Math 301 Assignment 6

These problems are due in class on Tuesday. If your homework takes up multiple pages, they must be stapled together. Your work must be legible, and any frills from notebook paper must be removed.

#1. For this problem use the encryption function $e(p) = (15p - 8)$ mod 26 and the alphanumeric values

$$A = 1, B = 2, C = 3, ..., M = 13, N = 14, ..., X = 24, Y = 25, Z = 0$$

a. Encrypt the message ATTACK

b. Decrypt the message IYP

#2. Why is $e(p) = (2p)$ mod 26 not a good choice for an encryption function? Specifically, what problem might occur?

Prove all the remaining statements **by mathematical induction**

#3. (Sum of the first n odd numbers) Show that for all $n \geq 1$

$$1 + 3 + 5 + ... + 2n - 1 = n^2$$

#4. (Sum of a geometric sequence) If $r \neq 1$, show that for any real number a and all integers $n \geq 0$

$$a + ar + ar^2 + ... + ar^n = \frac{a(r^{n+1} - 1)}{r - 1}$$

#5. Find a formula for

$$\frac{1}{1 * 2} + \frac{1}{2 * 3} + ... + \frac{1}{n(n+1)}$$

by evaluating the expression for small values of n. Then prove your formula is correct by induction.

#6. (Product of the first n odd numbers) Show that for all $n \geq 1$

$$1 * 3 * 5 * ... * (2n-1) = \frac{(2n)!}{2^n * n!}$$

(recall that $(k+1)! = (k+1) * k!$)

#7. (Powers of a square matrix) Let A be a square matrix. Then $A^2 = A * A$, $A^3 = A * A * A$, etc.

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Find a formula for A^n by evaluating the powers of A for small values of n. Then prove your formula is correct by induction.

#8. More to come...

Practice Problems

Section 4.6: # 1, 3, 5 (Warning: the book uses $A = 0, B = 1, ...Z = 25$, which is obviously the wrong way to do it)

Section 5.1: # 5, 7, 9, 11, 13, 15