Methods Toward Broadband dFT Spectroscopy

Joshua Chen, Dr. Carlos A. Ríos Ocampo, and Prof. Juejun Hu
Department of Materials Science & Engineering
Massachusetts Institute of Technology, Cambridge, MA 02139

Abstract

The novel on-chip digital Fourier transform (dFT) spectrometer has great potential to provide superior performance compared to traditional benchtop instruments and other existing alternatives. Previously shown to detect at a 20nm wavelength bandwidth with 25pm resolution, the design's unique exponential scaling of spectral channels and signal-to-noise quality indicate the possibility for broadband spectroscopy (500nm bandwidth and greater) that will offer massive advantages in terms of size, weight, power, and cost. The Ansys Lumerical INTERCONNECT simulations, theoretical calculations, and other computational methods are developed to enable this goal.

Background

- The dFT architecture is a reconfigurable Mach-Zehnder interferometer (MZI) that produces 2^j states, where j is the number of switches and each switch routes to a specific path.
- Each state is characterized by a unique optical path difference (OPD) that results in a different output transmission with varying periods due to destructive interference. OPD ∈ ΔL * [0, 1, 2, …, 2^j - 1], where ΔL is the unit path length.

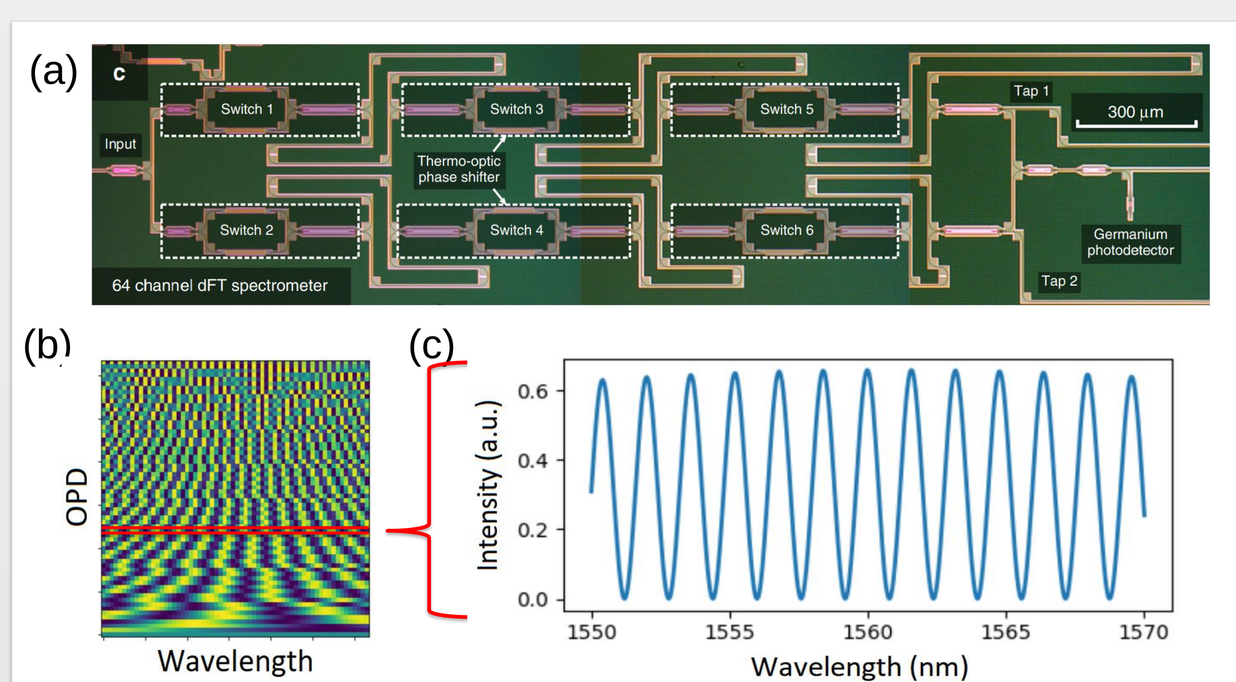
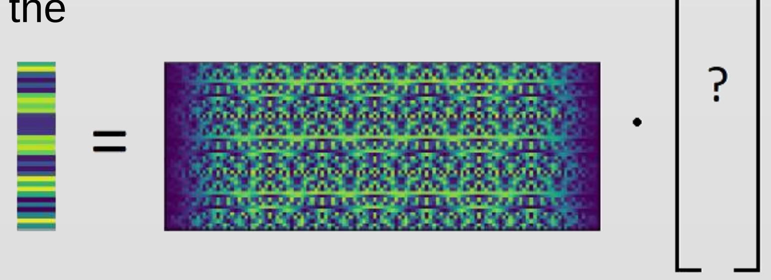


Fig. 1. (a) 6-switch dFT spectrometer. Each switch can route to two different possible path lengths[1]. (b) Transmission spectra for all 64 permutations of the 6 switches stacked vertically. (c) An example spectra corresponding to one switch configuration.

- **Fig. 1b** represents calibration matrix A. In narrowband, A has 64 OPD values (m rows) and 801 wavelength values (n cols) for 0.025 nm resolution.
- **Eq. 1** represents the *under-constrained* reconstruction problem to solve for the input spectrum, seen here as **x**.
  - **y:** m × 1
  - **A:** m × n
  - **x:** n × 1

$$\mathbf{y = Ax} \quad (1)$$



- Switches are turned on and off by thermo-optic phase shifters where each phase shift is wavelength-dependent according to **Eq. 2**. Broadband reconstruction requires multiple windows for proper reconstruction resolution.

$$\Delta\phi = \frac{\phi_0}{\lambda}\left[\lambda_{min} + \frac{\Delta\lambda_T}{N}\left(i - \frac{1}{2}\right)\right],\ \ i = 1, 2, \ldots, N-1 \quad (2)$$

- I have written a Python Library to simulate the dFT device in Lumerical Interconnect, accounting for a variety of parameters, designs, and tests.

Theory & Discussion

Derivation of Transmissions

Starting with the fundamental equations we can derive the characterization matrix A.

$$\vec{E}(\vec{r}, t) = Ae^{i(\vec{k}\vec{r} - \omega t)} \quad (3)$$

$$I \propto |E|^2$$

Combined intensity vs. wavelength after path difference L becomes, simply:

$$I_L(\lambda) \propto |E_1(\lambda) + E_2(\lambda)|^2 = \left|\frac{1}{\sqrt{2}}e^{i\frac{2\pi}{\lambda}D} + \frac{1}{\sqrt{2}}e^{i\frac{2\pi}{\lambda}(D+L)}\right|^2$$

$$= \boxed{\cos\left(\frac{2\pi L}{\lambda}\right) + 1} \quad (4)$$

Assumptions/Simplifications:

- The electric field of light in the waveguides of the dFT spectrometer can be described by the monochromatic plane wave solution **Eq. 3**
- There is perfect transmission of light (no waveguide loss)
- There is perfect switch phase shifting (no frequency-dependent phase shifting)

Using the simplified form **Eq. 4**, optimization calculations can be done 100x faster than in Lumerical, to then be checked in simulation.

Optimizing Characterization Matrix

Question: How can we create the most effective characterization matrix?

- Each row of the A matrix is linearly independent
- Consider the tradeoff of more noise versus extending the basis
- Must understand how the Elastic-D1 smoothing algorithm reacts to noise versus more data

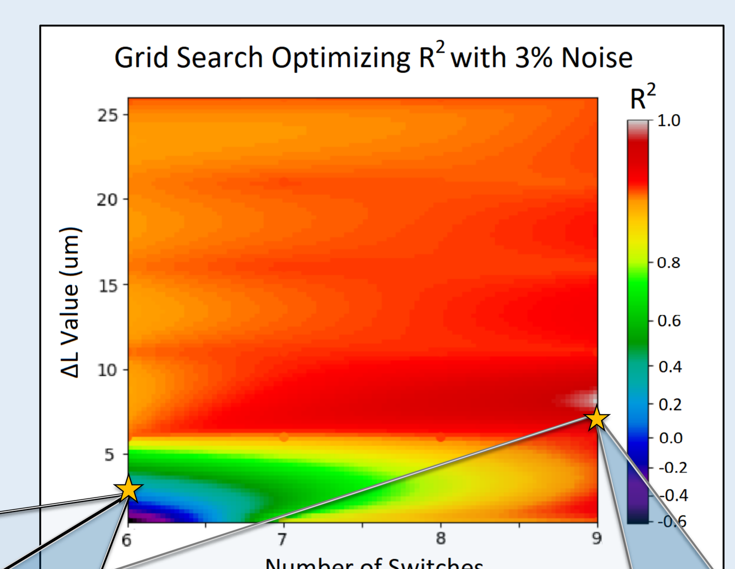
Can spend time deriving the above. Or, run a grid search:

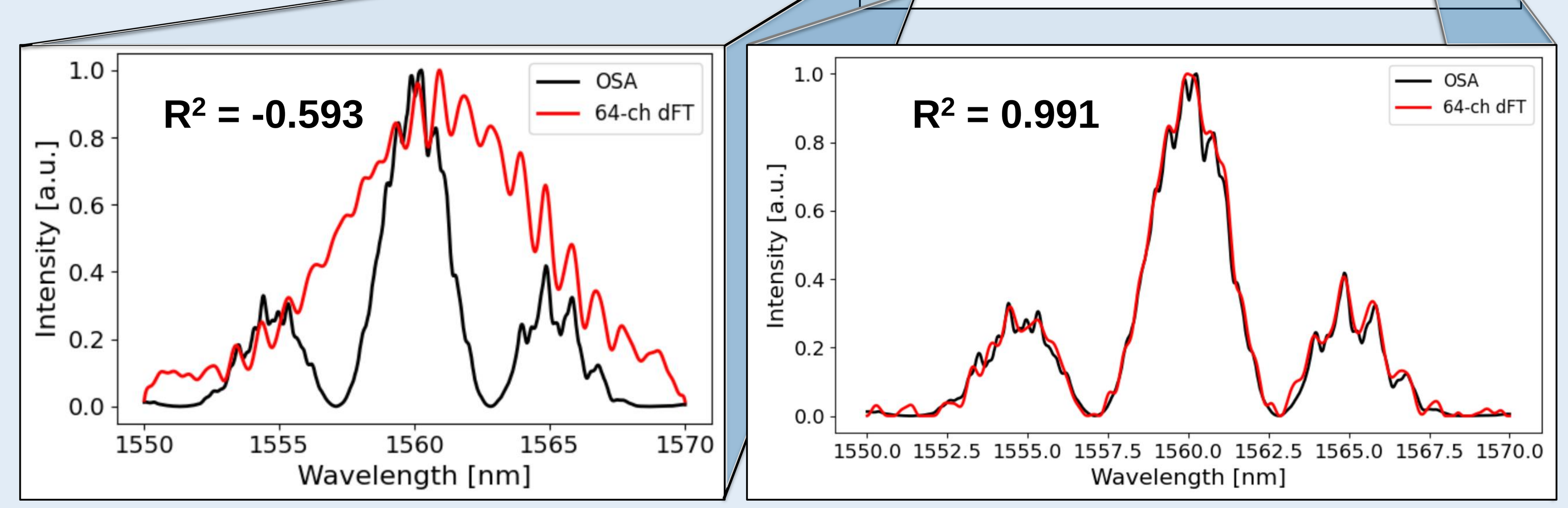
<u>HYPERPARAMETERS</u>:

- number of switches
- ΔL

<u>OPTIMIZING</u>:

- R^2 value of reconstruction via Elastic-D1 algorithm after 3% random Gaussian noise added
- *7 samples each*





Linear Independence & Condition Number

According to our derived transmission **Eq. 3**, each OPD provides a linearly independent function of the form

$$\cos\left(\frac{n}{\lambda}\right),\ \text{where}\ n \in 0, 1, 2, \ldots, 2^j - 1$$

and there are currently 2^j functions. However, for each *n* value, the function has no freedom to translate horizontally. Adding a π/2 phase shifter on one arm of the MZI allows translation, and subsequently adds, for every *n*, its sine counterpart. Thus, **we have effectively doubled our sample size**.

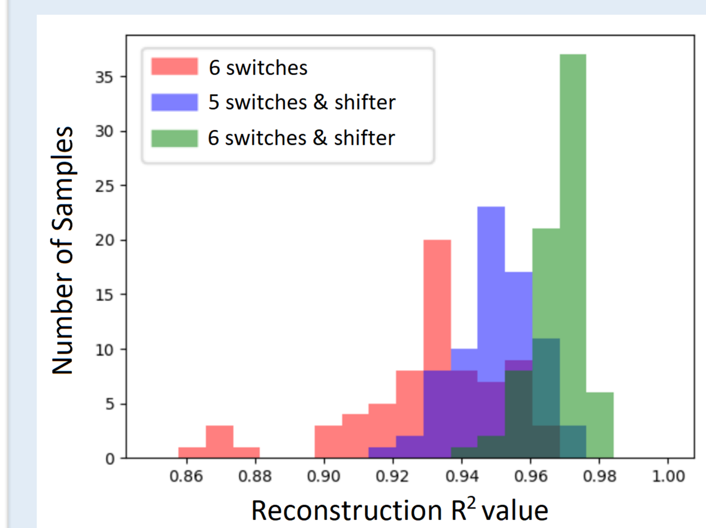


Fig. 2. Repeated sampling of reconstruction R^2 values for three dFT designs; the incorporation of one phase shifter consistently increases accuracy

The condition number of a matrix indicates the weight at which noise will propagate through a system; a good characterization matrix has a small condition number. Swapping a switch out for one phase shifter *lowers* the condition number of the system from 79.98 to 21.41 (surprising, since a switch consists of two phase shifters and a variety of other components). Furthermore, the reconstructed spectrum is notably more accurate (**Fig. 2**).

Conclusions

- The current barrier to broadband reconstruction is the Elastic-D1 algorithm. Runtime scales by power law with sample size—55 hrs for 500 nm.
- Performing a theoretical grid search prior to simulation and experiment provides an initial optimization 100x faster than using simulation alone.
- Adding a π/2 phase shifter on one arm effectively doubles the sample size and provides even greater reconstruction accuracy than adding switches.
- Future designs and experiments can continue to be tested and optimized using the dFT Python Library developed.

References

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