Understanding 2-Qubit and 3-Qubit GHZ States

This document provides an explanation of two fundamental quantum circuits designed to create entangled states: the 2-qubit Bell state (which is the simplest form of a GHZ state) and the 3-qubit Greenberger–Horne–Zeilinger (GHZ) state. Both circuits demonstrate the powerful phenomenon of quantum entanglement, where the state of multiple qubits becomes intrinsically linked, even when physically separated.

The Concept of a GHZ State

A Greenberger–Horne–Zeilinger (GHZ) state is a particular type of maximally entangled quantum state involving three or more qubits. The defining characteristic of a GHZ state is that all qubits are in a superposition, but their measurement outcomes are perfectly correlated. If one qubit is measured, the states of all other qubits in the GHZ state are instantly determined, regardless of distance.

The general form of an N-qubit GHZ state is:

$$\frac{1}{2}(|00\ldots0\rangle+|11\ldots1\rangle)$$

This means the system is in a superposition of all qubits being |0⟩ OR all qubits being |1⟩, with no other combinations possible once measured.

1. 2-Qubit Entanglement Circuit (Bell State)

The 2-qubit entanglement circuit, commonly known as a **Bell state**, is the simplest instance of a GHZ state. It is a fundamental building block for many quantum algorithms and communication protocols.

Circuit Construction:

1. **Initialization:** The circuit starts with both qubits in the |0⟩ state: |00⟩.
2. **Hadamard Gate on Qubit 0:**
   - The `qc.h(0)` operation applies a Hadamard gate to the first qubit (q0).
   - This transforms q0 from |0⟩ into a superposition state: 21(|0⟩+|1⟩).
   - The combined state of the two qubits becomes: 21(|00⟩+|10⟩).
3. **CNOT Gate (Controlled-NOT) on (Qubit 0, Qubit 1):**
   - The `qc.cx(0, 1)` operation applies a CNOT gate where q0 is the control qubit and q1 is the target qubit.
   - If the control qubit (q0) is |0⟩, the target qubit (q1) remains unchanged.
   - If the control qubit (q0) is |1⟩, the target qubit (q1) flips its state (from |0⟩ to |1⟩).
   - Applying this to the superposition:
     - The |00⟩ part of the superposition remains |00⟩.

    - The |10⟩ part of the superposition becomes |11⟩ (since `q0` is |1⟩, it flips `q1`).
  - The final state becomes the Bell state: 21(|00⟩+|11⟩).

Expected Outcomes:

When measured, this entangled state will probabilistically collapse into either |00⟩ or |11⟩, each with approximately 50% probability (for an ideal simulator). Crucially, you will *not* observe outcomes like |01⟩ or |10⟩, demonstrating the perfect correlation between the entangled qubits.

2. 3-Qubit GHZ State Circuit

The 3-qubit GHZ state extends the concept of entanglement to three qubits, demonstrating even stronger non-local correlations and serving as a key resource for various quantum computing applications like quantum error correction and distributed quantum computing.

Circuit Construction:

1. **Initialization:** The circuit starts with all three qubits in the |0⟩ state: |000⟩.
2. **Hadamard Gate on Qubit 0:**
   - `qc_ghz.h(0)` applies a Hadamard gate to `q0`, putting it into superposition: 21 (|0⟩+|1⟩).
   - The overall state is now: 21(|000⟩+|100⟩).
3. **First CNOT Gate on (Qubit 0, Qubit 1):**
   - `qc_ghz.cx(0, 1)` uses `q0` as control and `q1` as target.
   - This transforms the state to: 21(|000⟩+|110⟩).
   - At this point, `q0` and `q1` are entangled, similar to a Bell state, while `q2` is still in |0⟩.
4. **Second CNOT Gate on (Qubit 0, Qubit 2):**

   - `qc_ghz.cx(0, 2)` uses `q0` as control and `q2` as target.
   - If `q0` is |0⟩, `q2` remains |0⟩.
   - If `q0` is |1⟩, `q2` flips from |0⟩ to |1⟩.
   - Applying this to the state 21(|000⟩+|110⟩):
   - The |000⟩ component remains |000⟩.
   - The |110⟩ component becomes |111⟩ (because `q0` is |1⟩, flipping `q2` to |1⟩).
   - The final state is the 3-qubit GHZ state: 21(|000⟩+|111⟩).

Expected Outcomes:

Upon measurement, the 3-qubit GHZ state will collapse into either |000⟩ or |111⟩, each with roughly 50% probability (in an ideal simulation with many shots). No other measurement outcomes (like |010⟩, |101⟩, etc.) should be observed. This demonstrates that all three qubits are perfectly correlated: if the first qubit is measured as |0⟩, the other two must also be |0⟩, and similarly for |1⟩.

Importance and Applications

Both the 2-qubit and 3-qubit GHZ states are crucial for understanding and implementing various quantum computing concepts:

- **Entanglement Demonstration:** They provide clear, measurable evidence of entanglement, a non-classical correlation vital for quantum advantage.
- **Quantum Communication:** Bell states are used in quantum key distribution (QKD) and quantum teleportation.
- **Quantum Error Correction:** Larger GHZ states are fundamental to certain quantum error correction codes, where redundancy across entangled qubits helps protect quantum information from noise.
- **Quantum Sensing:** Entangled states can enhance the sensitivity of quantum sensors beyond classical limits.

By experimenting with these circuits in Qiskit, you can directly observe the probabilistic nature of quantum measurement and the powerful correlations inherent in entangled systems, laying a strong foundation for understanding more complex quantum phenomena.