

Math 301 Assignment 7

These problems are due in class on Tuesday. If your homework takes up multiple pages, they must be stapled together. Your work must be legible, and any frills from notebook paper must be removed.

Book Exercises

Section 5.2: # 10, 16 (For both problems, you should introduce a new variable in your explanation. #10 is similar to the stones problem we did in class, but the algebra is much simpler. For #16, use the (row, column) notation we used in class. For the proof, explain what the first player's response should be to what the second player does.)

Section 5.3: # 48, 50

Additional Exercises

#1. Use mathematical induction to show that for $n \geq 0$, $\text{cube}(n) = n^3$

```
procedure cube( $n$ : nonnegative integer)
    if  $n = 0$ 
        return 0
    else
        return  $\text{cube}(n - 1) + 3n^2 - 3n + 1$ 
```

#2. Consider the following recursive algorithm

```
procedure  $f(a, b$ : nonnegative integers)
    if  $a = 0$  and  $b = 0$ 
        return 1
    else if  $b = 0$ 
        return  $f(a - 1, b) \times 3$ 
    else
        return  $f(a, b - 1) + 5$ 
```

- Use induction to show that for $m \geq 0$, $f(m, 0) = 3^m$
- Use induction and the result from part a to show that for $m, n \geq 0$, $f(m, n) = 3^m + 5n$

#3. Use strong induction and proof by cases to show that $Prod(x, y) = xy$ for all real numbers x and all $y \geq 0$.

procedure $Prod(x, y$: x a real number, y a nonnegative integer)

if $y = 0$

return 0

else if y is even

return $2 \times Prod(x, \frac{y}{2})$

else

return $2 \times Prod(x, \frac{y-1}{2}) + x$

Practice Problems

Section 5.2: # 5, 7, 13, 15, 17, 35

Section 5.3: # 1, 3, 49, 51

Section 5.4: # 45, 49