Find the Bezout coefficients for 52 and 14

1) First find the gcd

gcd(52, 14)	= gcd(14, 10)	= gcd(10, 4)	= gcd(4, 2)	= 2
52 = 3*14 + 10	14 = 1*10 + 4	10 = 2*4 + 2	4 = 2*2 + 0	
52 mod 14 = 10	14 mod 10 = 4	10 mod 4 = 2	4 mod 2 = 0	
52 div 14 = 3	14 div 10 = 1	10 div 4 = 2		

2) Then solve a series of equations to get 2 = s*52 + t*14

52 = 3*14 + 10	10 = 52 - 3*14	2 = 10 - 2*4
14 = 1*10 + 4	4 = 14 - 1*10	2 = 10 - 2*(14-1*10)
10 = 2*4 + 2	2 = 10 - 2*4	= 10 -2*14 + 2*10
		= -2*14+3*10
		2 = -2*14+3*(52 - 3*14)
		= -2*14 + 3*52 - 9*14
		= 3*52 - 11*14

We get

$$2 = 3 * 52 - 11 * 14$$

so the Bezout coefficients are 3 and -11

We can also get this by multiplying matrices of the form

$$\begin{bmatrix} 0 & 1 \\ 1 & -q \end{bmatrix} \text{ by the vector } \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where q is one of the quotients we found in the Euclidean Algorithm (a div b)

Recall that we found 52 div 14 = 3, 14 div 10 = 1, and 10 div 4 = 2 (because 4 mod 2 = 0, we don't use 4 div 2)

$$\begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Multiplying from right to left, we get

$$\begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$$

And looking at the four vectors above, each one gives the coefficients for writing 2 (the gcd) using different values of a and b.

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad 2 = 0 * 4 + 1 * 2 \quad \begin{bmatrix} 1 \\ -2 \end{bmatrix} \quad 2 = 1 * 10 - 2 * 2 \quad \begin{bmatrix} -2 \\ 3 \end{bmatrix} \quad 2 = -2 * 14 + 3 * 10 \quad \begin{bmatrix} 3 \\ -11 \end{bmatrix} \quad 2 = 3 * 52 - 11 * 14$$