Information and Database Management Systems I

(CIS 4301 UF Online)

Fall 2019

Instructor: Dr. Markus Schneider

Homework 4

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Pledge (Must be signed according to UF Honor Code)

On my honor, I have neither given nor received unauthorized aid in doing this assignment.

Josh Smith

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Signature

For scoring use only:

	Maximum	Received
Exercise 1	35	
Exercise 2	20	
Exercise 3	35	
Exercise 4	10	
Total	100	

Exercise 1 [35 points]

1. [5 points] Consider the relation schema R = (A, B, C, D, E, F) with the functional dependencies FD = {A→B, D→E, A→C}. Which of the following sets of attributes functionally determine E and which sets are the candidate key? If no candidate key found, compute it. Show each step.
   - AD
   - BCD
   - AC
   - CD
   - AF

2. [5 points] Consider a relation schema R(X, Y, Z) with the functional dependencies XY→Z and Z→X. Can we conclude that Y→XZ holds? If yes, please argue why. If no, please argue why not by giving a counter example.

3. [5 points] Consider the relation schema R = (A, B, C, D, E, F, G, H) with functional dependencies FD = {A→B, CH→A, B→E, BD→C, EG→H, DE→F}. Which of the following FDs is also guaranteed to be satisfied by R? Show each step.
   - ADG→CH
   - CGH→BF
   - BFG→AE
   - ADE→CH

4. [5 points] Consider the relation schema R = (A, B, C, D, E, F, G, H, I, J) with functional dependencies FD = {B→E, E→FH, BCD→G, CD→A, A→J, I→BCDE, H→I}. Determine if B→J holds and list every candidate key. Show each step.

5. [15 points] We have a set of functional dependencies given as F = {A→B, B→C} for four attributes A, B, C, and D in a relation schema R. Write down all the functional dependencies in the closure F^+ of F and count them.

Exercise 1

1 The attributes that functionally determine E are:
AD, BCD and CD, because
$AD^+ = ADBEC$
$BCD^+ = BCDE$
$AC^+ = ACB$
$CD^+ = CDE$
$AF^+ = AFBC$
No candidate key found since there's no way to reach F with the functional dependencies. Assigning $F \rightarrow \phi$, the candidate key would be AD.

2. We cannot conclude that $Y \rightarrow XZ$ holds. The reason for this is because there's the counterexample where Y has the same value between iterations. For example, imagine we have $X_1 Y \rightarrow Z_1$ and $Z_1 \rightarrow X_1$ and also $X_2 Y \rightarrow Z_2$ and $Z_2 \rightarrow X_2$. We can assign Y to be y_1 in both instances, which would lead to a contradiction since $X_1 \neq X_2$ and $Z_1 \neq Z_2$.

3. The only FD satisfied by R is $ADG \rightarrow CH$, because
$ADG^+ = ADGBECHF$, which has CH
$CGH^+ = CGHABEH$, which has B but not F
$BFG^+ = BFGEH$, which has E, but not A
$ADE^+ = ADEBCF$, which has G but not H.

4. The FD $B \rightarrow J$ holds because $B^+ = BEFHICDGAJ$, which contains J. The keys are B, H and I because
$B^+ = BEFHICDGAJ = R$
$H^+ = HIBCDEFGAJ = R$
$I^+ = IBCDEFHGAJ = R$

FDs 5. There are 109 FD:

7 A^+: $A \to A$ $A \to AB$ $A \to BC$ $A \to ABC$
$A \to B$ $A \to AC$
$A \to C$

3 B^+: $B \to B$ $B \to C$ $B \to BC$

1 C^+: $C \to C$

1 D^+: $D \to D$

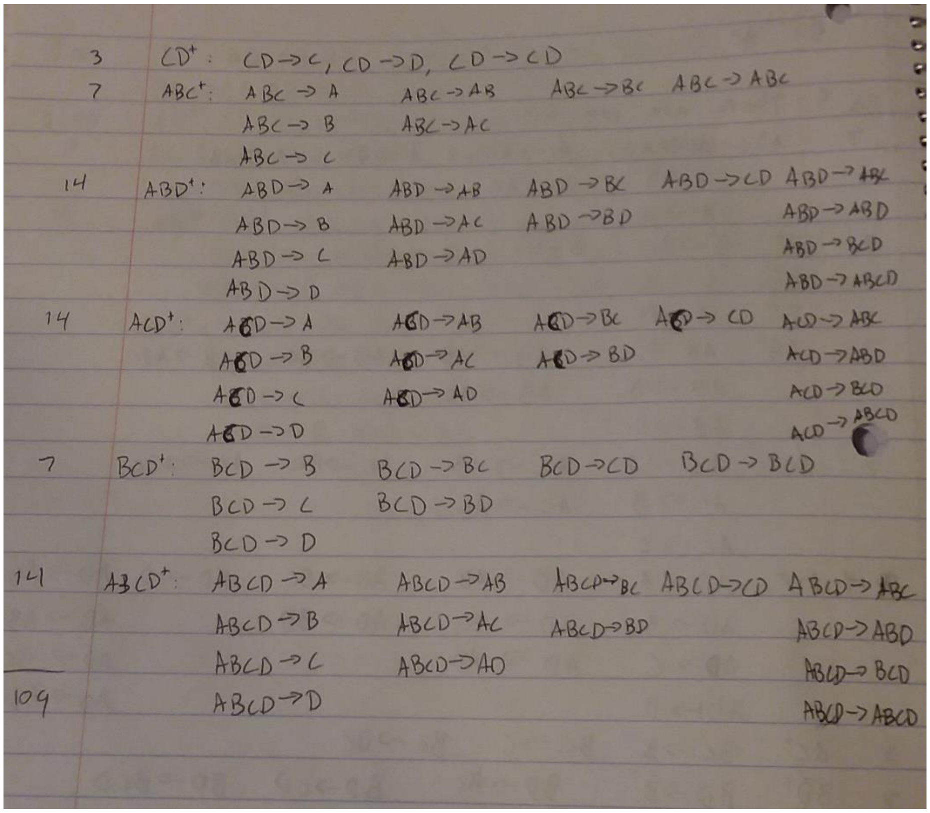
7 AB^+: $AB \to A$ $AB \to AB$ $AB \to BC$ $AB \to ABC$
$AB \to B$ $AB \to AC$
$AB \to C$

7 AC^+: $AC \to A$ $AC \to AB$ $AC \to BC$ $AC \to ABC$
$AC \to B$ $AC \to AC$
$AC \to C$

14 AD^+: $AD \to A$ $AD \to AB$ $AD \to BC$ $AD \to CD$ $AD \to ABC$
$AD \to B$ $AD \to AC$ $AD \to BD$ $AD \to ABD$
$AD \to C$ $AD \to AD$ $AD \to BCD$
$AD \to D$ $AD \to ABCD$

3 BC^+: $BC \to B$ $BC \to C$ $BC \to BC$

7 BD^+: $BD \to B$ $BD \to BC$ $BD \to CD$ $BD \to BCD$
$BD \to C$ $BD \to BD$
$BD \to D$



3	CD^+:	CD→C, CD→D, CD→CD				
7	ABC^+:	ABC→A	ABC→AB	ABC→BC	ABC→ABC	
		ABC→B	ABC→AC			
		ABC→C				
14	ABD^+:	ABD→A	ABD→AB	ABD→BC	ABD→CD	ABD→ABC
		ABD→B	ABD→AC	ABD→BD		ABD→ABD
		ABD→C	ABD→AD			ABD→BCD
		ABD→D				ABD→ABCD
14	ACD^+:	ACD→A	ACD→AB	ACD→BC	ACD→CD	ACD→ABC
		ACD→B	ACD→AC	ACD→BD		ACD→ABD
		ACD→C	ACD→AD			ACD→BCD
		ACD→D				ACD→ABCD
7	BCD^+:	BCD→B	BCD→BC	BCD→CD	BCD→BCD	
		BCD→C	BCD→BD			
		BCD→D				
14	$ABCD^+$:	ABCD→A	ABCD→AB	ABCD→BC	ABCD→CD	ABCD→ABC
		ABCD→B	ABCD→AC	ABCD→BD		ABCD→ABD
		ABCD→C	ABCD→AD			ABCD→BCD
109		ABCD→D				ABCD→ABCD

Exercise 2 [20 points]

1. [5 points] Consider the relation schema R = (A, B, C, D, E, F, G, H) with functional dependencies F = {A→C, AC→E, D→EH, F→G} and G = {A→BCE, AD→CFG, D→A, DE→GH, F→D}. Are the two sets F and G **equivalent**? Show each step.

2. [2.5 points each] Use the Armstrong axioms to prove the following deductions.
   (1) {X→Y, X∪Y→Z} ⇒ {X→Z}
   (2) {X→Z, Y→W} ⇒ {X∪Y→Z∪W}

3. [5 points] Consider the relation schema R = (A, B, C, D, E) with the set of functional dependencies F = {A→BC, CD→E, B→D, E→A}. List all candidate keys of R by using the Armstrong's Axioms. Show each step.

4. [5 points] For a relation scheme R = (A, B, C, D, E, F) and a set of functional dependencies given as F = {A→B, A→C, CD→E, CD→F, B→E}, use Armstrong's Axioms rules to find one candidate key for R. Show each step.

Exercise 2

1. F and G are equivalent:

LHS of F: A, AC, D, F

With respect to G:

A^+ : ABCE, $A \rightarrow C$ holds for A^+

AC^+ : ACBE, $AC \rightarrow E$ holds for AC^+

D^+ : DABCEFGH, $D \rightarrow EH$ holds for D^+

F^+ : FDABCEFGH, $F \rightarrow G$ holds for F^+

2. (1) $X \rightarrow Y \equiv X \vee X \rightarrow X \vee Y \equiv X \rightarrow X \vee Y$

Since $X \rightarrow X \vee Y$ and $X \vee Y \rightarrow Z$, then $X \rightarrow Z$

(2) $X \rightarrow Y \equiv X \vee Z \rightarrow Y \vee Z$

Also, $Z \rightarrow W \equiv Y \vee Z \rightarrow Y \vee W$

Since $X \vee Z \rightarrow Y \vee Z$ and $Y \vee Z \rightarrow Y \vee W$, then $X \vee Z \rightarrow Y \vee W$

3. A, E, CD and BC are keys.

$A \rightarrow BC$ is equivalent to $A \rightarrow \underline{B}$ and $A \rightarrow \underline{C}$.

Since $A \rightarrow B$ and $B \rightarrow \underline{D}$, we get $A \rightarrow D$

Also, $A \rightarrow CD$ and $CD \rightarrow E$, so $A \rightarrow \underline{E}$

Since $A \rightarrow B$, $A \rightarrow C$, $A \rightarrow D$ and $A \rightarrow E$, we get $A \rightarrow ABCDE$ (with a union), so A is a key.

Since $E \rightarrow A$ and $A \rightarrow ABCDE$, then $E \rightarrow ABCDE$ (which makes E a key).

Since $CD \rightarrow E$ and $E \rightarrow ABCDE$, we get $CD \rightarrow ABCDE$ (making CD a key).

Also, $B \rightarrow D \equiv BC \rightarrow CD$. So, $BC \rightarrow CD$ and $CD \rightarrow ABCDE$ $\equiv BC \rightarrow ABCDE$ (making BC a key).

4. AD is a candidate key.

$A \to A,\ A \to B,\ A \to C \equiv A \to ABC$

$A \to B$ and $B \to E \equiv A \to E$

$A \to ABC$ and $A \to E \equiv A \to ABCE$

$A \to ABCE \equiv AD \to ABCDE$

$A \to C \equiv AD \to CD$

$AD \to CD$ and $CD \to F \equiv AD \to F$

$AD \to ABCDE \cup AD \to F \equiv AD \to ABCDEF$

Exercise 3 [35 points]

1. [15 points] Find a minimal cover for the relation R = (A, B, C, D, E, F, G, H) with the set F = {A→B, ABCD→E, EF→GH, ACDF→EG} of functional dependencies. Show each step.

2. [10 points] Find a minimal cover for the relation R = (A, B, C, D, E) with the set F = {A→BC, CD→E, B→D, E→A} of functional dependencies. Show each step.

3. [10 points] Find a minimal cover for the relation R = (A, B, C, D, E, F) with the set F = {A→D, AC→DE, B→F, D→CE} of functional dependencies. Show each step.

Exercise 3

1. LHS

ABCD → E

$A^+ = AB$ ×

$B^+ = B$ ×

$C^+ = C$ ×

$D^+ = D$ ×

$ACD^+ = ACDBE$ ✓

EF → G

$E^+ = E$

$F^+ = F$

ACDF → EG

$ACDF^+ = ACDFBEGH$ ✓

RHS

A → B

$A^+ = A$ ×

ACD → E

$ACD^+ = ACDB$ ×

EF → GH

EF → G : $EF^+ = EFH$

EF → H : $EF^+ = EFG$

F_c = {A → B, ACD → E, EF → H, EF → G}

2. LHS

CD → E

$C^+ = C$ ×

$D^+ = D$ ×

RHS

A → BC

A → B: $A^+ = AC$ ×

A → C: $A^+ = ABD$ ×

CD → E

$CD^+ = CD$ ×

B → D

$B^+ = B$ ×

E → A

$E^+ = E$ ×

So, F_c = {A → BC, CD → E, B → D, E → A} (Already minimal)

3. LHS

$AC \to DE$

$A^+ = ADCE$ ✓ $A \to DE$

RHS $F_c = \{A \to D, A \to DE, B \to F, D \to CE\}$

$A \to D$

$A^+ = A$ ×

$A \to DE$

$A \to D$: $A^+ = ADE$ ✓

$A \to E$: $A^+ = ADCE$ ✓ $A \not\to \emptyset$

$B \to F$

$B^+ = B$ ×

$D \to CE$

$D \to C$: $D^+ = DE$ ×

$D \to E$: $D^+ = DC$ ×

$F_c = \{A \to D, B \to F, D \to CE\}$

Exercise 4 [10 points]

1. [5 points] Consider the relation schema R = (A, B, C, D, E, F) with a set of functional dependencies F = {CF→D, AE→F, D→A, AB→C}. List all candidate keys of R in a systematic manner (do not use the Armstrong's Axioms) and explain how you determine them. Show each step.

2. [5 points] Consider the relation schema R(A, B, C, D, E, F) with the functional dependencies FD = {D→C, CE→A, D→A, AE→D}. Determine all candidate keys of R in a systematic manner (do not use the Armstrong's Axioms) and explain how you determine them.

Exercise 4

1. The only attributes that are located to the left of a given FD is B and E. Checking $BE^+ = BE$, which is not the key. Next, lets try each attribute along with BE:

$ABE^+ = ABEFCD$ key

$CBE^+ = CBE$

$DBE^+ = DBEACF$ key

$FBE^+ = FBE$

So, the keys are ABE and DBE.

2. The only attributes not located to the right of any given FD are B and E. Checking if BE is a candidate key, we get $BE^+ = BE$. So, this is not the key. Trying every combinatio with the other attributes, we get:

$$\left.\begin{aligned} ABE^+ &= ABEDC \\ CBE^+ &= CBEAD \\ DBE^+ &= DBECA \end{aligned}\right\} \text{keys}$$

So, the keys are ABE, CBE and DBE.