Math 301 Assignment 3

These problems are due in class on Tuesday. If your homework takes up multiple pages, they must be stapled together. Your work must be legible, and any frills from notebook paper must be removed.

Book Exercises

Section 2.4: # 4, 10, 12, 14, 32

Additional Exercises

#1. For this exercise, let M be a 2×2 matrix and let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{w}_1, \mathbf{w}_2$ be vectors in $\mathbb{R}^2$ so that $M\mathbf{v}_1 = \mathbf{w}_1$ and $M\mathbf{v}_2 = \mathbf{w}_2$.

a. To get you started with these proofs, it will be helpful to write out what the matrix and vectors look like. So suppose that

$$M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \mathbf{v}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

First find a formula for $\mathbf{w}_1$ and $\mathbf{w}_2$ using these variables.

b. Show that $M(\mathbf{v}_1 + \mathbf{v}_2) = \mathbf{w}_1 + \mathbf{w}_2$

c. Show that for any constant c, $M(c\,\mathbf{v}_1) = c\,\mathbf{w}_1$

d. Show that for any constants c_1, c_2, if $\mathbf{u} = c_1\,\mathbf{v}_1 + c_2\,\mathbf{v}_2$, then $M\mathbf{u} = c_1\,\mathbf{w}_1 + c_2\,\mathbf{w}_2$

#2. A <u>diagonal matrix</u> has a 0 for every non-diagonal entry. For example, a 3×3 diagonal matrix has the form

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

(a) Find $\det(A)$

(b) Show that the product of two 3×3 diagonal matrices is a diagonal matrix. (This is true for any size diagonal matrix, but this restriction simplifies the proof).

#3. Let S be the parallelogram in $\mathbb{R}^2$ whose sides are formed by the vectors $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$, and let $M = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$. Sketch S and $f_M(S)$. Does the image have the same orientation as the original set?

#4. Find the 3×3 matrix M with corresponding matrix transformation f_M from $\mathbb{R}^3$ to $\mathbb{R}^3$ if

$$f_M\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}5\\-2\\3\end{bmatrix} \qquad f_M\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}5\\-9\\4\end{bmatrix} \qquad f_M\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\-7\\-5\end{bmatrix}$$

#5. Let $A = \begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}$.

a. Find det (A).

b. Find A^{-1}

c. Find the 2×2 matrix M with corresponding matrix transformation f_M from $\mathbb{R}^2$ to $\mathbb{R}^2$ if

$$f_M\left(\begin{bmatrix}2\\-1\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix} \qquad f_M\left(\begin{bmatrix}1\\3\end{bmatrix}\right) = \begin{bmatrix}-1\\5\end{bmatrix}$$

#6. Find R_{72° (the rotation matrix that turns vectors 72° counterclockwise), and use it to rotate the vectors $\mathbf{u} = \begin{bmatrix}0\\-1\end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix}-1\\3\end{bmatrix}$. Round all numbers to the nearest hundredth.

#7. Let $A = R_{120^\circ}$. Find A, A^2 and A^3. Give exact values for numbers.

#8. Let $\mathbf{b} = \begin{bmatrix}-5\\3\end{bmatrix}$ and $T_{\mathbf{b}}$ be the translation defined by $T_{\mathbf{b}}(\mathbf{v}) = \mathbf{v} + \mathbf{b}$.

(a) Find $R_{-45^\circ}\left(T_{\mathbf{b}}\left(\begin{bmatrix}2\\-4\end{bmatrix}\right)\right)$ and $T_{\mathbf{b}}\left(R_{-45^\circ}\left(\begin{bmatrix}2\\-4\end{bmatrix}\right)\right)$

(b) Find the 3×3 translation matrix $T_{\mathbf{b}}$, then find $R_{-45^\circ}T_{\mathbf{b}}$ and $T_{\mathbf{b}}R_{-45^\circ}$ (here R_{-45° is the transformation of $\mathbb{R}^3$ that rotates the xy-plane 45° clockwise).

Practice Problems

Section 2.4: # 1, 3, 9, 11, 13, 15, 29, 31
Section 2.6: # 1, 3, 7, 9, 11