

Unit 6: Trigonometric Identities

SLO: I can verify a trig identity numerically and graphically.
I can use trig identities to simplify trig expressions.

An *equation* is a statement of equality that may be satisfied by none, one, two, or more values of a variable.

An *identity* is an equation that is satisfied by ALL values of the variable that are in the domain of each side of the equation.

Ex: $2(x+1) = 2x+2$ $(x+3)^2 = x^2 + 6x + 9$ $\frac{x^2 - 3x - 10}{x - 5} = x + 2$ where $x \neq 5$

No matter what values of x we plug into the equation, the equation will be true.

A trigonometric identity is a trigonometric equation that is true for all permissible values of the variable in the expressions on both sides of the equation.

Trig functions are used to model behaviour in the physical world. Equivalent expressions for trig functions (identities) can be substituted to allow scientists to analyze data or solve a problem more efficiently. Trig identities allow us to transform many complicated trig expressions into a simpler, more compact form.

Trigonometric identities can be verified both numerically and graphically.

Try graphing the following functions on the same axes using the domain $-360^\circ \leq x \leq 360^\circ$ and a range of $-2 \leq y \leq 2$.

$$y = \sin x$$

$$y = \cos x \tan x$$

What do you notice about the graphs?

Verify the identity by choosing 2 values of x to substitute into the functions.

The Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

The Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

The Pythagorean Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Example: Simplify the expression, and determine the non-permissible values, in radians.

HINT #1:

- Re-write the expressions in terms of $\sin x$ and $\cos x$, then simplify.
- Perform the indicated operation: multiplication, division, addition, subtraction

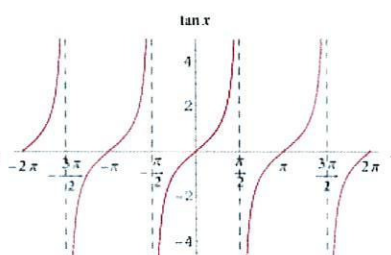
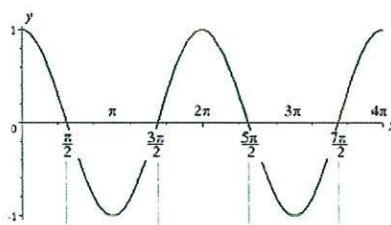
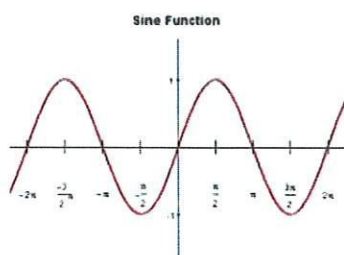
To determine the non-permissible values, set the denominators equal to 0.

Remember!

Visualize the graphs of $y = \sin x$ and $y = \cos x$ to help you determine the non-permissible values.

a) $\frac{\cot x}{\csc x \cos x}$

b) $\frac{\sec x}{\tan x}$



Example: By using the eight fundamental identities, transform each expression in column I into the equivalent expression in column II.

Column I	Column II
$\frac{\tan \theta}{\sin \theta}$	$\sec \theta$

Column I	Column II
$\sin \theta (\cot \theta + 1)$	$\cos \theta + \sin \theta$

Column I	Column II
$\cos^2 \theta + \sin^2 \theta + \tan^2 \theta$	$\sec^2 \theta$

Column I	Column II
$(\sec \theta - 1)(\sec \theta + 1)$	$\tan^2 \theta$

Column I	Column II
$(1 - \sin^2 \theta)(\sec^2 \theta - 1)$	$\sin^2 \theta$