r\omega\cos\theta + r\omega)\,\hat e_\theta$

$(\ddot{\vec r})_{rel} = (r\dot\omega\sin\theta + r\omega^2\cos\theta)\hat e_r + (r\dot\omega\cos\theta - r\omega^2\sin\theta + r\dot\omega)\hat e_\theta$

$\ddot{\vec r} = (\ddot{\vec r})_{rel} + \omega_{FRAME} \times \dot{\vec r}$

$$\vec a = \ddot{\vec r} = \begin{bmatrix} r(-\omega^2 + \dot\omega\sin\theta) \\ r\dot\omega(\cos\theta + 1) \\ 0 \end{bmatrix} = \begin{bmatrix} r(-\alpha^2 t^2 + \alpha\sin\theta) \\ r\alpha(\cos\theta + 1) \\ 0 \end{bmatrix} \longleftarrow$$

5)



$\vec r = R\hat e_r - l\sin\alpha\,\hat e_\theta - l\sin\psi\,\hat e_r + l\sin\alpha\cos\psi\,\hat e_z$

$\vec r = (R - l\sin\psi)\,\hat e_r + (-l\sin\alpha\cos\psi)\hat e_\theta + l\cos\alpha\cos\psi\,\hat e_z$

$\vec\omega_{FRAME} = v/R\ \hat e_z$

$\dot{\vec r} = (\dot{\vec r})_{rel} + \vec\omega_{FRAME} \times \vec r$ ← USE MATLAB SCRIPT TO COMPUTE

$\ddot{\vec r} = (\ddot{\vec r})_{rel} + \vec\omega_{FRAME} \times \dot{\vec r}$

SEE MATLAB FILE FOR ANSWER