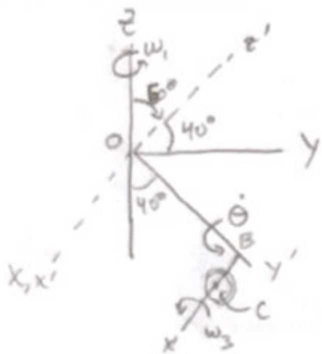
HOMEWORK #3 | ME EN 534 | JOSHUA CANRAS

1)



$\theta(t) = \frac{\pi}{20}\cos(2t)$ rad

$\omega_1 = 0.5$ rad/s

$\omega_2 = 7$ rad/s

$t = 3$ sec

$\dot{\theta} = -\frac{\pi}{10}\sin(2t)$

$\vec{\omega}_{C/O} = \vec{\omega}_{C/B} + \vec{\omega}_{B/O} + \vec{\omega}_O$

$\vec{\omega}_{C/B} = \omega_3\,\hat{i}'$

$\vec{\omega}_{B/O} = \dot{\theta}\,\hat{j}'$

$\vec{\omega}_O = \omega_1\,\hat{K}$

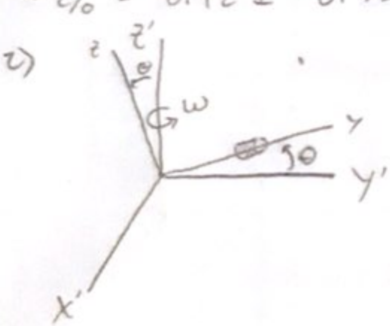
$\vec{\omega}_{C/O} = \omega_3\,\hat{i}' + \dot{\theta}\,\hat{j}' + \omega_1\,\hat{K}$

$$\begin{matrix} x \\ y \\ z \end{matrix} = [R]^T \begin{bmatrix} \omega_3 \\ \dot{\theta} \\ 0 \end{bmatrix}' + \begin{bmatrix} 0 \\ 0 \\ \omega_1 \end{bmatrix}$$

$[R] = [R_{y'}(\theta)][R_x(-50°)]$

$\vec{\omega}_{C/O} = 6.92\,\hat{I} - 0.75\,\hat{J} - 0.24\,\hat{K}$ rad/s $\longleftarrow$

2)



$\omega = 0.2$ rad/s

$\theta(t) = \frac{\pi}{6}\sin 2t$ rad

$\dot{\theta}(t) = \frac{\pi}{3}\cos 2t$ rad/s

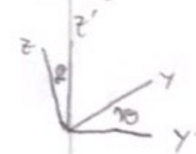
$\ddot{\theta}(t) = -\frac{2}{3}\pi\sin 2t$ rad/s²

@ $t = \pi$ sec

$y = 40$ cm

$\dot{y} = -30$ cm/s

$\ddot{y} = -4$ cm/s²



$\vec{\omega}_{FRAME} = \dot{\theta}\,\hat{i} + \omega\,\hat{K}'$

$\vec{\omega}_{FRAME} = \dot{\theta}\,\hat{i} + \omega\sin\theta\,\hat{j} + \omega\cos\theta\,\hat{k}$

$\vec{r} = y\,\hat{j}$

$\dot{\vec{r}} = (\dot{\vec{r}})_{rel} + \vec{\omega}\times\vec{r}$

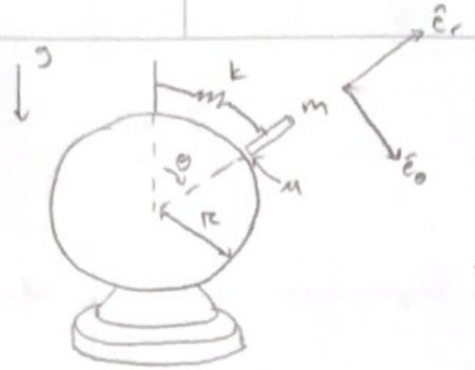
$\dot{\vec{r}} = -y\omega\cos\theta\,\hat{i} + \dot{y}\,\hat{j} + \dot{\theta}y\,\hat{k}$

$\dot{\vec{r}} = -8\,\hat{i} - 30\,\hat{j} + 41.89\,\hat{k}$ cm/s $\longleftarrow$

$\ddot{\vec{r}} = (\ddot{\vec{r}})_{rel} + \vec{\omega}\times\dot{\vec{r}}$

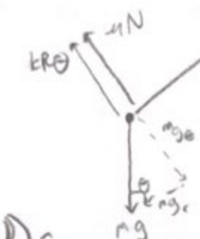
$\ddot{\vec{r}} = 12\,\hat{i} - 49.46\,\hat{j} - 62.83\,\hat{k}$ cm/s² $\longleftarrow$

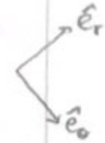
3)



$$\vec{\omega}_{FRAME} = \dot{\theta}\hat{e}_z$$

$$\vec{r} = R\hat{e}_r$$
$$\dot{\vec{r}} = (\dot{\vec{r}})_{rel} + \vec{\omega}\times\vec{r} = R\dot{\theta}\hat{e}_\theta$$
$$\ddot{\vec{r}} = (\ddot{\vec{r}})_{rel} + \vec{\omega}\times\dot{\vec{r}} = R\ddot{\theta}\hat{e}_\theta - R\dot{\theta}^2\hat{e}_r$$
$$\ddot{\vec{r}} = -R\dot{\theta}^2\hat{e}_r + R\ddot{\theta}\hat{e}_\theta$$





$$\Sigma F_\theta = 0 \Rightarrow$$

$$\vec{F} = (N - mg\cos\theta)\hat{e}_r + (mg\sin\theta - kR\theta - \operatorname{sgn}\mu N)\hat{e}_\theta$$

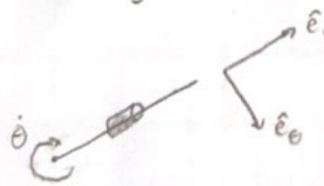
$$\hat{e}_r: \quad N - mg\cos\theta = -mR\dot{\theta}^2 \Rightarrow N = mg\cos\theta - mR\dot{\theta}^2$$

$$\hat{e}_\theta: \quad mg\sin\theta - kR\theta - \mu N) = mR\ddot{\theta}$$

$$mg\sin\theta - kR\theta - \mu mg\cos\theta\,\operatorname{sgn}(\dot{\theta}) + \mu mR\dot{\theta}^2\operatorname{sgn}(\dot{\theta}) = R\ddot{\theta}m$$

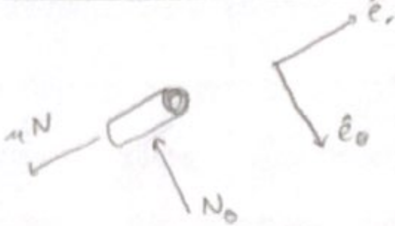
$$\ddot{\theta}Rm - \dot{\theta}^2\mu mR\operatorname{sgn}(\dot{\theta}) + \theta kR + \mu mg\cos\theta\,\operatorname{sgn}(\dot{\theta}) - mg\sin(\theta) = 0 \longleftarrow$$

4)



$\dot{\theta}$ = CONSTANT

$$\vec{r} = r\hat{e}_r \qquad \vec{\omega} = \dot{\theta}\hat{e}_z$$
$$\dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$
$$\ddot{\vec{r}} = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + r\cancel{\ddot{\theta}}^{\,0}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r$$
$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + 2\dot{r}\dot{\theta}\hat{e}_\theta$$



$N = \sqrt{N_\theta^2 + N_z^2}$

$N_z = mg$ b/c $\ddot{r}_z = 0$

$\vec{F} = -\mu N \hat{e}_r - N_\theta \hat{e}_\theta + (mg - N_z)\hat{e}_z$

$\hat{e}_r: \; -\mu N = m(\ddot{r} - r\dot{\theta}^2)$

$\hat{e}_z = \; mg - N_z = 0$
$N_z = mg$

$\hat{e}_\theta: \; -N_\theta = m 2\dot{r}\dot{\theta}$
$N_\theta = -m2\dot{r}\dot{\theta}$

$N = \sqrt{4m^2\dot{r}^2\dot{\theta}^2 + m^2g^2} = m\sqrt{4\dot{r}^2\dot{\theta}^2 + g^2}$

$t = 0:$
$r(0) = 5\text{ cm}$
$\dot{r}(0) = 0\text{ cm/s}$

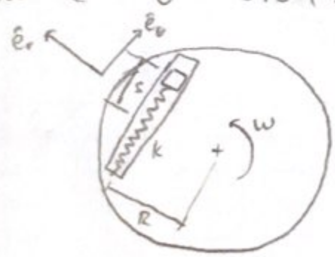
$-\mu \cancel{m}\sqrt{4\dot{r}^2\dot{\theta}^2 + g^2} = \cancel{m}(\ddot{r} - r\dot{\theta}^2)$

$\ddot{r} = r\dot{\theta}^2 - \mu\sqrt{4\dot{r}^2\dot{\theta}^2 + g^2}$

USE MATLAB'S ODE45 SOLVER:

$r(t) \approx 30\text{ cm}$ @ $t \approx 0.69\text{ sec}$ ⟵

5)



$\vec{r} = R\hat{e}_r + s\hat{e}_\theta$ $\qquad \omega_{FRAME} = -\omega\,\hat{e}_z$

$\dot{\vec{r}} = \dot{s}\hat{e}_\theta - R\omega\hat{e}_\theta + s\omega\hat{e}_r = s\omega\hat{e}_r + (\dot{s} - R\omega)\hat{e}_\theta$

$\ddot{\vec{r}} = (\dot{s}\omega + s\dot{\omega})\hat{e}_r + (\ddot{s} - R\dot{\omega})\hat{e}_\theta - s\omega^2\hat{e}_\theta + \dot{s}\omega\hat{e}_r - R\omega^2\hat{e}_r$

$\ddot{\vec{r}} = (2\dot{s}\omega + s\dot{\omega} - R\omega^2)\hat{e}_r + (\ddot{s} - R\dot{\omega} - s\omega^2)\hat{e}_\theta$

$\vec{F} = -N\hat{e}_r - ks\,\hat{e}_\theta$ $\qquad \vec{F} = m\ddot{\vec{r}}$

$\hat{e}_\theta: \; -ks = m(\ddot{s} - R\dot{\omega} - s\omega^2)$

$0 = \ddot{s} + (\frac{k}{m} - \omega^2)s - R\dot{\omega}$ ⟵ (a)

$\hat{e}_r: \quad -N = m(2\dot{s}\omega + s\dot{\omega} - R\omega^2)$

$$N = mR\omega^2 - 2m\dot{s}\omega - ms\dot{\omega} \longleftarrow (b)$$

(c) SEE MATLAB CODE AND PLOTS.

I USED ODE45 IN MATLAB. I DID NOT SPECIFY A STEP SIZE.

$\omega = \dot{\theta} = S\sin t \qquad \dot{\omega} = \ddot{\theta} = S\cos t$

$$\theta = \int_0^t S\sin t = -S\cos t\Big|_0^t = -S\cos t + S = S(1-\cos t)$$

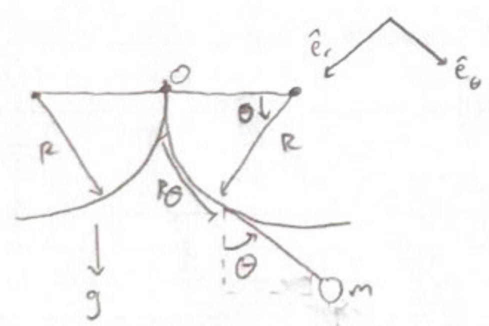
WHEN $\theta = 2\pi = S(1-\cos t)$

$$\Rightarrow t = \cos^{-1}\left(1 - \frac{2\pi}{S}\right)$$

d) $S_{max} = 0.32\ m \longleftarrow$

e) $N_{max} = 18.5\ N \longleftarrow$

6)



$\vec{r} = R\hat{e}_r + (2R - R\theta)\hat{e}_\theta \qquad \vec{\omega}_{FRAME} = \dot{\theta}\hat{e}_z$

$\dot{\vec{r}} = (\dot{\vec{r}}_{rel}) + \omega \times \vec{r}$

$= \cancel{-R\dot{\theta}\hat{e}_\theta} + \cancel{R\dot{\theta}\hat{e}_\theta} - \dot{\theta}(2R - R\theta)\hat{e}_r$

$\dot{\vec{r}} = (R\dot{\theta}\theta - 2\dot{\theta}R)\hat{e}_r = \dot{\theta}R(\theta - 2)\hat{e}_r$

$\ddot{\vec{r}} = (R\ddot{\theta}\theta + R\dot{\theta}^2 - 2\ddot{\theta}R)\hat{e}_r + \dot{\theta}^2 R(\theta - 2)\hat{e}_\theta$

$\vec{F} = mg\sin\theta\,\hat{e}_r + (mg\cos\theta - T)\hat{e}_\theta$

$\hat{e}_r: \quad R\ddot{\theta}\theta - 2\ddot{\theta}R + R\dot{\theta}^2 = mg\sin\theta$

$$\ddot{\theta}R(\theta - 2) + R\dot{\theta}^2 - mg\sin\theta = 0 \longleftarrow (a)$$

$$T_A + \cancel{V_A}^{\,0} = \cancel{T_B}^{\,0} + V_B$$

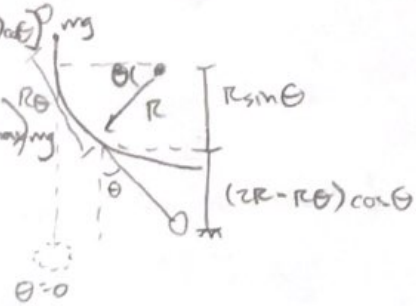
$$\tfrac{1}{2} m \dot{\vec{r}} \cdot \dot{\vec{r}} = mgh \qquad \dot{\vec{r}} \cdot \dot{\vec{r}} = \dot{\theta}^2 R^2 (\theta - 2)^2$$

$$\tfrac{1}{2} m \dot{\theta}^2 R^2 (\cancel{\theta_0}^{\,0} - 2)^2 = 2R - \left[R\sin\theta + (2R - R\theta)\cos\theta\right] mg$$

$$2m\dot{\theta}_0^2 R^2 = (2R - R\sin\theta_{max} - (2R - R\theta_{max})\cos\theta_{max}) mg$$

$\theta_0 = 0$

$\dot{\theta}_0 = \left(\frac{g}{2R}\right)^{1/2}$



$$h = 2R - \left[R\sin\theta + (2R - R\theta)\cos\theta\right]$$

$$\cancel{2m}\left(\frac{\cancel{g}}{\cancel{2R}}\right)\cancel{R^2} = (2\cancel{R} - \cancel{R}\sin\theta_{max} - (2\cancel{R} - \cancel{R}\theta_{max})\cos\theta_{max})\cancel{mg}$$

$$1 = 2 - \sin\theta_{max} - 2\cos\theta_{max} + \theta_{max}\cos\theta_{max}$$

USING WOLFRAM ~~ALPHA~~ SOLVER:

$$\theta_{max} = \frac{\pi}{2} \longleftarrow (b)$$