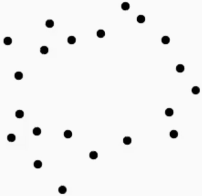
Applied and Computational Topology

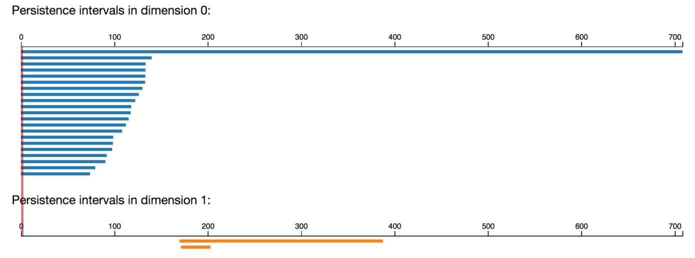
Advisor: Henry Adams

Joshua Mirth

Greenslopes – March 22, 2018

Basics – Persistent Homology





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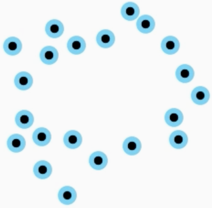

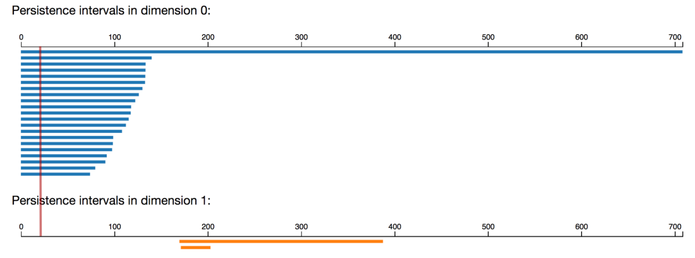

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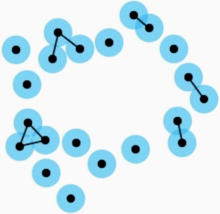

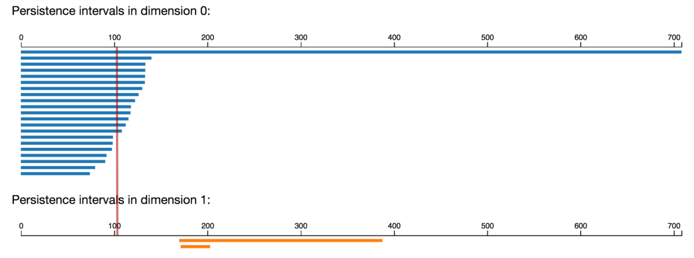

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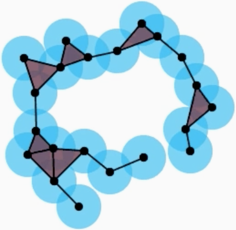

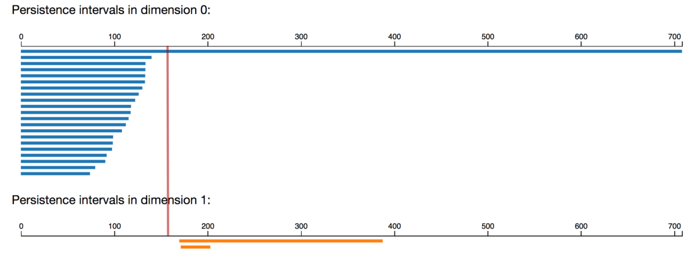

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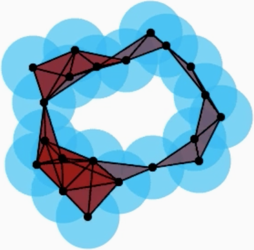

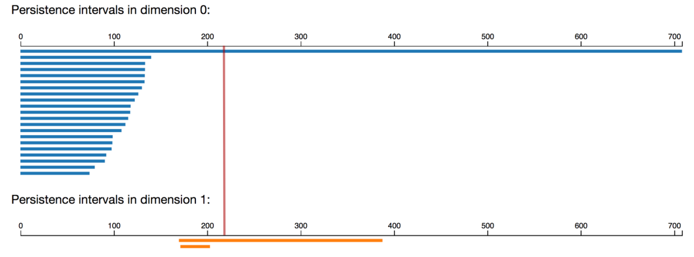

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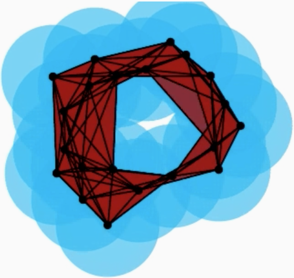

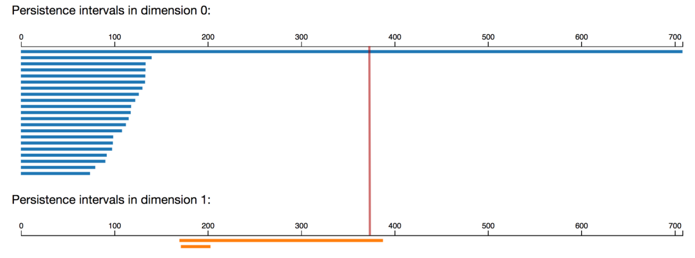

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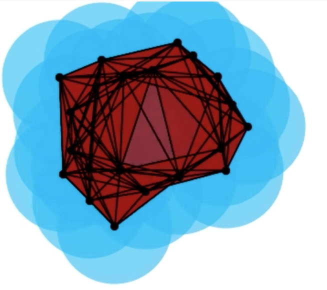

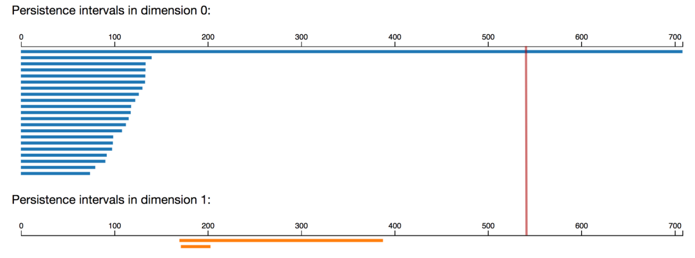

Problems

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  - Algebraic, combinatorial, and differential topology, metric geometry, ...

My Projects

Theorem:
For a metric space X and sufficiently small r, the Vietoris–Rips complex at radius r is homotopy equivalent to X.

(See Hausmann, Latschev, Adams and M.)

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The higher homotopy groups of the Vietoris–Rips complex successively vanish predictably as the radius is increased.

(Under study for geodesic spaces.)

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Persistent homology fractal dimension (with Pattern Analysis Lab).

For more information...

Faculty:

- Henry Adams,
- Amit Patel,
- Chris Peterson,
- Michael Kirby.
- Pattern Analysis Lab

Students:

- Johnathan Bush,
- Alex McCleary
- Joshua Mirth,
- Dustin Sauriol,
- Shannon Stiverson.

References:

- Henry Adams and Joshua Mirth. Metric thickenings of Euclidean submanifolds. arXiv:1709.02492, 2017.
- Justin Michael Curry. Topological data analysis and cosheaves. Japanese Journal of Industrial and Applied Mathematics, July 2015.
- Jeane-Claude Hausmann. On the Vietoris–Rips Complexes and a Cohomology Theory for Metric Spaces. In Prospects in Topology, number 138 in Annals of Mathematics Studies, pages 175187. 1995.
- Janko Latschev. Vietoris-Rips complexes of metric spaces near a closed Riemannian manifold. Archiv der Mathematik, 2001.
- Chad M. Topaz, Lori Ziegelmeier, and Tom Halverson. Topological Data Analysis of Biological Aggregation Models. PLOS ONE, May 2015.