EET 3422: Antenna Theory and Practice

Course Overview

Contact Hours: 45

Course Purpose: To equip students with a comprehensive understanding of the fundamental principles underlying transmitting and receiving antennas.

Expected Learning Outcomes: Upon successful completion of this course, students will be able to:

- Select appropriate antenna devices for various applications.
- Apply theoretical knowledge through practical laboratory experiences and industrial attachments.
- Design and construct basic antenna components.

Course Content

Module 1: Antenna Theory

- Radiation mechanism
- Antenna basic parameters and characteristics
- Types of antennas
- Properties of antennas
- Effects of antenna height, temperature, and front-to-back ratio
- Antenna field zones

Module 2: Antenna Arrays

- Pattern multiplication and array factor
- Broadside and end-fire arrays
- Feed networks: quarter-wave section, parasitic, and log-periodic

Module 3: Long-Wire and Aperture Antennas

- Radiation from long-wire antennas
- Aperture antennas: horn and reflector

- Antenna performance analysis

Module 4: Radio Wave Propagation

- Properties of the radio spectrum
- Wave propagation in various media
- Reflection, refraction, diffraction, and scattering, Atmospheric ducts and radio links
- Signal-to-noise ratio

Mode of Delivery

- Lectures
- Presentations
- Tutorials
- Group discussions
- Laboratory/workshop practice

Assessment

Type	**Weighting (%)**
Examination	60
Practicals	25
Continuous Assessment	10
Assignments	5
Total	**100**

Introduction to Antennas

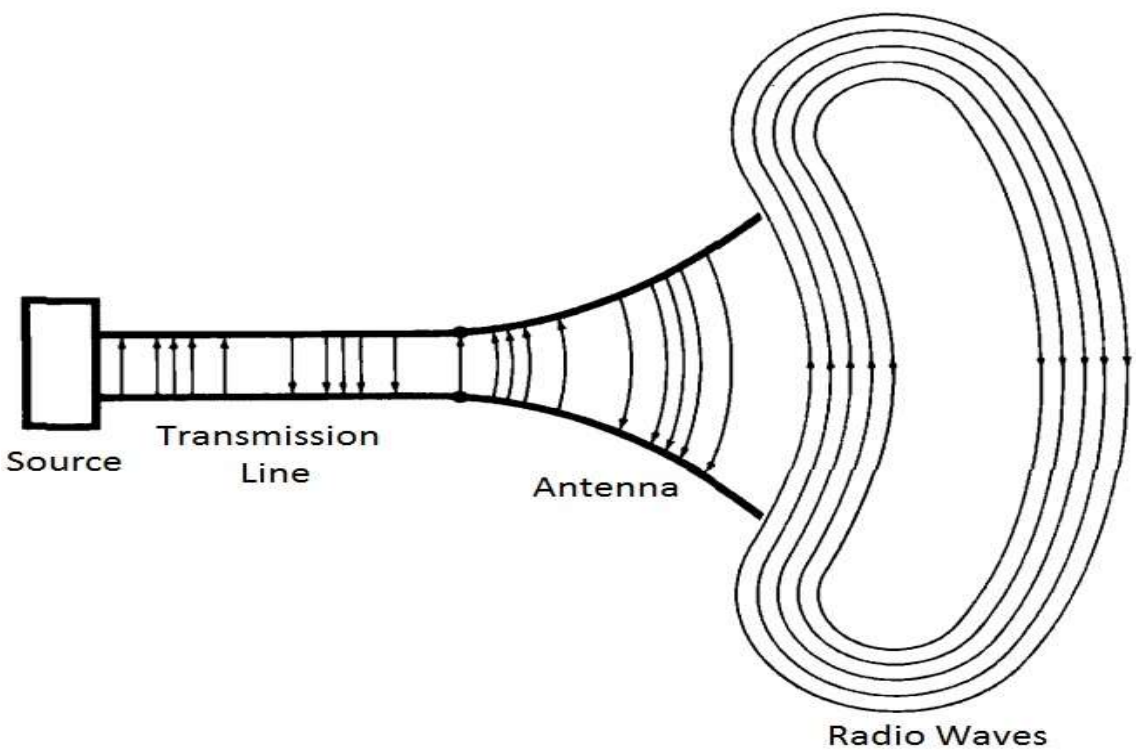
What is an Antenna?

An **antenna** is a device that converts electrical energy into electromagnetic waves, and vice versa. It is typically used in conjunction with a radio transmitter or receiver.   It is sometimes called an Aerial.

A signal from a transmission line like a co-axial cable, is given to an antenna, which then converts the signal into electromagnetic energy to be transmitted through space (hence the term free space).

Radio waves are a type of electromagnetic wave with wavelengths ranging from about 1 millimeter to 100 kilometers and frequencies ranging from 3 kHz to 300 GHz.

They are used primarily for communication purposes, such as in radio broadcasting, television, mobile phones, and Wi-Fi.



Antenna Construction

- Typically, an antenna consists of metallic conductors (elements) connected to a transmitter or receiver via a transmission line.

- Antennas act as interfaces between conducted waves and freely propagating electromagnetic waves.
- While most antennas are metallic, dielectric antennas (made from non-conductive materials) also exist.

Function of antennas

Antenna can be used for both Transmission and Reception of electromagnetic radiation i.e. a Transmitting Antenna receives electrical signals from a transmission line and converts them into radio waves whereas a Receiving Antenna does the exact opposite i.e. it accepts radio waves from the space and converts them to electrical signals and gives them to a transmission line.

Why Do We Need Antennas?

Antennas are essential for **wireless communication**, providing a means to transmit and receive signals where physical connections are impractical or impossible.

Scenario 1

consider an airplane communicating with air traffic control. A physical cable connecting the two would be absurd due to the aircraft's movement. Antennas enable wireless communication, making such interactions feasible.

Scenario 2

In order to contact a remote area, the wiring has to be laid down throughout the whole route along the valleys, the mountains, the tedious paths, the tunnels etc., to reach the remote location. The evolution of wireless technology has made this whole process very simple. Antenna is the key element of this wireless technology.

In the above image, the antennas help the communication to be established in the whole area, including the valleys and mountains. This process would obviously be easier than laying a wiring system throughout the area.

Radiation Mechanism

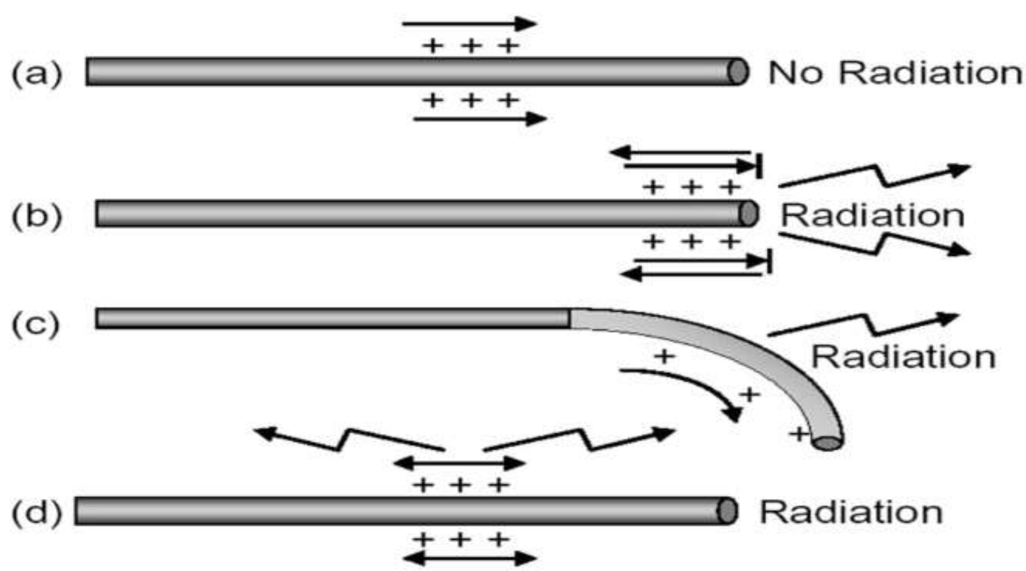
The sole functionality of an antenna is **power radiation** or **reception**. Antenna (whether it transmits or receives or does both) can be connected to the circuitry at the station through a transmission line.

- **Transmission Line**: A conductor that carries electric current over distances with minimal energy loss. Examples include coaxial cables and waveguides.
- **Antenna and Transmission Line**: The antenna is typically connected to the transmitter or receiver through a transmission line, which conveys electrical signals to or from the antenna.

However, a **uniform** transmission line by itself does not radiate power efficiently. For radiation to occur, certain conditions must be introduced to disrupt the steady flow of current, converting the electrical energy into electromagnetic waves.

Conditions That Cause Radiation:

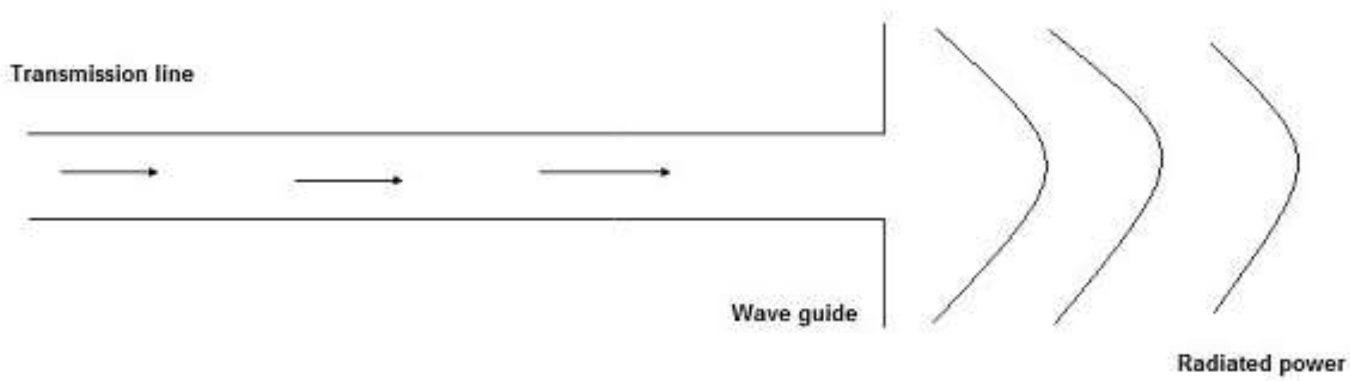
1. **Bending or Truncation of the Transmission Line**:
   - A **straight, uniform transmission line** with constant current and velocity does not radiate power because it *lacks any discontinuities*.
   - When you **bend** or **truncate** the transmission line, it causes a disruption in the current flow, which induces radiation.
   - **Example**: The tip of a truncated wire (antenna) or the junction where the transmission line bends radiates energy into space as electromagnetic waves.
2. **Accelerating or Decelerating Current**:
   - Even if the transmission line is not bent, **radiation can still occur if the current is not constant**.
   - When the **current accelerates or decelerates** along the line, it creates disturbances in the electric and magnetic fields, which generate radiation.
   - **Example**: An alternating current (AC) that changes direction or magnitude periodically will naturally accelerate and decelerate, producing a radiating electromagnetic field.



Antenna radiation mechanism

3. **Waveguides**:

   - **Waveguides** are special types of transmission lines designed to carry and radiate high-frequency electromagnetic waves (like microwaves). They are typically **hollow conductors** with specific dimensions that allow efficient wave propagation.
   - The electromagnetic waves are confined within the waveguide and radiate when an appropriate discontinuity or aperture is introduced.



The above diagram represents a waveguide, which acts as an antenna. The power from the transmission line travels through the waveguide which has an aperture, to radiate the energy.

Applications of Antennas

Antennas are essential components in a wide range of devices and systems, including:

- Broadcasting (radio and television)
- Two-way radio communication
- Radar systems
- Mobile phones and satellite communication
- Wireless devices (Bluetooth, Wi-Fi, etc.)

Wireless Communication and Antennas

In wireless communication systems, antennas are responsible for both transmitting and receiving signals. They radiate electromagnetic waves into space, and a portion of this energy is captured by receiving antennas.

Essentially, antennas serve as transitional elements between free space and guided wave structures like transmission lines or waveguides.

Antenna Theory - Basic Parameters

In wireless communication, antennas play a crucial role in transmitting and receiving electromagnetic waves.

Understanding the basic parameters that define an antenna's performance is key to optimizing wireless communication systems.

The following parameters are fundamental to antenna design and operation:

- Frequency
- Wavelength
- Impedance matching
- VSWR & reflected power
- Bandwidth
- Percentage bandwidth
- Radiation intensity

Frequency

The frequency of a wave refers to the number of oscillations or cycles per second, measured in Hertz (Hz)

In simple terms, frequency refers to how often an event occurs.

A periodic wave repeats itself every 'T' seconds (time period). The frequency of a periodic wave is the reciprocal of the time period (T).

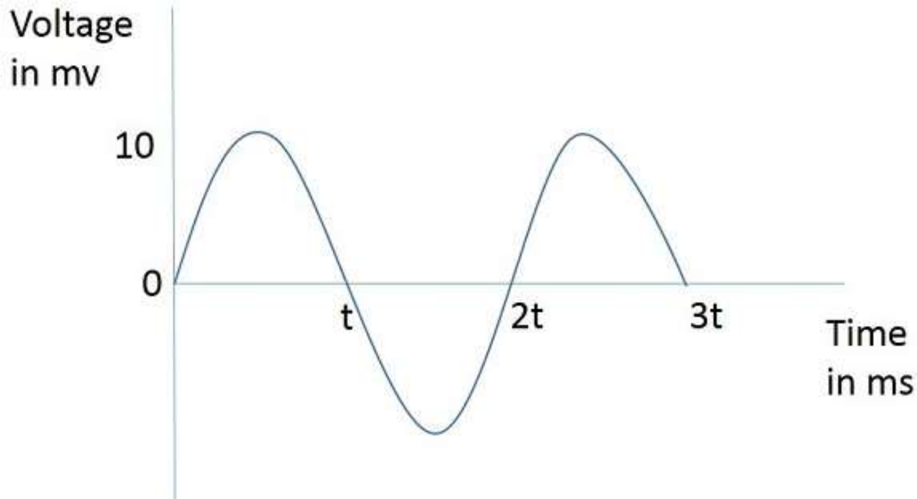
Mathematical Expression:

$$f = \frac{1}{T}$$

Where:

- f is the frequency of the periodic wave.
- T is the time period at which the wave repeats.

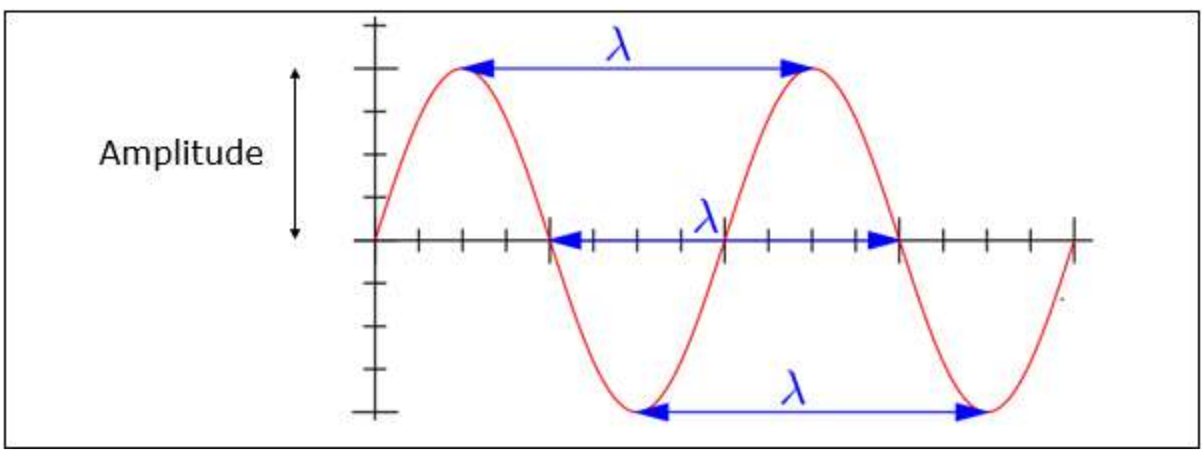
Example: The figure below shows a sine wave representing voltage in millivolts against time in milliseconds. The wave repeats every 2t milliseconds. Therefore, the time period T=2t milliseconds, and the frequency $f = \frac{1}{2t}$ KHz.



Wavelength

Definition: The distance between two consecutive maximum points (crests) or two consecutive minimum points (troughs) is known as the wavelength.

In simpler terms, the wavelength is the distance between two consecutive positive peaks or two consecutive negative peaks of a wave.



Mathematical Expression:

$$\lambda = \frac{c}{f}$$

Where:

- Λ is the wavelength.
- c is the speed of light (3×10^8 meters/second).
- f is the frequency.

Units: Wavelength (λ) is expressed in units of length, such as meters, feet, or inches, with meters being the most used.

Impedance Matching

Definition: Impedance matching is the process of ensuring that the impedance of the antenna matches the impedance of the transmission line and the load (typically 50 ohms in most communication systems).

Importance:

- **Efficient Power Transfer**: Proper impedance matching ensures maximum power transfer from the transmission line to the antenna without reflection or loss.
- **Reflection Reduction**: When impedance is mismatched, a portion of the power is reflected toward the transmitter, leading to energy loss.

Necessity of Matching:

- A resonant device, such as an antenna, delivers better output at a narrow band of frequencies if impedance is matched.
- For a receiver antenna, the antenna's output impedance should match the input impedance of the receiver amplifier circuit.
- For a transmitter antenna, the antenna's input impedance should match the transmitter amplifier's output impedance, along with the transmission line impedance.

VSWR & Reflected Power

VSWR is the ratio of the maximum to the minimum voltage in a standing wave pattern along the transmission line. It measures the level of impedance mismatch.

A stationary (or standing) wave is a wave formed by the superposition of two progressive waves of the same frequency and amplitude travelling in the opposite direction

VSWR is a key indicator of how well the impedance of different components in a transmission line (such as the antenna, transmission line, and source) match.

A mismatch in impedance between these components causes some of the power that's sent through the line to reflect toward the source rather than being fully transmitted.

Ideally, you want a VSWR of 1:1, which implies no reflected power (100% power transfer).

VSWR Formula:

$$VSWR = \frac{Vmax}{Vmin}$$

Here, Vmax and Vmin are the maximum and minimum voltages along the transmission line.

- If VSWR = 1, it indicates perfect matching (no reflection).
- A higher VSWR means a larger mismatch, leading to more reflected power.

Reflected Power

Reflected power results from impedance mismatch, and it's the portion of the signal that's not absorbed or radiated by the antenna, but instead travels back toward the transmitter. If too much power is reflected, it can:

- **Damage equipment**: Especially in high-power transmission systems, significant reflected power can cause overheating and damage to transmitters.
- **Inefficiency**: The reflected power represents wasted energy that could have been used for communication or signal transmission, reducing the system's effectiveness.

Power Reflection Coefficient (Γ)

The amount of power reflected back is characterized by the reflection coefficient:

$$\Gamma = \frac{ZL - Z0}{ZL + Z0}$$

Where:

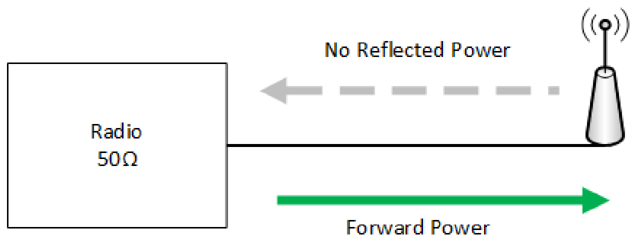
- ZL is the load impedance (e.g., the antenna).
- Z0 is the characteristic impedance of the transmission line.

The power reflected is related to VSWR by the following formula:

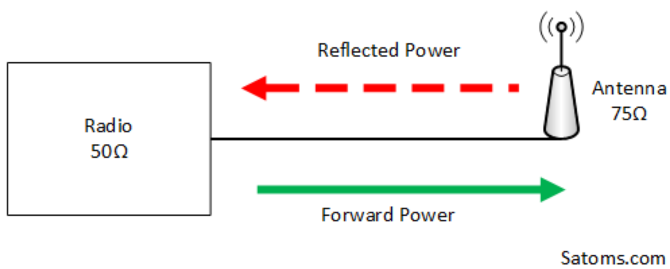
$$VSWR = \frac{1+|\ \Gamma\ |}{1-|\ \Gamma\ |}$$

- **If |Γ|=0** (perfect impedance match), then VSWR = 1.
- **If |Γ|=1** (complete reflection), then VSWR tends to infinity.

VSWR – Matched Impedance



VSWR – Mismatched Impedance



A perfectly tuned antenna system has a VSWR of 1.0, this means that no power is being reflected back along the transmission line to the radio.

However, as you can see in the VSWR table, having a VSWR of just 3 means that 25% of the Tx power is being reflected back to the radio transmitter. A VSWR of 6 and over 50% is going straight back into the radio.

VSWR and Reflected Power (%)

VSWR	% of Reflected Power
1.0	0
1.5	4
2.0	11.1
3.0	25
4.0	36
5.0	44
6.0	51
7.0	56.3
8.0	60.5
9.0	64
10	66.9
15	76.6
20	81.9

The typical impedance of a transmission systems is **50Ω or 75Ω**.
An impedance mismatch can result in poor performance and high reflected power back into the radio. The transmitter might not be able to transmit at full power and could result in damage.

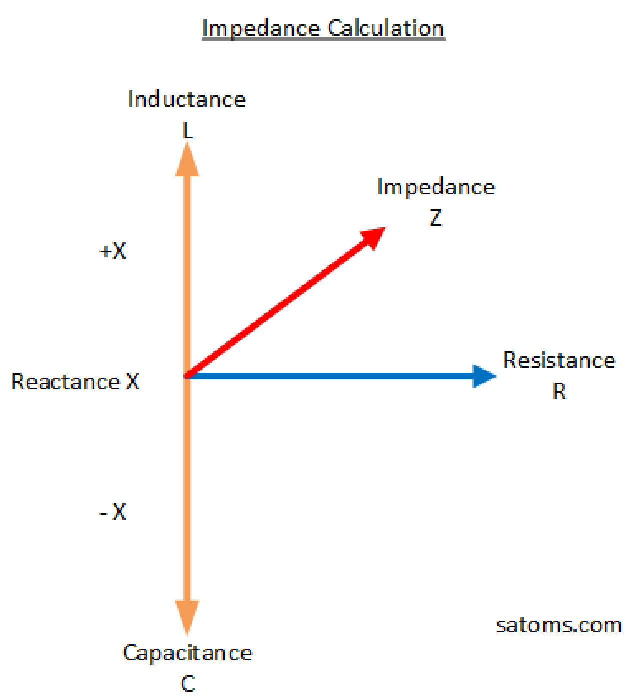
Factors affecting VSWR

- Frequency
- Antenna ground
- Nearby metal objects
- Type of antenna construction
- Temperature

Matching the system impedance can be achieved by trimming (or tuning) the antenna for the correct frequency or automatic tuning equipment.

Impedance

It refers to the opposition that a circuit presents to the flow of alternating current (AC) and is a combination of both resistance and reactance (inductive and capacitive components).



In an electrical circuit, impedance is the total opposition to the flow of an AC signal, and it is measured in ohms (Ω).

$$Z = R + jX$$

Where:

- R is the **resistance** (real part), representing energy losses (e.g., due to heat). This does not change with frequency.
- X is the **reactance** (imaginary part), representing the energy stored in the system's electric and magnetic fields. This does change with frequency and consists of Inductance (L) and Capacitance (C)
- *j* is the imaginary unit.

Bandwidth

Bandwidth refers to the range of frequencies over which an antenna can operate effectively.

In practical terms, it defines the frequency limits within which the antenna can transmit or receive signals while maintaining acceptable performance levels (such as gain, impedance matching, and radiation pattern).

It is a critical factor in determining an antenna’s suitability for a specific application, particularly in wireless communication.

- **Bandwidth** is the difference between the highest and lowest frequencies that an antenna can effectively transmit or receive.

$$Bandwidth = f_{high} - f_{low}$$

- Where:
  - f_{high} is the highest frequency the antenna can handle,
  - f_{low} is the lowest frequency the antenna can handle.

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2. Types of Bandwidth:

Absolute Bandwidth: The simple difference between high and low frequency, providing the total width of the frequency range in Hertz (Hz).

$$Absolute\ Bandwidth = f_{high} - f_{low}$$

- **Relative Bandwidth**: A normalized measure, expressed as a percentage of the center frequency (fcenter).

$$Relative\ Bandwidth = \left(\frac{fhigh - flow}{fcenter}\right) \times 100$$

$$Where: fcenter = \frac{fhigh + flow}{2}$$

  - **Narrowband Antennas**: Antennas with low relative bandwidth, meaning they are designed for a small range of frequencies. Example: GPS antennas.
  - **Broadband Antennas**: Antennas with high relative bandwidth, capable of handling a wide range of frequencies. Example: Log-periodic antennas.

Basic Types of Antennas

Antennas may be divided into various types depending upon −

- The physical structure of the antenna.
- The frequency ranges of operation.
- The mode of applications etc.

Physical structure

Following are the types of antennas according to the physical structure. You will learn about these antennas in later chapters.

- Wire antennas
- Aperture antennas
- Reflector antennas
- Lens antennas
- Micro strip antennas
- Array antennas

Frequency of operation

Following are the types of antennas according to the frequency of operation.

- Very Low Frequency (VLF)
- Low Frequency (LF)
- Medium Frequency (MF)
- High Frequency (HF)
- Very High Frequency (VHF)
- Ultra-High Frequency (UHF)
- Super High Frequency (SHF)
- Microwave
- Radio wave

Mode of Applications

Following are the types of antennas according to the modes of applications −

- Point-to-point communications
- Broadcasting applications
- Radar communications
- Satellite communications

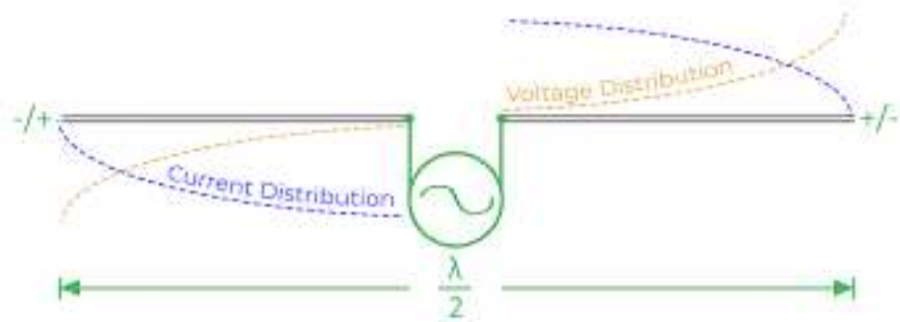
Types of Antennas

1. **Wire Antennas:**
    - **Dipole Antenna:**
        - Consists of two conductive elements, usually metal rods.
        - Most common type of antenna, often used in radio and television broadcasting.
        - The term "dipole" literally means "two poles," and that's exactly what a dipole antenna is: two similar lengths of conductive material, typically metal rods or wires.
        - **Construction:** It consists of two identical conductive elements separated by an insulator at their center.

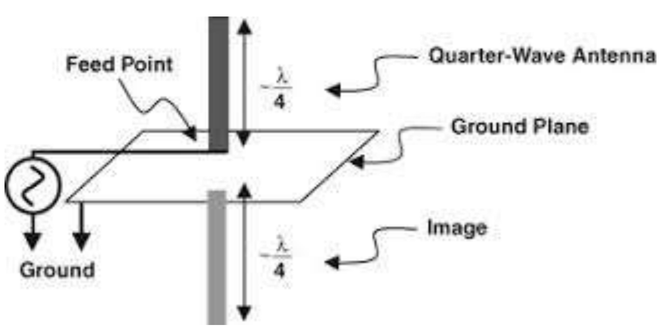
▪ **Operation:** Voltage and current flowing through the elements create radio waves (electromagnetic waves).

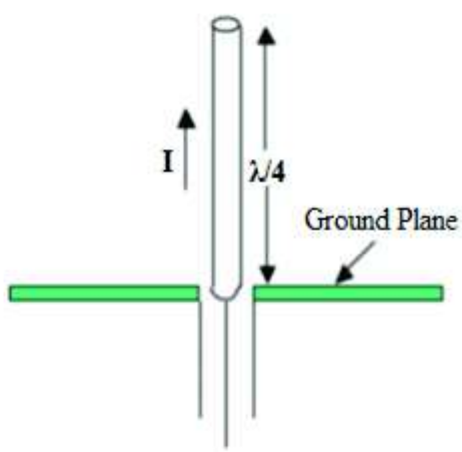
Current and Voltage Distribution:

- The center point (feed point) experiences **minimum voltage** and **maximum current**
- The ends of the elements exhibit **maximum voltage** and **minimum current**. This creates a standing wave pattern of current and voltage along the antenna length.

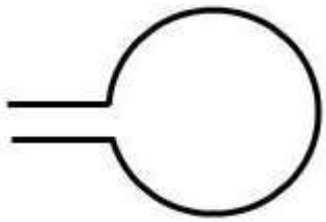


- **Monopole Antenna:**
  - A single rod or wire with one end connected to the ground plane.
  - Often used in car radios and mobile communication.





- **Loop Antenna:**
  - A loop of wire or conductor, either single or multiple turns.
  - Commonly used in AM radio receivers.



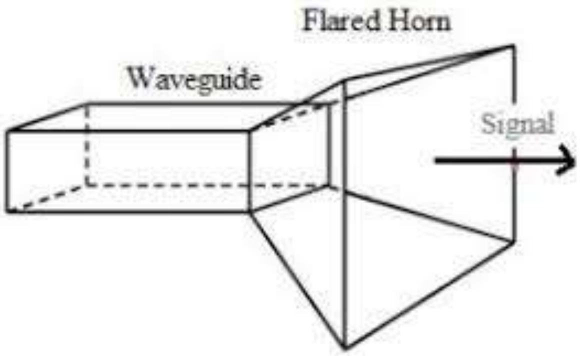
Loop

Fig 1: Circular loop antenna

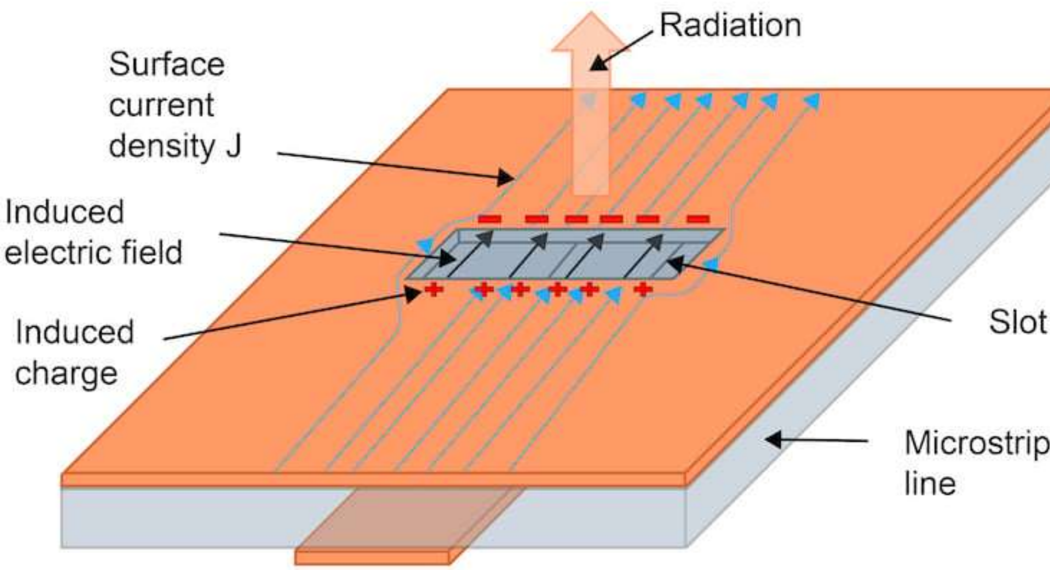
Fig 2: Square loop antenna

2. **Aperture Antennas:**
   - **Horn Antenna:**
     - Flared metal waveguide used in microwave frequencies.
     - The end of the antenna is widened or in the horn shape. Because of this structure, there is larger directivity so that the emitted signal can be easily transmitted to long distances.
     - Horn antennas operate in microwave frequency, so the frequency range of these antennas is super high or ultra-high which ranges from 300 MHz – 30 GHz.

- Commonly used in radar and satellite communication.



- **Slot Antenna:**
  - A slot cut in a metal surface, acting as a radiating element.
  - Often used in aircraft and missile applications.



3. **Array Antennas:**
   - **Phased Array Antenna:**
     - Composed of multiple radiating elements, which can be electronically steered.
     - Used in radar, satellite communication, and advanced communication systems.

Principle of Operation

A phased array antenna is a group of individual antenna elements that can be electronically controlled to produce a directional beam of radio waves.

The fundamental principle behind phased arrays is the superposition of waves. When multiple sine waves of the same frequency but different phases are combined, the resulting wave has a new amplitude and phase. By carefully controlling the phase and amplitude of the signals fed to each antenna element, the overall radiation pattern can be manipulated.

Beam Steering and Shaping

Beam Steering: By adjusting the phase of the signals fed to each antenna element, the direction of the main beam can be electronically steered without physically moving the antenna.

Beam Shaping: The width and power of the main beam can be controlled by adjusting the amplitude and phase of the signals.

Analog Phased Array System

An older method for implementing a phased array involves using analog components. The RF signal from the transmitter is divided into multiple paths, each feeding a different antenna element. Attenuators and phase shifters are used to control the amplitude and phase of the signal on each path.

Key Components:

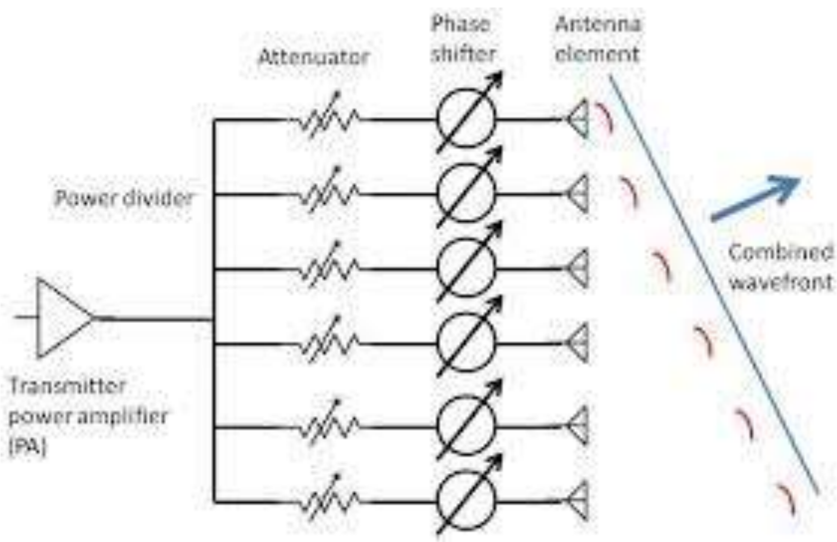
Power amplifier (PA): Amplifies the RF signal.

Power divider: Splits the RF signal into multiple equal parts.

Attenuators: Reduce the signal amplitude on specific paths.

Phase shifters: Adjust the phase of the signal on specific paths.

Antenna elements: Individual antennas that radiate the RF signal.



- **Yagi-Uda Antenna:**
  - Consists of multiple parallel elements in a line, usually a driven element and additional reflectors and directors.
  - Commonly used in TV reception and amateur radio.

4. **Reflector Antennas:**
   - **Parabolic Reflector Antenna:**
     - A parabolic-shaped reflector that directs radio waves to a focal point.
     - Widely used in satellite dishes, radio telescopes, and microwave communication.

- **Corner Reflector Antenna:**
    - Consists of a dipole antenna with a corner reflector behind it to enhance directivity.
    - Often used in UHF television reception.

5. **Microstrip Antennas (Patch Antennas):**
    - **Patch Antenna:**
        - A flat, rectangular or circular metal patch mounted over a ground plane.
        - Commonly used in mobile phones, GPS devices, and Wi-Fi routers due to their compact size and ease of integration.
6. **Log-Periodic Antennas:**
    - **Log-Periodic Dipole Array (LPDA):**
        - A wideband antenna composed of a series of dipole elements of varying lengths.
        - Used in TV reception, communication systems, and as broadband antennas.
7. **Helical Antennas:**
    - **Helical Antenna:**
        - A wire wound into a helical shape, radiating circularly polarized waves.
        - Used in satellite communication, GPS, and space telemetry.
8. **Dielectric Antennas:**
    - **Dielectric Resonator Antenna (DRA):**
        - Uses dielectric materials to produce resonant modes.
        - Often used in microwave and millimeter-wave applications.
9. **Biconical Antennas:**
    - **Biconical Antenna:**
        - Consists of two conical conductors facing each other.

- Used in wideband applications, such as EMC testing and VHF measurements.

Each type of antenna is designed to serve specific applications, offering unique characteristics such as directivity, frequency range, and polarization.

Properties of Antenna

Antenna Impedance

Antenna impedance is the electrical characteristic representing the load presented by an antenna to a transmitter or the source impedance for a receiver. It's composed of two primary components:

- **Radiation resistance:** This virtual resistance quantifies the power transformed into electromagnetic radiation. It's calculated by multiplying the square of the antenna's RMS current by the radiated power.
- **Ohmic resistance:** This is the actual resistance of the antenna's conductive elements, resulting in power loss as heat.

Antenna Efficiency

The efficiency of an antenna is determined by the ratio of radiated power to total input power.

$$efficiency = \frac{P_{Rad}}{P_A}$$

Mathematically, it's also expressed as:

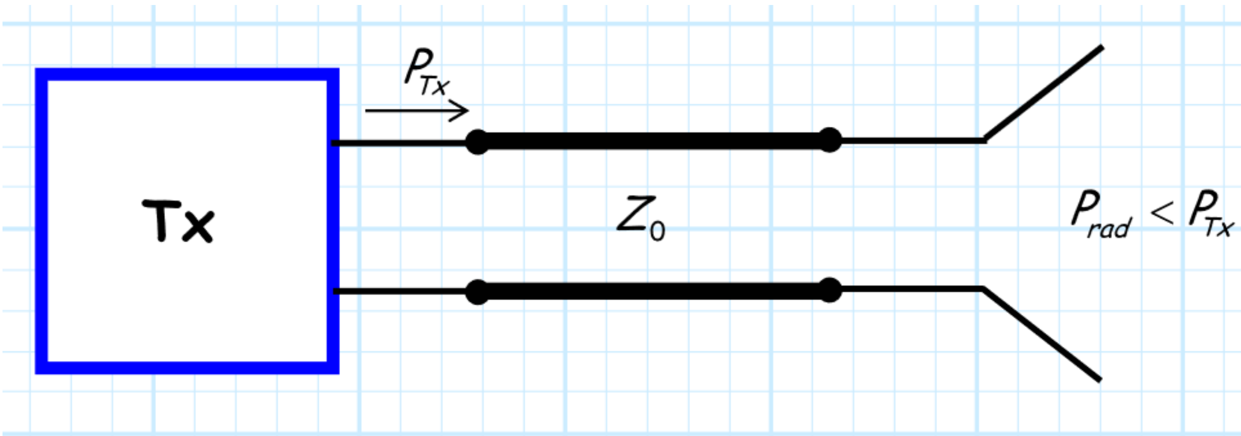
$$Antenna\ Efficiency = e\ = \left(\frac{Rr}{Rr\ +\ Rl}\right)\ *\ 100\%$$

A higher radiation resistance relative to ohmic resistance leads to a more efficient antenna.

Power Transfer and Radiated Power:

- **Radiated Power (Prad)**: Ideally, the radiated power should be equal to the available power from the transmitter (Ptx).

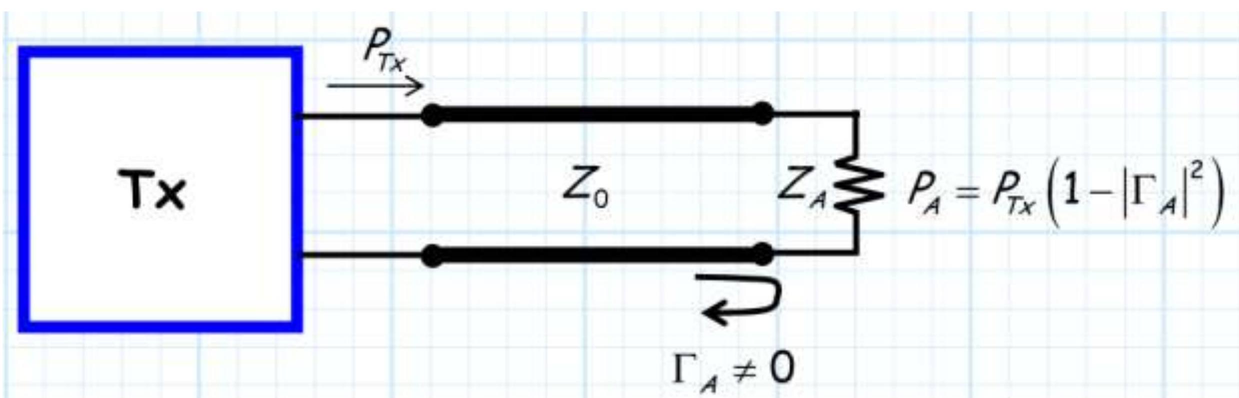
  However, $Prad$ is always slightly less than Ptx due to inefficiencies.



- **Reason for Inequality (Ptx ≠ Prad):** The discrepancy arises because:
    1. **Power Reflection:** Power is reflected at the antenna if the antenna impedance (ZA) is not matched to the transmission line impedance (Z0).
    2. **Power Dissipation:** Power is lost as heat due to the lossy nature of the antenna materials.

Antenna Impedance and Reflection:

- The antenna impedance acts as the load at the end of a transmission line.
- **Matching Impedance:** If ZA≠Z0, then power will be reflected, and the power delivered to the antenna (PA) will be less than the transmitter available power (Ptx).



$$ZA = R_A + jX_A$$

$$R_A = R_i + R_r$$

To avoid power reflection and ensure that all available power is delivered to the antenna, the antenna impedance (ZA) must match the characteristic impedance (Z0) of the transmission line:

$$ZA = Z0 \implies \Gamma = 0$$

- **Reflection Coefficient (Γ):** When ZA does not equal Z0, some of the power is reflected, leading to reduced power being delivered to the antenna:

$$P_A = P_{tx}(1 - |\Gamma|^2)$$

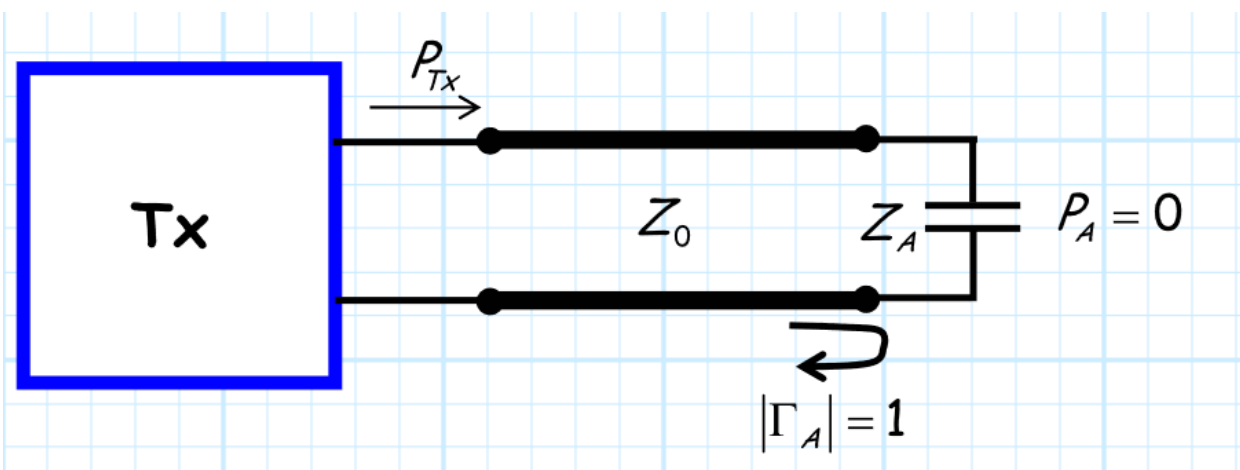
Resistive vs. Reactive Impedance:

- **Real vs. Reactive Impedance:**
  - **Resistive Impedance:** An ideal antenna impedance is purely resistive, which allows it to absorb power.

    * A resistor will convert its absorbed power into heat.

    * An antenna will (ideally) convert its absorbed power into a propagating, spherical, electromagnetic wave!
  - **Reactive Impedance:** A real load can absorb incident energy, whereas a purely reactive load cannot

    If the antenna impedance is purely reactive, all incident power would be reflected, leading to zero power being absorbed by the antenna (PA=0).



Thus, it is imperative that the impedance of an antenna have a real component if we wish for it to absorb energy, with maximum power transfer occurring when $Z_A = Z_0$

- **Ideal Antenna Functionality:**
  - **Absorbed Power:** A good antenna absorbs power and radiates it as an electromagnetic wave. This is in contrast to a resistor, which dissipates absorbed power as heat.

In summary

$P_{rad} = e * PA$ $\quad P_A = P_{tx}(1-|\,\Gamma\,|^2)$ $\quad P_{rad} = e * P_{tx}(1-|\,\Gamma\,|^2)$

Question 1:

A transmission line with a characteristic impedance of Z0=75 Ω is connected to an antenna with an impedance ZA=50 Ω. The transmitter provides Ptx=100 W.

- Calculate the reflection coefficient Γ.
- Determine the fraction of the power reflected back toward the transmitter.
- How much power is delivered to the antenna?

Given:

- Characteristic impedance of the transmission line $Z_0 = 75\ \Omega$
- Antenna impedance $Z_A = 50\ \Omega$
- Transmitter power $P_{tx} = 100$ W

Step 1: Calculate the Reflection Coefficient Γ

The reflection coefficient Γ is given by the formula:

$$\Gamma = \frac{Z_A - Z_0}{Z_A + Z_0}$$

Substitute the given values of $Z_A = 50\ \Omega$ and $Z_0 = 75\ \Omega$:

$$\Gamma = \frac{50 - 75}{50 + 75}$$

$$\Gamma = \frac{-25}{125} = -0.2$$

Thus, the reflection coefficient is:

$$\Gamma = -0.2$$

Step 2: Determine the Fraction of Power Reflected

The fraction of the power reflected is given by $|\Gamma|^2$. Since $\Gamma = -0.2$, we calculate:

$$|\Gamma|^2 = (-0.2)^2 = 0.04$$

This means that **4%** of the power is reflected back toward the transmitter.

Step 3: Calculate the Power Delivered to the Antenna

The power delivered to the antenna is given by:

$$P_A = P_{tx} \times (1 - |\Gamma|^2)$$

Substitute the values of $P_{tx} = 100$ W and $|\Gamma|^2 = 0.04$:

$$P_A = 100 \times (1 - 0.04) = 100 \times 0.96 = 96 \text{ W}$$

Thus, **96 W** of power is delivered to the antenna.

Radiation Pattern, Beam Area, and Beam Efficiency

1. Radiation Pattern

1.1 Definition and Importance

- **Radiation Pattern**: The radiation pattern of an antenna is a graphical representation of the **distribution of radiated powe**r from the antenna as a function **of direction in space**.
  - It describes how the power varies with the direction away from the antenna.
  - It is usually represented in polar or Cartesian coordinates.

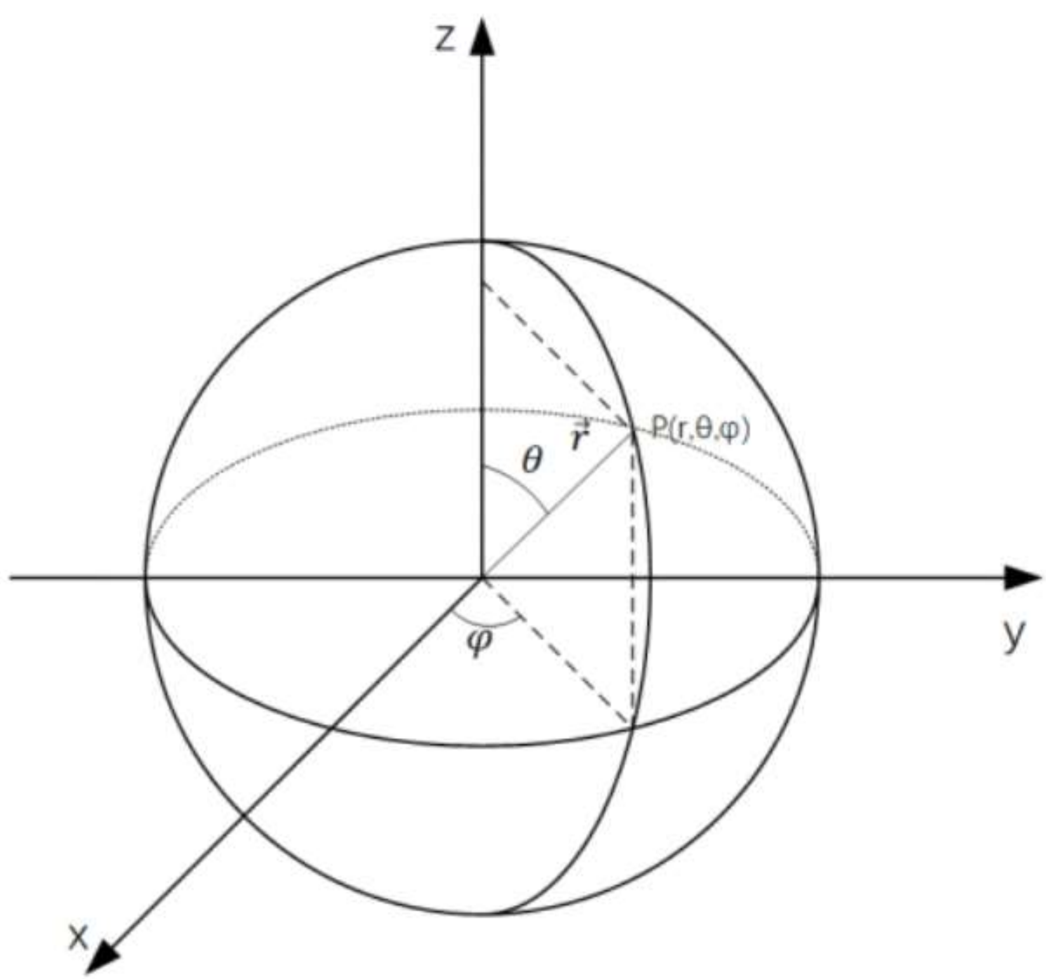
It displays the strength of the radiated filed or power density as a function of direction i.e. $(\theta \quad and \quad \phi)$

Spherical Coordinate System

The spherical coordinate system is commonly used to describe the radiation pattern of antennas. It provides a way to define the position of a point in three-dimensional space using three parameters: the radial distance r, the elevation angle θ(theta), and the azimuth angle φ (phi).

- **Radial Distance (r)**: The distance from the origin (typically the antenna location) to the point of interest in space.
- **Theta (θ)**: The elevation angle or polar angle, measured from the positive z-axis down to the point in question. It ranges from 0∘ to 180∘. It indicates how far up or down the point is relative to the z-axis (typically the vertical axis).
- **Phi (φ)**: The azimuth angle, measured in the x-y plane from the positive x-axis to the projection of the point onto the x-y plane. It ranges from 0∘ to 360∘. It indicates how far around the point is in the horizontal plane.

The following coordinate system shows the directions



Classification of Radiation Patterns

Power pattern: The trace of the angular variation of the received/radiated power at a constant radius from the antenna.

Amplitude field pattern: The trace of the spatial variation of the magnitude of the electric (or magnetic) field at a constant radius from the antenna.

Often, the field and power patterns are normalized with respect to their maximum value, yielding normalized field and power patterns.

The power pattern is usually plotted on a logarithmic scale or more commonly in decibels (dB).

For an antenna:

- **Field pattern (in linear scale)** typically represents a plot of the magnitude of the electric or magnetic field as a function of the angular space.
  It can be $|\ E(\theta,\phi)\ |\ \ or\ \ |\ H(\theta,\phi)\ |$ |
- **Power pattern (in linear scale)** typically represents a plot of the square of the magnitude of the electric or magnetic field as a function of the angular space.

$$P(\theta,\phi) = |\ E(\theta,\phi)\ |^2\ \ or\ \ |\ H(\theta,\phi)\ |^2$$

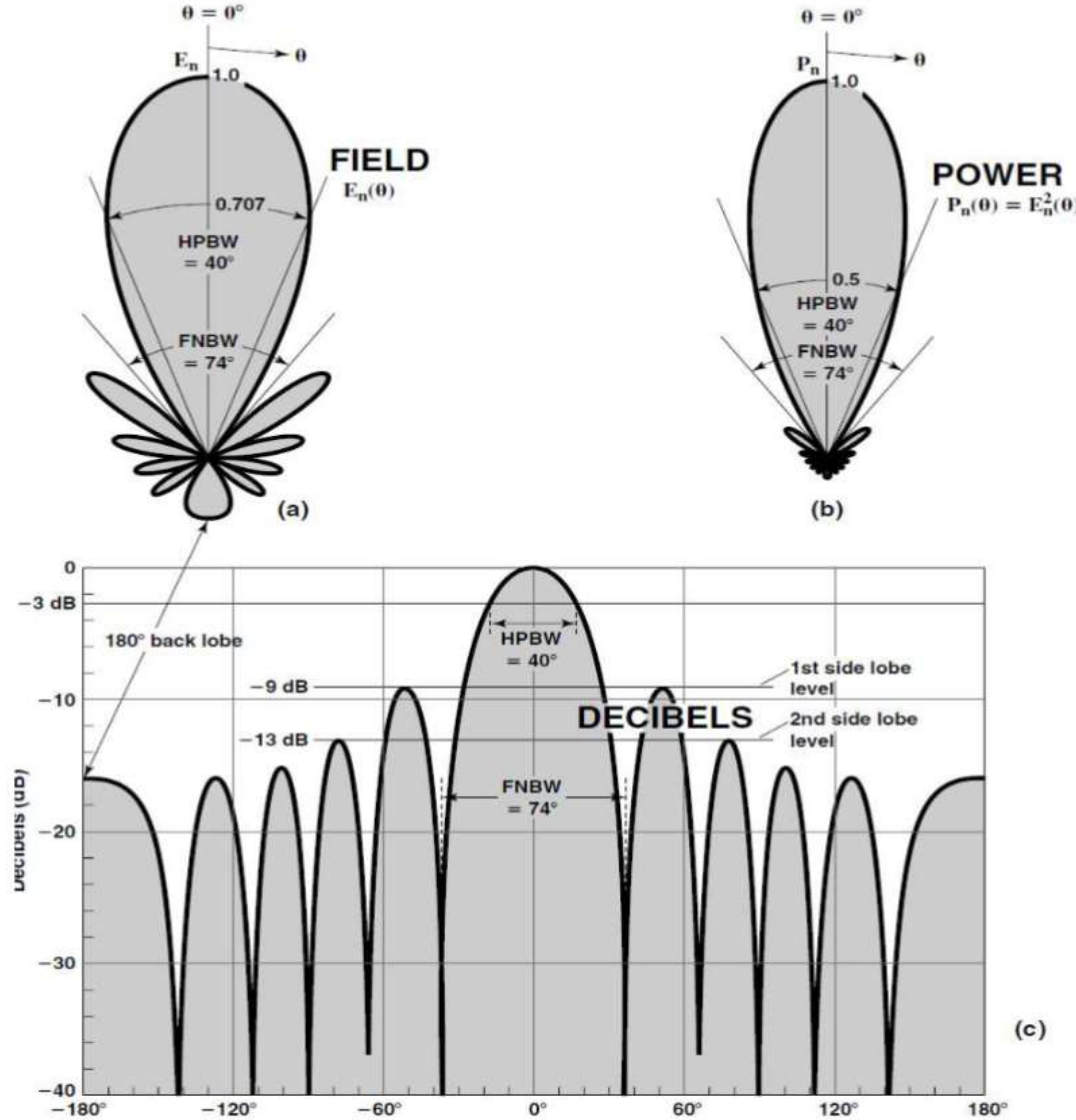
- **Power pattern (in dB)** represents the magnitude of the electric or magnetic field, in decibels, as a function of angular space.

  The conversion to decibels for power is done using the following formula:

$$P_{dB} = 10\log_{10}\left(\frac{P(\theta,\phi)}{Pref}\right)$$

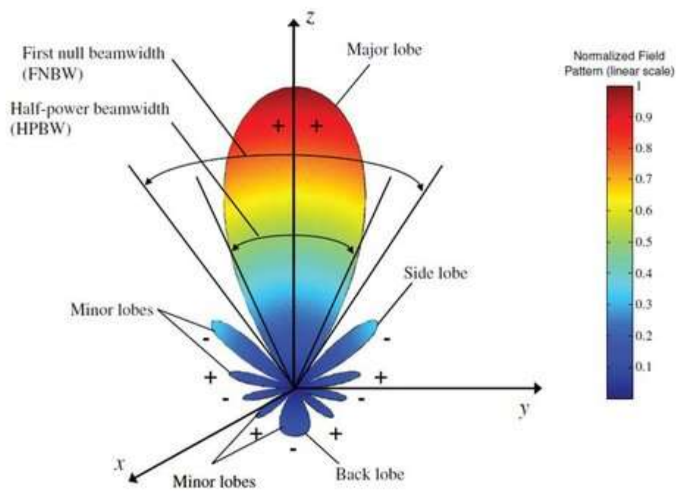
  where:

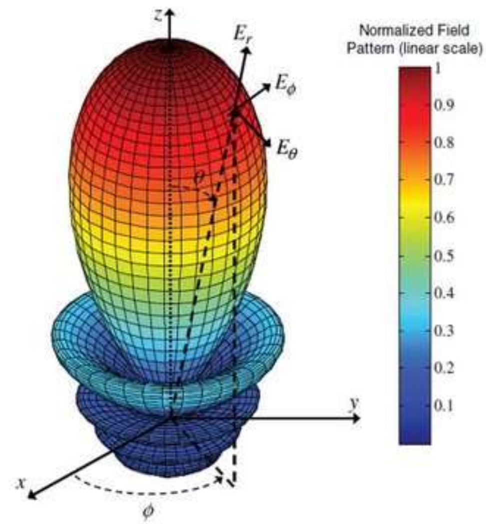
  - P(θ,φ) is the power pattern at a given angle.
  - Pref is a reference power, often the maximum power or the power at a specific reference point.



3D Radiation Patterns

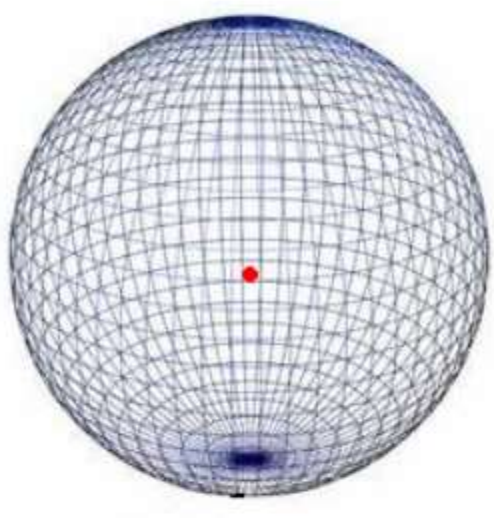
A **3D radiation pattern** provides a complete picture of how an antenna radiates in all directions, encompassing both the azimuth and elevation planes. These patterns are often represented as three-dimensional plots showing the distribution of radiated energy in space around the antenna.



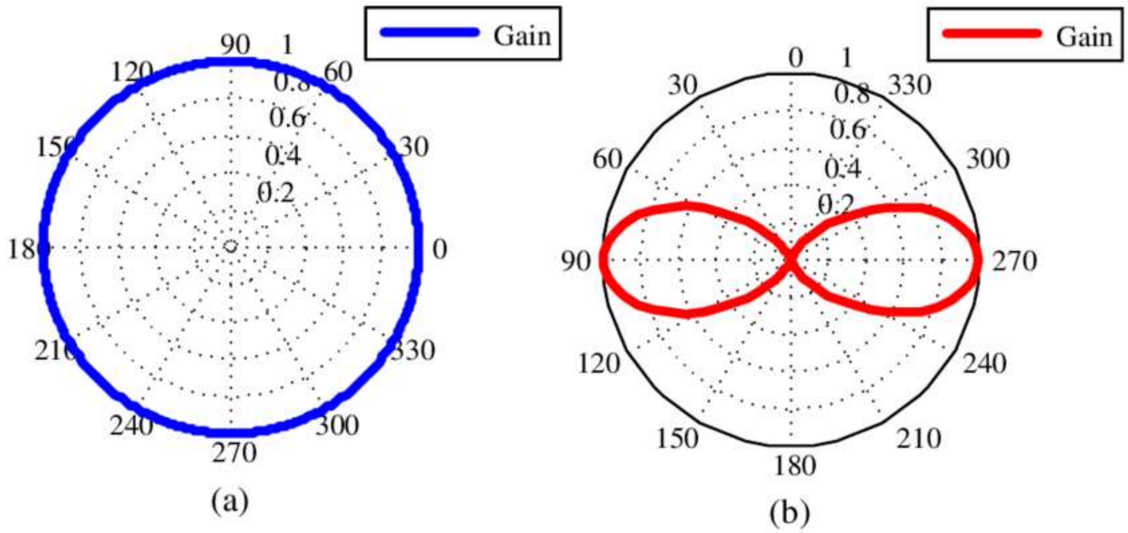


1.2 Types of Radiation Patterns

- **Isotropic Pattern**:
  - An **isotropic antenna** is an idealized antenna that radiates power uniformly in all directions.
  - It serves as a reference for comparing real antennas.



- **Directional Pattern**:
  - A **directional antenna** radiates more power in specific directions than others.
  - Examples include Yagi-Uda antennas, parabolic dish antennas, and horn antennas.



Isotropic (a) vs directional (b) radiation patterns

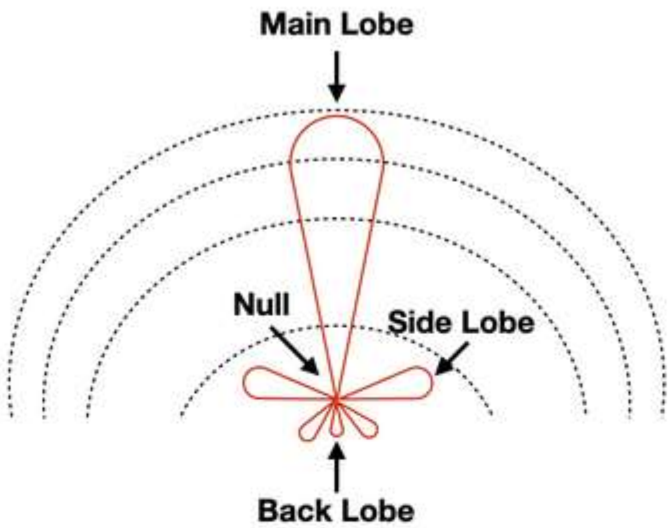
1.3 Radiation Pattern Lobes

A radiation pattern describes how an antenna radiates energy into space.

The radiation pattern typically contains lobes, which are ***regions in space where the radiated power is concentrated.***

These lobes help to understand the directionality and performance of an antenna.

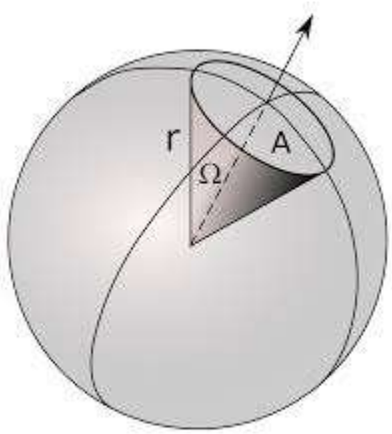
- **Main Lobe**:
  - The lobe that contains the maximum power and points in the desired direction of radiation.
- **Side Lobes**:
  - Smaller lobes that appear at angles away from the main lobe, representing power radiated in unintended directions.
  - High side lobes can lead to interference and reduced efficiency.
- **Back Lobe**:
  - The lobe that radiates power in the opposite direction to the main lobe.
  - Ideally, this should be minimized.
- **Nulls**: Areas with minimal or no radiation.



Solid angle

It is the angle subtended by any part of a spherical surface of unit radius at its center.

It is represented as Ω



A **steradian** (symbol: **sr**) is the SI unit of **solid angle**

Mathematically, the solid angle Ω in steradians can be expressed as:

$$\Omega = \frac{A}{r^2}$$

where:

- Ω = solid angle in steradians
- A = area of the surface on the sphere

- r = radius of the sphere

The total solid angle surrounding a point in three-dimensional space is 4π steradians. This corresponds to the surface area of a sphere, where the surface area is $4\pi r^2$.

2. Beam Area (ΩA)

- **Beam Area (ΩA)**: The beam area is the solid angle through which all the power of the antenna would be radiated if it were concentrated in the main lobe.
  - It is expressed in steradians (sr), which is the SI unit for measuring solid angles.
  - Beam area gives a measure of the antenna's directivity and how concentrated its radiation is in the main lobe.
- **Beam Area Formula**: The beam area is calculated as:

$$\Omega_A = \int_{\theta=0}^{\pi} \int_{\Phi=0}^{2\pi} P(\theta, \Phi) d\Omega = \int_{\theta=0}^{\pi} \int_{\Phi=0}^{2\pi} P(\theta, \varPhi) sin\theta d\theta d\phi$$

Where:

- P(θ,φ) = power radiated per unit solid angle in the direction specified by the angles θ (elevation) and φ (azimuth).
- θ = elevation angle (0 to π)
- φ = azimuth angle (0 to 2π)

2.3 Beam Area and Directivity

- **Relation to Directivity**:
  - The beam area is inversely proportional to the directivity of the antenna.
  - **Directivity (D)** is defined as the ratio of the maximum radiation intensity in the main lobe to the average radiation intensity over all directions:

Mathematically:

$$D(\theta, \phi) = \frac{U(\theta, \phi)}{U_{avg}} = 4\pi \frac{U(\theta, \phi)}{P_{rad}}$$

Where:

- D is the directivity.
- U is the radiation intensity in a given direction (W/steradian).

- U_{avg} is the average radiation intensity (total power radiated divided by 4π).

It is also given by:

$$D = \frac{4\pi}{\Omega_A}$$

Since the directivity is a dimensionless ratio that may vary greatly, it is common to give a logarithmic measure, in decibel over isotropic (dB).

$$logarithmic\ measure\ of\ D\ =\ 10\log_{10}(D(\theta,\phi))$$

- Directivity is a dimensionless quantity.
- A high directivity means the antenna is effective in radiating energy in a specific direction, making it useful for applications like point-to-point communication.
- A smaller beam area indicates a more concentrated radiation pattern and higher directivity.

Example questions

An antenna radiates a maximum radiation intensity of U(θ,φ)=200 W/sr in its main lobe, while the total power radiated by the antenna is Prad=50 W. Calculate the directivity D(θ,φ) of the antenna and the beam area.

Step 1: Calculate Directivity $D(\theta, \phi)$

The formula for directivity D is given by:

$$D(\theta, \phi) = \frac{U(\theta, \phi)}{U_{avg}}$$

Where U_{avg} is the average radiation intensity over all directions and can be calculated as:

$$U_{avg} = \frac{P_{rad}}{4\pi}$$

Calculation of U_{avg}:

$$U_{avg} = \frac{50\ \text{W}}{4\pi} \approx \frac{50}{12.5664} \approx 3.97887\ \text{W/sr}$$

Now, calculate $D(\theta, \phi)$:

$$D(\theta, \phi) = \frac{200\ \text{W/sr}}{3.97887\ \text{W/sr}} \approx 50.31$$

The beam area Ω_A is related to the directivity D by the formula:

$$D = \frac{4\pi}{\Omega_A}$$

Rearranging this gives:

$$\Omega_A = \frac{4\pi}{D}$$

Substituting the calculated value of D:

$$\Omega_A = \frac{4\pi}{50.31} \approx \frac{12.5664}{50.31} \approx 0.249\ \text{sr}$$

2.4 Practical Significance of beam area in antenna design

- Antennas with smaller beam areas are typically used in applications requiring focused energy in specific directions, such as satellite communication and radar.
- Larger beam areas are found in antennas designed for broader coverage, such as broadcast antennas.

3. Beam Efficiency (ηB)

Beam Efficiency (ηB): Beam efficiency is the ratio of the power radiated in the main lobe to the total power radiated by the antenna.

- It is a measure of how effectively an antenna concentrates its power in the desired direction.
- Mathematically, it is expressed as:

$$\eta_B = \frac{\text{Power in Main Lobe}}{\text{Total Radiated Power}}$$

Since beam efficiency is a ratio of two powers, it is a unitless quantity and is often expressed as a percentage:

$$\eta_B\,(\%) = \left(\frac{P_{main\ lobe}}{P_{total}}\right) \times 100$$

Importance in Applications

- **High Beam Efficiency**:
  - Desired in applications where it is crucial to direct as much power as possible in a **specific direction**, such as in radar systems and satellite communications.
- **Low Beam Efficiency**:
  - May be acceptable in applications where **wide coverage** is needed, and some power can be allowed to radiate in directions other than the main lobe.

3.4 Beam Efficiency and Antenna Gain

- **Relationship to Gain**:
  - **Antenna Gain** is the product of the directivity and the efficiency of the antenna.

Mathematically:

$$G = \eta_B \times D$$

Where:

- G is the gain of the antenna.
- η_B is the beam efficiency
- D is the directivity.

Interpretation:

- Gain represents how effectively an antenna converts input power into radiated power in the desired direction.
- Higher beam efficiency contributes to higher gain, meaning that more power is radiated in the intended direction.
- Gain is a dimensionless quantity and is frequently expressed in decibels (dB):

$$G_{dB} = 10log10(G)$$

Question:

An antenna has the following characteristics:

- Total power input to the antenna: Pinput=100 W
- Total power radiated by the antenna: Prad=70 W
- Power radiated in the main lobe: Pmain lobe=50 W
- Beam area of the antenna: ΩA=0.25sr

1. Calculate the beam efficiency (ηB) and antenna efficiency(ηA).
2. Determine the directivity (D) of the antenna
3. Calculate the gain of the antenna

1. Calculate the Beam Efficiency (η_B) and Antenna Efficiency (η_A)

Beam Efficiency (η_B) is calculated as the ratio of the power radiated in the main lobe to the total power radiated by the antenna:

$$\eta_B = \frac{P_{main\,lobe}}{P_{rad}}$$

Substituting the values:

$$\eta_B = \frac{50\text{ W}}{70\text{ W}} \approx 0.7143\text{ (or 71.43\%)}$$

Antenna Efficiency (η_A) is calculated as the ratio of the total power radiated by the antenna to the total power input:

$$\eta_A = \frac{P_{rad}}{P_{input}}$$

Substituting the values:

$$\eta_A = \frac{70\text{ W}}{100\text{ W}} = 0.7\text{ (or 70\%)}$$

2. Determine the Directivity (D) of the Antenna

Directivity (D) is calculated using the beam area with the formula:

$$D = \frac{4\pi}{\Omega_A}$$

Substituting the value of beam area:

$$D = \frac{4\pi}{0.25} = \frac{4 \cdot 3.14159}{0.25} = 50.27$$

3. Calculate the Gain of the Antenna

Gain (G) is calculated using the relationship between gain, beam efficiency, and directivity:

$$G = \eta_B \times D$$

Substituting the calculated values:

$$G = 0.7143 \times 50.27 \approx 35.93$$

Beam Width

Beam width is a crucial factor in the radiation pattern of an antenna, representing the aperture angle from which most of the power is radiated.

In the context of an antenna's radiation pattern, the **main lobe** (or main beam) is the primary area where the maximum and most consistent energy is radiated.

There are two main considerations related to beam width:

1. **Half Power Beam Width (HPBW)**
2. **First Null Beam Width (FNBW)**

Half-Power Beam Width (HPBW)

It is he angular separation, in which the magnitude of the radiation pattern decreases by 50% (or -3 dB) from the peak of the main beam.

In simpler terms, the beam width indicates the area where most of the power is radiated, aligning with the peak power output.

HPBW specifically refers to the angle within which the relative power remains above 50% of the peak power in the effective radiated field of the antenna.

To find the points where the pattern achieves its half-power (-3 dB points), relative to the maximum value of the pattern:

- Field pattern at **0.707** value of its maximum
- Power pattern (in linear scale) at its **0.5** value of its maximum
- Power pattern (in dB) at **-3** dB value of its maximum

Decibels for Power

The decibel (dB) is a logarithmic unit used to express ratios of power, voltage, or intensity. The formula for power in decibels is:

$$dB = 10\log_{10}\left(\frac{P}{P0}\right)$$

where P is the power being measured and P0 is a reference power level.

Decibels for Voltage

When expressing voltage levels, the formula for decibels is slightly different because power is proportional to the square of the voltage in a resistive circuit. The formula for voltage is:

$$dB = 20log10\left(\frac{V}{V0}\right)$$

Where:

- V = Voltage level being measured (in volts).
- V0 = Reference voltage level (in volts).
- The factor of 20 is used because power is proportional to the square of voltage, so the logarithmic ratio must be multiplied by 2.

First-Null Beamwidth (FNBW):

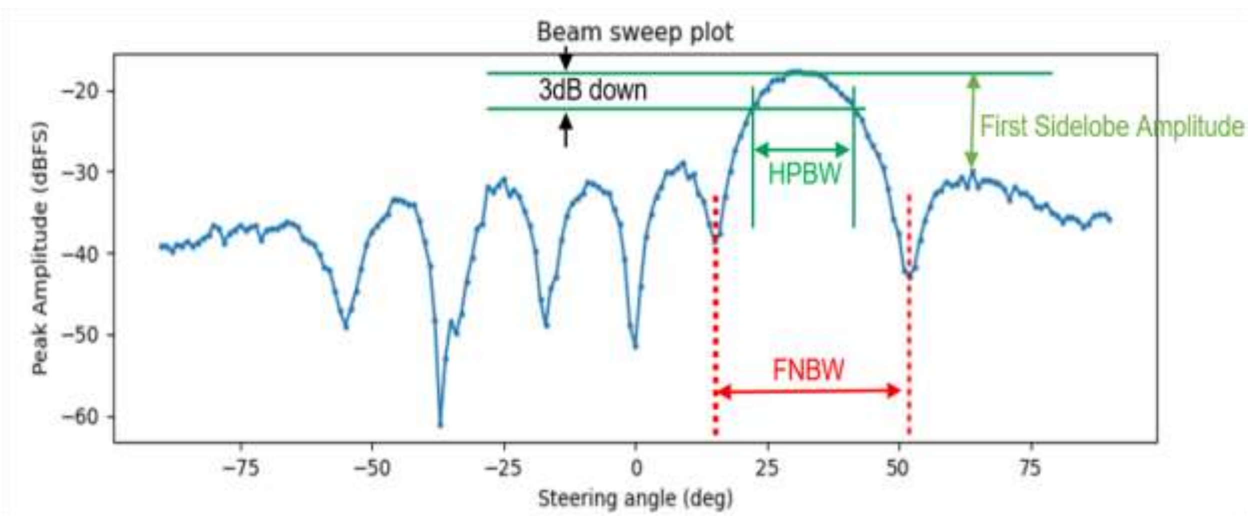
It is the angular separation between the first nulls if the pattern.

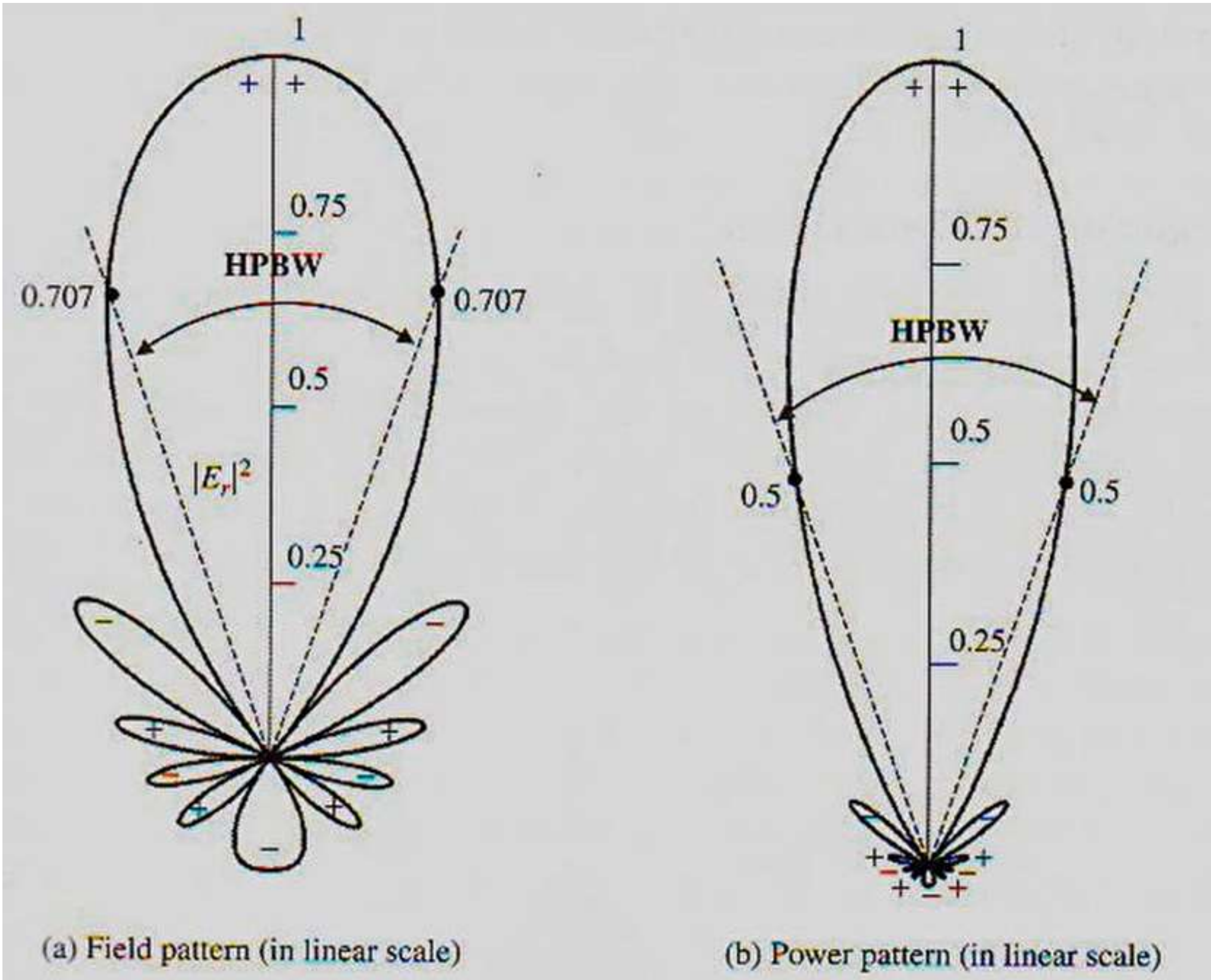
Often

$$FNBW \approx 2HPBW$$

Units

The unit of FNBW and HPBW are **radians** or **degrees**.





Front-to-Back Ratio (F/B Ratio)

The **Front-to-Back Ratio (F/B Ratio)** is an important parameter in antenna performance, particularly for directional antennas. It represents the ratio of the power radiated in the main forward direction (front) of the antenna to the power radiated in the opposite (back) direction. It is usually expressed in decibels (dB).

- **Definition**: The F/B Ratio is calculated by comparing the strength of the signal radiated in the forward direction to the signal strength radiated in the backward direction.

$$F/B - ratio\ (dB) = 10 \times \log_{10}\left(\frac{P_{forward}}{P_{backward}}\right)$$

Where:

$Pforward$ = Power radiated in the forward direction.

$Pbackward$ = Power radiated in the backward direction.

Significance: A high F/B Ratio indicates that the antenna effectively radiates energy in the desired forward direction while minimizing radiation in the undesired backward direction. This is particularly useful in reducing interference and improving signal clarity in communication systems.

Antenna Field Zones

The space around an antenna is typically divided into three distinct zones: the **Near-Field (Reactive Near-Field and Radiating Near-Field) Zone**, and the **Far-Field Zone**. These zones are defined based on the distance from the antenna and the characteristics of the electromagnetic fields present.

1. **Reactive Near-Field Zone**:
    - **Location**: Very close to the antenna, usually within a distance of less than $\lambda/2\pi$ where λ is the wavelength).

    $$distance < \lambda/2\pi$$

    - **Characteristics**: The electromagnetic fields in this zone are predominantly reactive, meaning they store energy but do not radiate it efficiently. These fields are primarily electric or magnetic, depending on the type of antenna. The field strength drops off rapidly with distance.
2. **Radiating Near-Field Zone (Fresnel Zone)**:
    - **Location**: This zone lies between the reactive near-field and the far-field zones, typically from a distance of $\lambda/2\pi$ to $2D^2/\lambda$

      where D is the largest dimension of the antenna.

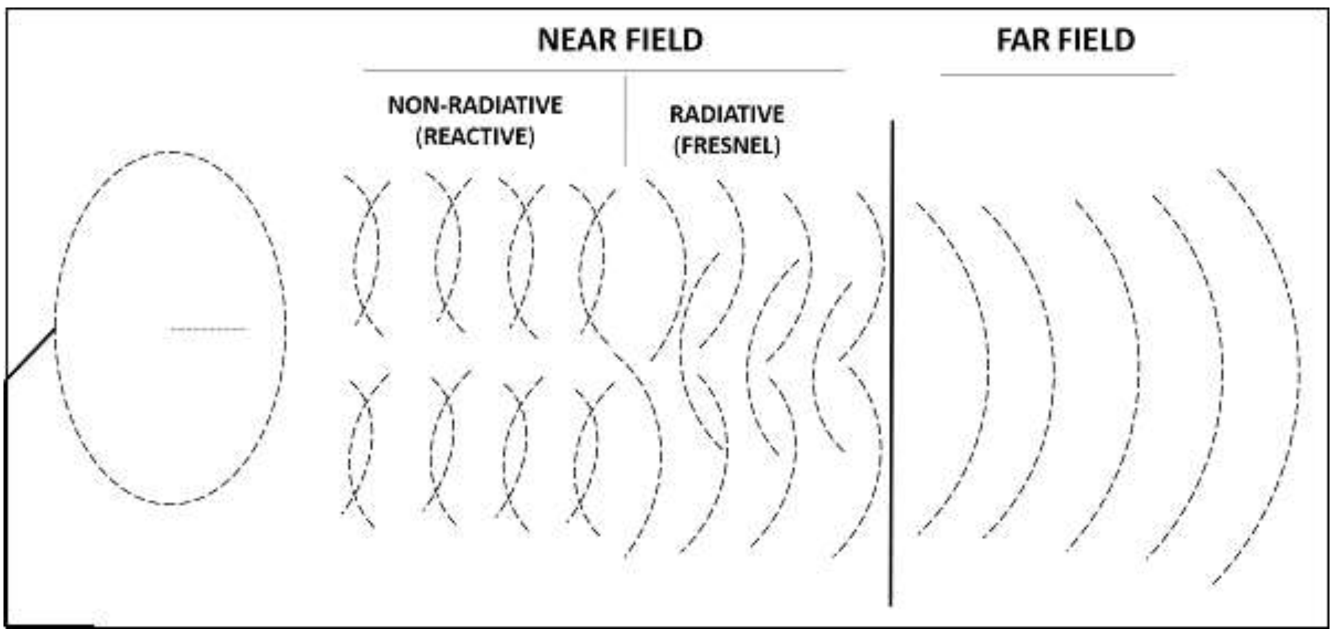
    $$\frac{\lambda}{2\pi} < distance < \frac{2D^2}{\lambda}$$

    - **Characteristics**: In this region, the electromagnetic fields begin to radiate energy away from the antenna. However, the field patterns are still complex, with both reactive and radiating components. This zone is characterized by non-uniform field distribution.
3. **Far-Field Zone (Fraunhofer Zone)**:

- **Location**: Beyond the radiating near-field zone, typically at distances greater than $\frac{2D^2}{\lambda}$.

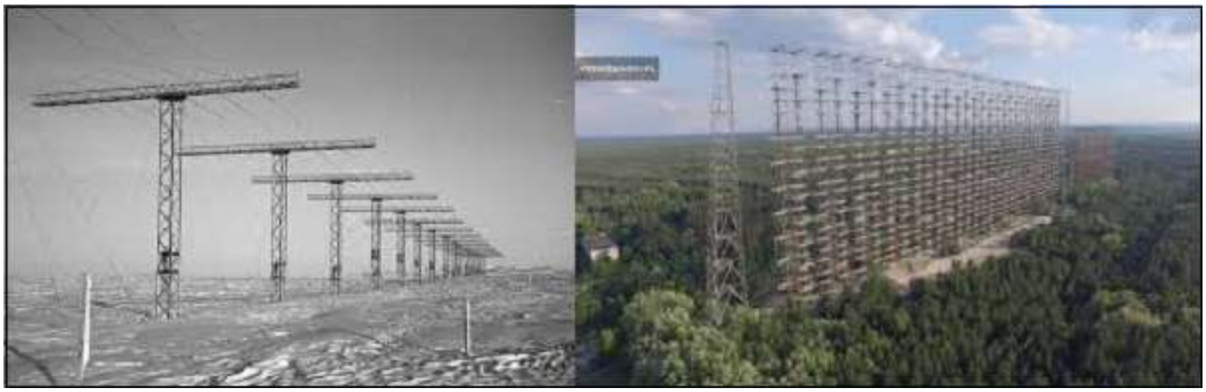
$$distance > \frac{2D^2}{\lambda}$$

- **Characteristics**: In the far-field zone, the electromagnetic fields become radiative, and the wavefronts are essentially planar. The electric and magnetic fields are perpendicular to each other and to the direction of propagation. The field strength decreases inversely with distance.



Antenna Arrays

An individual antenna can radiate energy in a particular direction, resulting in better transmission. But what if more elements are added to it to produce a more efficient output? This idea led to the invention of antenna arrays.



An **antenna array** is a radiating system composed of multiple individual radiators or elements. Each radiator has its own induction field, and the elements are placed so closely together that they lie within each other's induction fields. Therefore, the radiation pattern produced by an antenna array is the vector sum of the individual elements' radiation patterns.

The spacing between elements and the length of the elements relative to the wavelength are important design considerations for antenna arrays.

When antennas radiate individually, they produce separate radiation patterns, but when combined into an array, the radiation from all elements sums up to form a single radiation beam with high gain, high directivity, and better performance, while minimizing losses.

Advantages of Antenna Arrays

- **Increased Signal Strength:** The combined output of multiple elements results in a stronger signal.
- **High Directivity:** The radiation can be focused in a specific direction, improving communication.
- **Reduction of Minor Lobes:** Minimizes the unwanted radiation in directions other than the main beam.
- **High Signal-to-Noise Ratio:** Improves the clarity and quality of the received signal.
- **High Gain:** Concentrates the energy more effectively, increasing the power in the desired direction.
- **Reduced Power Wastage:** More efficient energy usage by directing power where it is needed.
- **Improved Performance:** Overall enhancement of the antenna's capabilities.

Disadvantages of Antenna Arrays

- **Increased Resistive Losses:** More elements can lead to higher resistive losses.
- **Mounting and Maintenance Challenges:** Arrays can be complex and harder to install and maintain.
- **Large Space Requirement:** Significant external space is needed to accommodate the array.

Applications of Antenna Arrays

- **Satellite Communications:** For focused and reliable communication with satellites.
- **Wireless Communications:** Improves signal strength and quality in wireless networks.
- **Military Radar Communications:** Essential for precise detection and tracking.
- **Astronomical Studies:** Used in radio astronomy to detect weak cosmic signals.

Antenna arrays enhance the performance of communication systems across various fields by leveraging the combined effects of multiple radiating elements.

Linear Antenna Arrays

- A **linear antenna array** is characterized by its elements being equally spaced along a straight line.
- If all elements are fed with currents of equal magnitude and have a progressive, uniform phase shift, the array is termed a **uniform linear array**. This type of configuration is common because it provides a predictable and controllable radiation pattern.

Control Parameters for Shaping Antenna Patterns

- In an array consisting of identical elements, several parameters can be adjusted to shape the overall radiation pattern of the antenna:

  1. **Geometrical Configuration:** The physical layout of the array can vary, including linear, circular, rectangular, spherical, and other shapes, affecting how the radiation pattern is formed.
  2. **Relative Displacement Between Elements:** The spacing between the individual elements impacts how the waves interfere with each other, influencing directivity and side lobes.
  3. **Excitation Amplitude:** The current's strength fed to each element can be varied to shape the radiation pattern and control the side lobe levels.
  4. **Excitation Phase:** The phase of the current fed to each element can be adjusted to steer the main beam in desired directions, allowing for beam steering and directionality.
  5. **Relative Pattern of Individual Elements:** The inherent radiation pattern of each element also contributes to the overall array pattern, affecting its performance.

Common Types of Antenna Arrays

- **Broadside Array (BSA):** In this configuration, the array is designed to radiate perpendicular to the axis of the array elements. This results in a broad beam directed outward from the sides of the array.
- **End-Fire Array (EFA):** Here, the array radiates along the axis of the elements, with the main beam pointing in the same direction as the alignment of the elements, typically towards the end of the array.

- **Collinear Array:** Elements are arranged in a straight line along the same axis, and the array radiates in a direction perpendicular to this line. This configuration is used to maximize gain in a specific direction.
- **Parasitic Array:** This type includes both driven and parasitic elements, where parasitic elements are not directly connected to the power source but are excited by mutual coupling. This configuration is commonly seen in Yagi-Uda antennas.

Broadside Array (BSA)

- **Definition and Arrangement:** A broadside array consists of multiple identical antennas arranged parallel to each other, with the direction of maximum radiation being perpendicular to the plane of the array elements. A typical broadside array configuration is illustrated in Figure below.

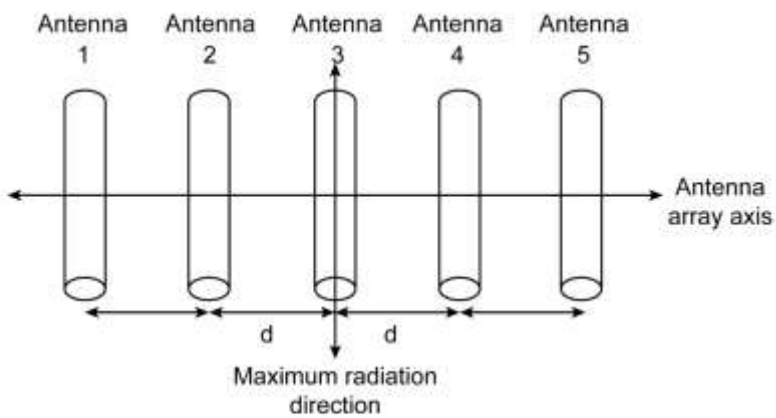


Figure 1: Broadside Array

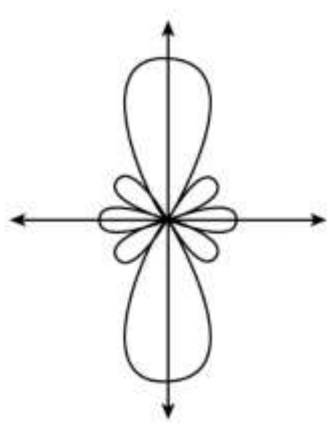
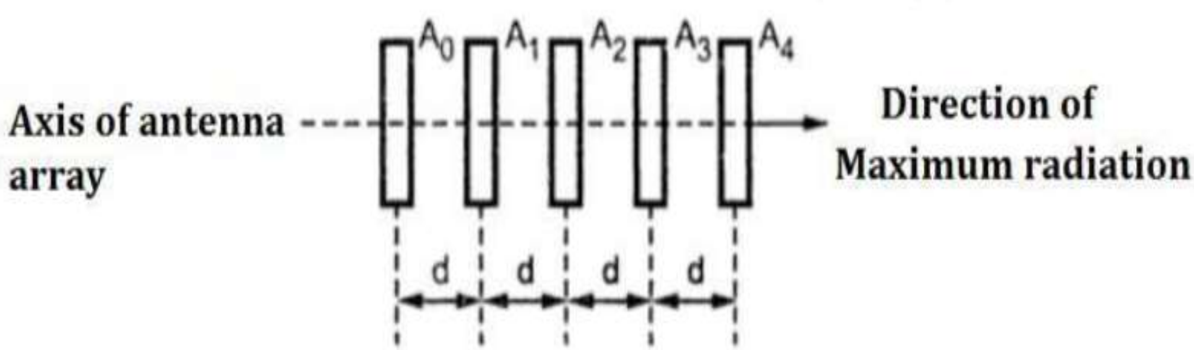


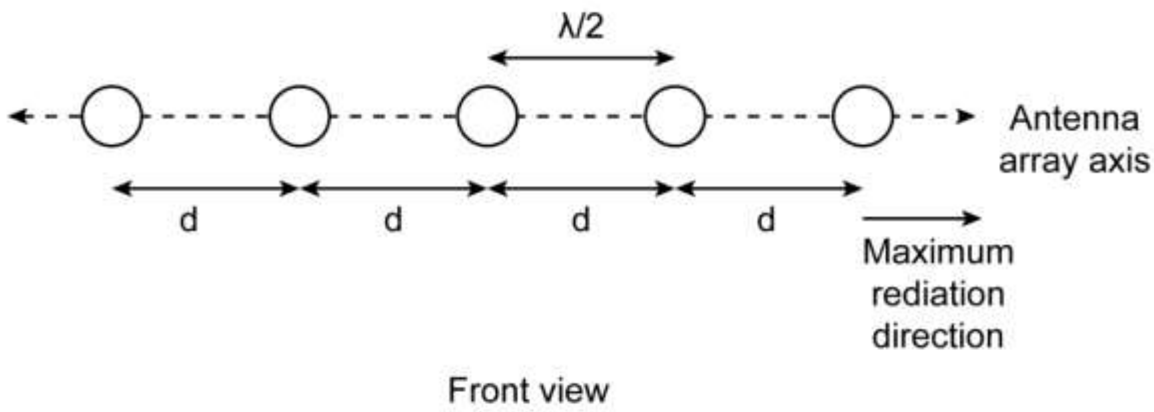
Figure 2: Radiation pattern of BSA

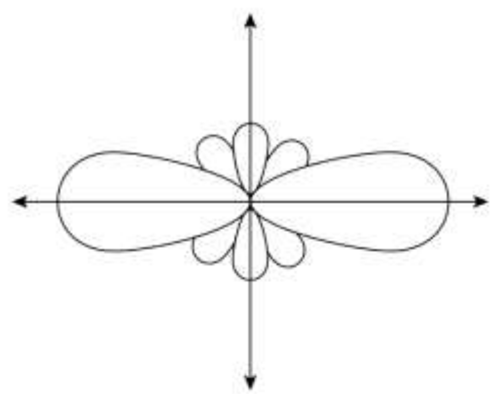
- **Element Placement:** The antennas are aligned along a straight line, which is called the axis of the antenna array. Each individual antenna element is positioned perpendicularly to this axis. All elements in the array are spaced equally along the axis, and the spacing between any two adjacent elements is denoted by *d*.
- **Feeding the Elements:** In a broadside array, all antenna elements are fed with currents that have equal magnitude and are in phase. This feeding configuration ensures that the maximum radiation is directed perpendicularly to the line of the array's axis.
- **Radiation Pattern:** The radiation pattern of a broadside array is bidirectional, meaning it radiates strongly in two opposite directions perpendicular to the axis of the array. This pattern is ideal for applications requiring radiation in a plane orthogonal to the array's length.

End-Fire Array (EFA)

- **Definition and Arrangement:** The end-fire array is similar to the broadside array regarding the arrangement of elements. However, the key difference lies in the direction of maximum radiation. In a broadside array, the maximum radiation is perpendicular to the axis of the array, whereas, in an end-fire array, the maximum radiation is along the axis of the array. This configuration is shown in Figure below.





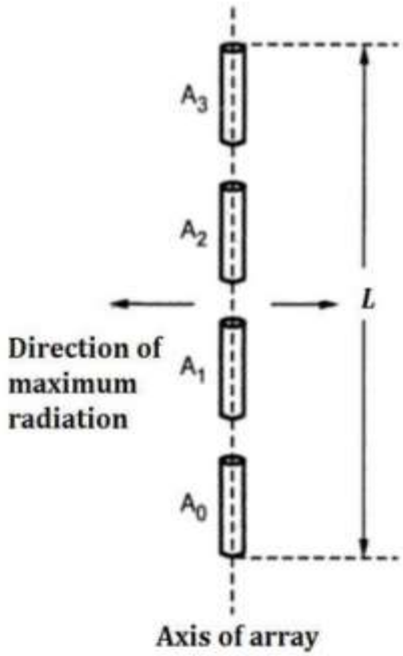


Radiation pattern

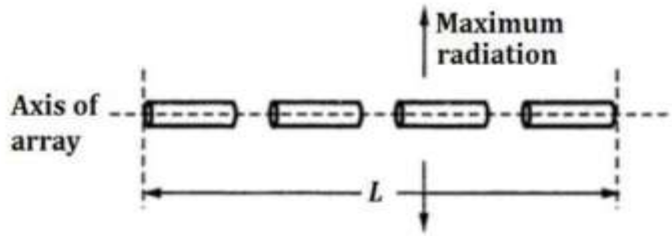
- **Element Placement:** The end-fire array consists of a number of identical antennas spaced equally along a straight line, similar to the broadside array. However, the design intention is to direct the radiation pattern along the line of the array.
- **Feeding the Elements:** Each antenna in the end-fire array is fed with currents of equal magnitude, but their phases vary progressively along the line. This phase variation is crucial for achieving unidirectional radiation, where the maximum radiation is directed along the axis of the array.
- **Radiation Pattern:** The end-fire array produces a unidirectional radiation pattern, meaning the radiation is focused along the axis of the array. This unidirectional nature makes it effective for applications requiring focused energy in a specific direction.

Collinear Array

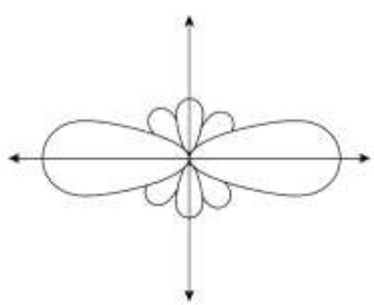
- **Definition and Arrangement:** As the name suggests, in a collinear array, antennas are arranged coaxially. This means the antennas are placed end-to-end along a single line, forming a straight-line array. This arrangement is shown in Figures (a) and (b).



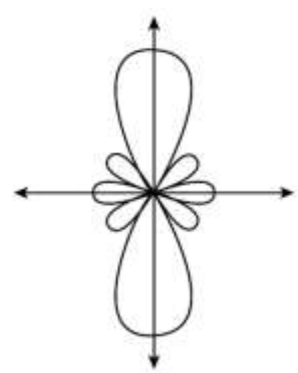
(a) Vertical



(b) Horizontal



Radiation pattern for (a)



Radiation pattern for (b)

- **Feeding the Elements:** The individual elements in a collinear array are fed with currents that are equal in both magnitude and phase. This feeding condition is similar to that of a broadside array.

- **Radiation Pattern:** In a collinear array, the direction of maximum radiation is perpendicular to the axis of the array. This orientation allows the array to achieve high gain in the desired direction, making it suitable for applications where broadside radiation patterns are needed.

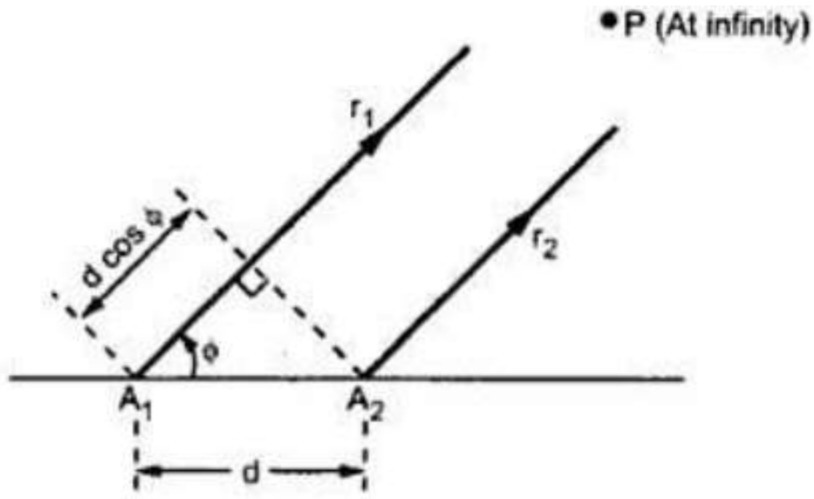
Array of Point Sources

- **Definition:** An array of point sources consists of isotropic radiators that occupy zero volume. These are idealized antennas that radiate equally in all directions. Analyzing a large number of point sources in an array is complex, so the simplest scenario involves two point sources. This basic case provides foundational insights that can be extended to arrays with more sources.
- **Basic Configuration:** The simplest array involves two isotropic point sources separated by a distance *d* and having the same polarization.

$$distance\ between\ elements = d, length = \frac{wavelength}{2}$$

- **Conditions for Analysis:** The behavior of the array can be studied under the following conditions:
    1. Two point sources with equal current magnitudes and the same phase.
    2. Two point sources with equal current magnitudes but opposite phases.
    3. Two point sources with unequal current magnitudes and any phase relationship.
- **Equal Magnitude and Phase:**
    - When two point sources are separated by a distance *d* and fed with currents of equal magnitude and phase, they create a specific radiation pattern.
    - Assuming far-field observations, the distance from a far point P to each source, *r1* and *r2*, can be approximated as

$$r1 \approx r2 \approx r$$



- The wave from point source 2 reaches *P* earlier than the wave from point source 1 due to the path difference. The radiation path difference is:

$$path\ difference = d\cos\phi$$

Path Difference:
The path difference between the waves from two point sources can be expressed in terms of the wavelength (λ) as:

$$Path\ difference = dcos\phi$$

Here, *d* is the distance between the two sources, and ϕ is the angle of observation.

Phase Difference:
The corresponding phase difference (ψ) can be derived as:

$$Phase\ difference\ \psi = \frac{2\pi}{\lambda} \times path\ difference = \frac{2\pi}{\lambda} \times dco\,s\,\phi = kd\cos\phi$$

Here, $k = \frac{2\pi}{\lambda}$ is the wave number.

Field Components from Point Sources:
Let the electric field component from point source 1 be represented as:

$$E_1 = E_0 e^{-\frac{j\psi}{2}}$$

Similarly, the electric field component from point source 2 is:

$$E_2 = E_0 e^{\frac{j\psi}{2}}$$

where E0 is the amplitude of the fields from each source.

phases of the two electric fields are expressed as +ψ/2 and −ψ/2 to account for the relative phase difference caused by the path difference between the waves from the two sources

This setup means that when we add the fields E1 and E2, the phase difference ψ between them reflects only the effect of the path difference.

Total Far-Field at Point P:
The total electric field (ET) at a distant point P due to both point sources is the sum of E1 and E2:

$$ET = E1 + E2 = E_0 e^{-\frac{j\psi}{2}} + E_0 e^{\frac{j\psi}{2}}$$

Simplifying this expression gives:

$$ET = E0\left(e^{-\frac{j\psi}{2}} + e^{\frac{j\psi}{2}}\right) = E_0 \times 2cos\left(\frac{\psi}{2}\right)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Thus

$$2\cos(\theta) = e^{j\theta} + e^{-j\theta}$$

Substituting ψ from Equation previously:

$$ET = 2E_0 cos\left(\frac{kdcos\phi}{2}\right)$$

This equation represents the total field intensity at point P due to two point sources with currents of the same amplitude and phase.

Normalized Pattern:

When the total amplitude of the field at point P is 2E0 and the phase shift is $\left(\frac{kdcos\phi}{2}\right)$, the pattern is said to be normalized by setting 2E0=1.

Maxima Direction

- From the previous Equations, the total electric field ET is maximum when the cosine function is maximized, i.e., $cos\left(\frac{kdcos\phi}{2}\right)$ is at its peak.

- The maximum value of the cosine function is ±1\pm1±1. Therefore, the condition for maxima is given by:

$$\frac{kdcos\phi}{2} = \pm n\pi, where\ n = 0,1,2, \ldots$$

- If n=0:

$$\frac{kdcos(\phi_{max})}{2} = 0$$

$$cos(\phi_{max}) = 0$$

$$\Longrightarrow\ \phi max = 90° \ or\ 270°$$

This shows that the direction of maximum radiation occurs at angles $90°$ and $270°$, which are perpendicular to the axis of the array.

Minima Direction

Condition for Minima:

From the equation for the total field (Equation 3.5), the field is minimum when the cosine term $cos(kd\ cos\ \varphi\ /\ 2)$ is minimum. The minimum value of a cosine function is 0. Therefore, the condition for minima is:

$$cos\left(\frac{kdcos\varphi}{2}\right) = 0$$

If the spacing between the two point sources is λ/2, we can substitute d = λ/2 and k=2π/λ into the equation:

$$cos\left(\frac{\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right)cos\varphi}{2}\right) = 0$$

Simplifying, we get:

$$cos(\frac{\pi}{2}\ cos\ \varphi) = 0$$

Solving for ϕ:

To find the angles ϕ where the minima occur, we take the inverse cosine of both sides:

$$\frac{\pi}{2}\ cos\ \varphi = \cos^{-}(0) = \pm(2n + 1)\frac{\pi}{2}\ ,\ \ where\ n = 0, 1, 2, \ldots$$

Dividing by π and taking the inverse cosine again:

$$cos\ \varphi\ =\ \pm(2n\ +\ 1)$$

For n = 0:

$$cos\ \varphi\ =\ \pm 1$$

This gives us the minima directions:

$$\varphi_{min}\ =\ 0°\ or\ 180°$$

Half-Power Point Directions

Condition for Half-Power Points:

A half-power point is the angle where the radiated power is half of the maximum power. This corresponds to the voltage or current being 1/√2 times the maximum value. Therefore, the condition for half-power points is:

$$cos\left(\frac{kdcos(\varphi_{HPPD})}{2}\right)\ =\ \pm\frac{1}{\sqrt{2}}$$

Spacing of λ/2:

Substituting d = λ/2 and k=$2\pi/\lambda$ into the equation:

$$cos\left(\frac{\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right)cos\varphi_{HPPD}}{2}\right)\ =\ \pm\frac{1}{\sqrt{2}}$$

Simplifying, we get:

$$cos(\frac{\pi}{2}\ cos\ \varphi_{HPPD})\ =\ \pm\frac{1}{\sqrt{2}}$$

Solving for ϕ:

$$\frac{\pi}{2}\ cos\ \varphi_{HPPD}\ = \cos^{-}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$\frac{\pi}{2}\ cos\ \varphi_{HPPD}\ = \pm\frac{\pi}{4}$$

$$cos\ \varphi_{HPPD} = \pm\frac{1}{2}$$

$$\cos\,\varphi_{HPPD} = \cos^{-}\frac{1}{2}$$

This gives us the half-power point directions:

$$\varphi_{HPPD}\ =\ \pm 60°\ or\ \pm 120°$$

Interpretation:

- When the spacing between the sources is λ/2, the half-power points occur at angles of ±60° and ±120° relative to the line connecting the sources.
- This means that the radiation pattern will have significant power levels at these angles.

Field Pattern:

The field pattern for a two-element array with λ/2 spacing is bidirectional, resembling a figure-of-eight. When rotated about the axis, it forms a three-dimensional doughnut-shaped pattern.

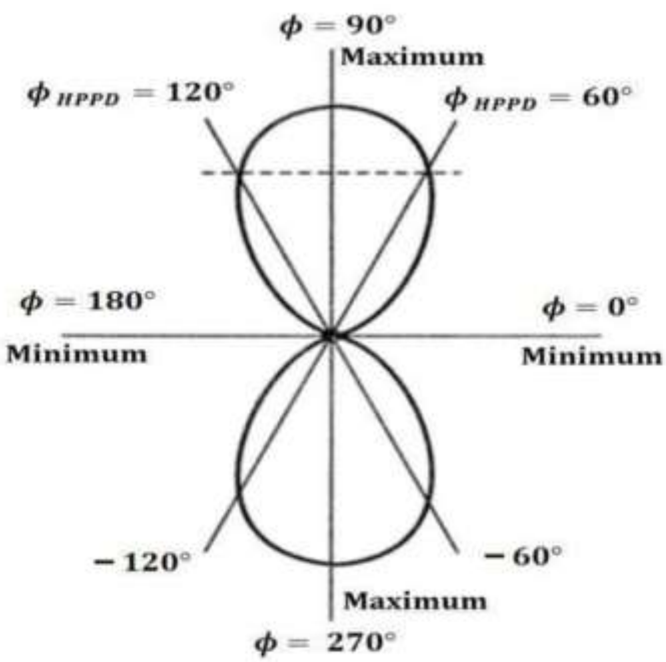
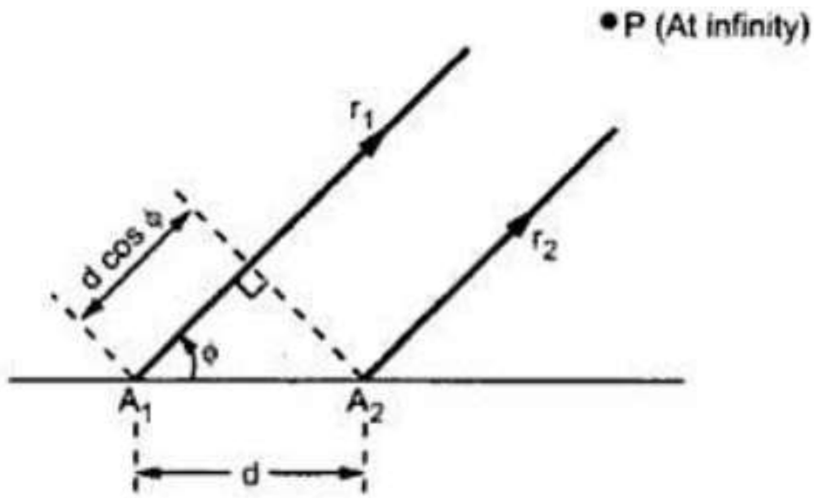


Fig. 3-5 Field pattern for two point source with $d = \lambda/2$ and fed with currents equal in magnitude and phase.

Two Point Sources with Currents Equal in Magnitude and Opposite Phase

Setup:

- Consider two point sources separated by distance 'd' and supplied with currents of equal magnitude and opposite phase (180° phase difference).



Total Field:

The total field at a far point P is given by:

$$E_T = -E1 + E2$$

Field Expressions:

Assuming equal magnitudes of currents, the fields at point P due to the point sources 1 and 2 can be written as:

$$E1 = E0 * e^{-\frac{j\psi}{2}}$$

$$E2 = E0 * e^{\frac{j\psi}{2}}$$

Total Field:

Substituting these expressions into the equation for ET:

$$E_T = -E0 * e^{-\frac{j\psi}{2}} + E0 * e^{\frac{j\psi}{2}}$$

$$= E0 * \left(e^{\frac{j\psi}{2}} - e^{-\frac{j\psi}{2}}\right)$$

$$= j2 * E0 * \left(\frac{e^{\frac{j\psi}{2}} - e^{-\frac{j\psi}{2}}}{j2}\right)$$

$$= j2 * E0 * sin\left(\frac{\psi}{2}\right)$$

$$sin\left(\frac{\psi}{2}\right) = \frac{e^{\frac{j\psi}{2}} - e^{-\frac{j\psi}{2}}}{j2}$$

Phase Angle:

As in the previous case, the phase angle ψ can be written as:

$$\psi = kd\ cos\ \varphi$$

Normalized Pattern:

Substituting the value of ψ into the equation for E_T:

$$E_T = j2E_0 * sin\left(\frac{kdcos\varphi}{2}\right)$$

By setting (j2E0) = 1, the pattern is normalized.

Interpretation:

- The radiation pattern for two point sources with equal magnitude and opposite phase is also bidirectional.
- However, the nulls and maxima occur at different angles compared to the in-phase case.
- The pattern is primarily determined by the sine function, resulting in a different shape compared to the cosine-based pattern in the in-phase case.

Maxima Direction:

- **Condition:** The total field is maximum when sin(kd cos ϕ / 2) is maximum.
- **Maximum Value:** The maximum value of the sine function is ±1.
- **Condition for Maxima:**

$$sin\left(\frac{kdcos\varphi}{2}\right) = \pm 1$$

If the spacing between the two point sources is λ/2, we can substitute d = λ/2 and $k = \frac{2\pi}{\lambda}$ into the equation:

$$sin\left(\frac{\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right)cos\varphi}{2}\right) = \pm 1$$

$$sin\left(\frac{\pi}{2}cos\varphi\right) = \pm 1$$

- **Solving for ϕ:**

$$\frac{\pi}{2}cos\ \varphi = \pm(2n + 1)\frac{\pi}{2} \qquad where\ n = 0,1,2, \dots ..$$

For n = 0:

$$\frac{\pi}{2}cos\ \varphi = \pm\frac{\pi}{2}$$

$$cos\ \varphi = \pm 1$$

$$\varphi_{max} = 0° \ or\ 180°$$

Minima Direction:

- **Condition:** The total field is minimum when **sin(kd cos ϕ / 2)** is minimum.
- **Minimum Value:** The minimum value of the sine function is 0.
- **Condition for Minima:**

$$sin\left(\frac{kdcos\varphi}{2}\right) = 0$$

If the spacing between the two point sources is λ/2, we can substitute d = λ/2 and k=$2\pi/\lambda$ into the equation:

$$sin\left(\frac{\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right)cos\varphi}{2}\right) = 0$$

$$sin\left(\frac{\pi}{2}cos\varphi\right) = 0$$

- **Solving for ϕ:**

$$\frac{\pi}{2}cos\varphi = \pm n\pi \quad where\ n = 0,1,2, \dots$$

For n = 0:

$$\frac{\pi}{2}cos\varphi = 0$$

$$\cos\varphi_{min} = 0$$

Therefore, the minimum directions are:

$$\varphi_{min} = 90° \; or \; 270°$$

Interpretation:

- **Maxima:** The maximum radiation occurs along the line connecting the two sources (0° and 180°).
- **Minima:** The radiation is minimum perpendicular to the line connecting the sources (90° and 270°).
- This pattern results in a single main lobe centered along the line connecting the sources, with nulls perpendicular to this line.

Half-Power Point Directions in a Two-Element Array (Opposite Phase)

Condition for Half-Power Points:

Similar to the previous case, the condition for half-power points is:

$$sin\left(\frac{kdcos\varphi}{2}\right) = \pm\frac{1}{\sqrt{2}}$$

Substituting d = λ/2 and k=$2\pi/\lambda$ into the equation:

$$sin(\frac{\pi}{2}\; cos\; \varphi) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\; cos\; \varphi = (2n + 1)\frac{\pi}{4} \qquad where\; n = 0,1,2, ...$$

For n = 0:

$$cos\; \varphi_{HPPD} = \pm\frac{1}{2}$$

This gives us the half-power point directions:

$$\varphi_{HPPD} = \pm 60° \; or \; \pm 120°$$

Interpretation:

- The half-power points occur at the same angles (±60° and ±120°) as in the case of equal phase.
- This indicates that the radiation pattern has a similar shape, but with nulls and maxima located differently due to the phase difference.

Field Pattern:

The field pattern for a two-element array with λ/2 spacing and opposite phase will have a single main lobe centered along the line connecting the sources, with nulls perpendicular to this line. The half-power points will be located at ±60° and ±120°, as calculated above.

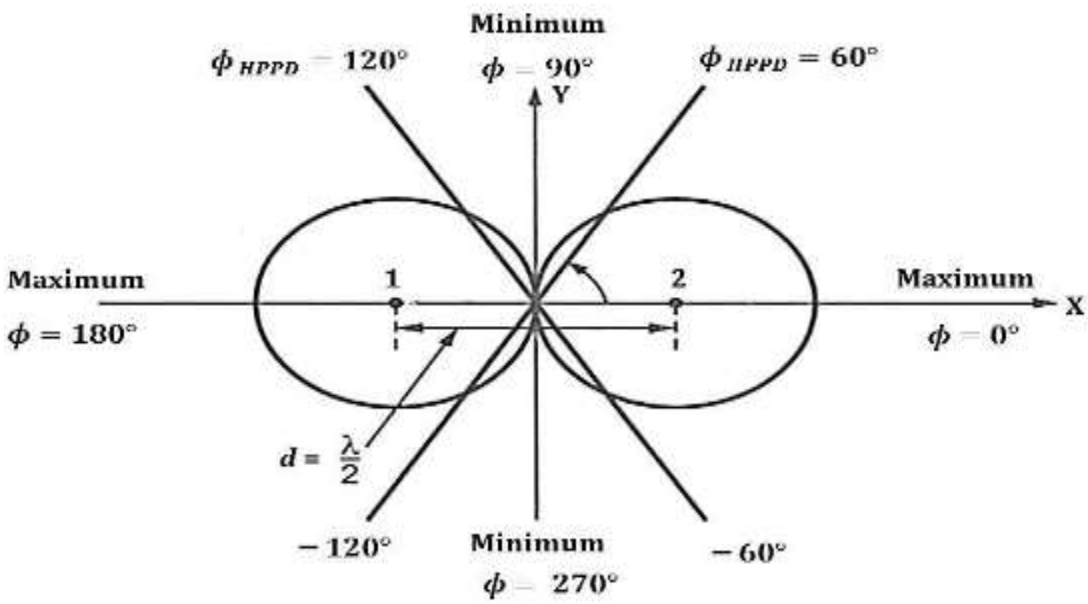


Fig. 3-6 Field pattern for two point source with $d = \lambda/2$ and fed with currents equal in magnitude and out of phase.

Antenna Feed Networks

Feed networks play a crucial role in antenna systems, especially in complex arrangements like antenna arrays. They ensure that the signal from the transmitter is efficiently delivered to each antenna element or, conversely, that the received signal from each element is correctly combined and delivered to the receiver. The design and implementation of feed networks significantly impact the performance, efficiency, and pattern characteristics of the antenna system.

What is a Feed Network?

- A **feed network** refers to the configuration of transmission lines, waveguides, or other components that distribute RF signals to and from antenna elements. It serves as the interface between the transmission source (e.g., transmitter or receiver) and the radiating elements of the antenna array.
- Feed networks can be simple or complex depending on the application. They might consist of a single feed line for a basic antenna or involve a sophisticated arrangement of multiple transmission lines, power dividers, and phase shifters in large antenna arrays.

Functions of Feed Networks

1. **Impedance Matching:**
    - One of the primary functions of a feed network is to match the impedance of the transmission line to the antenna. Impedance matching ensures maximum power transfer and minimizes reflections, which can lead to power loss and potential signal distortion.
2. **Signal Distribution:**
    - In antenna arrays, feed networks distribute the signal equally or in a controlled manner to each element. This distribution is crucial for forming the desired radiation pattern, controlling beam direction, and improving directivity.
3. **Phase Control:**
    - Feed networks can incorporate phase shifters to control the phase of the signal fed to each antenna element. By adjusting the phase, the feed network can steer the beam direction of the antenna array without physically moving the antennas, a process known as electronic beam steering.
4. **Power Division:**

- In multi-element arrays, feed networks must divide the input power appropriately among the elements. Power dividers are used to split the signal into multiple paths, ensuring each antenna element receives the correct amount of power to contribute to the overall radiation pattern.

5. **Isolation and Filtering:**

    - Feed networks may also include components that provide isolation between elements to prevent mutual coupling and reduce interference. Filters can be integrated to remove unwanted frequencies, ensuring that only the desired signal is transmitted or received.

Types of Feed Networks

1. **Corporate Feed Networks:**

    - Also known as a parallel feed network, the corporate feed distributes the signal from a single input to multiple outputs using a series of power dividers. This type of feed network offers high isolation between ports and equal path lengths, making it suitable for large, phased array antennas.

2. **Series Feed Networks:**

    - In a series feed, the signal passes through each antenna element sequentially. This arrangement simplifies the design but may introduce phase errors due to varying path lengths and higher insertion losses. Series feed networks are typically used in smaller arrays or where precise phase control is less critical.

3. **Hybrid Feed Networks:**

    - A combination of corporate and series feed techniques, hybrid feed networks balance the benefits of both types. They are designed to optimize power distribution, phase control, and minimize losses, making them suitable for medium-sized arrays with moderate complexity.

4. **Waveguide Feed Networks:**

    - For high-frequency applications, waveguides are used in feed networks. They offer low loss and high power-handling capability, making them ideal for microwave and millimeter-wave antennas. Waveguide feeds can be designed in various configurations, including parallel and series, similar to coaxial and microstrip feeds.

5. **Coaxial and Microstrip Feed Networks:**

- Coaxial cables and microstrip lines are common in low to medium-frequency applications. They offer flexibility in design and ease of integration with other electronic components. Microstrip feed networks are often used in printed antenna arrays due to their compact size and planar structure.

Components of Feed Networks

1. **Power Dividers and Combiners:**
   - Essential for distributing and combining signals in feed networks, power dividers split the input signal into multiple outputs with specific phase and amplitude characteristics. Power combiners perform the opposite function, merging signals from multiple elements into a single output.
2. **Phase Shifters:**
   - Phase shifters adjust the phase of the signal in each branch of the feed network. By controlling the phase, they enable electronic beam steering, allowing the antenna array to change its radiation pattern and direction dynamically.
3. **Transmission Lines:**
   - Transmission lines, such as coaxial cables, microstrip lines, and waveguides, are used to carry RF signals from the source to the antenna elements. The choice of transmission line depends on factors such as frequency, power handling, and physical layout constraints.
4. **Matching Networks:**
   - These networks are designed to match the impedance of the transmission line to the antenna input impedance. Matching networks use components like inductors, capacitors, and transformers to achieve the desired impedance transformation.

Quarter-Wave Section in Feed Networks

A **quarter-wave section** is a fundamental element in feed network design due to its impedance transformation properties. Its unique ability to match impedances makes it a versatile tool in achieving efficient power transfer and minimizing signal reflections.

What is a Quarter-Wave Section?

- A **quarter-wave section** is a segment of a transmission line that has an electrical length equal to one-quarter of the signal's wavelength (λ/4\lambda/4λ/4). It acts as an impedance transformer by using the properties of wave propagation along the line.

How Does a Quarter-Wave Section Work?

1. **Impedance Transformation Property:**
   - When a quarter-wave section is inserted between two impedances, it transforms the load impedance ZL to a different impedance Zin at the input according to the relation:

$$Zin = \frac{Z0^2}{ZL}$$

   where Z0 is the characteristic impedance of the quarter-wave section.

   - This property allows designers to match a high impedance to a low impedance or vice versa by carefully selecting the characteristic impedance Z0Z_0Z0 of the quarter-wave transformer.

2. **Reflection Minimization:**
   - Impedance mismatches in transmission lines cause reflections, leading to signal loss and potential interference. A quarter-wave section minimizes reflections by matching impedances, thereby ensuring that most of the signal power is delivered to the load or received from the antenna.

3. **Frequency Dependence:**
   - The effectiveness of a quarter-wave section is inherently frequency-dependent. It is designed to operate at a specific frequency where the length of the section equals one-quarter of the wavelength. Deviations from this frequency can reduce the impedance matching effectiveness.

Applications of Quarter-Wave Sections

1. **Antenna Feed Networks:**
   - In antenna arrays, quarter-wave sections are used to match the impedance between the feed line and the individual antenna elements. This ensures efficient radiation and reception by each element, optimizing the overall array performance.

2. **Filter Design:**
    - Quarter-wave sections are employed in RF filter design to create band-pass and band-stop filters. By using multiple quarter-wave sections, designers can achieve sharp cut-off frequencies and precise filtering characteristics.
3. **Baluns and Impedance Matching Networks:**
    - Quarter-wave transformers are often part of balun designs to match the balanced output of antennas to the unbalanced input of transmission lines. They are also used in general impedance-matching networks to interface different parts of RF systems.

Design Considerations

- **Choosing the Characteristic Impedance Z0:**

The characteristic impedance of the quarter-wave section should be chosen based on the impedances to be matched. An ideal Z0Z0 is the geometric mean of the source and load impedances:

$$Z_0 = \sqrt{Zin \times ZL}$$

Physical Length and Dielectric Material:

The physical length of the quarter-wave section must account for the dielectric constant of the material in which the transmission line is embedded. The effective wavelength in the medium is shorter than in free space, so the actual length of the quarter-wave section is calculated as:

$$l = \frac{\lambda_0}{4\epsilon r}$$

where λ0 is the free-space wavelength, and ϵr is the relative permittivity of the material.

Bandwidth and Frequency Range:

While quarter-wave sections are effective at their design frequency, they have a limited bandwidth. The impedance matching properties degrade as the operating frequency moves away from the design frequency, making them suitable for narrowband applications.

Aperture Type Antennas: Horn and Reflector Antennas

Aperture antennas are a category of antennas that radiate or receive electromagnetic waves through an opening or aperture. These antennas are widely used in microwave and satellite communication systems due to their high gain, directivity, and efficiency. The two most common types of aperture antennas are **horn antennas** and **reflector antennas**. Both play crucial roles in various applications, including radar, radio telescopes, and communication links.

1. Horn Antennas

Horn antennas are a type of aperture antenna that uses a flaring metal waveguide structure to direct radio waves in a specific direction. The structure's shape resembles a horn, hence the name. These antennas are commonly used in microwave frequency applications due to their simple design, ease of fabrication, and high directivity.

Types of Horn Antennas

1. **Rectangular Horn Antennas:**
    - These have a rectangular aperture, and their waveguide feed is also rectangular.
    - Commonly used in applications where linear polarization is required.
2. **Pyramidal Horn Antennas:**
    - A special type of rectangular horn where the aperture flares out in both the E-plane and H-plane.
    - It offers better gain and directivity compared to a simple rectangular horn.
3. **Circular Horn Antennas:**
    - These have a circular cross-section and are fed by a circular waveguide.
    - They are used in applications where circular polarization or a symmetrical radiation pattern is desired.
4. **Conical Horn Antennas:**
    - The aperture of the horn flares out in a conical shape.
    - Suitable for applications requiring broad bandwidth and circular polarization.

Design and Working Principle of Horn Antennas

- **Waveguide Feeding:** Horn antennas are typically fed by a waveguide. The waveguide acts as a transmission line that guides the electromagnetic wave to the horn's throat. The wave gradually transitions from the waveguide mode to a radiating mode as it moves through the horn.
- **Flare Angle and Aperture Size:** The horn's flare angle and aperture size determine the radiation pattern and impedance matching characteristics. A larger aperture provides higher directivity and gain but also makes the horn more directional.
- **Impedance Matching:** The horn's gradual flare reduces impedance mismatch between the waveguide and free space. This reduces reflection and increases the efficiency of power transmission.

Performance Characteristics of Horn Antennas

1. **Directivity and Gain:**
    - The directivity of a horn antenna is a measure of how concentrated the radiation pattern is in a particular direction. For a rectangular horn, the directivity D can be approximated by: $D = \frac{4\pi A}{\lambda^2}$ where A is the area of the aperture, and λ is the wavelength of the signal.
    - The gain G of the horn is related to its directivity by the efficiency factor η:

      $$G = \eta D$$

      Typically, η ranges from 0.5 to 0.8.
2. **Beamwidth:**
    - The beamwidth of a horn antenna is the angular width of the main lobe of the radiation pattern. It is inversely proportional to the aperture dimensions:

      $$Beamwidth \approx \frac{70\lambda}{d}$$

      where d is the larger dimension of the aperture.
3. **Radiation Pattern:**
    - Horn antennas have a directional radiation pattern, with most energy concentrated along the axis of the horn. The shape of the horn and the size of the aperture determine the exact pattern.

4. **Bandwidth:**
   - Horn antennas generally have wide bandwidth. The bandwidth depends on the horn's shape and size, but they typically operate effectively over a range of frequencies.
5. **Polarization:**
   - The polarization of the horn antenna is determined by the feed structure. Linear polarization is common, but circular and elliptical polarizations can be achieved using appropriate feed structures.

- **Aperture Efficiency ηa:**

$$\eta a = \frac{Effective\ aperture\ area}{Physical\ aperture\ area}$$

   - Indicates how effectively the physical aperture area is utilized for radiation.

- **Directivity for Pyramidal Horn:**

$$D = \frac{32\pi^2 ab}{\lambda^2}$$

where a and b are the horn's aperture dimensions along the E-plane and H-plane, respectively.

2. Reflector Antennas

Reflector antennas are another type of aperture antenna that use a reflective surface to direct radio waves. The most common reflector antennas are parabolic reflectors, which have a parabolic-shaped reflecting surface. They focus electromagnetic waves into a narrow beam or, conversely, collect incoming waves to a focal point.

Types of Reflector Antennas

1. **Parabolic Reflector Antennas:**
   - The most widely used reflector antenna type. The parabolic shape ensures that all incoming parallel rays (from a distant source) are reflected to a common focal point. Conversely, a source placed at the focal point radiates a collimated beam.
2. **Cassegrain Reflector Antennas:**
   - This configuration uses a main parabolic reflector and a secondary hyperbolic reflector. The primary reflector focuses waves onto the secondary reflector, which then directs them to a feed located near the main reflector's vertex.

Cassegrain antennas are used in satellite communication and deep space network antennas due to their compact feed design.

3. **Gregorian Reflector Antennas:**
   - Similar to the Cassegrain, but uses an elliptical secondary reflector to achieve certain beam-shaping characteristics. It offers improved performance in terms of reduced side lobes and aberrations.
4. **Offset Parabolic Reflector:**
   - In this design, the feed is placed off-axis to prevent feed blockage and shadowing. This configuration improves the antenna's efficiency and is often used in satellite TV dishes.

Design and Working Principle of Reflector Antennas

- **Focusing Property of Parabolas:** Reflector antennas rely on the geometric property of parabolas, where all rays originating from the focus are reflected parallel to the parabolic axis, forming a highly directional beam.
- **Feed Mechanism:** The feed, typically a horn or dipole, is positioned at the focal point of the parabola. The reflector directs the waves radiated by the feed into a collimated beam, achieving high gain and directivity.
- **Reflector Shape and Size:** The shape and size of the reflector determine the beamwidth, gain, and overall performance of the antenna. A larger reflector size results in a narrower beamwidth and higher directivity.

Performance Characteristics of Reflector Antennas

1. **Gain and Directivity:**
   - Reflector antennas provide very high gain and directivity, making them suitable for long-distance communication. The gain G of a parabolic reflector is given by:

$$G = \eta \left(\frac{\pi D}{\lambda}\right)$$

   where D is the diameter of the reflector, λ is the wavelength, and η is the efficiency factor.
2. **Beamwidth:**
   - The half-power beamwidth (HPBW) of a parabolic reflector is approximately given by:

$$HPBW \approx \frac{70\lambda}{D}$$

A larger diameter D results in a narrower beam.

3. **Side Lobes and Back Lobes:**
   - Reflector antennas have side lobes, which are smaller lobes of radiation off the main beam direction. The design aims to minimize side lobes to reduce interference. Back lobes (radiation in the opposite direction) are also minimized.
4. **Polarization:**
   - The polarization of a reflector antenna is determined by the feed. It can be linear, circular, or elliptical depending on the application.
5. **Aperture Efficiency:**
   - Reflector antennas have high aperture efficiency, typically between 50% and 80%, depending on the feed illumination and reflector surface accuracy.
6. **Bandwidth:**
   - Reflector antennas can operate over a wide frequency range. However, practical bandwidth is limited by the feed and reflector shape, which must maintain the same focal properties over the desired frequency range.

- **Effective Aperture Aeff:**

$$A_{eff} = \eta_a \times Aphysical$$

where ηa is the aperture efficiency, and $Aphysical$ is the physical area of the reflector.

Questions (Horn Antenna & Reflector Antenna)

Question 1

Given: A rectangular horn antenna with dimensions of 20 cm by 10 cm, operating at a frequency of 10 GHz.

Calculate:

- *The aperture area of the antenna in square meters.*
- *The wavelength of the signal in meters.*
- *The directivity of the antenna.*

- *Assuming an efficiency factor of 0.7, the gain of the antenna in decibels.*

Step 1: Calculate the Aperture Area

The aperture area A of a rectangular horn antenna can be calculated using the formula:

$$A = \text{length} \times \text{width}$$

Given the dimensions:

- Length = 20 cm = 0.2 m
- Width = 10 cm = 0.1 m

Now substituting the values:

$$A = 0.2\,\text{m} \times 0.1\,\text{m} = 0.02\,\text{m}^2$$

Step 2: Calculate the Wavelength

The wavelength λ can be calculated using the formula:

$$\lambda = \frac{c}{f}$$

where:

- c is the speed of light (3×10^8 m/s),
- f is the frequency in Hz.

Given that the frequency is 10 GHz:

$$f = 10 \times 10^9 \text{ Hz}$$

Now substituting the values:

$$\lambda = \frac{3 \times 10^8}{10 \times 10^9} = \frac{3}{10} \times 10^{-1} = 0.03\,\text{m (or 3 cm)}$$

Step 3: Calculate the Directivity

The directivity D of a rectangular horn antenna can be approximated using the formula:

$$D \approx \frac{4\pi A}{\lambda^2}$$

Substituting the values we calculated:

$$D \approx \frac{4\pi(0.02)}{(0.03)^2} = \frac{4\pi(0.02)}{0.0009}$$

Calculating the numerical values:

$$D \approx \frac{0.2513}{0.0009} \approx 279.25$$

Step 4: Calculate the Gain

The gain G of the antenna can be calculated using the formula:

$$G = \eta D$$

where η is the efficiency factor. Given that $\eta = 0.7$:

$$G = 0.7 \times 279.25 \approx 195.475$$

To convert gain from a linear scale to decibels (dB):

$$G_{\text{dB}} = 10 \log_{10}(G)$$

Calculating the gain in dB:

$$G_{\text{dB}} = 10 \log_{10}(195.475) \approx 10 \times 2.291 = 22.91 \text{ dB}$$

Question 2

Given: A horn antenna with a larger dimension of 30 cm, operating at a frequency of 5 GHz.

Calculate:

- *The wavelength of the signal in meters.*
- *The approximate beamwidth of the antenna in degrees.*

Step 1: Calculate the Wavelength

The wavelength (λ) can be calculated using the formula:

$$\lambda = \frac{c}{f}$$

where:

- c is the speed of light (3×10^8 m/s),
- f is the frequency in Hz.

Given that the frequency is 5 GHz:

$$f = 5 \times 10^9 \text{ Hz}$$

Now substituting the values:

$$\lambda = \frac{3 \times 10^8}{5 \times 10^9} = \frac{3}{5} \times 10^{-1} = 0.06 \text{ meters (or 6 cm)}$$

Step 2: Calculate the Beamwidth

The approximate beamwidth can be calculated using the formula:

$$\text{Beamwidth} \approx \frac{70\lambda}{d}$$

where d is the larger dimension of the aperture.

Given that the larger dimension $d = 30$ cm $= 0.3$ m:

$$\text{Beamwidth} \approx \frac{70 \times 0.06}{0.3} = \frac{4.2}{0.3} = 14 \text{ degrees}$$

Question 1

Given: *A parabolic reflector antenna with a diameter of 5 meters, operating at a frequency of 2 GHz.*

Calculate:

1. *The wavelength of the signal.*
2. *The half-power beamwidth (HPBW) of the antenna in degrees.*

Step 1: Calculate the Wavelength

The wavelength (λ) can be calculated using the formula:

$$\lambda = \frac{c}{f}$$

where:

- c is the speed of light (3×10^8 m/s),
- f is the frequency in Hz.

Given that the frequency is 2 GHz:

$$f = 2 \times 10^9 \text{ Hz}$$

Now substituting the values:

$$\lambda = \frac{3 \times 10^8}{2 \times 10^9} = \frac{3}{2} \times 10^{-1} = 0.15 \text{ meters (or 15 cm)}$$

Step 2: Calculate the Half-Power Beamwidth (HPBW)

The HPBW can be approximated using the formula:

$$\text{HPBW} \approx \frac{70\lambda}{D}$$

where D is the diameter of the antenna.

Given that the diameter $D = 5$ meters, we can substitute the values:

$$\text{HPBW} \approx \frac{70 \times 0.15}{5} = \frac{10.5}{5} = 2.1 \text{ degrees}$$

Question 4

Given: *A parabolic reflector antenna with a diameter of 8 meters, operating at a frequency of 8 GHz. Assume an aperture efficiency of 75%.*

Calculate:

1. *The physical area of the reflector.*
2. *The effective aperture of the antenna.*

Step 1: Calculate the Physical Area of the Reflector

The physical area A of a parabolic reflector (circular) is calculated using the formula:

$$A = \frac{\pi D^2}{4}$$

where:

- D is the diameter of the reflector.

Given that the diameter $D = 8$ meters:

$$A = \frac{\pi(8^2)}{4} = \frac{\pi \cdot 64}{4} = 16\pi \approx 50.27\,\mathrm{m}^2$$

Step 2: Calculate the Effective Aperture

The effective aperture A_e can be calculated using the formula:

$$A_e = \eta A$$

where η is the aperture efficiency. Given that the aperture efficiency $\eta = 0.75$:

$$A_e = 0.75 \times A = 0.75 \times 50.27 \approx 37.70\,\mathrm{m}^2$$

Question 3

Given: *A parabolic reflector antenna with a diameter of 10 meters, operating at a frequency of 12 GHz. Assume an aperture efficiency of 60%.*

Calculate:

1. *The wavelength of the signal.*
2. *The gain of the antenna in decibels.*
3. *The directivity of the antenna.*

Step 1: Calculate the Wavelength

The wavelength (λ) can be calculated using the formula:

$$\lambda = \frac{c}{f}$$

where:

- c is the speed of light (3×10^8 m/s),
- f is the frequency in Hz.

Given that the frequency is 12 GHz:

$$f = 12 \times 10^9 \text{ Hz}$$

Now substituting the values:

$$\lambda = \frac{3 \times 10^8}{12 \times 10^9} = \frac{3}{12} \times 10^{-1} = 0.025 \text{ meters (or 25 mm)}$$

Step 2: Calculate the Directivity

The directivity (D) of a parabolic reflector can be calculated using the formula:

$$D \approx \frac{4\pi A}{\lambda^2}$$

where A is the area of the aperture. The area A for a circular aperture is given by:

$$A = \frac{\pi D^2}{4}$$

Given the diameter $D = 10$ meters:

$$A = \frac{\pi(10^2)}{4} = \frac{100\pi}{4} = 25\pi \approx 78.54 \text{ m}^2$$

Now, substituting into the directivity formula:

$$D \approx \frac{4\pi(25\pi)}{(0.025)^2} = \frac{100\pi^2}{0.000625} \approx \frac{100 \times 9.87}{0.000625} \approx 1583200$$

Step 3: Calculate the Gain

The gain (G) of the antenna can be calculated using the directivity and the aperture efficiency (η):

$$G = \eta D$$

Given the aperture efficiency $\eta = 0.6$:

$$G = 0.6 \times 1583200 \approx 949920$$

Step 4: Convert Gain to Decibels

To convert the gain from a linear scale to decibels (dB), use the formula:

$$G_{\mathrm{dB}} = 10 \log_{10}(G)$$

Calculating the gain in dB:

$$G_{\mathrm{dB}} = 10 \log_{10}(949920) \approx 10 \times 5.978 = 59.78 \mathrm{~dB}$$

Electromagnetic Waves

Introduction:

Electromagnetic waves are a form of energy that can propagate through both matter and the vacuum of space, unlike mechanical waves, which require a medium (like air or water) to travel. These waves are formed by the interplay of electric and magnetic fields, which are intrinsically linked: a changing magnetic field induces a changing electric field, and vice versa. This interaction creates electromagnetic waves.

Properties of Electromagnetic Waves:

- **No Medium Required:** Electromagnetic waves can travel through a vacuum, meaning they can move through empty space without needing a physical medium.
- **Linked Electric and Magnetic Fields:** The wave is formed when changing magnetic fields generate electric fields and changing electric fields generate magnetic fields. This mutual induction allows the wave to propagate.

Historical Development:

- **James Clerk Maxwell:** In the 1860s and 1870s, the Scottish scientist James Clerk Maxwell developed a comprehensive theory to describe electromagnetic waves. He demonstrated that electric and magnetic fields can couple together to form these waves. Maxwell summarized the principles of electromagnetism into a set of four equations, now famously known as **Maxwell's Equations**. These equations form the foundation of classical electromagnetism, optics, and electric circuits, describing how electric and magnetic fields are generated and altered by each other and by charges and currents.
- **Heinrich Hertz:** Building on Maxwell's theoretical work, German physicist Heinrich Hertz experimentally demonstrated the existence of electromagnetic waves. Hertz's experiments in the late 19th century led to the production and detection of radio waves, a type of electromagnetic wave. His findings confirmed Maxwell's theory by showing that radio waves travel at the speed of light, proving that radio waves are a form of light.

Key Discoveries:

1. **Speed of Electromagnetic Waves:** Hertz's experiments showed that the speed of radio waves is identical to the speed of light. This discovery confirmed that radio waves and light waves are both types of electromagnetic waves.
2. **Electromagnetic Waves as Free Fields:** Hertz also demonstrated that electromagnetic waves can exist independently of wires. By creating electric and magnetic fields that could detach from their source, he showed that these fields could propagate through space as electromagnetic waves, further validating Maxwell's predictions.

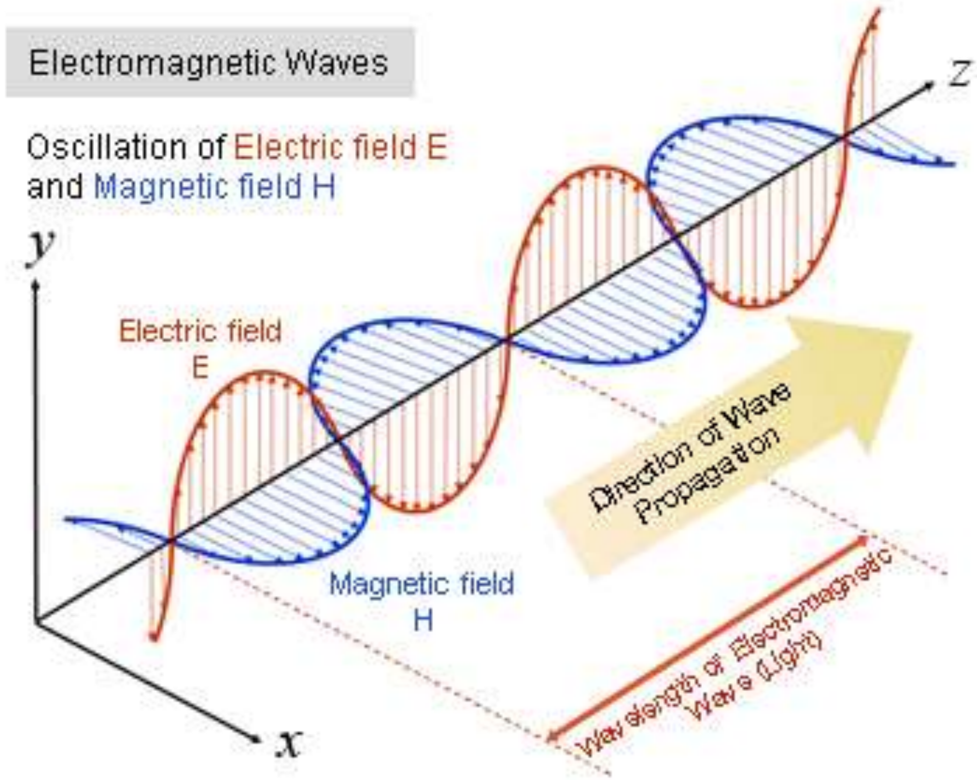
- A spatially-varying electric field generates a time-varying magnetic field, and vice versa. This interdependence is the essence of Maxwell's equations.

Oscillating Fields:

- An oscillating electric field generates an oscillating magnetic field.
- In turn, the oscillating magnetic field generates an oscillating electric field.
- This continuous interaction results in the propagation of electromagnetic waves.

Formation of Electromagnetic Waves:

- The mutual generation of oscillating electric and magnetic fields leads to the creation of electromagnetic waves. These waves consist of electric and magnetic fields oscillating perpendicular to each other and the direction of wave propagation.



Speed of EM waves

- In the studies of electricity and magnetism, experimental physicists had determined two physical constants - the electric (ε0) and magnetic (μ0) constant in vacuum.
- These two constants appeared in the EM wave equations, and Maxwell was able to calculate the velocity of the wave (i.e., the speed of light) in terms of the two constants:

$$c = \frac{1}{\sqrt{\varepsilon 0 \mu 0}} \approx 3.0 \times 10^{8}\, m/s$$

$$\varepsilon 0 = 8.8542 \times 10^{-12} C^2 s^2/kgm^3\ (permittivity\ of\ vacuum)$$

$$\mu 0 = 4\pi \times 10^{-7}\, kgm/A^2 s^2\ (permeability\ of\ vacuum)$$

Therefore, the three experimental constants, ε0, μ0, and c, previously thought to be independent are now related in a fixed and determined way.

Electromagnetic Wave Equations

The cross product $\vec{E} \times \vec{B}$ always gives the direction of travel of the wave.

The cross products of the Cartesian unit vectors a_x, a_y & a_z follow a specific pattern in a right-handed coordinate system.

In a right-handed Cartesian coordinate system, these unit vectors obey:

1. **Basic Rule:**

$$\hat{a}_x \times \hat{a}_y = \hat{a}_z$$

2. **Cyclic Permutations:** The pattern follows cyclic permutations, meaning if we shift each unit vector "one step forward," the same cross product rule applies:

$$\hat{a}_y \times \hat{a}_z = \hat{a}_x$$

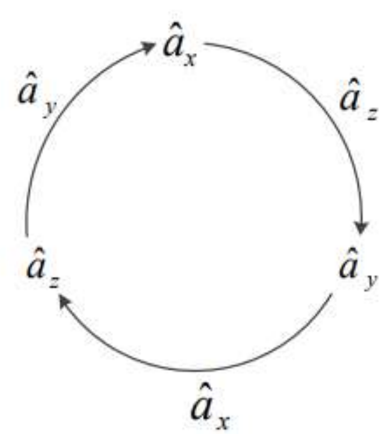
$$\hat{a}_z \times \hat{a}_x = \hat{a}_y$$

3. **Anti-Cyclic or Negative Permutations:** When the order of multiplication is reversed, we get the negative direction (since reversing the order in the cross product introduces a negative sign):

$$\hat{a}_y \times \hat{a}_x = -\hat{a}_z$$

$$\hat{a}_z \times \hat{a}_y = -\hat{a}_x$$

$$\hat{a}_x \times \hat{a}_z = -\hat{a}_y$$



Summary of Cross Products in a Right-Handed System

Putting it all together, the mutual cross products of the Cartesian unit vectors are:

Cross Product	Result
$\hat{a}_x \times \hat{a}_y$	$\hat{a}_z$
$\hat{a}_y \times \hat{a}_z$	$\hat{a}_x$
$\hat{a}_z \times \hat{a}_x$	$\hat{a}_y$
$\hat{a}_y \times \hat{a}_x$	$-\hat{a}_z$
$\hat{a}_z \times \hat{a}_y$	$-\hat{a}_x$
$\hat{a}_x \times \hat{a}_z$	$-\hat{a}_y$

Assume that the EM wave is traveling in the positive direction of an x-axis, that the electric field is oscillating parallel to the y-axis, and that the magnetic field is oscillating parallel to the z-axis:

$$E = E_0 \sin(kx - \omega t)\, a_y$$

$$B = B_0 \sin(kx - \omega t)\, a_z$$

E0 = amplitude of the electric field, B0 = amplitude of the magnetic field, ω = angular frequency of the wave, k = angular wave number of the wave

The angular frequency of the wave and angular wave number of the wave are given as follows.

$$\omega = 2\pi f \ \ rads \qquad k = \frac{2\pi}{\lambda}$$

At any specified time and place: E/B=E0/B0=c=speed of electromagnetic wave

Question 1

An electromagnetic wave travels along the x-axis with a frequency $f = 6 \times 10^{14}\,\mathrm{Hz}$ and wavelength $\lambda = 500\,\mathrm{nm}$. The wave's angular frequency is ω, the wave number is k, and the speed of light is $c = 3 \times 10^8\,\mathrm{m/s}$.

(a) Calculate the wave number k of the wave.

(b) Calculate the angular frequency ω of the wave.

Given:

- Frequency $f = 6 \times 10^{14}\,\mathrm{Hz}$
- Wavelength $\lambda = 500\,\mathrm{nm} = 500 \times 10^{-9}\,\mathrm{m}$
- Speed of light $c = 3 \times 10^{8}\,\mathrm{m/s}$

(a) Calculate the wave number k.

The wave number k is given by:

$$k = \frac{2\pi}{\lambda}$$

Substituting $\lambda = 500 \times 10^{-9}\,\mathrm{m}$:

$$k = \frac{2\pi}{500 \times 10^{-9}} = \frac{2 \times 3.14159}{500 \times 10^{-9}} \approx 1.2566 \times 10^{7}\,\mathrm{m}^{-1}$$

(b) Calculate the angular frequency ω.

The angular frequency ω is given by:

$$\omega = 2\pi f$$

Substituting $f = 6 \times 10^{14}\,\mathrm{Hz}$:

$$\omega = 2 \times 3.14159 \times 6 \times 10^{14} \approx 3.7699 \times 10^{15}\,\mathrm{rad/s}$$

Question 2

In an electromagnetic wave traveling along the x-axis, the electric field oscillates parallel to the y-axis, given by $\vec{E} = E_0 \sin(kx - \omega t)\hat{a}_y$, and the magnetic field oscillates parallel to the z-axis, given by $\vec{B} = B_0 \sin(kx - \omega t)\hat{a}_z$. The amplitude of the electric field is $E_0 = 300\,\mathrm{V/m}$.

(a) Calculate the corresponding magnetic field amplitude B_0.

(b) Write the full expressions for $\vec{E}$ and $\vec{B}$ as functions of x and t using the values of E_0, B_0, and $\omega = 2\pi f$.

Given:

- $E_0 = 300\,\mathrm{V/m}$
- Wave traveling in the x-axis
- $\vec{E} = E_0 \sin(kx - \omega t)\hat{a}_y$
- $\vec{B} = B_0 \sin(kx - \omega t)\hat{a}_z$
- Frequency $f = 6 \times 10^{14}\,\mathrm{Hz}$

(a) Calculate the corresponding magnetic field amplitude B_0.

Since $E_0 = cB_0$, we have:

$$B_0 = \frac{E_0}{c} = \frac{300}{3 \times 10^8} = 1 \times 10^{-6}\,\mathrm{T} = 1\,\mu\mathrm{T}$$

(b) Write the full expressions for $\vec{E}$ and $\vec{B}$ as functions of x and t.

Using $\omega = 3.7699 \times 10^{15}\,\mathrm{rad/s}$ and $k = 1.2566 \times 10^7\,\mathrm{m}^{-1}$:

$$\vec{E}(x,t) = 300 \sin(1.2566 \times 10^7\, x - 3.7699 \times 10^{15}\, t)\hat{a}_y\,\mathrm{V/m}$$

$$\vec{B}(x,t) = 1 \times 10^{-6} \sin(1.2566 \times 10^7\, x - 3.7699 \times 10^{15}\, t)\hat{a}_z\,\mathrm{T}$$

Electromagnetic waves and Energy transmission

Electromagnetic wave represents the transmission of energy.

In an electromagnetic wave, the energy transmitted is divided equally between the electric and magnetic fields.

1. **Electric Field Energy Density**: The energy density u_E associated with the electric field E in free space is given by:

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

where:

- ϵ_0 is the permittivity of free space ($\approx 8.854 \times 10^{-12}\,\mathrm{F/m}$),
- E is the magnitude of the electric field.

2. **Magnetic Field Energy Density**: The energy density u_B associated with the magnetic field B in free space is given by:

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

where:

- μ_0 is the permeability of free space ($\approx 4\pi \times 10^{-7}\,\mathrm{N/A^2}$),
- B is the magnitude of the magnetic field.

3. **Total Energy Density**: The total energy density u of the electromagnetic wave is the sum of the energy densities from both the electric and magnetic fields:

$$u = u_E + u_B$$

Since the electric and magnetic field energy densities are equal in an electromagnetic wave ($u_E = u_B$), we have:

$$u \;=\; 2\,u_E \;=\; 2\,u_B$$

Thus, the total energy density, u, is twice the energy density of either the electric or magnetic component alone. This balance between u_E and u_B is a key characteristic of energy transmission in electromagnetic waves, where the energy is equally split between the oscillating electric and magnetic fields.

Q1

An electromagnetic wave has an electric field with a peak amplitude of $E_0 = 100\,\mathrm{V/m}$. Given the permittivity of free space $\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$ and the permeability of free space $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$:

(a) Calculate the energy density u_E associated with the electric field.

(b) Find the corresponding magnetic field amplitude B_0.

(c) Calculate the energy density u_B associated with the magnetic field.

(d) Determine the total energy density u and verify that $u = 2u_E = 2u_B$.

(a) Calculate the energy density u_E associated with the electric field.

$$u_E = \frac{1}{2}\epsilon_0 E_0^2$$

Substituting the values:

$$u_E = \frac{1}{2} \times 8.854 \times 10^{-12}\,\mathrm{F/m} \times (100)^2\,\mathrm{V/m^2}$$

$$u_E = \frac{1}{2} \times 8.854 \times 10^{-12} \times 10^4$$

$$u_E \approx \frac{1}{2} \times 8.854 \times 10^{-8} \approx 4.427 \times 10^{-8}\,\mathrm{J/m^3}$$

(b) Find the corresponding magnetic field amplitude B_0.

Using the relationship $E_0 = cB_0$, we can rearrange it to find B_0:

$$B_0 = \frac{E_0}{c}$$

Substituting $c = 3 \times 10^8\,\mathrm{m/s}$:

$$B_0 = \frac{100}{3 \times 10^8} \approx 3.33 \times 10^{-7}\,\mathrm{T}$$

(c) Calculate the energy density u_B associated with the magnetic field.

$$u_B = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

Substituting the values:

$$u_B = \frac{1}{2}\frac{(3.33 \times 10^{-7})^2}{4\pi \times 10^{-7}}$$

Calculating $(3.33 \times 10^{-7})^2$:

$$(3.33 \times 10^{-7})^2 \approx 1.1089 \times 10^{-13}$$

Now, substituting into the equation for u_B:

$$u_B = \frac{1}{2}\frac{1.1089 \times 10^{-13}}{4\pi \times 10^{-7}} \approx \frac{1.1089 \times 10^{-13}}{2.5133 \times 10^{-6}} \approx 4.41 \times 10^{-8}\,\mathrm{J/m^3}$$

(d) Determine the total energy density u and verify that $u = 2u_E = 2u_B$.

$$u = u_E + u_B = 4.427 \times 10^{-8} + 4.41 \times 10^{-8} \approx 8.837 \times 10^{-8}\,\mathrm{J/m^3}$$

Verifying:

$$2u_E = 2 \times 4.427 \times 10^{-8} = 8.854 \times 10^{-8}\,\mathrm{J/m^3}$$

$$2u_B = 2 \times 4.41 \times 10^{-8} = 8.82 \times 10^{-8}\,\mathrm{J/m^3}$$

The total energy density u is approximately equal to $2u_E$ and $2u_B$, confirming the relationships.

Q2

In a different electromagnetic wave, the magnetic field amplitude is $B_0 = 5 \times 10^{-6}\,\mathrm{T}$. Given $\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$, $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$, and $c = 3 \times 10^8\,\mathrm{m/s}$:

(a) Calculate the energy density u_B associated with the magnetic field.

(b) Determine the corresponding electric field amplitude E_0.

(a) Calculate the energy density u_B associated with the magnetic field.

$$u_B = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

Substituting the values:

$$u_B = \frac{1}{2}\frac{(5\times10^{-6})^2}{4\pi\times10^{-7}} = \frac{1}{2}\frac{25\times10^{-12}}{4\pi\times10^{-7}}$$

Calculating $4\pi\times10^{-7}\approx1.2566\times10^{-6}$:

$$u_B = \frac{1}{2}\frac{25\times10^{-12}}{1.2566\times10^{-6}} \approx \frac{25\times10^{-12}}{2.5132\times10^{-6}} \approx 9.93\times10^{-6}\,\mathrm{J/m^3}$$

(b) Determine the corresponding electric field amplitude E_0.

Using the relationship:

$$E_0 = cB_0$$

Substituting the values:

$$E_0 = 3\times10^{8}\times5\times10^{-6} = 1500\,\mathrm{V/m}$$