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CSCE 440

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Introduction

Interpolation is numerical method that constructs a new set of data within a set of known data points. It is often used when there is a set of data points that are obtained by sampling or experimentation to estimate or in this case, interpolate the value of that function. When it comes to interpolation, the function can be interpolated with a given set of points (x_i, y_i) considering that there are no two x_i values that are equal and that y_i corresponds with x_i. There are many methods to interpolate data, such as Lagrange Polynomial interpolation, Newton's Divided Differences and Neville's Method. This paper investigates these methods to interpolate data and are implemented in MATLAB.

Lagrange Polynomial Interpolation

Given a set of $(x_{\mathrm{j}}, y_{\mathrm{j}})$ data points, the Lagrange polynomial interpolation formula can be calculated by first finding the product of the Lagrange polynomials,

$$L_i(x) = \prod_{j=0, j\neq i}^{n} \left(\frac{x - x_j}{x_i - x_j} \right)$$

Once the product of that sequence is found, the interpolation can be calculated by multiplying $L_i(x)$ with its corresponding y_i value to return a polynomial expression,

$$P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

Where, $f(x_i) = y_i$

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Newton's Divided Differences

Newton's divided differences is a recursive interpolation method and is computed using the following formula,

$$p_n(x) = a_0 + \sum_{i=1}^{n} \left(a_i \cdot \prod_{j=0}^{i-1} (x - x_j) \right)$$

Where, $a_i = f[x_0, x_1, \dots, x_i]$ is called divided difference of order i for f

Neville's Method

Neville's method is like Newton's divided differences as it is based on that method. Given a set of (x_j, y_j) data points, Neville's method evaluates the polynomial at a specific x value and its formula is as follows,

$$p_{0,\cdots,n}(x) = \left(\frac{x - x_j}{x_i - x_j} \right) p_{0,\cdots,j-1,j+1,\cdots,n}(x) + \left(\frac{x_i - x}{x_i - x_j} \right) p_{0,\ldots,i-1,i+1,\cdots,n}(x)$$

Methodology

To test these methods, I used temperature and pressure data from between 4/26 and 4/29 in Lincoln, Nebraska. As interpolation provides approximate values within an interval, I obtained the highest and lowest temperatures each day and the temperatures recorded within that range. I also rounded the temperatures to the nearest whole number and since no two x_i values can be equal, I calculated the average atmospheric pressure at where $x_i = x_j$.To verify if the interpolated values are correct, I used alternate values along within the interval and set aside the other values to determine the mean square error of each method.

Initially, I intended to use temperature and humidity as data points but found that humidity does not correspond with temperature. Instead I found that it is atmospheric pressure that corresponds with temperature and can be used at different temperatures within the interval to find close approximates of the corresponding atmospheric pressure.

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Date	Temperature,C	Pressure, hPa
26-Apr	2	1016
26-Apr	7	1016.4
26-Apr	13	1018.4
26-Apr	19	1015.6
26-Apr	21	1015.1
27-Apr	1	1017.9
27-Apr	8	1017.5
27-Apr	16	1014.7
27-Apr	24	1014.1
27-Apr	27	1011.8
28-Apr	1	1021.4
28-Apr	6	1019.9
28-Apr	11	1022.7
28-Apr	16	1021.1
28-Apr	20	1019.8
29-Apr	10	1020
29-Apr	13	1020.4
29-Apr	16	1019.2
29-Apr	19	1011.1
29-Apr	21	1013.1
29-Apr	24	1012.2

Table 1:Data used to produce polynomial interpolation

Date	Temperature,C	Pressure, hPa
26-Apr	3	1015.9
26-Apr	4	1015.9
26-Apr	5	1019
26-Apr	8	1017.8
26-Apr	9	1019.2
26-Apr	12	1015.9
26-Apr	15	1015.5
26-Apr	16	1017.5
26-Apr	18	1016.2
26-Apr	20	1015.9
27-Apr	2	1018.6
27-Apr	3	1018.6
27-Apr	4	1017.8
27-Apr	9	1017.2
27-Apr	10	1016.8
27-Apr	13	1015.9
27-Apr	14	1016.9
27-Apr	17	1016.4
27-Apr	18	1013.5
27-Apr	21	1015.7
27-Apr	22	1012.6
27-Apr	23	1015
27-Apr	25	1013.4
27-Apr	26	1011.4
28-Apr	2	1020.4
28-Apr	5	1018.7
28-Apr	8	1020.3
28-Apr	12	1023.2
28-Apr	14	1021.1
28-Apr	17	1019.3
28-Apr	18	1021.6
28-Apr	19	1019.9
29-Apr	11	1019.9
29-Apr	12	1020.4
29-Apr	18	1018.8
29-Apr	20	1015.1
29-Apr	22	1011.4
29-Apr	23	1013.3

Table 2: Data used to compare interpolation methods

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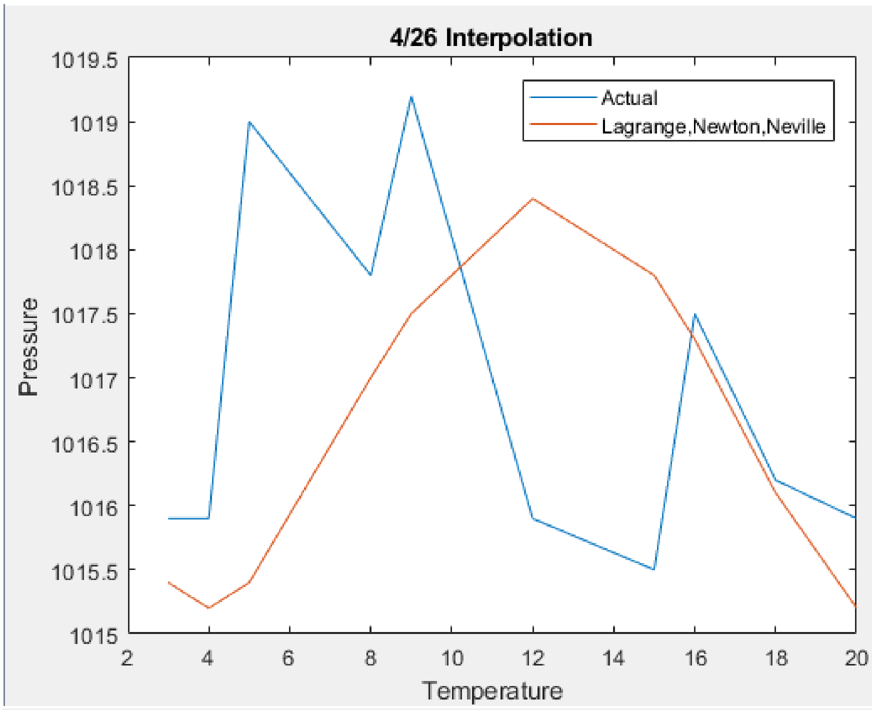
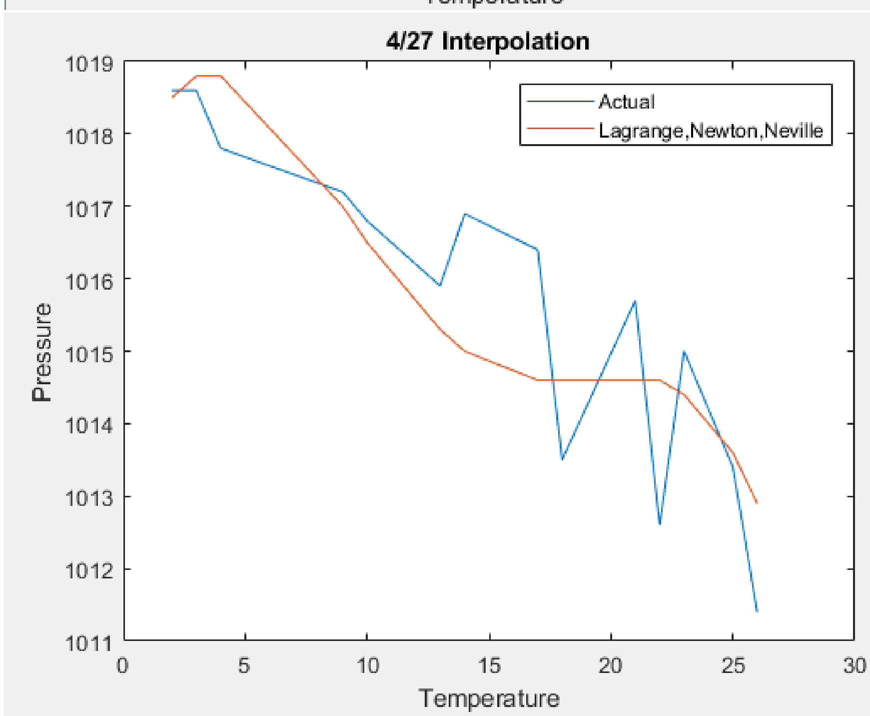
Results





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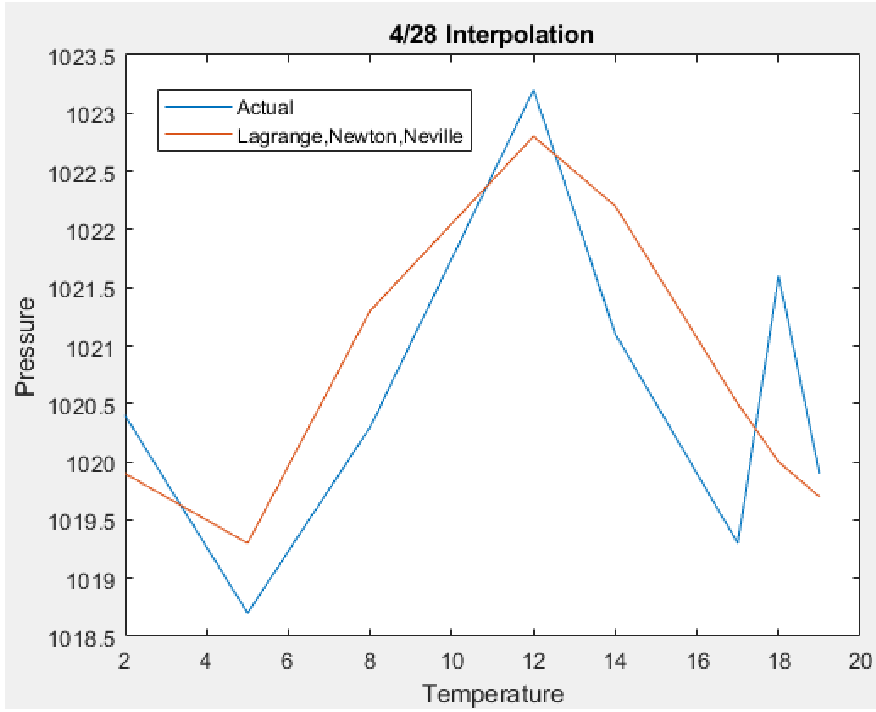

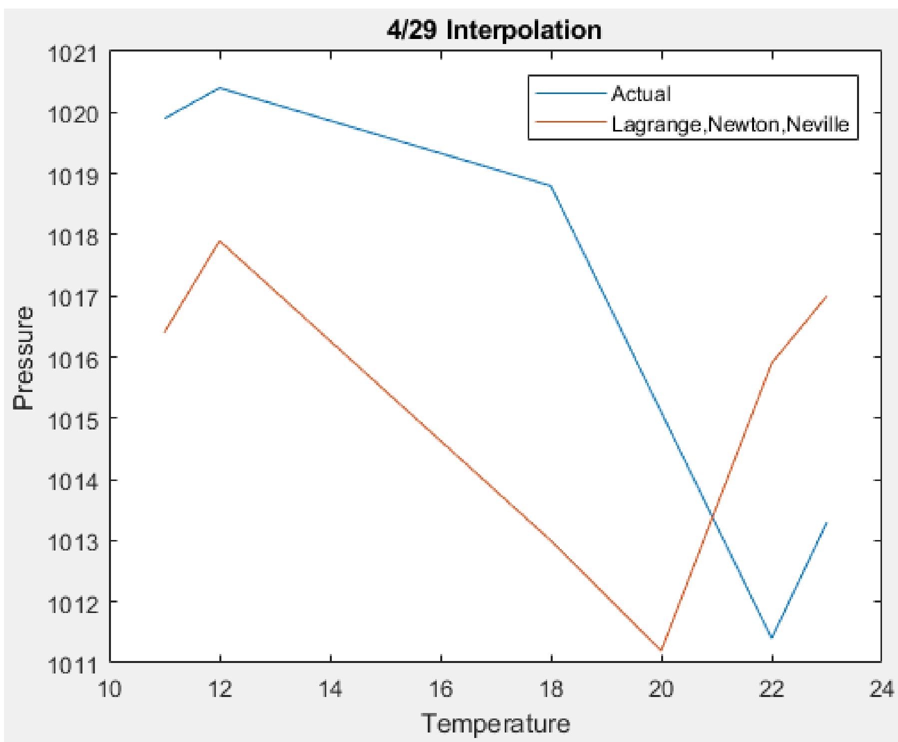

Interpolation Method	*Absolute Mean Error*
Lagrange	0.448
Newton's	0.448
Neville's	0.448

Table 3: Error of different interpolation methods

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Conclusions

All three interpolation methods would yield the same approximate, but the absolute mean error is too high. One factor that could have influenced the high error is the unpredictable weather that could potentially fluctuate at random points. If one of these interpolation methods need to be chosen, either one could be used as it yields the same approximates. Although interpolation helps yield close approximates, there are many other factors that influences the weather.

References

- "Weather History for KLNK - April, 2018." Weather Underground (10.226.236.170), www.wunderground.com/history/airport/KLNK/2018/4/26/DailyHistory.html?req_city=&req_state=&req_statename=&reqdb.zip=&reqdb.magic=&reqdb.wmo=.
- "Weather History for KLNK - April, 2018." *Weather Underground (10.226.238.177)*, www.wunderground.com/history/airport/KLNK/2018/4/27/DailyHistory.html?req_city=&req_state=&req_statename=&reqdb.zip=&reqdb.magic=&reqdb.wmo=.
- "Weather History for KLNK - April, 2018." *Weather Underground (10.226.224.163)*, www.wunderground.com/history/airport/KLNK/2018/4/28/DailyHistory.html?req_city=&req_state=&req_statename=&reqdb.zip=&reqdb.magic=&reqdb.wmo=.
- "Weather History for KLNK - April, 2018." *Weather Underground (10.226.239.135)*, www.wunderground.com/history/airport/KLNK/2018/4/29/DailyHistory.html?req_city=&req_state=&req_statename=&reqdb.zip=&reqdb.magic=&reqdb.wmo=.