Context-Free Languages

Now refer to the description of **Walks** on page 22.

Let SAW be the language of strings over the alphabet {N, S, E, W} that represent self-avoiding walks.

Problem 8. [8 marks]

Prove or disprove: The language SAW is context-free.

Your submission can be typed or hand-written, but it must be in PDF format and saved as a file `prob8.pdf`.

Prove that SAW is not context free.
Assume SAW is context free Then it has a CFG G in Chomsky Normal Form. Let k be the number of non-terminal symbols in G. Let n be any positive integer such that n>2^k.

Let w be the string (N^n , N, S, E, W). which can be written like this;

(NN……...N,N, S, E , W)

n times

It satisfies the definition of the strings in SAW, so it belongs to the language. Therefore by pumping lemma for CFL, there exists strings u,v,x,y,z such that w = uvxyz and |vxy|<= 2^k, and v and y are not both empty, and u v^i x y^i z is in the language SAW for all i >=0.

Observe that |vxy| < n, since |vxy| < 2^k and 2^k < n. Consider the implication sof this for the location of vxy within the string w. They could both be within one of the strings of N or they could lie across (N….N, N, S, E, W).

If either v or y causes the walk to walk into itself then the word will no longer be in the language of SAW.

If N is chosen, then due to NSEW, it will end up not being in the language of SAW.

If N…N,N,S,E,W is chosen, then no matter how many times you pump up or down it will end up not being in the language of SAW.

So, every possible location of vxy leads to a contradiction with the conclusion of the pumping lemma for context free languages.
So the initial assumption that SAW is context-free is wrong.