Path Loss Analysis using Curve-Fitting for a CBRS System

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1 Introduction

Propagation path-loss models are used to predict the coverage of a given network initially and then engineered for capacity. These models can also be used to determine key system parameters such as transmission power, frequency, antenna height, etc. Received signal power decreases as a receiver moves away from a transmitter, this is known as signal attenuation or path loss. Attenuation is proportional to the second power of distance in the free space but can vary according to up to the fourth or fifth power in some dense urban areas, especially when inside a building. The exponent of increase in attenuation with increasing distance, known as path loss exponent, is a key observation we can get from analyzing the received signal data at various locations within a network.

We are trying to perform propagation analysis on the receiver data captured within the Citizens Broadband Radio Service (CBRS) network deployed by South Bend Community School Corporation (SBCSC) in and around a few schools in the city of South Bend.

2 Related Work

We can start with the most simple case, that of a direct wave propagating in free space. When a wave propagates directly from a transmitting antenna to a receiving antenna, which means no obstacles are blocking the direct path between the transmit and receive antennas, it is known as line-of-sight propagation. We further consider the transmit antenna to be omnidirectional with uniform radiation power, in all directions. The received power in such cases, using the Friis transmission formula [1], is given by

$$P_r = P_t G_t G_r (\frac{\lambda}{4\pi d})^2$$

where P_t is the transmit power, λ is the wavelength, d is the distance from the transmitter, G_t and G_r are the respective transmitter and receiver antenna gains.

Suppose we aren't given the transmit power but we know where the transmitter is located and we know the received power P_0 at some reference distance d_0, the path loss $\frac{P_r}{P_0}$ will then be $(\frac{d_0}{d})^2$. The exponent of path loss, in this case, is 2. But, line-of-sight propagation in free space from an omnidirectional antenna is a very ideal case and cannot be observed in practical settings. Thus, the path loss exponent, say γ, will not exactly be 2.

Assuming we are given some data containing received powers at corresponding locations within a network, we pick some reference distance d_0 and note the corresponding power P_0. When we take the log of path loss (ratio), we end up with

$$\log P_r = \gamma \log(\frac{d_0}{d}) + \log P_0$$

Which simplifies to

$$\log P_r = -\gamma \log d + \log P_0 + \gamma \log d_0$$

Suppose we know the received power at a distance of 1 meter from the transmitter, the above equation further simplifies to

$$\log P_r = -\gamma \log d + \log P_0 \tag{1}$$

Generally, power in a signal is calculated in decibels and thus we know $\log P_r$. We can notice that we have a line equation with the slope being $-\gamma$ or negative of path loss exponent and the intercept is power in decibels at a distance of one meter. Hence, the task of finding the path loss exponent in a network is of fitting a line to the data of received power in dB vs log of the corresponding distance in meters.

Received power varies even for the same distance between the transmitter and the receiver, due to changing environments. Assuming there are many random factors affecting the received power, we introduce a random variable in (1) having Gaussian distribution and mean zero. Thus, we can realize that it is the average of the received power at a particular distance, that satisfies (1). This is known as lognormal shadowing.

Various empirical models based on the log distance path loss exist for different environments such as indoor, outdoor, urban, suburban, rural, etc. Selecting a propagation path loss model to determine network parameters is a very critical aspect of designing a network as the wrong selection may lead to coverage issues, high interference, etc. Some popular empirical models used widely are Okumura-Hata [2], Walfisch-Bertoni [3], etc.

3 Approach

We had the data consisting of received power in decibels and the latitude-longitude coordinates of measurement locations. Data was collected primarily for three base stations, as shown in fig 1, by tracking their carrier frequencies.

Using the latitude and longitude coordinates of the measurement locations and of the corresponding base stations (according to the carrier frequencies), we calculated the distances between transmitter and receiver using the haversine formula,

$$d = 2r \arcsin\left(\sqrt{\sin^2\left(\frac{\alpha_2 - \alpha_1}{2}\right) + \cos(\alpha_2)\cos(\alpha_1)\sin^2\left(\frac{\beta_2 - \beta_1}{2}\right)}\right)$$

where, r is the radius of the earth (at South Bend), (α_1, β_1) and (α_2, β_2) are the lat-long

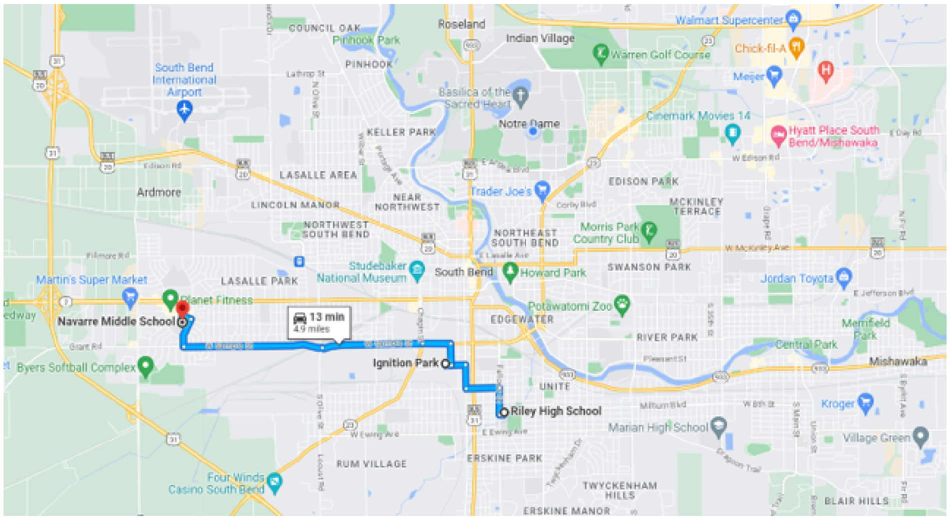

Figure 1: Base Station Locations in South Bend

coordinates of the base stations and the measurement locations.

We use simple linear regression to fit a line $\hat{y} = \hat{m}x + \hat{c}$ to our data, where the log of distance is the independent variable x and the received power in dB y depends on the distance. Our objective is to minimize the sum of squared errors,

$$\min_{m,c} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2 = \min_{m,c} \sum_{i=1}^{n} (mx_i + c - y_i)^2 \tag{2}$$

where, n is the number of data points and y_i is the measured received power at location i. The optimal values of m and c can be calculated by taking the partial derivatives of the objection function, and are given as,

$$\hat{m} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \quad \text{and} \quad \hat{c} = \bar{y} - \hat{m}\bar{x} \tag{3}$$

the negative of $\hat{m}$ will be the path loss exponent.

4 Results

Fig 2 shows the RSRP vs log dist data for the Riley School. The observed path loss exponent is 2.399 and the average sum squared error is 63.3. While fig 3 shows the plot for Ignition Park with path loss exponent 2.194 and error 66.167.

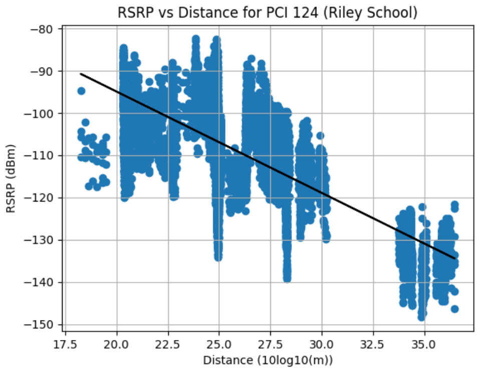

Figure 2: RSRP vs Dist for Riley High School

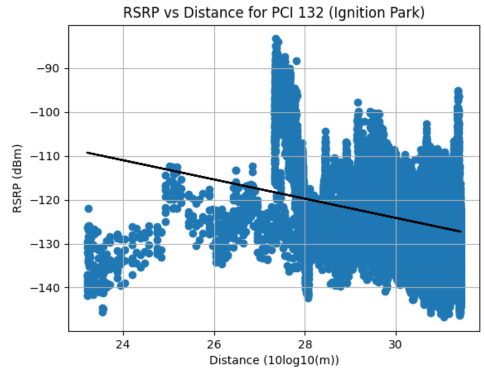

Figure 3: RSRP vs Dist for Ignition Park

Fig 4 shows the plot for Navarre School with path loss exponent 2.09 and error 99.992. For fig 5, we randomly shuffle the Navarre school data and extract the first 3/4th data points and calculate the path loss exponent for it. We repeat the process 100 times and average all the path loss exponents. Then we plot the RSRP vs Dist data with random 1/4th data points and plot a line with the average path loss exponent, we reduce the error to 98.675. This process helps a little bit as we can notice many empty and less dense regions in fig 4.

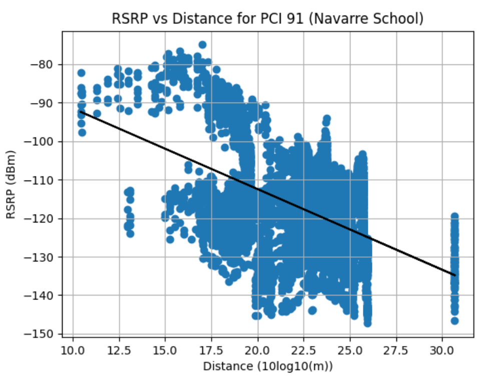

Figure 4: RSRP vs Dist for Navarre Middle School

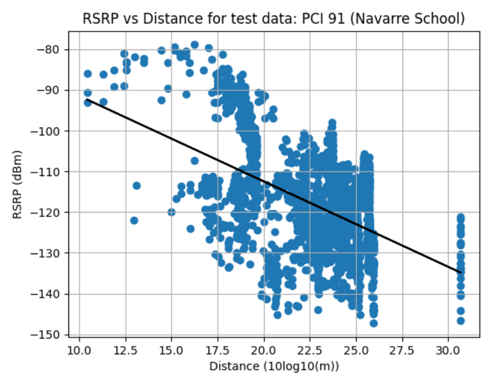

Figure 5: Testing over Navarre School Data

For the fig 6 we divide the log of distance into many small equidistant intervals. We then take the average of all the received power values in dBm for each of the intervals and plot them. We can see the effect of lognormal shadowing here. We picked up one specific interval and noted all the received power values for it. We then plot a histogram (fig 7) of these values and compare the distribution with a Gaussian distribution curve.

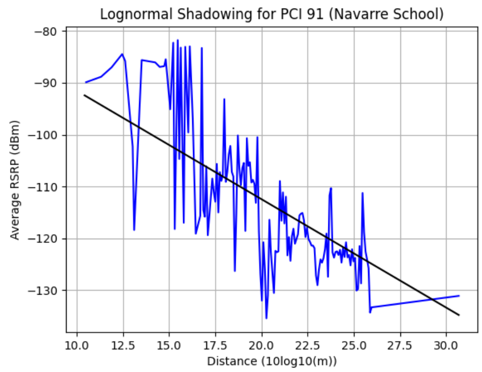

Figure 6: Lognormal Shadowing for Navarre School Data

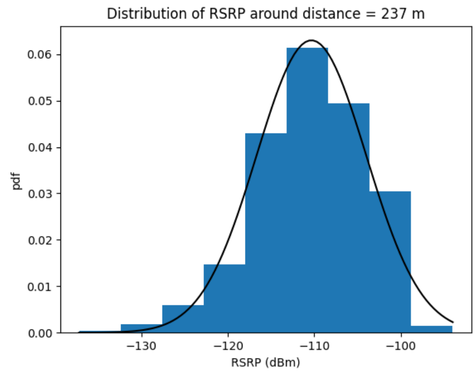

Figure 7: RSRP Distribution over an Area

5 Conclusion

We tried to analyze signal propagation in a practical communication system by trying to fit a path loss model to the received power vs distance data. A very basic simple linear regression using ordinary least squares was implemented and a key component of the path loss model, the path loss exponent, could be deduced using this process. We observed plots for three different base stations to get three different path loss exponents, each representing the environment around these base stations to some degree.

To improve performance we tried a machine learning-like training testing approach. Though the improvement wasn't much significant, it was aligned with the intuition behind the approach. We further tried to observe the effect of lognormal shadowing through our data. We could also confirm a Gaussian-like distribution of the received power values in a small radial distance range.

Apart from the simple linear regression, an attempt to use orthogonal regression, a special case of Deming regression [4], did not give fruitful results. Deming regression assumes error in both the independent and the dependent variables. And if we assume equal error variances for both, we end up minimizing the sum of squared perpendicular distance instead of the sum of the squared vertical distance involving just the dependent variable. In our result, a line almost parallel to the horizontal axis was just reducing the vertical errors and even doing worse than in the ordinary case. Also, polynomial regression was implemented to observe the effect of each power of distance on the path loss, but the results were inconclusive.

References

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