Time series analysis using wavelets

Thesis · September 2012

1 author:

Anastasia Panori
Panteion University of Social and Political Sciences

HELLENIC REPUBLIC
UNIVERSITY OF MACEDONIA

Department of Economics

MSc Thesis

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Panori Anastasia

(Translated from Greek)

Supervisor: Ass. Prof. Aikaterini Kyrtsou

Thessaloniki, 2012

I would like to express my special thanks to the Assistant Professor Mrs. Aik. Kurtsou, for the confidence she showed me with the assignment of this work and for all valuable advice she gave me during its development. Also, thanks I want to express to my family and my friends who stood by me all this time.

CONTENTS

Introduction

Analysis of economic time series is of particular interest in economics, in order to provide valuable insights about the behaviour of financial markets. One of the promising methods of analysis, which in recent years seems to have been highly developed, is wavelet analysis.

From a mathematical point of view, wavelets are an evolution of the Fourier transform. Through their use, scientists have managed to overcome the problem of the limited range of time series that until now could be analyzed with the Fourier transform, giving a new impetus to economic research. This, coupled with the parallel development of new mathematical tools and computational methods, made wavelets one of the most useful modern methods of analysis.

The main purpose of this study is to apply the wavelet analytical method to the time series of three markets: United States, United Kingdom and Korea. Using data derived from the daily returns of these exchanges and covering a long period of time, from January 1973 to August 2002, we will try to find hidden information that may exist at different frequencies and give us attributes as to the structure of each market. Therefore, a simultaneous analysis in terms of time and frequency will be performed, exploiting the main differentiation characteristic of the wavelet transform compared to Fourier analysis.

Initially, there will be a first comparison between characteristics of the time series resulting from wavelet analysis, such as kurtosis, skewness and autocorrelation, and then we will move on to the causes of their differentiation.

In addition, we choose to use the theory of effective markets as the main theoretical basis upon which the economic analysis and the empirical application is designed and performed. Fama (1969) was the first to claim that the effectiveness of a market always lies in the fact that the overall information available to it, is directly reflected in its prices. Since the time of its formulation, this theory has acquired a wide range of supporters who, through their research, try to find elements to verify it, either in its original form or in some similar way (Grossman & Stiglitz, 1980).

However, in recent years, due to the further development of computational tools, and hence methods of analysing time series, it is now possible to observe a number of abnormalities in the behaviour of markets, which contradict the hitherto theories of their effectiveness, raising various questions about their actual structure and the mechanisms that operate within them. In a related article, Jensen (1978) emphasizes that even though there is no other theory in economics, apart from effective markets, which indicates so many empirical outcomes to support it, there have been some new evidence which start to show an incompatibility with it.

Therefore, the first question emerging from the above, and which we will try to answer through the first part of the analysis carried out in this thesis, is whether and to which extent the markets being investigated are effective. On a second level, in case evidence indicates that there is inefficiency in the markets, we will try to identify the various factors that cause it, as well as identify the type of their structure.

Existing literature has started to develop new theories about the structures that govern stock markets in recent years. A commonplace of much of these theories is the possibility of non-linear structures and heterogeneity of investors, characteristics that when observed are largely responsible for diversifying the behavior of markets when receiving similar information.

In our case, testing for the existence of such chaotic structures in the selected time series is achieved through the implementation of the BDS Test. Correspondingly, GARCH analysis (1.1) helps us study the variability observed in time series along all frequencies in the three markets, aiming at exploring the heterogeneity of investors in these markets.

Regarding its structure, the study is divided into two parts. The first chapter describes the methodological framework upon which the wavelet theory is based, as well as a series of financial applications that have been carried out to date, alongside with their main conclusions. In the second part, we present the empirical analysis that has been implemented in this study, as well as the main outcomes we have obtained from it. The conclusions that follow, and an extensive theoretical commentary on them, are presented at the end of Chapter 2. Finally, all tables presenting the results referring to the BDS statistics, GARCH analysis coefficients (1,1) and the diagrams of autocorrelation coefficients are given in the annexes.

Chapter 1: Theoretical analysis

1.1 Introduction

In mathematics there are several tools for the analysis of time series, but in recent years the use of wavelets has been largely developed, particularly in the field of economic science. Starting from the analysis of the word wavelets, we see that it refers to "small" waves, which extend over a finite time. This is in contrast with the Fourier analysis, which uses cosine and sine functions that extend into the (-∞, +∞) interval.

According to Percival & Walden (2000), let us consider a function *ψ(·)* which is defined in the real axis (-∞, +∞) and to which the following applies:

- its average is zero:

$$\int_{-\infty}^{+\infty} \psi(u)du = 0 \qquad (1)$$

- and the integral of its square is equal to the unit:

$$\int_{-\infty}^{+\infty} \psi^2(u)du = 1 \qquad (2)$$

From the above two conditions, we observe that while (2) tells us that function *ψ(·)* should be different from zero at some points, (1) stresses that any positive deviation from zero should be neutralized by a corresponding negative, so that the average remains zero.

In addition, let's suppose a number *ε*, for which it holds that 0< *ε* <1. Then we can consider a certain interval [-T, T] such that:

$$\int_{-\mathrm{T}}^{\mathrm{T}} \psi^2(u)du > 1 - \varepsilon \qquad (3)$$

When relation (3) applies, it shows that for very small *ε*, all the activity of *ψ(·)* is limited within the defined interval [-T, T], which in relation to the original (-∞, +∞) it can be considered too small. Consequently, conditions (1), (2) and (3) define a function representing a "small" wave, i.e. a wavelet.

The first and the simplest wavelet defined in the literature is the Haar wavelet, which was named after Haar who used it in 1910, and is defined as follows:

$$\psi^{(\mathrm{H})}(u) \equiv \begin{cases} -1/\sqrt{2}, & -1 < u \le 0; \\ 1/\sqrt{2}\ , & 0 < u \le 1; \\ 0\ , & \text{αλλού} \end{cases} \qquad (4)$$

In addition, another very important condition that must be met, in order to be able to characterize a wavelet suitable for the analysis of time series, is the admissibility condition. The Fourier transform of a wavelet is given by relation (5):

$$\Psi(f) \equiv \int_{-\infty}^{+\infty} \psi(u) e^{-i2\pi fu} du \qquad (5)$$

Thus, we consider that a wavelet is admissible when for its Fourier transform holds that the relation:

$$C_{\psi} \equiv \int_{0}^{+\infty} \frac{|\Psi(f)|^2}{f} df \qquad (6)$$

satisfies the condition: $0 < C_{\psi} < +\infty \quad (7)$

The above condition assures the possibility of reconstituting our initial $x(\cdot)$ signal from the Continuous Wavelet Transform (CWT), which will be discussed later in this chapter.

One of the most important wavelet analyses that have been performed is that of Morlet wavelet transform in the late 1970s. Morlet dealt with the analysis of geophysical signals for the extraction of oil and gas, which he wanted to decompose in high and low frequency components. He considered the following as his initial wavelet:

$$\psi(u) = Ce^{-i\omega_o u}(e^{-u^2/2} - \sqrt{2}e^{-\omega_o^2/4}e^{-u^2}) \qquad (8)$$

Where C and ω_o are constant variables. In addition, the following applies:

$$\int_{-\infty}^{+\infty} e^{-i\omega_o u} e^{-u^2/2} du = \sqrt{2\pi} e^{-\omega^2/2} \qquad (9)$$

It follows from (9) that the condition (1) is satisfied, while at the same time the choice of C and ω_o is made so that the relation (2) is also satisfied. As the ω_o increases, we notice that the negative term of equation (8) decreases continuously and ends to be negligible. Finally, for high values of ω_o it is true that:

$$\psi(u) \approx \psi_{\omega_o}^{(\mathrm{M})} \equiv \pi^{-1/4} e^{-i\omega_o u} e^{-u^2/2} \qquad (10)$$

The wave given by equation (10) is often called Morlet wavelet, and a schematic representation of this is given in Figure 1.1. In the vertical axis we have the wavelet values, while the horizontal axis represents time.

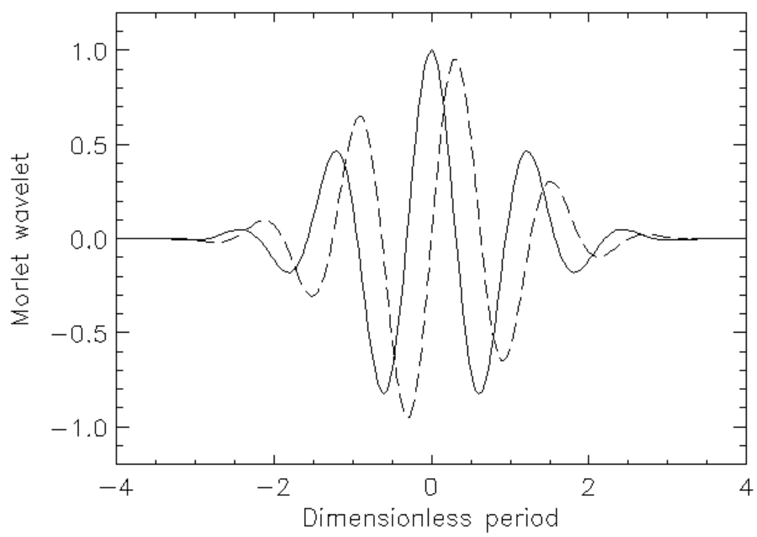

Figure 1.1: Morlet Wavelet. The continuous line represents the real part while the dotted part of the wavelet is dotted.

1.2 Continuous Wavelet Transform (CWT)

In the previous section, we analyzed the basic properties of wavelets, which make them suitable tools for analyzing time series. In this section, our goal will be to show how this analysis is achieved, as well as a deeper understanding of its mechanisms.

Initially, according to Percival & Walden (2000) we assume a real function $x(\cdot)$, which we consider to be a function of time t and we will refer to it as an input signal. The following relation is defined as a mean of this function in interval [a, b]:

$$\frac{1}{b-a}\int_a^b x(u)du \equiv \alpha(a,b) \qquad (11)$$

For which we assume that it is well defined and that it holds a < b. Moreover, for convenience we take $x(\cdot)$ as a step function which can be expressed in space [a, b] in the form of:

$$x(t) = x_j \quad \gamma\iota\alpha \quad a + \frac{j}{N}(b-a) < t \leq a + \frac{j+1}{N}(b-a) \quad \kappa\alpha\iota \ \gamma\iota\alpha \quad j = 0, \ldots, N-1 \qquad (12)$$

Based on the definition of Riemann integral, relation (11) can also be written as a sum:

$$\frac{1}{b-a}\int_a^b x(u)du = \frac{1}{b-a}\sum_{j=0}^{N-1} x_j \frac{b-a}{N} = \frac{1}{N}\sum_{j=0}^{N-1} x_j \ (13)$$

In order to understand the methodology of wavelet analysis it would be more useful to modify relation (13), which represents the mean of function $x(\cdot)$. It is defined as the length of space [a, b] where $\lambda \equiv b - a$, with mean t = ((a + b)/2. By replacing in (11) we get the following relation:

$$A(\lambda, t) \equiv \alpha(t - \frac{\lambda}{2}, t + \frac{\lambda}{2}) = \frac{1}{\lambda}\int_{t-\frac{\lambda}{2}}^{t+\frac{\lambda}{2}} x(u)du \qquad (14)$$

So, we call A (λ, t) as the mean of signal x (·) with scale λ centered on point t. Specifically, changes of this mean indicate particular interest, showing the extent to which, it remains constant over a time frame that we have defined through variable λ. Changes in the mean of the signal x (·) between two time frames is given by the following relation:

$$D(\lambda, t) \equiv A(\lambda, t + \frac{\lambda}{2}) - A(\lambda, t - \frac{\lambda}{2}) = \frac{1}{\lambda}\int_t^{t+\lambda} x(u)du - \frac{1}{\lambda}\int_{t-\lambda}^{t} x(u)du \qquad (15)$$

To understand the process by which we arrived at this result, we initially consider a scale λ=1, which may correspond to any time unit depending on the case we study (e.g. day, hour). Therefore, the difference of the means in the case where λ=1, and in the time interval starting at t-1/2, will be given based on the previous relationship (14) as follows:

$$D(1, t - \frac{1}{2}) \equiv A(1, t) - A(1, t-1) = \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} x(u)du - \int_{t-\frac{3}{2}}^{t-\frac{1}{2}} x(u)du \qquad (16)$$

The above relationship can be shifted to time t in order to take the form:

$$D(1, t) \equiv A(1, t + \frac{1}{2}) - A(1, t - \frac{1}{2}) = \int_t^{t+1} x(u)du - \int_{t-1}^{t} x(u)du \qquad (17)$$

A schematic representation of (17) could for example show us how the average daily temperature changes from day to day. Finally, by using (17) we can get the general expression of (15), which in the example mentioned before, would show us the change of the average daily temperature to any other scale. The way in which this analysis relates to wavelets will be understood immediately below.

Since in relation (15) there is no overlap of time between the two integrals of the right member, we could compile them into an integral and write it with the help of a wavelet as follows:

$$D(\lambda,t)=\int_{-\infty}^{+\infty}\psi_{\lambda,t}(u)x(u)du \tag{18}$$

Where:

$$\psi_{\lambda,t}(u)\equiv\begin{cases}-1/\lambda & , \quad t-\lambda<u<t;\\ 1/\lambda & , \quad t<u<t+\lambda;\\ 0 & , \quad \text{αλλού}\end{cases} \tag{19}$$

(19) is a generic expression, which in the specific case where λ = 1 and t = 0, it becomes:

$$\psi_{1,0}(u)\equiv\begin{cases}-1 & , \quad -1<u<0;\\ 1 & , \quad 0<u<1;\\ 0 & , \quad \text{αλλού}\end{cases} \tag{20}$$

We notice that relation (20) can also be written in the form $\psi_{1,0}(u)=\sqrt{2}\psi^{(\mathrm{H})}(u)$, where $\psi^{(\mathrm{H})}(u)$ is the Haar wavelet of relation (4). Therefore, (18) can be written alternatively as a function of the Haar wavelet:

$$\int_{-\infty}^{+\infty}\psi^{(\mathrm{H})}(u)x(u)du\equiv W^{(H)}(1,0) \tag{21}$$

We see, therefore, that by using the Haar wavelet we can extract information from function x (·) about the difference of the means on a scale of two unitary periods, having as a starting point t=0. To be able to obtain more general results regarding both the magnitude of the period and the starting point, we should use the general form of Haar wavelet, given by relation (22):

$$\psi_{\lambda,t}^{(H)}(u) \equiv \frac{1}{\sqrt{\lambda}}\psi^{(\mathrm{H})}\left(\frac{u-t}{\lambda}\right) = \begin{cases} -\frac{1}{\sqrt{2\lambda}} & , \quad t-\lambda < u \leq t; \\ \frac{1}{\sqrt{2\lambda}} & , \quad t < u \leq t+\lambda; \\ 0 & , \quad \text{αλλού} \end{cases} \qquad (22)$$

Moreover, function $\psi_{\lambda,t}^{(H)}(\cdot)$ satisfies the initial relationships (1) and (2) which are necessary for us to be able to use it for wavelet analysis. Finally, using the Haar wavelet as the basis, we get the Haar CWT transformation, which can be interpreted as the difference between two adjacent means of scale *λ*, the first one of which starts on point *t*, while the second ends at this point. The CWT transformation is the collection of variables $W^{(H)}(\lambda,t)$ for λ = 0 and -∞ < t < +∞.

Instead of the Haar wavelet we can use any other corresponding wavelet as the basis for getting the CWT transformation, according to the relevant application. The general form of the CWT transformation is defined as follows:

$$W(\lambda,t) \equiv \int_{-\infty}^{+\infty} \psi_{\lambda,t}(u)x(u)du \quad \text{όπου} \quad \psi_{\lambda,t}(u) \equiv \frac{1}{\sqrt{\lambda}}\psi\left(\frac{u-t}{\lambda}\right) \qquad (23)$$

The use of different types of wavelets essentially changes the type of means, and the ways in which they are combined to produce CWT transformation.

Moreover, if the function $\psi_{\lambda,t}(\cdot)$ satisfies the admissibility condition of relation (6) and even for the signal x (·) it is true that:

$$\int_{-\infty}^{+\infty} x^2(t)dt < +\infty \qquad (24)$$

Then we can restore the original signal x (·) from its CWT transform by the relation:

$$x(t) = \frac{1}{C_\psi}\int_0^{+\infty}\left[\int_{-\infty}^{+\infty} W(\lambda,u)\frac{1}{\sqrt{\lambda}}\psi\left(\frac{t-u}{\lambda}\right)du\right]\frac{d\lambda}{\lambda^2} \qquad (25)$$

Where C_ψ is given by (7). Also, for the energy of the signal x (·) it is true that:

$$\int_{-\infty}^{+\infty} x^2(t) = \frac{1}{C_\psi}\int_0^{+\infty}\left[\int_{-\infty}^{+\infty} W^2(\lambda,t)dt\right]\frac{d\lambda}{\lambda^2} \tag{26}$$

From (26) we can see that $W^2(\lambda,t)/\lambda^2$ is a probability density function, which decomposes the energy of the original signal into various scales and times.

The main feature of CWT transformation is that it enables us to be able to "see" within our original signal and find information that was not visible by simply displaying the signal over time. Also, the fact that it is possible to fully recover the original signal from its CWT transformation means that both the function of the signal in the time domain and its CWT constitute the same mathematical entity.

1.3 Discrete Wavelet Transform (DWT)

Discrete Wavelet Transform (DWT) is essentially a procedure for processing our original signal through a set of high-pass and low-pass filters.

Initially, DWT is an ortho-normal transformation with coefficients $\{W_n : n = 0,\ldots,N-1\}$, using which we can write it as follows:

$$\mathbf{W} = W\mathbf{X} \tag{27}$$

Where:

W: is a column table of length N = 2^J, whose n-elements are the DWT coefficients $\mathbf{W}_n$ and

W: is a table of dimension *N x N* with real coefficients, which defines the DWT and satisfies the condition $W^T W = I_N$.

In addition, for the transformation to be orthogonal it should be the case where $\mathbf{X}=W^T\mathbf{W}$ and $\|\mathbf{W}\|^2 = \|\mathbf{X}\|^2$.

Generally, the elements of table *W* are grouped, so that each group contains $N/2^j$ elements with $j = 1, \ldots, J$. To make this process more comprehensible, we can take the Haar DWT transformation using N = 16 as an example. In this case, the lines that will constitute the "boundary" between the different groups will be those corresponding to n = 0, 8, 12, 14 and 15, and the corresponding line-vectors are given below:

$$W_{0\bullet}^{T}=\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\underbrace{0,\ldots\ldots,0}_{14}\right]$$

$$W_{8\bullet}^{T}=\left[-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2},\underbrace{0,\ldots\ldots,0}_{12}\right]$$

$$W_{12\bullet}^{T}=\left[\underbrace{-\frac{1}{\sqrt{8}},\ldots,-\frac{1}{\sqrt{8}}}_{4},\underbrace{\frac{1}{\sqrt{8}},\ldots,\frac{1}{\sqrt{8}}}_{4},\underbrace{0,\ldots\ldots,0}_{8}\right]$$

$$W_{14\bullet}^{T}=\left[\underbrace{-\frac{1}{4},\ldots,-\frac{1}{4}}_{8},\underbrace{\frac{1}{4},\ldots,\frac{1}{4}}_{8}\right]$$

$$W_{15\bullet}^{T}=\left[\underbrace{\frac{1}{4},\ldots\ldots,\frac{1}{4}}_{16}\right]$$

The remaining 11 lines left to complete table *W* will be obtained with the aid of table *T*, which constitutes an ortho-normal transformation and for which we have that:

$$T\mathbf{B}\equiv\left[B_{N-1},B_0,B_1,\ldots,B_{N-3},B_{N-2}\right]^T$$

And

$$T^{-1}\mathbf{B}\equiv\left[B_1,B_2,B_3,\ldots,B_{N-2},B_{N-1},B_0\right]^T$$

Where B is N-dimensional column-table. With the help of table *T*, therefore, the remaining lines appear as follows:

$$W_{1\bullet} = T^2 W_{0\bullet}$$
$$W_{2\bullet} = T^4 W_{0\bullet}$$
$$\ldots\ldots$$
$$W_{7\bullet} = T^{14} W_{0\bullet}$$

και

$$W_{9\bullet} = T^4 W_{8\bullet}$$
$$W_{10\bullet} = T^8 W_{8\bullet}$$
$$W_{11\bullet} = T^{12} W_{8\bullet}$$

και

$$W_{13\bullet} = T^8 W_{12\bullet}$$

In this way, table *W* will be orthogonal.

Moreover, based on the above, relation (27) can be rewritten as follows:

$$\mathbf{W} = \begin{bmatrix} W_0 \\ . \\ . \\ . \\ W_7 \\ W_8 \\ . \\ . \\ W_{11} \\ W_{12} \\ W_{13} \\ W_{14} \\ W_{15} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}(X_1 - X_0) \\ . \\ . \\ . \\ \frac{1}{\sqrt{2}}(X_{15} - X_{14}) \\ \frac{1}{2}(X_3 + X_2 - X_1 - X_0) \\ . \\ . \\ \frac{1}{2}(X_{15} + X_{14} - X_{13} - X_{12}) \\ \frac{1}{\sqrt{8}}(X_7 + \ldots + X_4 - X_3 - \ldots - X_0) \\ \frac{1}{\sqrt{8}}(X_{15} + \ldots + X_{12} - X_{11} - \ldots - X_8) \\ \frac{1}{4}(X_{15} + \ldots + X_8 - X_7 - \ldots - X_0) \\ \frac{1}{4}(X_{15} + \ldots + X_0) \end{bmatrix} \tag{28}$$

Also using the formula:

$$\overline{X}_t(\lambda) \equiv \frac{1}{\lambda}\sum_{l=0}^{\lambda-1} X_{t-l}$$

we can define $\overline{X}_t(\lambda)$ as the sampling mean of the scale λ in a set of values [t-λ+1, t], and thereby, rewrite relationship (28) in a simpler and more comprehensible form, which is given by (29).

$$\mathbf{W} = \begin{bmatrix} W_0 \\ . \\ . \\ . \\ W_7 \\ W_8 \\ . \\ . \\ W_{11} \\ W_{12} \\ W_{13} \\ W_{14} \\ W_{15} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}}(\overline{X}_1(1) - \overline{X}_0(1)) \\ . \\ . \\ . \\ \frac{1}{\sqrt{2}}(\overline{X}_{15}(1) - \overline{X}_{14}(1)) \\ (\overline{X}_3(2) - \overline{X}_1(2)) \\ . \\ . \\ (\overline{X}_{15}(2) - \overline{X}_{13}(2)) \\ \sqrt{2}(\overline{X}_7(4) - \overline{X}_3(4)) \\ \sqrt{2}(\overline{X}_{15}(4) - \overline{X}_{11}(4)) \\ 2(\overline{X}_{15}(8) - \overline{X}_7(8)) \\ 4\overline{X}_{15}(16) \end{bmatrix} \tag{29}$$

Thus, the first *N/2* elements will refer to unit scale changes, the next *N/4* to double-scale changes, until we finally reach the last term that will be the average of the total signal.

The first *N-1* coefficients of table **W** are called *wavelet coefficients*, while the last coefficient W_{N-1} is the *scaling coefficient*. In addition, all rows of tale *W* which constitute a group, i.e. refer to a particular scale and produce the corresponding wavelet coefficients, are essentially circular displacements of one another.

Based on this separation and generalizing the case of DWT, we can write the table of DWT coefficients for a signal **X** of length $N=2^J$ as given below:

$$W\mathbf{X} = \begin{bmatrix} W_1 \\ W_2 \\ . \\ . \\ . \\ W_J \\ V_J \end{bmatrix} \mathbf{X} = \begin{bmatrix} W_1\mathbf{X} \\ W_2\mathbf{X} \\ . \\ . \\ . \\ W_J\mathbf{X} \\ V_J\mathbf{X} \end{bmatrix} = \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \\ . \\ . \\ . \\ \mathbf{W}_J \\ \mathbf{V}_J \end{bmatrix} = \mathbf{W} \qquad (30)$$

Where:

- table W_j has dimension $N/2^j$ x N
- Vj is a vector 1 x N
- $\mathbf{W}_j$ is a column-vector of length $N/2^j$
- $\mathbf{V}$j includes the last element of $\mathbf{W}$

1.3.2 Wavelet Filter

So far, a qualitative analysis of the DWT transformation process has been given and only the Haar DWT has been described in detail. In the following paragraphs, we will present the process by which DWT transformation is created through an algorithm, also known as the pyramid algorithm, which was first used in wavelet study by Mallat (1989).

More specifically, this algorithm uses linear filters indicating extraordinarily high speed, which exceeds that of the Fast Fourier Transform (FFT).

Initially, to begin the analysis, we consider a real wavelet filter $\{h_l$: l = 0,…, L-1$\}$. For this filter to have width L, it should be $h_0 \neq 0$ and $h_{L-1} \neq 0$. Moreover, we define that its values outside interval [0, L-1] should be zero, that is, $h_l = 0$ for $l < 0$ and $l \geq L$.

A wavelet filter should also satisfy the following conditions:

- its total sum being zero: $$\sum_{l=0}^{L-1} h_l = 0 \qquad (31)$$
- its energy to be unitary: $$\sum_{l=0}^{L-1} h_l^2 = 1 \qquad (32)$$
- to be perpendicular to its even displacements: $$\sum_{l=0}^{L-1} h_l h_{l+2n} = 0 \qquad (33)$$

Conditions (32) and (33) are necessary for the wavelet filters to be orthogonal. In addition, we also define the transfer function H(f) of the filter as follows:

$$H(f) \equiv \sum_{l=-\infty}^{+\infty} h_l e^{-i2\pi fl} = \sum_{l=0}^{L-1} h_l e^{-i2\pi fl} \qquad (34)$$

According to the theory mentioned in the previous paragraph, the N / 2 wavelet coefficients for the unit scale are:

$$W_{1,t} \equiv 2^{1/2} W_{1,2t+1} \;\; \text{για} \;\; t = 0,..,\frac{N}{2}-1 \qquad (35)$$

And

$$2^{1/2} W_{1,t} \equiv \sum_{l=0}^{L-1} h_l X_{t-l \bmod N} \;\; \text{για} \;\; t = 0,..,N-1 \qquad (36)$$

By replacing (36) within (35) we get:

$$W_{1,t} = \sum_{l=0}^{L-1} h_l X_{2t+1-l \bmod N} = \sum_{l=0}^{L-1} h_l^o X_{2t+1-l \bmod N} \;\; \text{για} \;\; t = 0,..,\frac{N}{2}-1 \qquad (37)$$

Relation (37) shows that we are filtering the original signal $\{X_t\}$ with a wavelet filter $\{h_t\}$ to get the wavelet coefficients for the unit scale. In addition, we have defined h^o_l in the case where L ≤ N as follows:

$$h_l^o = \begin{cases} h_l \;, & 0 \le l \le L-1 \\ 0 \;, & L \le l \le N-1 \end{cases} \qquad (38)$$

The study of two specific Daubechies wavelet filters, Haar and D(4), has led to the conclusion that these are high pass filters with a rated lower frequency for which $|f| \in [1/4, 1/2]$.

1.3.3 Scaling Filter

While using the wavelet filter, we can get the first N/2 coefficients of table *W* given in (30), the scaling filter helps us to get table V_1, with dimension N/2xN of the same relationship. This process is performed so that we can finally calculate the last N/2 lines of table W. It should be stressed that, except in the case where N = 2 so that it is J = 1, the lines of table V_1 do not constitute the final lines of table W, that is, the vector V_J.

Filter $\{g_l\}$ is fact a quadrature mirror of the wavelet filter analyzed in the previous paragraph. To derive it we use the following relationship:

$$g_l \equiv (-1)^{l+1} h_{L-1-l} \qquad (39)$$

By reversing the above relationship, we get:

$$h_l = (-1)^l g_{L-1-l} \qquad (40)$$

The basic properties of the scaling filter are equivalent to those of the wavelet filter. In this case, we can say that this is true for:

- its total sum being zero: $\sum_{l=0}^{L-1} g_l = \sqrt{2}$ ή $\sum_{l=0}^{L-1} g_l = -\sqrt{2}$
- its energy to be unitary: $\sum_{l=0}^{L-1} g_l^2 = 1$
- to be perpendicular to its even displacements: $\sum_{l=0}^{L-1} g_l g_{l+2n} = \sum_{l=-\infty}^{+\infty} g_l g_{l+2n} = 0$

Also, the transfer function of the scaling filter is given by (41), which connects it to the transfer function of the corresponding wavelet filter:

$$G(f) \equiv \sum_{l=-\infty}^{+\infty} g_l e^{-i2\pi fl} = \sum_{l=0}^{L-1} g_l e^{-2\pi fl} = e^{-2\pi f(L-1)} H\left(\frac{1}{2} - f\right) \qquad (41)$$

If we consider for both the wavelet and the scaling filter that $\mathcal{H}(f) \equiv |H(f)|^2$ and $\mathcal{G}(f) \equiv |G(f)|^2$ where $\mathcal{H}$(f) and $\mathcal{G}$(f) are the squared profit functions for the filters, we can write, using relationship (41), that:

$$\mathcal{G}(f) = \mathcal{H}\left(\frac{1}{2} - f\right) \qquad (42)$$

Using the squared $\mathcal{H}$(f) and $\mathcal{G}$(f) gain functions we can briefly rewrite a relationship for each of the last two properties of the scaling filter and the wavelet filter as follows:

$$H(f)+H\left(f+\frac{1}{2}\right)=2 \qquad \text{και} \qquad G(f)+G\left(f+\frac{1}{2}\right)=2 \qquad (43)$$

Based on (42) we can get:

$$G(f)+H(f)=2 \quad \text{for all f} \quad (44)$$

From the above analysis, we can conclude that a scaling filter $\{g_l\}$ behaves like a low-pass filter with a frequency spectrum [-1/4,1/4]. Finally, the N/2 first scaling coefficients are defined similarly, as in the case of wavelet coefficients, as:

$$V_{1,t} \equiv 2^{1/2} V_{1,2t+1} \qquad \gamma\iota\alpha \qquad t=0,\ldots,\frac{N}{2}-1 \quad (45)$$

Where:

$$2^{1/2} V_{1,2t+1} \equiv \sum_{l=0}^{L-1} g_l X_{t-l \bmod N} \qquad \gamma\iota\alpha \qquad t=0,\ldots,N-1 \qquad (46)$$

By replacing it follows that:

$$V_{1,t} \equiv \sum_{l=0}^{L-1} g_l X_{2t+1-l \bmod N} = \sum_{l=0}^{N-1} g_l^o X_{2t+1-l \bmod N} \qquad \gamma\iota\alpha \qquad t=0,\ldots,\frac{N}{2}-1 \qquad (47)$$

As before, in correspondence with $\{h^o_l\}$, $\{g^o_l\}$ is the filter $\{g_l\}$ but limited in length N. In many cases, the scaling filter is also referred to as "father wavelet", whereas the wavelet filter as "mother wavelet ".

1.3.4 Pyramid Algorithm

Pyramid algorithm is a process by which DWT is finally transformed into a signal. In the previous two sections, we have basically described the first step of this algorithm.

Percival & Walden (2000) consider that for an input signal **X** of length N = 2^J there are *J -1* stages. Each stage *j (j = 0,1, ..., J*) transforms table $\mathbf{V}_{j-1}$ of length $N/2^{j-1}$ (calculated in the previous step), in the vectors $\mathbf{V}_j$ and $\mathbf{W}_j$ of length $N/2^j$. Therefore, at each stage *j* vector $\mathbf{V}_{j-1}$ represents the input signal of the stage and is filtered by the filters $\{h_t\}$ and $\{g_l\}$ respectively. Finally, in the final stage J we can form table **W** of the DWT, which consists of the individual tables $\mathbf{W}_j$ of each stage and the table $\mathbf{V}_J$ of the final stage. The first step of the algorithm and the overall process followed, are given in Figures 1.2 and 1.3.

1st Stage of the Pyramid Algorithm

Creation of tables $\mathbf{W}_1$ and $\mathbf{V}_1$ including the N/2 wavelet and the N/2 scaling coefficients of the 1st stage.

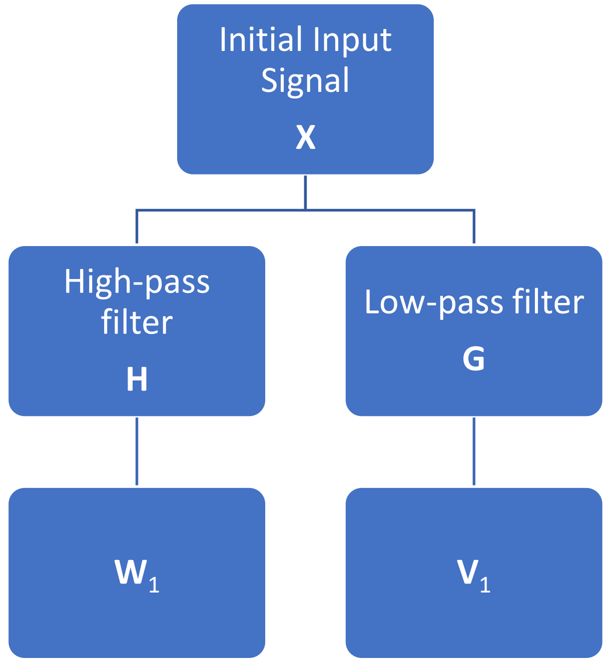

Figure 1.2: Graphical representation of the 1st stage of the Pyramid algorithm. The total number of samples obtained is N/2, after the signal passes from the filters. This is the number of samples that act as inputs of the 2nd stage.

Successive stages of the Pyramid algorithm

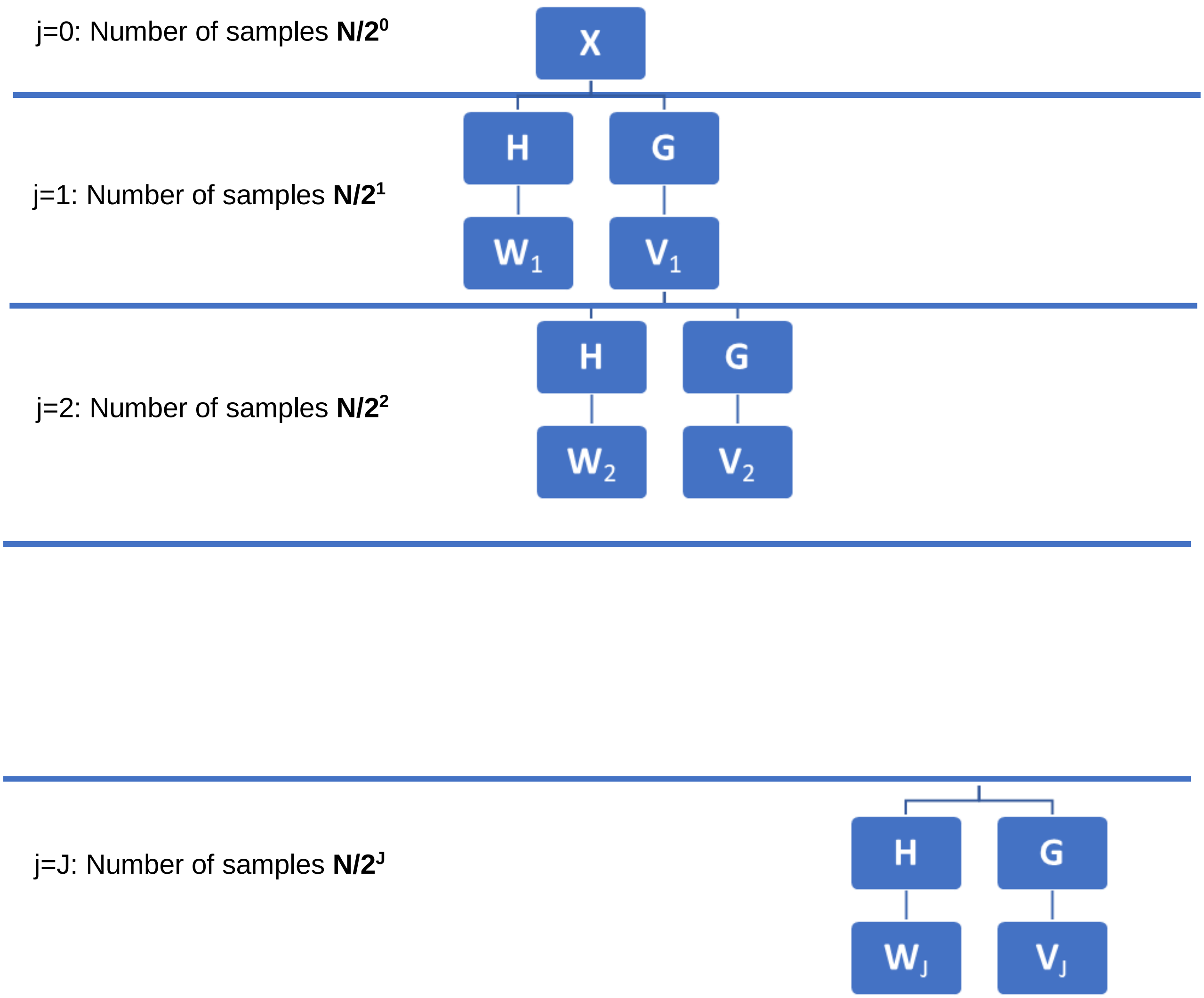

Figure 1.2: Schematic representation of the overall Pyramid algorithm process. The final DWT is derived from the $\mathbf{W}_j$ tables in each stage and the $\mathbf{V}_J$ table which is created in the final stage.

1.4 Comparison to Fourier Transform

In order to understand the basic differences between the Fourier Transform and wavelet transformation we should make a reference to the evolution of the first.

Fourier Transform was introduced as a mathematical tool that would help in analyzing signals, with the primary goal of revealing information in the frequency domain, which was not apparent in the time representation of the input signal. Through the Fourier Transform, this was ultimately achieved, with a major disadvantage that any time information is lost in the frequency-width representation.

In addition, sinusoidal functions of different frequencies are used as basis for Fourier transform, reaching infinity. This results in the Fourier inverse transformation being applied only in cases where there are infinite, periodic and static input signals, which is a major problem for the analysis of transient phenomena that may appear in the signals. Also, in this way of analysis, local characteristics of the signal become in the end intrinsic features of the Fourier transform.

The introduction of the short-time Fourier Transform (STFT) tries to solve this problem, i.e. the extraction of local information from the original signal. The main concept of STFT is that we have a "window" base function, which is shifted all the time, so the "window" separates our original signal into smaller parts. Assuming that each of these small parts of the signal is stationary, makes it possible to apply the Fourier transform.

Regarding the size of the "window" used to allocate the input signal, this is subject to Heisenberg's inexplicable principle. This means that we can know only time intervals where there are specific frequencies, that is, the more we reduce the size of the "window", the more the range of frequencies increases that can be accurately determined and vice versa.

The main disadvantage of STFT is the fact that the "window" we use remains constant in size, which makes STFT unable to detect high frequency signals, as well as to detect signals at low frequencies. A graphical representation of STFT or Windowed Fourier Transform (WFT) is shown in Figure 1.4. This disadvantage of STFT has led gradually to the evolution of the wavelet transform.

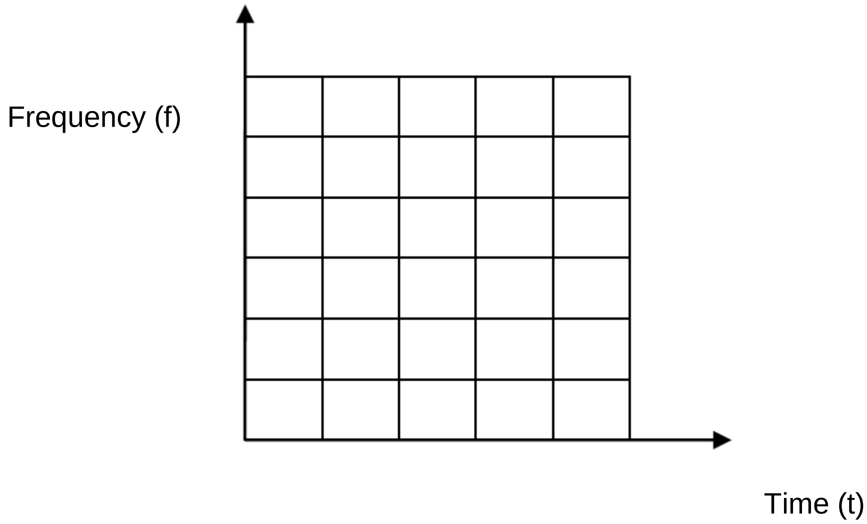

Figure 1.4: STFT- Short-term Fourier Transform. We notice that "windows" remain constant throughout the process.

The central concept of wavelet transformation is the use of a "window" function of a base function whose size can be varied at the different frequencies. This idea is illustrated in Figure 1.5, where we see that at low frequencies the width and frequency of "windows" are different from the corresponding high frequency sizes.

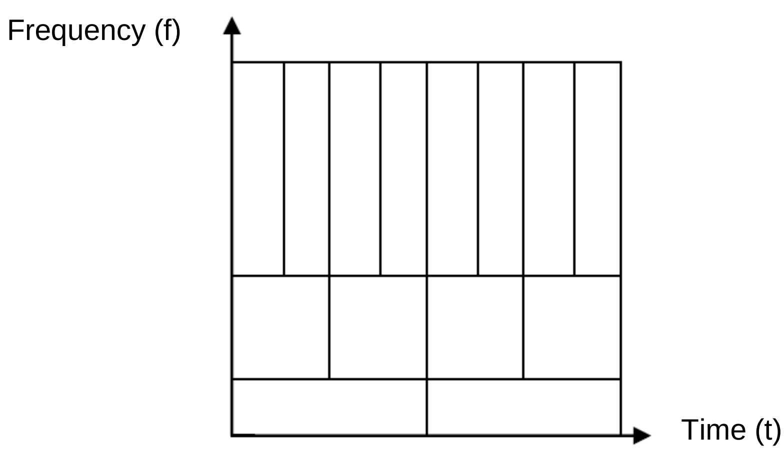

Figure 1.5: DWT - Discrete Wavelet Transformation. The "windows" in this case vary both in frequency and in width.

In addition, this possibility of varying the size of the "window" base functions used in the wavelet transform allows for the identification of short-duration and high-frequency phenomena, also known as transient phenomena, that are likely to occur in our original signal. Also, in the use of the wavelet transform there are no additional limitations concerning the nature of the input signal. That is, it can be of any form, for example not necessarily stagnant as it happens in the Fourier transform.

Summing up, we can say that the basic difference between the Fourier Transform and the wavelet transform lies in the fact that different base functions are used in their analysis. This results in the Fourier transform have a limited range of applications, being effective only when the input signal is stationary, and also, they cannot be used in the analysis of transient phenomena, as opposed to the wavelet transform which can be used to thoroughly analyze non-stationary signals.

1.5 Applications of the Wavelet Transform

In general, research and literature on the use of wavelets includes a variety of applications, which began with broader scientific fields, such as geology and telecommunications, and later expanded into the field of economic analysis. Given the thematic focus of this thesis, reference is made to the most important applications related to the field of economics.

In a more general technical framework, a substantial and extensive mathematical analysis of the use of wavelets for the analysis of time series is made in the article by Chiann et al. (1998). Below are the books by Percival & Walden (2000) and Gençay et al. (2002). Finally, a good mathematical introduction to the wavelet theory is given by Crowley (2007). Regarding wavelet applications in economic science, Ramsey & Zhang (1996) are two of the first to use this method to study the daily exchange rates between the Japanese yen, the German mark and the US dollar. Through wavelet libraries, they identify correlations between the frequency domain parities, which are not visible in the time domain.

In another study, Ramsey (1996), wishing to make a categorization of wavelet applications in the field of economics, indicates four categories. First, he defines papers referring to the ability of wavelets to be able to analyze non-stationary and highly complex signals. The second group represents those articles dealing with the structures of time series and the local phenomena that may appear, while in the third group, he includes studies that have as their main objective the analysis of signals at various time scales. Finally, the fourth category refers to the prognosis of time series. Subsequently, in their article Ramsey & Lampart (1998) focuses on highlighting the importance of scale-time analysis and how through this can be interpreted various anomalies in time series ignored by classical analysis. They also emphasize the importance of signal phase fluctuations in economic analysis.

Moving on to various economic applications, Gençay et al. (2000) focus on exporting intraday seasonalities from the original input signal. This is done by using wavelets to analyze the signal on the high and low frequency components, which in turn are filtered to extract a final signal free of intraday seasonalities. A little later (Gençay et al., 2003) proposed a new method for assessing systemic risk in a CAPM. According to this method, the wavelet transformation of market performance is calculated on each scale, as well as the covariance between the wavelet transformation of market performance and this portfolio. Finally, via these two sizes, it is possible to determine the beta of the portfolio. The empirical results of their research show that the relationship between the performance of the portfolio and its beta becomes stronger as the scale of wavelet transformation increases, so the CAPM model predictions are more relevant over a long-term scale than one short-term. A similar analysis for assessing systemic risk is also made in a subsequent article (Gençay et al., 2005).

Correspondingly, Kim & In (2005) examine the Fisher case, based on which there is a positive relationship between nominal stock returns and inflation. As a result of their research, based on the wavelets analysis, this relationship between stock returns and inflation is positive in cases where we have very long (128-month) or very small scales (1-month period) while a negative correlation appears in the intermediate scales. Correlations between futures markets and stock exchanges are also a very interesting subject that In & Kim (2006) discusses in their next study. In this article, the wavelets analysis shows both the type of correlations between the markets and their size, which is increasing as the scale of the time on which the analysis is made increases. They also study the risk hedging made through futures markets in relation to its effectiveness over various time scales.
In a similar philosophy, the next article of Kim & In (2007), which refers to the relationship that stock price fluctuations and changes in bond yields in G7 countries can have, is moving. In this case, the results of the survey show that the correlation between these changes varies from country to country and from scale to scale. In addition, stock price changes and bond yields are not moving together in most of the G7 countries except Japan.

Moving on, Fernandez (2006) deals with asset pricing and investor heterogeneity and concludes through wavelet analysis in conclusions that are consistent with existing CAPM theories. In particular, he creates an international version of CAPM, taking data on the stock indices of seven emerging countries in Latin America and Asia (for the period 1999-2004), which represents market risk and exchange rate risk, and then decomposes the model this through the wavelet transform. The conclusions reached are as follows:

- The results of the estimates depend on the world market portfolio, and in particular, the markets under study appear to be more interconnected with corresponding emerging economies and not with already developed.
- Value-at-Risk of a portfolio depends on the time horizon of the investor, where the potential losses in the short term are larger than the long-term horizon.
- Finally, further exposure to specific stock indices increases the Value-at-Risk range, always in relation to the time horizon of the investment.

Staying in the stock market analysis, Gallegati (2007) performs a wavelet analysis and finds a correlation between stock market yields and the US economy's economic activity. The results show that stock market returns tend to precede the level of economic activity in the economy, but only in the higher time scales corresponding to periods of 16 months and over.

Another interesting study is that of Tabak & Feitosa (2008), which applies a Multiresolution Analysis to check whether the Brazilian bond spreads curve contains information content with predictive value for the course of inflation. Initially, they divide the total time period into two subperiods: before and after the implementation of the strategy inflation targeting (IT). They then apply the wavelet analysis on the two input signals, the inflation rate and the bond rate, so as to get the corresponding time series of each for the individual time periods. Taqak & Feitosa (2008) conclude that before the IT application the predictive capacity of the spread is insignificant in both time scales studied. The effect, however, is changing for the period following the IT implementation, where there is some predictive capacity of spreading over the course of inflation, which is very important for determining the economic policy that a government should follow.

Rua & Nunes (2009) deal with wavelet analysis of the movements of various stock markets and how they relate to each other. More specifically, the analysis focuses on the US, UK, Japanese and German markets, and on all the possible correlations that may arise between them. Their analysis indicates that, firstly, Germany seems to be at low frequencies to follow largely the US and UK traffic throughout the sample. This coexistence between the countries since the late 1990s seems to be expanding to all frequencies, indicating that there has been some fundamental change in the structure of the markets at that time. Secondly, between the United States and the United Kingdom, there appears to be a strong correlation in stock market movements at all frequencies except very high for periods of less than one year, where again there are periods where the correlation is growing. Finally, the Japanese market seems to have little correlation with all other markets, as has been shown in other previous studies.

The next article of Rua (2010) moves on the same idea, but this time focuses on the analysis of the correlation of the economic circles of the large Eurozone countries. In this case, there is an increased correlation between countries at low frequencies, except for Germany and Italy. In particular, with respect to the typical frequencies of economic cycles (2-8 years), there appears to be an increase and stronger relationship since the mid-1990s, while for frequencies of less than 2 years movement associations seem to be weakening. In addition, it is stressed that a possible interpretation of the increase in economic cycle correlations in recent years is the creation of the Eurozone, which has in fact largely implemented the integration of the European market through the common monetary policy.

Naccache (2010) studies oil circles and the relationship between oil prices and Morgan Stanley Capital International (MSCI) Index for the World. Through a multi-resolution analysis, the study period 1946-2008 is divided into 9 analytical frequencies of the original signal. The results of the analysis have identified all shocks in the oil market. In the 2 to 4 months (D1) frequency, the shock of 1974 and 1985, in the frequency of 8-16 months (D3), the shock of 1991, also associated with the Gulf War, is shown, while at lower frequencies (D7 and D8) there are smaller shocks affecting the price of oil. With to the oil circles, D1 to D6 levels include oil cycles lasting from 2 months to 10 years, which can be matched with the real economic cycles of the macroeconomic and appear in the early 1970s. Correspondingly, D7 and D8 levels include much longer cycles, which are associated with energy investment cycles and are unlike the previous ones throughout the sample. Finally, at D9 level there appears to be only one cycle lasting 50 years and related to technological progress.

Chapter 2: Empirical application

This chapter presents the methodology followed in this thesis, as well as the results obtained from the analysis of the data.

2.1 Methodology

The basic wavelet methodology used is that of the Circular Maximal Overlap Discrete Wavelet Transform (MODWT), which is essentially a variation of the already known DWT.

Some of the basic differences of the two transformations are as follows:

- The MODWT transformation follows, like DWT, a stable pyramid algorithm, but it presents some structural differences compared to the classic DWT transformation. At each level, the pyramid algorithm applies not only to the original input signal **X** but also to its cyclic shift $T\mathbf{X}$. So, in this way, we avoid the sensitivity of DWT in choosing the starting point of a chronological order.
- Another basic difference of MODWT is the fact that it is not ortho-normal, unlike DWT.
- Also, in the case of DWT J_0 level, our sample is limited to a size that should be equal to an integer multiple of 2^{J0}, while in the case of MODWT our sample is well defined for any size N. This difference results in the number of calculations required for DWT being $O(N)$, while the corresponding calculations for MODWT are of the order of $O(N\log_2 N)$. Therefore, we observe that the use of MODWT carries an additional computational load, but this is equivalent to that of the fast Fourier transform.

In addition, the analysis is carried out based on the studies of Naccache (2010), Kim & In (2007) and In & Kim (2006), and thus, the least asymmetric filter LA(8) has been used.

2.2 Data

We use data referring to daily returns from the US, United Kingdom and Korean stock markets, starting from January 1st, 1973, until August 23rd, 2002. The choice of using daily returns was made based on Kim & In (2005) and In & Kim (2006) studies. In addition, with this choice of data, we wanted to have a difference in the types of markets analyzed (US and UK markets are developed, while Korea is emerging) so that we can identify their differences in the short-term and long-term investment horizons.

Each time series is of the size N = 7734, based on which the level of the MODWT transformation was determined. This level is given by formula (48):

$$J_0 < \log_2 7734 \qquad (48)$$

Thus, in our case J_0 =12.

So, we have 12 levels of frequency that are given below, in correspondence with the periods of time they express:

- Fr1: T1=2-4 days
- Fr2: T2=4-8 days
- Fr3: T3=8-16 days
- Fr4: T4=16-32 days
- Fr5: T5=1-2 months
- Fr6: T6=2-4 months
- Fr7: T7=4-8.5 months
- Fr8: T8=8.5-17 months
- Fr9: T9=1.4-2.8 years
- Fr10: T10=2.8-5.6 years
- Fr11: T11=5.6-11.2 years
- Fr12: T12=11.2-22.5 years

For ease of listing the results, the names of the frequencies will be shown on the diagrams rather than the time periods to which they refer.

2.3 Results

The results obtained from the analysis of the data are set out in the following paragraphs by category. Prior to their presentation, a small theoretical introduction precedes the conclusions.

2.3.1 Kurtosis

First, we consider a distribution function f(x), which corresponds to the variable X. The average, or otherwise, the expected value of the variable is given according to Greene (1993) from relationship (49):

$$E[x]=\begin{cases}\sum_x xf(x) & \text{in case x is discrete}\\ \int_x xf(x)\mathrm{dx} & \text{in case x is continuous}\end{cases} \tag{49}$$

The mean is usually denoted by *μ,* and thus, the variability of the variable X, the formula of which is given by the relation (50), can be determined:

$$Var[x]=E\left[(x-\mu)^2\right]=\begin{cases}\sum_x (x-\mu)^2 f(x) & \text{in case x is discrete}\\ \int_x (x-\mu)^2 f(x)dx & \text{in case x is continuous}\end{cases} \tag{50}$$

The variance is defined as the second central moment of the variable X, and correspondingly, the other central moments according to the general formula (51) are also derived:

$$\mu_r=E\left[(x-\mu)^r\right] \tag{51}$$

Kurtosis is the fourth central moment, expressing how thin or wide the tails of the distribution function f(x) are. The corresponding formula is given below:

$$\mu_4=E\left[(x-\mu)^4\right] \tag{52}$$

Its value in the normal distribution is 3. In the case where the curve of a sample is greater than 3, we consider that we have finiteness, which suggests the existence of heteroskedasticity and hence speculation. In addition, in cases where very high values are present, there is a higher probability of extreme returns.

Our analysis has been performed in EViews7. The following charts (Figures 2.1, 2.2 and 2.3) for the three countries were obtained, taking as a baseline the Korean case, which indicates the highest values. What we observe by making an analysis between the different frequencies

(Figures 2.1, 2.2 and 2.3) is that at high frequencies, that is to say in the short term, kurtosis presents higher values than the lower frequencies in all three time series under investigation.

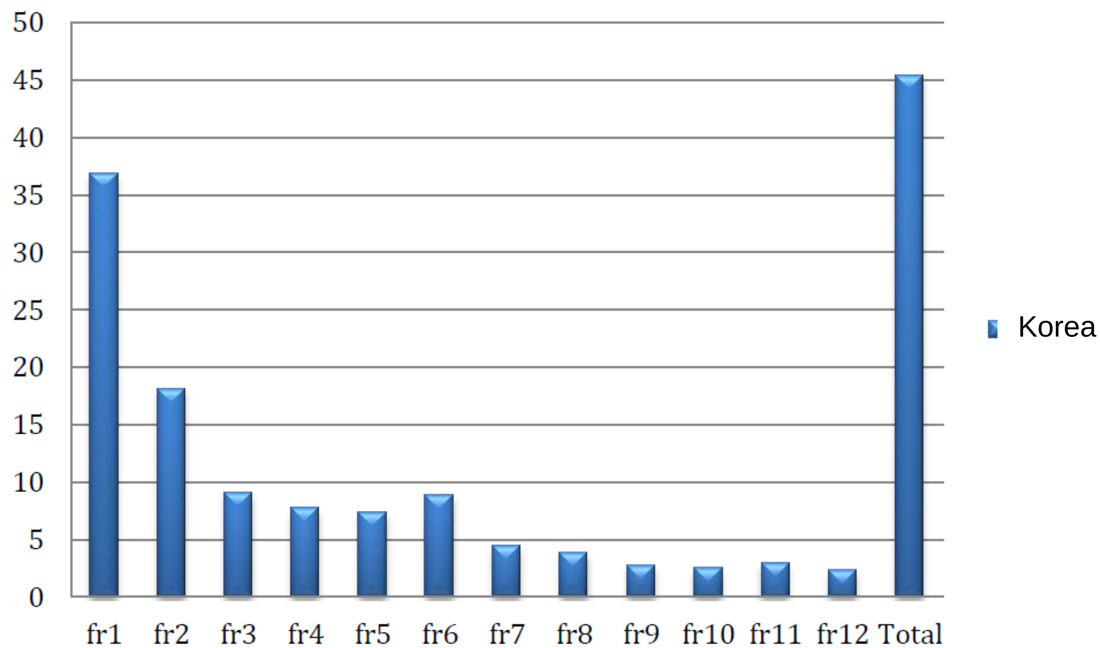

Figure 2.1: Kurtosis values at different frequencies for Korea.

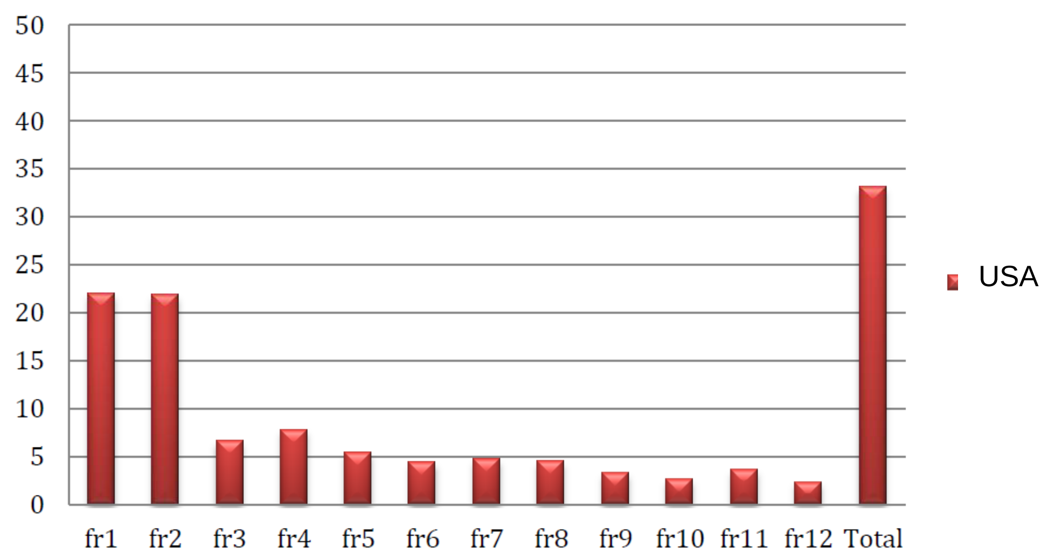

Figure 2.2: Kurtosis values at different frequencies for USA.

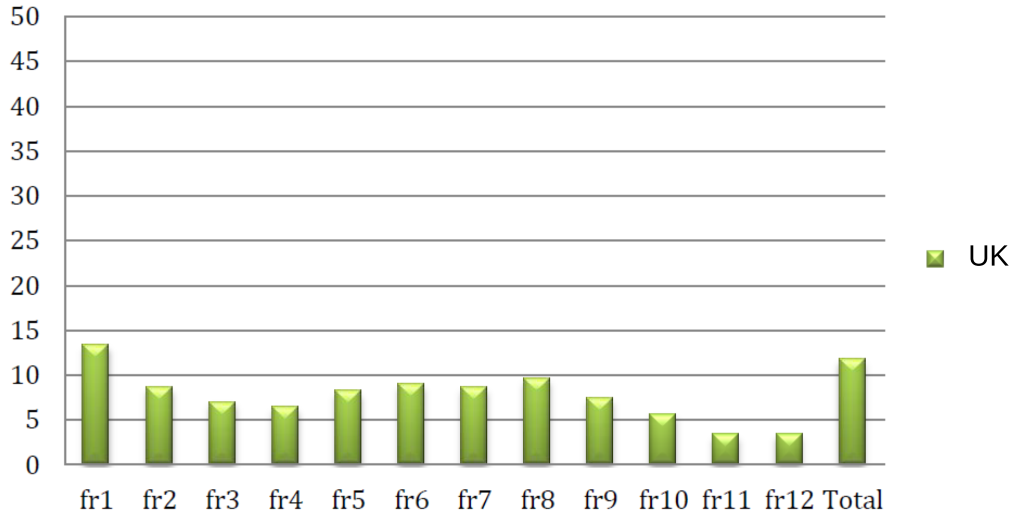

Figure 2.3: Kurtosis values at different frequencies for the UK.

Making a comparison between the different markets (Figure 2.4) strongly suggests that in the case of Korea, an emerging market, kurtosis values in both the overall sample and the high frequencies are much higher. We also notice that in the case of the United States, there is a fairly large variation in short-term bending and long-term investment horizons, unlike the United Kingdom, which shows a more stable picture.

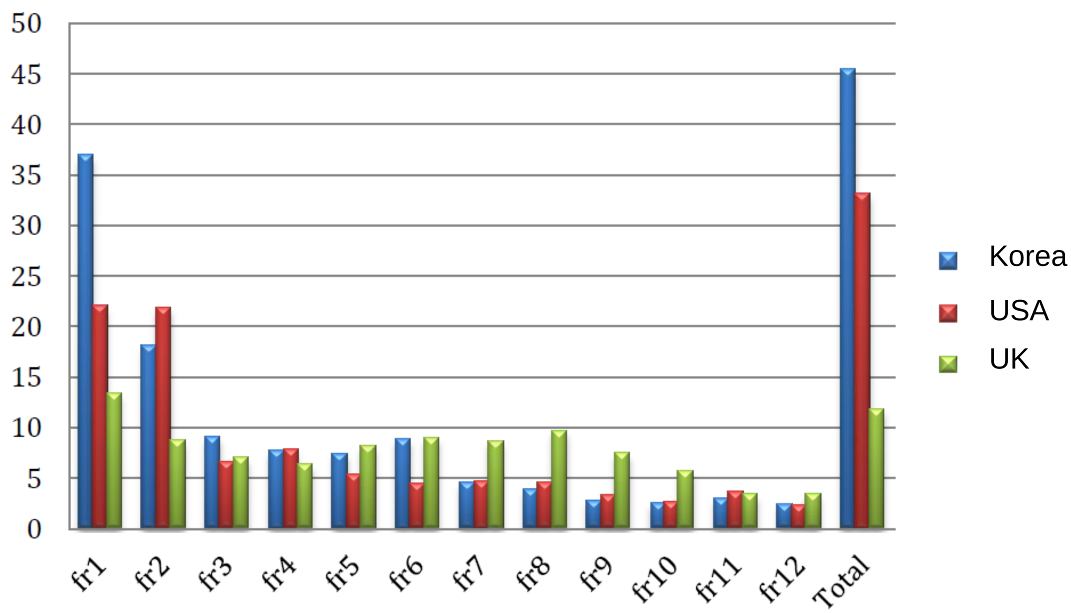

Figure 2.4: Kurtosis values at different frequencies for the three countries.

2.3.2 Skewness

In addition to the analysis made in the previous section, the third central moment of a variable **X** defines the asymmetry which, as its name implies, expresses the asymmetry of a distribution function. Its generic formula is given by relation (53):

$$\text{skewness} = \mu_3 = E\left[\left(x-\mu\right)^3\right] \qquad (53)$$

In addition, for symmetric distributions, it is true that:

$$f\left(\mu - x\right) = f\left(\mu + x\right) \qquad (54)$$

In the normal distribution the asymmetry value is 0. Also, for asymmetric distributions the asymmetry value will be positive in the case where its "long" tail is on the right of the mean *μ*, while vice versa, when we have negative asymmetry values its "long" tail is left of the mean *μ*. This latter case can be interpreted economically as the existence of more extreme negative observations, suggesting that the market is going through a bear market period.

From the wavelet analysis we can observe that in the case of Korea (Figure 2.5), we have negative asymmetry in almost all the frequencies, while in the case of the United States, and the United Kingdom (Figures 2.6 and 2.7 respectively) show both positive and negative values. However, we can say that in all three cases there are significant negative returns, since in the initial aggregate samples the asymmetry is negative. In order of magnitude, Korea has a more pronounced overall asymmetry, followed by the United States. and finally, the United Kingdom.

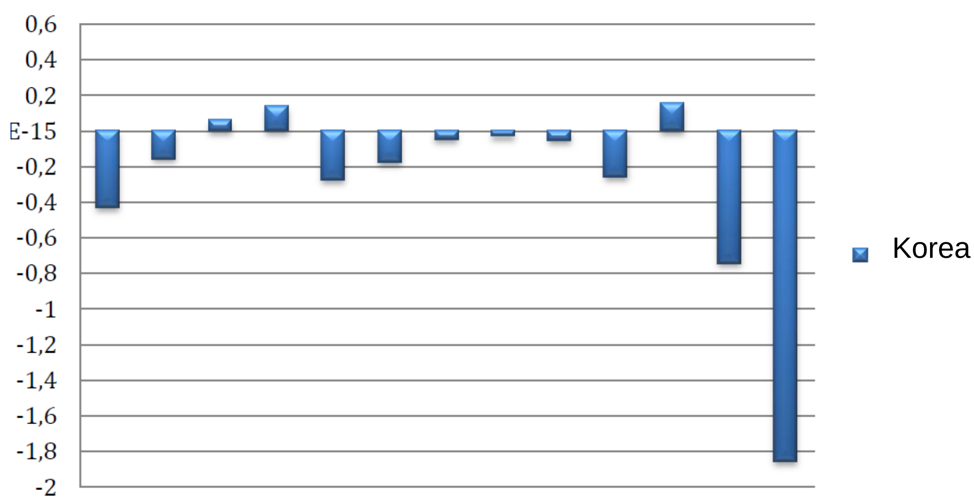

Figure 2.5: Skewness values at different frequencies for Korea.

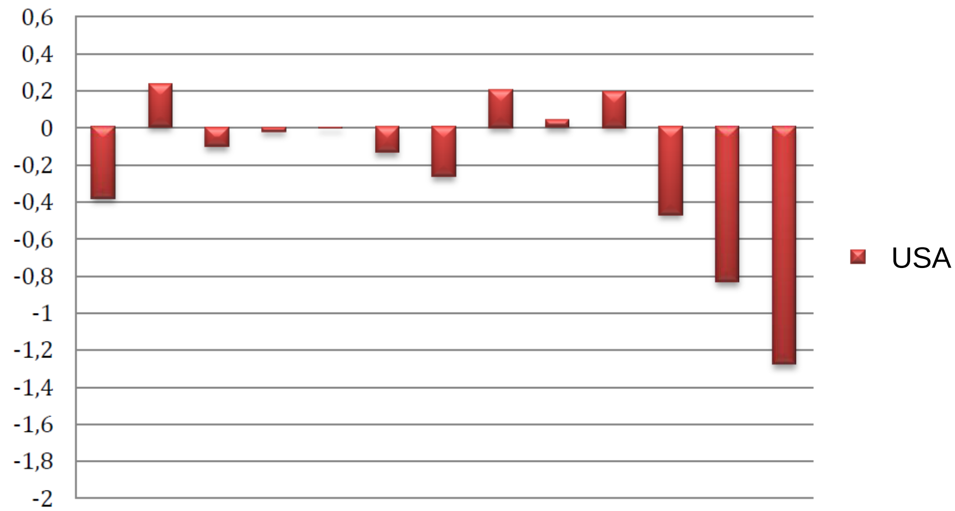

Figure 2.6: Skewness values at different frequencies for USA.

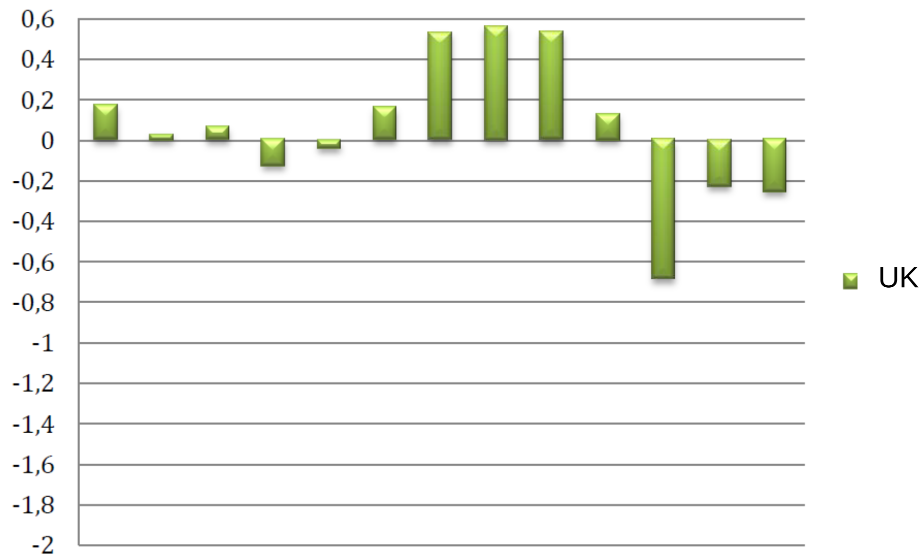

Figure 2.7: Skewness values at different frequencies for the UK.

2.3.3 Autocorrelation

According to Kendall & Buckland (1971) autocorrelation is defined as "the association between the members of a series of observations, either in time (as in time series) or in space (as in cross-sectional data)".

In particular, the autocorrelation function of degree k of a signal *X* with *N* number of observations is given by the general formula (55):

$$r_k = \frac{\sum_{i=1}^{N-k}\left(X_i - \overline{X}\right)\left(X_{i+k} - \overline{X}\right)}{\sum_{i=1}^{N}\left(X_i - \overline{X}\right)^2} \qquad (55)$$

As we can see, it consists of a set of autocorrelation coefficients, which show us the relation of a value of variable X at time *t* with its corresponding value at time *t-τ*. If the autocorrelation coefficient is positive, we can say that the variable X at time *t* has the same behavior as the moment *t-τ*, respectively, if it has a negative value, its behavior will be opposite.

Below are the charts with autocorrelation coefficients for each country at all frequencies. What we observe is that in all three markets the behavior of autocorrelation is similar. However, in the case of Korea for high frequencies, autocorrelation values are lower (Figure 2.8). As we move to smaller frequencies, we see a change in performance behavior. They now acquire a long-term memory, having large autocorrelation coefficients in all lags (Figures 2.8, 2.9, 2.10).

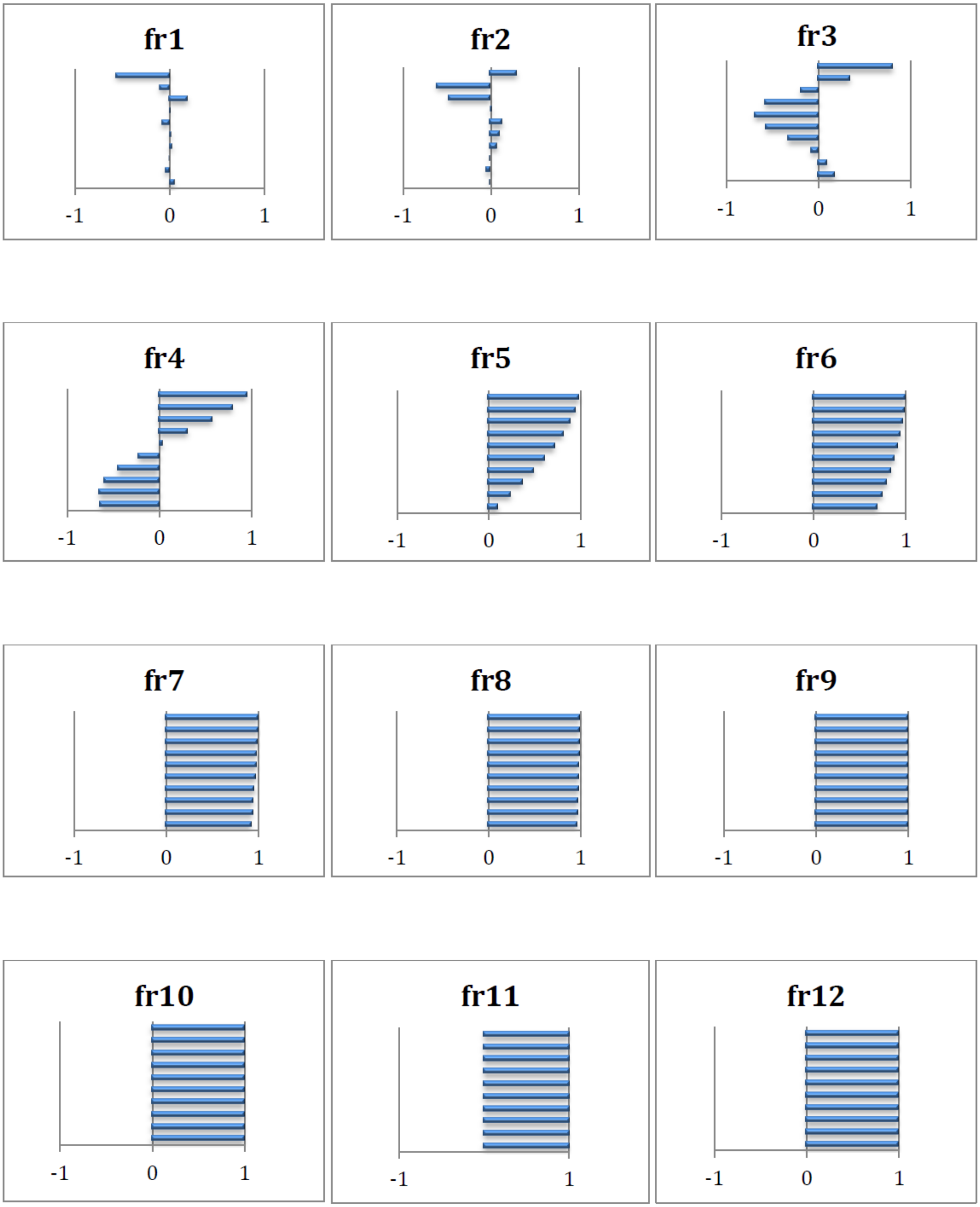

Figure 2.8: Autocorrelation coefficients at different frequencies for Korea.

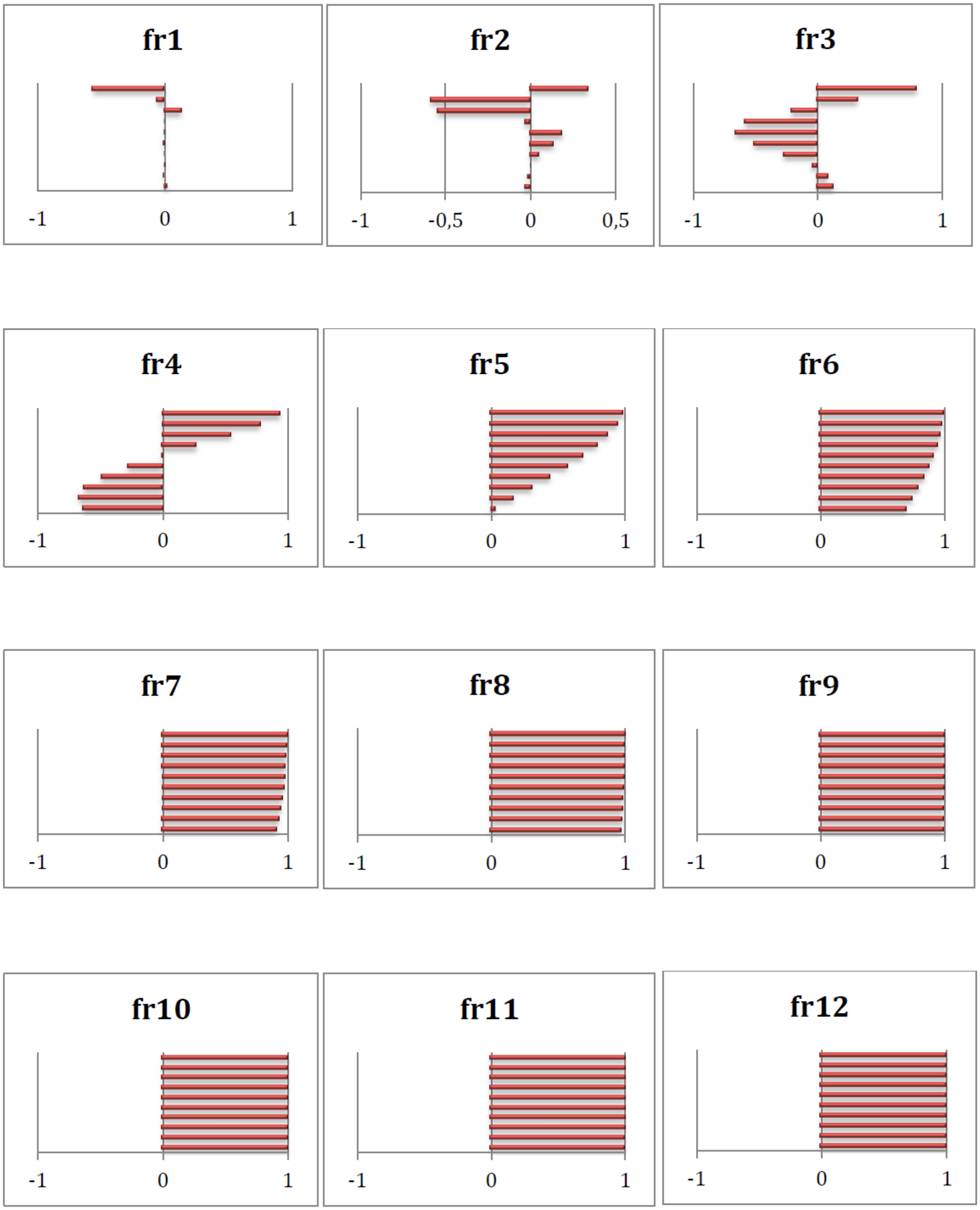

Figure 2.9: Autocorrelation coefficients at different frequencies for the US.

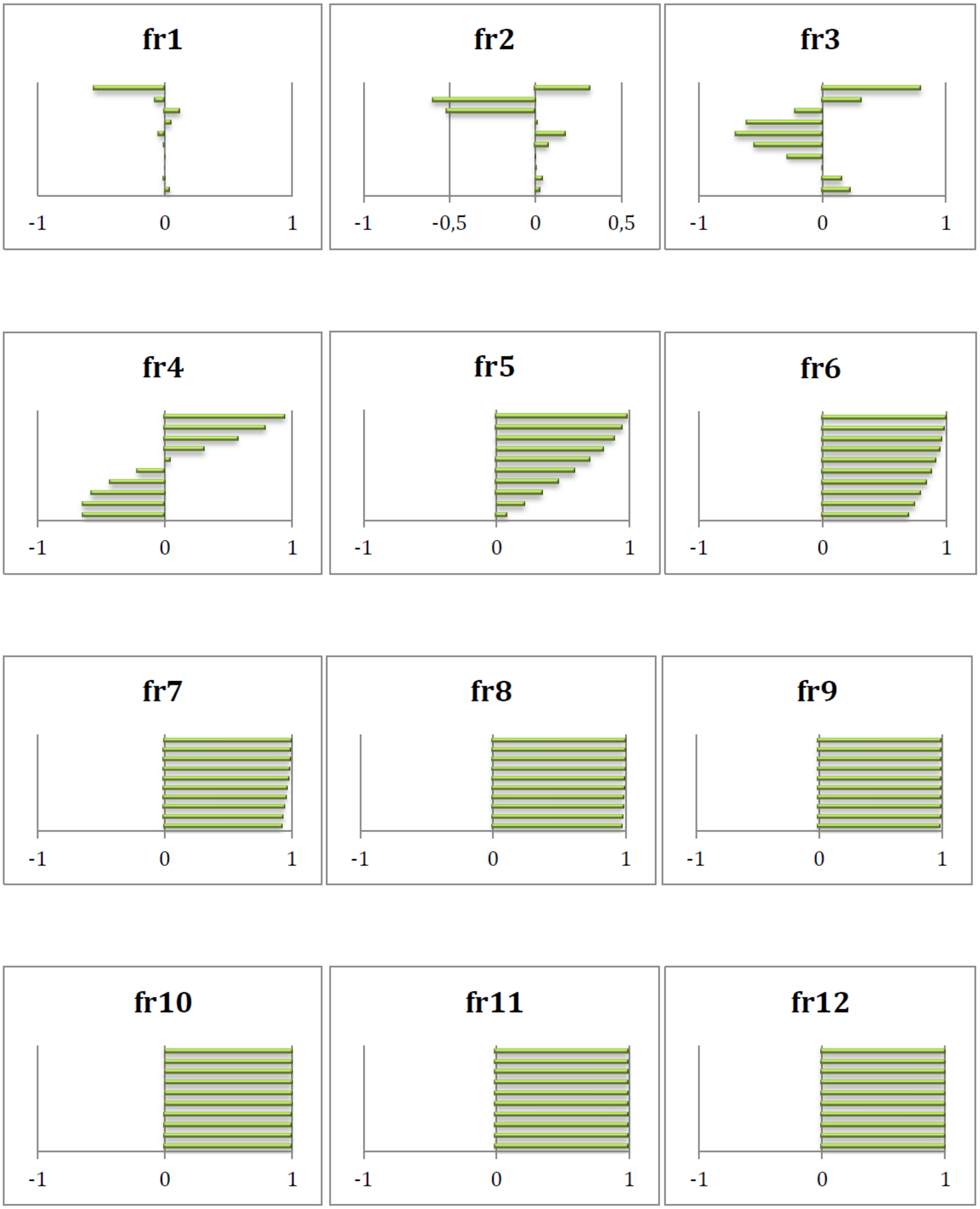

Figure 2.10: Autocorrelation coefficients at different frequencies for the UK.

2.3.4 BDS test

The BDS test according to Brock et al. (1987) is essentially a non-parametric method for controlling the existence of serial dependence and non-linear structure in a chronological order. By applying this method in a chronological order, we check the null hypothesis that our order is Independently and Identically Distributed (IID), thus a random chronological order. Also, the BDS test can also be used as a measure to test the goodness of fit of a model for residual dependence, by applying it to the model's residuals.

The correlation integral is a method originally introduced by Grassberger & Procaccia (1983) and helps us to identify the frequency with which some patterns can be repeated in a chronological order. Initially, assuming a chronological order x_t and taking its history of size *m,* given as $x_t^m = (x_t, x_{t-1}, \ldots, x_{t-m+1})$, the correlation integral that incorporates dimension *m* is given by (56):

$$C_{m,n}(\varepsilon) = \frac{2}{\binom{n}{2}} \sum_{1 \le s < t \le n} \sum \chi_\varepsilon \left(\left\| u_s^m - u_t^m \right\| \right) \qquad (56)$$

Where: $C_m(\varepsilon) = \lim_{n \to \infty} C_{m,n}(\varepsilon)$

If the data has been derived from a strictly stationary and stochastic process, this limit exists. In the case where this process is also independent, we also have:

$$C_m(\varepsilon) = C_1(\varepsilon)^m \qquad (57)$$

Moreover, we consider variable K, whose definition is given by the relation (58):

$$K(\varepsilon) = \int \left[F(u+\varepsilon) - F(u-\varepsilon) \right]^2 dF(u) \qquad (58)$$

We finally find that if our initial chronological order x_t is IID and $K(\varepsilon) > C(\varepsilon)^2$ for $m \geq 2$ relation (59):

$$\sqrt{n} \frac{T_{m,n}(\varepsilon)}{V_m} \qquad (59)$$

converges to a normal distribution N (0,1) and its asymptotic variance is given by (60):

$$\frac{1}{4} V_m^2 = m(m-2) C^{2m-2} \left(K - C^2 \right) + K^m - C^2 + 2 \sum_{j=1}^{m-1} \left[C^{2j} \left(K^{m-j} - C^{2m-2j} \right) - m C^{2m-2} \left(K - C^2 \right) \right] \quad (60)$$

In addition, the variable $V_{m,n}$ converges almost surely to V_m. Based on the above relationships, Dechert et al. (1996) finally define the BDS statistics as follows:

$$W_{m,n}(\varepsilon) = \sqrt{n}\frac{T_{m,n}(\varepsilon)}{V_{m,n}(\varepsilon)} \qquad (61)$$

In this study, the BDS test has been applied to all time series resulting from wavelet analysis, and it was observed that in all cases the statistical BDS (given in the tables as z-statistic) was statistically significant. In addition, we cannot say that BDS statistics have a steady, either increasing or decreasing trend, while the highest BDS values occur in the case of the United States and Korea. Therefore, the zero hypothesis of the existence of the IID process is rejected in all cases under investigation, and thus, in all cases we assume that the returns are not IID. The analytical results of the BDS Test are given in tables in Appendix A of this thesis.

2.3.5 GARCH(1,1) Analysis

An additional analysis applied to our data is GARCH (1,1). The implementation of the GARCH(1,1) model aims to investigate and study the behavior of the variability of each time series, so that some conclusions can be drawn about the type of markets we are studying. Below is a quick presentation of the method based on Gujarati (2004) and Brooks (2008).

Suppose we initially consider a basic linear model of the form:

$$y_t = \beta x_t + u_t \qquad (62)$$

For which holds that $u_t \sim N(0,\sigma_t^2)$. In the case where we assume that the variance of the error term is not constant and time dependent, then we are talking about existence of heteroskedasticity, and in this case, we define the conditional variance of the residuals of the model as follows:

$$\sigma_t^2 = \operatorname{var}\left(u_t \middle| u_{t-1}, u_{t-2}, ...\right) = \mathrm{E}\left[\left(u_t - \mathrm{E}(u_t)\right)^2 \middle| u_{t-1}, u_{t-2}, ...\right] \qquad (63)$$

Assuming that $E(u_t) = 0$, (63) becomes:

$$\sigma_t^2 = \operatorname{var}\left(u_t \middle| u_{t-1}, u_{t-2}, ...\right) = \mathrm{E}\left[u_t^2 \middle| u_{t-1}, u_{t-2}, ...\right] \qquad (64)$$

The abovementioned relationship indicates that the conditional variance of the variable u_t, with normal distribution and zero mean, is equal to the conditionally expected square value of u_t. In the case of the ARCH(1) model, we consider that this conditional variance of the error term depends on the square of the previous error term, u_{t-1}, which can be represented by (65):

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 \qquad (65)$$

A more generalized case of the above ARCH(1) model is the GARCH(1,1) model, developed by Bollerslev (1986) and Taylor (1986). This model is given by (66):

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \qquad (66)$$

In this case we observe that the conditional variance of the error term depends not only on the square of the previous error term, u_{t-1}, but also on the conditional variation of the error term of the previous period, σ_{t-1}^2.

We can also generalize this model in the form of GARCH(p, q), where there are p hysteresis terms of residual squares and q hysteresis terms of conditional variance squares. Finally, according to Gujarati (2004), a GARCH(p, q) model is essentially equivalent to an ARCH(p + q) model, but GARCH models are used to a greater extent than the corresponding ARCH, as they are more sparing and they avoid overfitting (Brooks, 2008).

In the context of this thesis, the GARCH(1,1) model has been applied to the data obtained after wavelet analysis at all frequencies for all three countries. Then, the corresponding GARCH Variance series was obtained, from which the variability eventually emerged for each time series.

The results, regarding the volatility of the time series, indicate that as the frequency decreases, i.e. as we move from short to long periods, volatility is declining in all three countries. Accordingly, by comparing the three economies, we observe that in the case of Korea we have higher volatility values, while in the other two cases values are close.

Figures 2.11, 2.12 and 2.13 below show some of the volatility results for each country, while the tables with the model coefficients and all the diagrams are given in Annexes B and C, respectively. In addition, it should be stressed here that all charts have been adapted to the same scale, so that they can be compared.

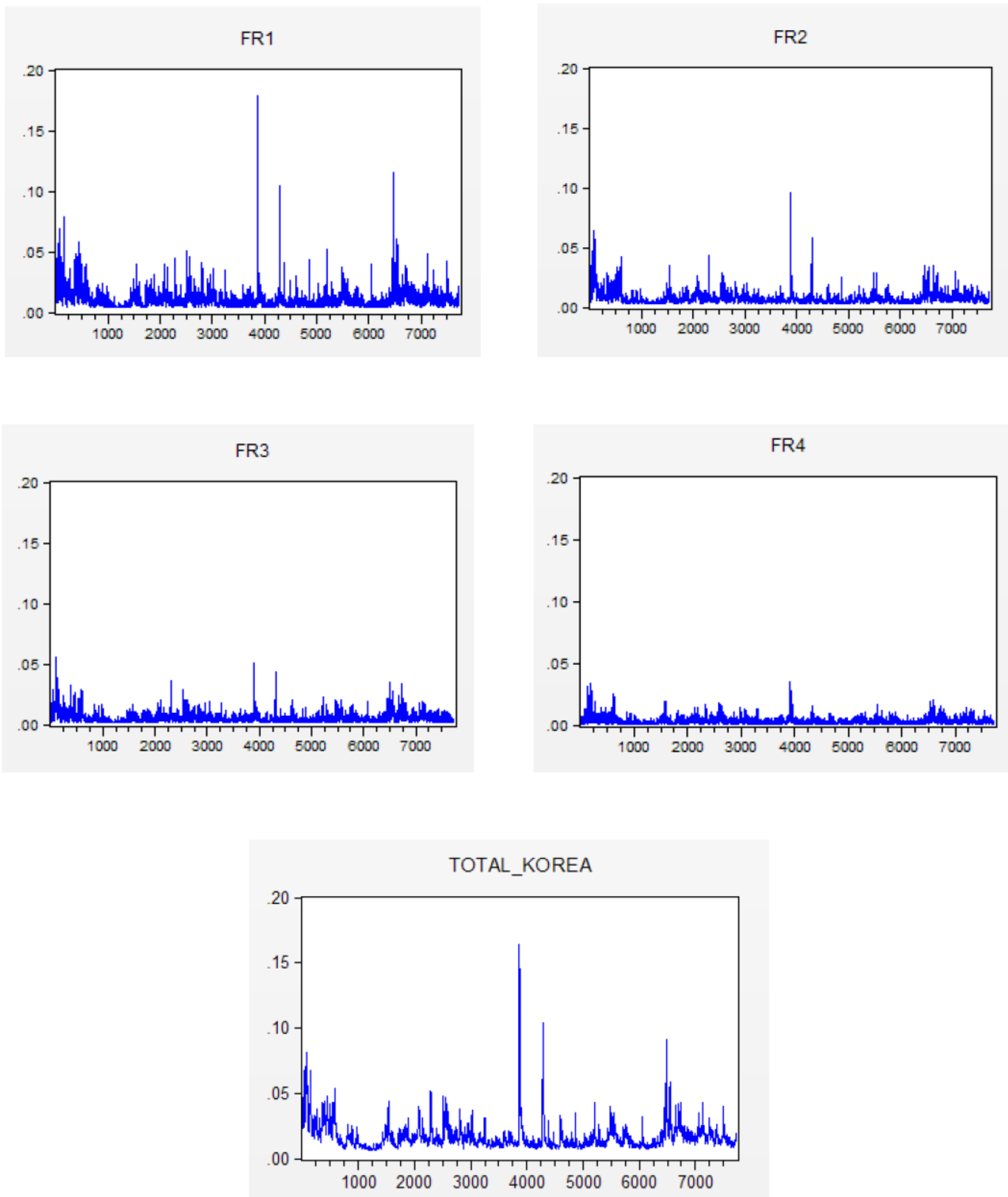

Figure 2.11: Volatility diagrams at different frequencies for Korea.

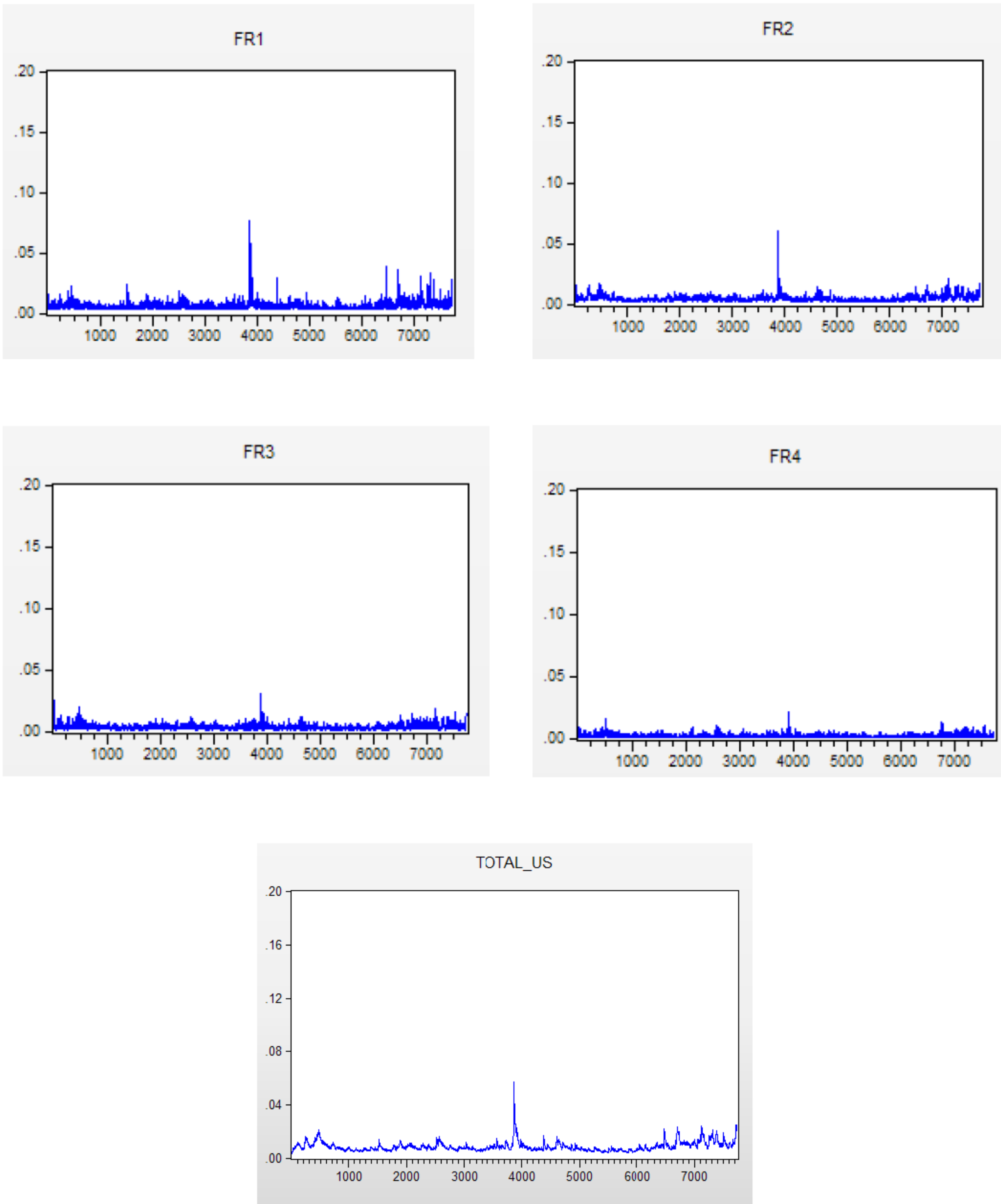

Figure 2.12: Volatility diagrams at different frequencies for the US.

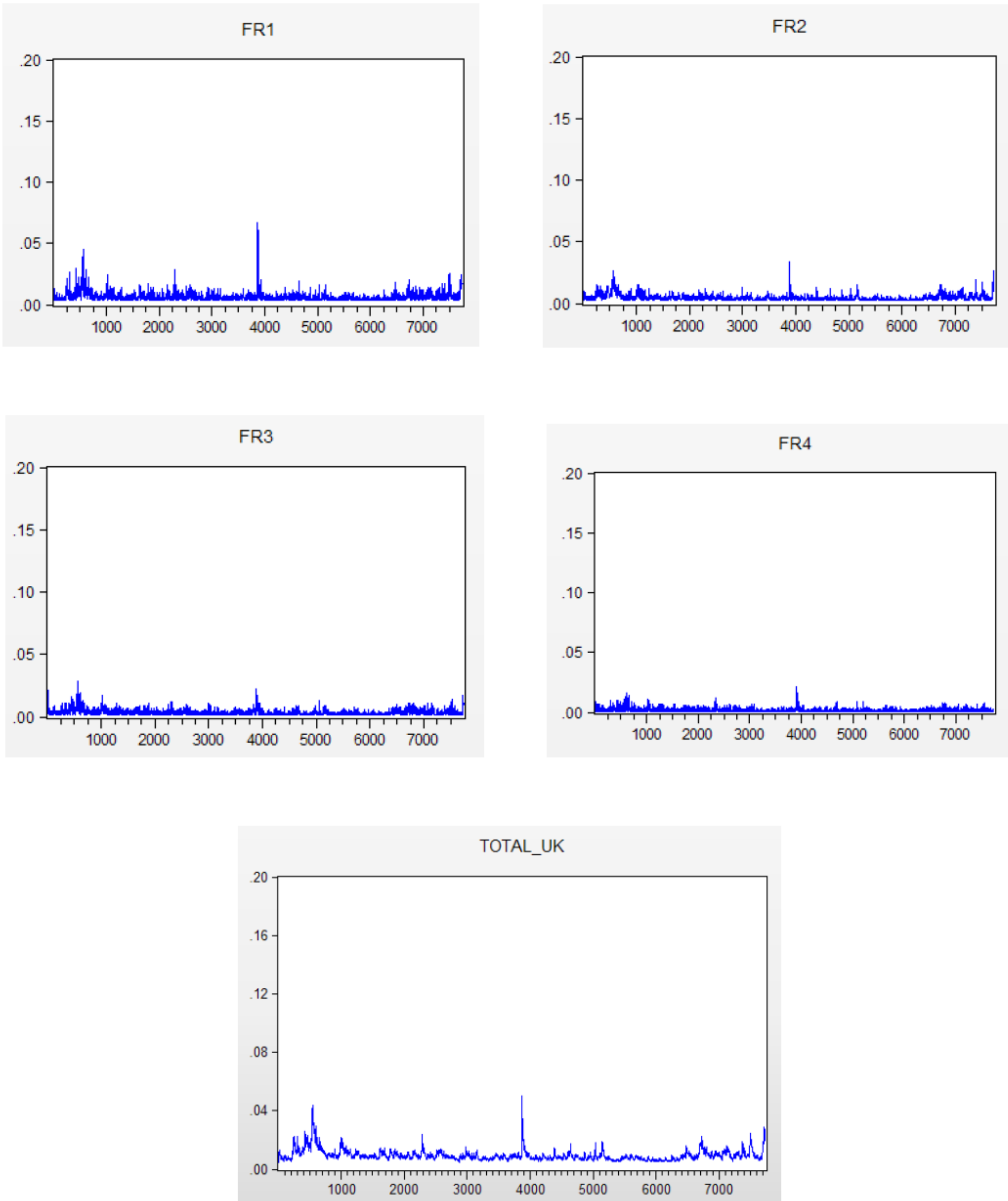

Figure 2.13: Volatility diagrams at different frequencies for the UK.

From of the above, some very interesting observations may emerge. At first, looking at the charts for the country's total samples, we observe that all of them exhibit some pronounced variability in the sample area corresponding to the October 1987 period. Recalling the historical events, we found that at that time, and in particular on 19th October 1987 the was a major stock market crisis, also known as Black Monday. According to Carlson (2006), the main causes that led to the Black Monday were:

- the difficulty in gathering information into a system that was rather chaotic and rapidly changing,
- the very large volume of transactions that were being attempted simultaneously,
- the uncertainty that prevailed and resulted on retreating many investors,
- the lack of liquidity observed on the market due to the record of margin calls observed on that day.

From chart data, we see that a strong volatility was first caused in the United States. on October 20th. This crisis was broadcasted the next day in the UK, and finally, on October 23rd it was felt by the Korean market as well. In addition, we notice that the sharp change in volatility was quite short and the market quickly returned to stability levels, which is also in line with the results of Ding et al. (1993).

Besides the '87 crisis, wavelet analysis at different frequencies helps us to identify some additional elements. From the analysis of frequencies, we can see that the most decisive role in the formation of volatility was played in all three cases by short-term investors, since in the case of low frequencies the volatility values are much lower. In particular, we are able to collect evidence about the different behavior of markets in relation to the same information, and whether this contradicts the theory of efficient markets.

Finally, we can distinguish other economic crises that were presented during the time from the results, including:

- February 10th, 1975: a political event, the election of Margaret Thatcher as the Conservative leader in the United Kingdom, has resulted in volatility in the UK's market. From the charts, we can see that in this case, investors at the lowest frequencies (fr2 to fr5) also play an important role in the final formation of volatility levels apart for very short-term investors (fr1).

- June 2nd, 1989: there is a strong volatility in Korea, which is due according to Chang et al. (1998) to large trade surpluses, which in turn led to increased liquidity, forcing the government to take measures to curb it.
- October 28th, 1997: General economic crisis in Korea, which according to Chang et al. (1998) was caused as a result of changes in the structure of the Korean market. In particular, key structural changes such as the liberalization of interest rates from state control, the increase in banks' autonomy and the reduction of entry conditions, the liberalization of foreign exchange and the opening of the market to foreign investors have resulted in a sharp increase the debt of the country, leading respectively to a debt crisis.
- July 15th, 2002: Fears about the solvency of British insurance companies also led to a UK market crisis.

2.4 Conclusions

All the above-mentioned time series analysis using wavelets, as well as the study of the characteristics of each of them through the EViews7 program, helped us to come up with some basic conclusions, which are presented in the following paragraphs.

Initially, starting from the characteristics of the time series resulting from the wavelet analysis, we can see that in all three countries there is a general tendency for a decreasing kurtosis, as we go to the lower frequencies, which is in line with the conclusions of Lux (1996). This, on an economic basis, can verify that we expect to find more speculation in the case of investors with a short-term investment horizon, compared to long-term investors. In addition, using the type of markets as a basis of comparison, we see that in the developing market of Korea kurtosis values at the first two frequencies are much higher, leading to a significant gap with the subsequent periods. This is followed by the US market, which also has a big difference in kurtosis between the first two frequencies and the next, while the UK market is more stable and freer of speculative effects, smoother fluctuation of kurtosis values over the entire spectrum of frequencies.

Moreover, the behavior of the three countries in terms of autocorrelation, where there are similarities in its form and differences in its size, is of particular interest. The results show that while at high frequencies the autocorrelation coefficients have very low values and indicate a lack of intense memory in the chronological order, as the frequency decreases, they show a constantly increasing trend, which ultimately leads them to unity, which suggests the existence of long-term memory.

The results presented above can give us some general insight into the characteristics of the markets we have examined, but it is not enough to see if our results verify existing economic theories. Thus, for this purpose, we will use the results obtained from the GARCH(1,1) analysis and relate to the volatility of the returns of the three markets.

In general, one of the most important theories that attempt to interpret the function of financial markets is the Efficient Market Hypothesis, according to which, stock prices in one market at all times fully reflect and instantly incorporate the whole available information on it. However, in many cases there is a different behavior of markets, such as anomalies, which may sometimes be caused by the market structure itself and can be seen as a consequence of speculative behavior. The results that have emerged from our study, in fact, help us to identify some deviations from the theory of efficient markets and to observe how information is disseminated in the three countries under investigation.

Initially, according to Lux (1996), an important element that breaks down the theory of efficient markets is the existence of chaotic processes in a market, which proves that time series that express the performance of markets, while at first may seem like stochastic processes, are in fact governed by deterministic laws. To test for the existence of IID procedures, the BDS test was used, the results of which are given in Appendix A for all frequencies in all three markets. Through the BDS test, the zero hypothesis that the chronological series in question were IID processes was rejected and a common course of three countries was generally observed. In addition, in the case of the United Kingdom we identify the lowest BDS values, which may indicate the greater stability of this market compared to the other two.

Also, according to Mishkin (2004), another element that is opposed to the theory of efficient markets is the existence of volatility in market returns, which has also become apparent in our analysis. We have seen that in all three cases returns are volatile and, in the case of Korea, this is even more evident than the other two markets.

In addition to looking for evidence to prove inefficiency in the markets, an important part of the study is also the identification of the reasons why different reactions to different events are observed and in what ways these are manifested.

More specifically, Kyrtsou & Malliaris (2009) talk about the difference in the dynamic behavior of time series and emphasize that different reactions of each market to some information is not only due to quantity and type information, but also to the mechanism by which this market operates. As Kyrtsou & Terraza (2002) mentions, "*markets are complex systems characterized by a variety* *of agents who perceive different new information differently*". It may be that the same information can cause different reactions in markets that differ in the process through which investors make their decisions.

Peters (1994) and Guillaume et al. (1997) also highlight the heterogeneity of investors as an essential factor. In turn, they point out that, in many markets, this is often caused due to the variations in investment times, geographic locations, the risk profile of investors and the institutional frameworks that dominate each market. We therefore expect, since the three markets examined in our case, are quite different in terms of their geographic location and the institutional frameworks in which they operate, that there is a difference in their dynamic behavior.

In our case, we notice that investors in each country have a different attitude in the way they make decisions, thus changing the dynamics by which information is adapted within each market. For example, in the case of Korea, which is still a growing market, investors are likely not to have developed highly effective ways of "filtering" information. This differentiates the structure of this market from the other two, resulting on Korea's investors to react more intensely to new information introducing the market. In contrast, in the case of the United States and the United Kingdom, we can argue that investors have developed a better judgment with regard to new information, so they can react more calmly, leading to lower levels of volatility.

Furthermore, the level of investor education plays an important role in the efficiency of a market. According to Daniel et al. (2001), investors who have an economic background and have been provided with an MBA type training can make more sound decisions about portfolio formation, which reduces market inefficiency. This happens in the United States and the United Kingdom.

All the above-mentioned data related to the diversification of the structure of the markets come to be verified through the GARCH(1,1) analysis carried out. The most notable example is the economic crisis of '87, for which our results indicate that the reactions of the three countries were different. Apart from the fact that there is a phase difference between their reactions, there is also heterogeneity in the intensity with which they react to the same information, as well as in the type of investors involved in shaping the final behavior of the market. In particular, the first reaction, as expected according to Kwan et al. (1995) and Eun & Shim (1989), is from the U.S. and it is similar to that of the United Kingdom, while Korea's reaction, which is more intense, defers. In addition, we notice that in the case of Korea those mainly responsible for high volatility are short-term investors (fr1), which differ quite a bit from the corresponding short-term investors of the other two countries. This is an important element that indicates that investor heterogeneity can play an important role in the different functioning of markets, making them more or less effective.

Discussion

Looking at the initial question raised in the introduction to this thesis, we can say that we have managed to respond to this clearly through the analysis that has been performed.

Initially, the methodology applied was the MODWT transformation (Maximal Overlap Discrete Wavelet Transform) at the $J_0 = 12$ level of analysis, on the US, United Kingdom and Korean stock markets, using the least asymmetric filter LA(8). Then, using the outputs that emerged, for each country respectively, an analysis and comparison of its econometric characteristics with those of the others was carried out. This comparison has been performed using simple characteristics of time series, such as curvature and asymmetry, as well as using the results of the BDS test and the GARCH(1,1) analysis.

Finally, we have concluded that all three markets have different behavior at high and low frequencies, i.e. at the level of short and long-term investors. In addition, market inefficiencies and investor heterogeneity have emerged, which are more pronounced in the case of the least-developed Korean market.

We can see that this work confirms the fact that the use of the wavelets method for time series analysis is a valuable tool for highlighting additional features. Through simultaneous analysis of frequency and time, not only much of the information of the original signal is not getting lost, but some extra features of the time series, especially at very high or very low frequencies, can be made clearer.

Finally, considering the usefulness of this method of analyzing time series, the significant results it has given us, as well as the fact that it is still in the early stages of its application in economic terms, seems to be an appropriate ground for further studies in this subject. For example, a next step could be to investigate the correlations between the three markets' performances across the frequency spectrum so that, if possible, some useful conclusions can be drawn as to the ways in which they affect each other and the time at which this influence takes place.

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Appendix A

This appendix includes tables with the results of the BDS Test for each country at all frequencies, as well as in the total sample. The values outside brackets are the z-statistic values, while the probability for each one of them is given in brackets.

- Korea

A.0 BDS test coefficients for Korea – Total sample

m/ε	0,5σ	1σ	1,5σ	2σ
2	18,374	20,961	23,417	24,635
	0,000	0,000	0,000	0,000
3	25,158	27,595	29,035	29,677
	0,000	0,000	0,000	0,000
4	31,935	33,032	33,024	32,77
	0,000	0,000	0,000	0,000
5	39,461	37,753	36,022	34,812
	0,000	0,000	0,000	0,000

A.1 BDS test coefficients for Korea – Fr1

m/ε	0,5σ	1σ	1,5σ	2σ
2	60,244	56,527	52,717	48,937
	0,000	0,000	0,000	0,000
3	87,772	67,925	58,445	52,699
	0,000	0,000	0,000	0,000
4	114,743	74,198	60,015	52,704
	0,000	0,000	0,000	0,000
5	150,741	81,225	61,116	52,106
	0,000	0,000	0,000	0,000

A.2 BDS test coefficients for Korea – Fr2

m/ε	0,5σ	1σ	1,5σ	2σ
2	32,860	33,983	34,054	33,227
	0,000	0,000	0,000	0,000
3	81,755	63,902	56,044	51,967
	0,000	0,000	0,000	0,000
4	118,037	78,136	63,785	57,263
	0,000	0,000	0,000	0,000
5	167,128	89,661	67,716	58,810
	0,000	0,000	0,000	0,000

A.3 BDS test coefficients for Korea – Fr3

m/ε	0,5σ	1σ	1,5σ	2σ
2	135,250	108,203	95,567	89,032
	0,000	0,000	0,000	0,000
3	198,697	116,618	93,995	84,482
	0,000	0,000	0,000	0,000
4	307,945	136,307	98,209	84,175
	0,000	0,000	0,000	0,000
5	487,200	162,447	104,729	85,579
	0,000	0,000	0,000	0,000

A.4 BDS test coefficients for Korea – Fr4

m/ε	0,5σ	1σ	1,5σ	2σ
2	275,919	176,370	144,709	132,627
	0,000	0,000	0,000	0,000
3	384,929	190,209	141,433	123,979
	0,000	0,000	0,000	0,000
4	608,499	212,554	140,074	116,669
	0,000	0,000	0,000	0,000
5	1064,238	250,361	143,337	112,622
	0,000	0,000	0,000	0,000

A.5 BDS test coefficients for Korea – Fr5

m/ε	0,5σ	1σ	1,5σ	2σ
2	359,269	204,957	163,935	148,834
	0,000	0,000	0,000	0,000
3	557,119	233,116	164,697	141,427
	0,000	0,000	0,000	0,000
4	922,566	269,409	166,337	134,769
	0,000	0,000	0,000	0,000
5	1654,525	321,315	171,438	130,633
	0,000	0,000	0,000	0,000

A.6 BDS test coefficients for Korea – Fr6

m/ε	0,5σ	1σ	1,5σ	2σ
2	397,376	224,725	183,254	169,556
	0,000	0,000	0,000	0,000
3	657,117	260,303	185,131	161,100
	0,000	0,000	0,000	0,000
4	1161,831	307,137	188,158	153,528
	0,000	0,000	0,000	0,000
5	203,899	374,238	195,149	148,827
	0,000	0,000	0,000	0,000

A.7 BDS test coefficients for Korea – Fr7

m/ε	0,5σ	1σ	1,5σ	2σ
2	472,562	268,947	216,590	202,973
	0,000	0,000	0,000	0,000
3	861,326	330,000	225,623	195,885
	0,000	0,000	0,000	0,000
4	1707,096	415,572	237,004	189,734
	0,000	0,000	0,000	0,000
5	3662,828	544,039	254,638	187,055
	0,000	0,000	0,000	0,000

A.8 BDS test coefficients for Korea – Fr8

m/ε	0,5σ	1σ	1,5σ	2σ
2	631,652	313,69	233,016	215,300
	0,000	0,000	0,000	0,000
3	1206,086	393,244	244,936	208,700
	0,000	0,000	0,000	0,000
4	2524,220	507,426	259,825	203,080
	0,000	0,000	0,000	0,000
5	5748,677	682,391	282,114	201,178
	0,000	0,000	0,000	0,000

A.9 BDS test coefficients for Korea – Fr9

m/ε	0,5σ	1σ	1,5σ	2σ
2	833,052	400,572	309,847	283,565
	0,000	0,000	0,000	0,000
3	1699,417	521,547	332,886	276,655
	0,000	0,000	0,000	0,000
4	3839,113	702,700	361,665	271,062
	0,000	0,000	0,000	0,000
5	9491,201	991,015	402,954	270,468
	0,000	0,000	0,000	0,000

A.10 BDS test coefficients for Korea – Fr10

m/ε	0,5σ	1σ	1,5σ	2σ
2	1248,148	469,416	339,406	296,638
	0,000	0,000	0,000	0,000
3	572,918	618,837	366,021	290,267
	0,000	0,000	0,000	0,000
4	5883,369	845,755	399,329	285,291
	0,000	0,000	0,000	0,000
5	14736,060	1211,586	446,933	285,608
	0,000	0,000	0,000	0,000

A.11 BDS test coefficients for Korea – Fr11

m/ε	0,5σ	1σ	1,5σ	2σ
2	538,047	307,865	259,340	271,245
	0,000	0,000	0,000	0,000
3	1048,722	394,188	278,629	265,175
	0,000	0,000	0,000	0,000
4	2248,141	521,032	302,698	260,364
	0,000	0,000	0,000	0,000
5	5255,330	719,555	337,238	260,369
	0,000	0,000	0,000	0,000

A.12 BDS test coefficients for Korea – Fr12

m/ε	0,5σ	1σ	1,5σ	2σ
2	557,450	308,524	263,852	251,329
	0,000	0,000	0,000	0,000
3	967,817	378,813	289,750	253,706
	0,000	0,000	0,000	0,000
4	1815,916	477,477	322,428	257,692
	0,000	0,000	0,000	0,000
5	3678,414	625,802	368,700	267,101
	0,000	0,000	0,000	0,000

- USA

A.0 BDS test coefficients for USA – Total sample

m/ε	0,5σ	1σ	1,5σ	2σ
2	8,341	11,296	14,055	15,854
	0,000	0,000	0,000	0,000
3	11,482	15,355	18,569	20,272
	0,000	0,000	0,000	0,000
4	14,105	18,143	21,224	22,706
	0,000	0,000	0,000	0,000
5	16,999	20,866	23,621	24,916
	0,000	0,000	0,000	0,000

A.1 BDS test coefficients for USA – Fr1

m/ε	0,5σ	1σ	1,5σ	2σ
2	58,092	57,168	55,133	53,076
	0,000	0,000	0,000	0,000
3	84,849	67,919	58,869	54,301
	0,000	0,000	0,000	0,000
4	110,070	73,948	59,754	53,415
	0,000	0,000	0,000	0,000
5	141,360	80,754	61,118	52,905
	0,000	0,000	0,000	0,000

A.2 BDS test coefficients for USA – Fr2

m/ε	0,5σ	1σ	1,5σ	2σ
2	27,557	29,558	31,082	30,832
	0,000	0,000	0,000	0,000
3	86,282	66,196	57,344	52,917
	0,000	0,000	0,000	0,000
4	125,423	82,780	66,807	59,865
	0,000	0,000	0,000	0,000
5	179,077	96,295	71,626	62,092
	0,000	0,000	0,000	0,000

A.3 BDS test coefficients for USA – Fr3

m/ε	0,5σ	1σ	1,5σ	2σ
2	148,535	120,787	105,180	97,668
	0,000	0,000	0,000	0,000
3	236,964	131,106	101,380	90,347
	0,000	0,000	0,000	0,000
4	383,044	159,132	107,857	90,247
	0,000	0,000	0,000	0,000
5	623,377	195,353	117,416	93,110
	0,000	0,000	0,000	0,000

A.4 BDS test coefficients for USA – Fr4

m/ε	0,5σ	1σ	1,5σ	2σ
2	309,518	196,110	157,654	144,048
	0,000	0,000	0,000	0,000
3	437,160	212,479	154,432	134,586
	0,000	0,000	0,000	0,000
4	710,182	239,811	153,316	126,623
	0,000	0,000	0,000	0,000
5	1271,429	286,792	157,598	122,211
	0,000	0,000	0,000	0,000

A.5 BDS test coefficients for USA – Fr5

m/ε	0,5σ	1σ	1,5σ	2σ
2	484,998	260,430	200,127	183,067
	0,000	0,000	0,000	0,000
3	787,604	306,487	204,182	174,543
	0,000	0,000	0,000	0,000
4	1381,306	367,685	209,664	166,919
	0,000	0,000	0,000	0,000
5	2658,637	456,906	219,905	162,450
	0,000	0,000	0,000	0,000

A.6 BDS test coefficients for USA – Fr6

m/ε	0,5σ	1σ	1,5σ	2σ
2	528,741	280,169	213,966	193,939
	0,000	0,000	0,000	0,000
3	940,974	340,444	221,774	186,443
	0,000	0,000	0,000	0,000
4	1806,076	423,809	231,708	179,880
	0,000	0,000	0,000	0,000
5	3738,155	547,431	247,504	176,630
	0,000	0,000	0,000	0,000

A.7 BDS test coefficients for USA – Fr7

m/ε	0,5σ	1σ	1,5σ	2σ
2	478,787	262,233	209,663	198,992
	0,000	0,000	0,000	0,000
3	867,447	319,775	218,337	192,119
	0,000	0,000	0,000	0,000
4	1707,187	399,855	229,260	186,157
	0,000	0,000	0,000	0,000
5	3634,922	519,409	246,209	183,599
	0,000	0,000	0,000	0,000

A.8 BDS test coefficients for USA – Fr8

m/ε	0,5σ	1σ	1,5σ	2σ
2	505,599	274,201	215,228	198,953
	0,000	0,000	0,000	0,000
3	935,250	336,798	224,771	192,374
	0,000	0,000	0,000	0,000
4	1887,166	424,660	236,752	186,701
	0,000	0,000	0,000	0,000
5	4132,802	556,777	255,115	184,442
	0,000	0,000	0,000	0,000

A.9 BDS test coefficients for USA – Fr9

m/ε	0,5σ	1σ	1,5σ	2σ
2	611,310	313,696	244,398	236,474
	0,000	0,000	0,000	0,000
3	1186,900	398,694	260,413	230,607
	0,000	0,000	0,000	0,000
4	2533,008	522,574	280,359	225,822
	0,000	0,000	0,000	0,000
5	5892,758	715,016	309,307	225,197
	0,000	0,000	0,000	0,000

A.10 BDS test coefficients for USA – Fr10

m/ε	0,5σ	1σ	1,5σ	2σ
2	1164,105	515,103	324,096	293,707
	0,000	0,000	0,000	0,000
3	2360,869	675,726	352,198	287,146
	0,000	0,000	0,000	0,000
4	5298,403	918,310	387,457	281,953
	0,000	0,000	0,000	0,000
5	13008,870	1307,328	437,560	281,972
	0,000	0,000	0,000	0,000

A.11 BDS test coefficients for USA – Fr11

m/ε	0,5σ	1σ	1,5σ	2σ
2	395,749	212,597	179,395	210,970
	0,000	0,000	0,000	0,000
3	713,566	258,334	189,848	206,924
	0,000	0,000	0,000	0,000
4	1398,264	321,755	202,839	203,851
	0,000	0,000	0,000	0,000
5	2967,929	416,259	221,962	204,574
	0,000	0,000	0,000	0,000

A.12 BDS test coefficients for USA – Fr12

m/ε	0,5σ	1σ	1,5σ	2σ
2	364,939	296,030	278,551	234,416
	0,000	0,000	0,000	0,000
3	593,865	361,071	306,613	237,802
	0,000	0,000	0,000	0,000
4	1033,884	451,738	342,079	242,791
	0,000	0,000	0,000	0,000
5	1930,909	587,264	392,267	253,034
	0,000	0,000	0,000	0,000

- UK

A.0 BDS test coefficients for UK – Total sample

m/ε	0,5σ	1σ	1,5σ	2σ
2	17,349	19,470	22,289	24,752
	0,000	0,000	0,000	0,000
3	22,459	24,741	27,594	30,000
	0,000	0,000	0,000	0,000
4	26,919	28,610	30,860	33,116
	0,000	0,000	0,000	0,000
5	31,975	32,367	33,357	34,975
	0,000	0,000	0,000	0,000

A.1 BDS test coefficients for UK – Fr1

m/ε	0,5σ	1σ	1,5σ	2σ
2	58,769	57,471	56,377	55,010
	0,000	0,000	0,000	0,000
3	85,210	68,422	61,060	57,554
	0,000	0,000	0,000	0,000
4	109,498	74,114	61,874	56,621
	0,000	0,000	0,000	0,000
5	138,713	80,160	62,760	55,765
	0,000	0,000	0,000	0,000

A.2 BDS test coefficients for UK – Fr2

m/ε	0,5σ	1σ	1,5σ	2σ
2	30,445	31,752	32,438	32,044
	0,000	0,000	0,000	0,000
3	89,636	68,939	59,476	54,712
	0,000	0,000	0,000	0,000
4	130,369	86,515	69,566	61,984
	0,000	0,000	0,000	0,000
5	186,845	101,211	75,028	64,461
	0,000	0,000	0,000	0,000

A.3 BDS test coefficients for UK – Fr3

m/ε	0,5σ	1σ	1,5σ	2σ
2	146,810	118,757	103,287	95,692
	0,000	0,000	0,000	0,000
3	234,453	129,385	100,455	89,798
	0,000	0,000	0,000	0,000
4	382,737	157,501	106,979	89,825
	0,000	0,000	0,000	0,000
5	630,433	194,192	116,529	92,473
	0,000	0,000	0,000	0,000

A.4 BDS test coefficients for UK – Fr4

m/ε	0,5σ	1σ	1,5σ	2σ
2	297,214	191,657	155,819	142,930
	0,000	0,000	0,000	0,000
3	419,497	208,661	153,174	133,875
	0,000	0,000	0,000	0,000
4	677,717	2355,822	152,401	126,253
	0,000	0,000	0,000	0,000
5	1206,913	281,399	156,709	121,987
	0,000	0,000	0,000	0,000

A.5 BDS test coefficients for UK – Fr5

m/ε	0,5σ	1σ	1,5σ	2σ
2	426,224	231,852	179,853	160,619
	0,000	0,000	0,000	0,000
3	670,956	265,830	180,730	152,126
	0,000	0,000	0,000	0,000
4	1127,728	310,017	182,572	144,477
	0,000	0,000	0,000	0,000
5	2062,976	373,173	188,258	139,557
	0,000	0,000	0,000	0,000

A.6 BDS test coefficients for UK – Fr6

m/ε	0,5σ	1σ	1,5σ	2σ
2	484,135	252,049	195,702	172,877
	0,000	0,000	0,000	0,000
3	818,551	294,493	197,820	163,941
	0,000	0,000	0,000	0,000
4	1484,984	350,872	201,197	155,938
	0,000	0,000	0,000	0,000
5	2892,076	432,102	208,840	150,877
	0,000	0,000	0,000	0,000

A.7 BDS test coefficients for UK – Fr7

m/ε	0,5σ	1σ	1,5σ	2σ
2	461,989	242,923	189,156	167,872
	0,000	0,000	0,000	0,000
3	794,539	284,579	191,819	159,933
	0,000	0,000	0,000	0,000
4	1473,033	340,161	195,747	152,837
	0,000	0,000	0,000	0,000
5	2941,734	420,555	203,899	148,577
	0,000	0,000	0,000	0,000

A.8 BDS test coefficients for UK – Fr8

m/ε	0,5σ	1σ	1,5σ	2σ
2	342,134	202,480	165,105	149,476
	0,000	0,000	0,000	0,000
3	553,590	229,938	165,488	142,102
	0,000	0,000	0,000	0,000
4	957,218	265,487	166,783	135,491
	0,000	0,000	0,000	0,000
5	1774,111	316,064	171,448	131,408
	0,000	0,000	0,000	0,000

A.9 BDS test coefficients for UK – Fr9

m/ε	0,5σ	1σ	1,5σ	2σ
2	311,595	184,966	151,390	139,427
	0,000	0,000	0,000	0,000
3	494,886	208,769	152,522	133,925
	0,000	0,000	0,000	0,000
4	837,610	239,397	154,547	129,061
	0,000	0,000	0,000	0,000
5	1516,984	282,883	159,780	126,563
	0,000	0,000	0,000	0,000

A.10 BDS test coefficients for UK – Fr10

m/ε	0,5σ	1σ	1,5σ	2σ
2	366,990	179,042	136,792	129,258
	0,000	0,000	0,000	0,000
3	592,380	203,062	138,938	125,989
	0,000	0,000	0,000	0,000
4	1021,674	234,098	141,992	123,272
	0,000	0,000	0,000	0,000
5	1888.78	278.241	148.145	122.822
	0	0	0	0

A.11 BDS test coefficients for UK – Fr11

m/ε	0,5σ	1σ	1,5σ	2σ
2	598,856	283,325	210,502	204,541
	0,000	0,000	0,000	0,000
3	1107,572	353,927	223,690	199,220
	0,000	0,000	0,000	0,000
4	2035,331	454,892	240,101	194,826
	0,000	0,000	0,000	0,000
5	4898,330	609,157	264,044	194,016
	0,000	0,000	0,000	0,000

A.12 BDS test coefficients for UK – Fr12

m/ε	0,5σ	1σ	1,5σ	2σ
2	497,963	391,039	313,682	324,522
	0,000	0,000	0,000	0,000
3	931,111	521,822	343,229	316,023
	0,000	0,000	0,000	0,000
4	1902,858	723,095	380,449	309,043
	0,000	0,000	0,000	0,000
5	4226,025	1051,829	433,185	307,752
	0,000	0,000	0,000	0,000

Appendix B

This annex presents the values of the coefficients derived from the GARCH(1,1) analysis, which were applied to the data of each country and to all frequencies.

In the first row the values of the coefficients are given, and from below just the probability that has been calculated for each of them. We notice that in most cases the coefficients are statistically significant, but there are also some values, especially in small frequencies, for which the probability is greater than 0.05.

As given in equation (51) in paragraph 2.3.5, coefficients a0 refer to the equation constant, coefficients a1 correspond to the term u_{t-1}^2, while coefficients a2 are the coefficients for the term σ_{t-1}^2.

GARCH(1,1) model estimates for Korea (fr1-fr6)

Συντελεστές υποδείγματος	**Συχνότητα**					
	fr1	**fr2**	**fr3**	**fr4**	**fr5**	**fr6**
a0	5,70E-06	2,60E-06	1,42E-06	8,86E-07	1,68E-07	2,42E-08
	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)
a1	0,4647	0,3904	0,7226	0,9451	0,9814	0,9739
	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)
a2	0,5711	0,6187	0,3672	0,1117	0,0490	0,0476
	(0,000)	(0,000)	(0,000)	(0,000)	(0,3053)	(0,4263)

GARCH(1,1) model estimates for Korea (fr7-fr12)

Συντελεστές υποδείγματος	**Συχνότητα**					
	fr7	**fr8**	**fr9**	**fr10**	**fr11**	**fr12**
a0	4,67E-09	8,10E-12	2,66E-13	9,84E-14	3,30E-15	8,22E-15
	(0,000)	(0,9282)	(0,5856)	(0,5931)	(0,4358)	(0,0712)
a1	0,9854	0,8008	0,2793	0,3044	0,3245	0,2632
	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)
a2	0,0219	0,2433	0,6650	0,6837	0,6791	0,6442
	(0,8218)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)

GARCH(1,1) model estimates for USA (fr1-fr6)

Συντελεστές υποδείγματος	Συχνότητα					
	fr1	**fr2**	**fr3**	**fr4**	**fr5**	**fr6**
a0	4,54E-06	1,30E-06	8,66E-07	3,07E-07	7,21E-08	1,27E-08
	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,2120)
a1	0,4338	0,2976	0,6838	0,9401	0,9895	1,0472
	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)
a2	0,4966	0,6658	0,3538	0,1061	-0,0109	-0,0835
	(0,000)	(0,000)	(0,000)	(0,000)	(0,9004)	(0,6529)

GARCH(1,1) model estimates for USA (fr7-fr12)

Συντελεστές υποδείγματος	Συχνότητα					
	fr7	**fr8**	**fr9**	**fr10**	**fr11**	**fr12**
a0	8,94E-10	3,19E-12	2,37E-12	1,59E-15	5,81E-16	6,34E-13
	(0,4005)	(0,0681)	(0,0447)	(0,4173)	(0,4304)	(0,0242)
a1	0,9892	0,7937	0,7506	0,2598	0,3083	0,2046
	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)
a2	0,0120	0,2196	0,2420	0,6496	0,6673	0,6228
	(0,9127)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)

GARCH(1,1) model estimates for the UK (fr1-fr6)

Συντελεστές υποδείγματος	Συχνότητα					
	fr1	**fr2**	**fr3**	**fr4**	**fr5**	**fr6**
a0	4,70E-06	1,29E-06	6,51E-07	3,60E-07	6,24E-08	1,34E-08
	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)
a1	0,3967	0,2996	0,6576	0,9538	0,9715	0,9674
	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)
a2	0,5232	0,6666	0,3967	0,0739	0,0410	0,0346
	(0,000)	(0,000)	(0,000)	(0,000)	(0,4917)	(0,7913)

GARCH(1,1) model estimates for the UK (fr7-fr12)

Συντελεστές υποδείγματος	**Συχνότητα**					
	fr7	**fr8**	**fr9**	**fr10**	**fr11**	**fr12**
a0	1,81E-09	5,81E-11	1,91E-12	1,16E-14	9,33E-14	3,25E-14
	(0,7296)	(0,3419)	(0,0780)	(0,4773)	(0,3399)	(0,3698)
a1	1,1064	0,9335	0,3022	0,3101	0,2933	0,3035
	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)	(0,000)
a2	-0,1153	0,0932	0,6758	0,6798	0,6753	0,6782
	(0,5065)	(0,1966)	(0,000)	(0,000)	(0,000)	(0,000)

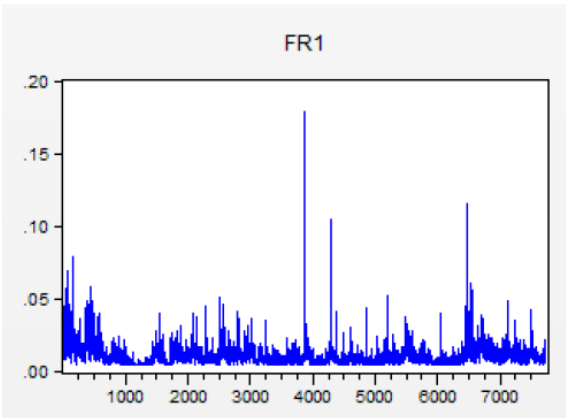
GARCH(1,1) model estimates for the total sample

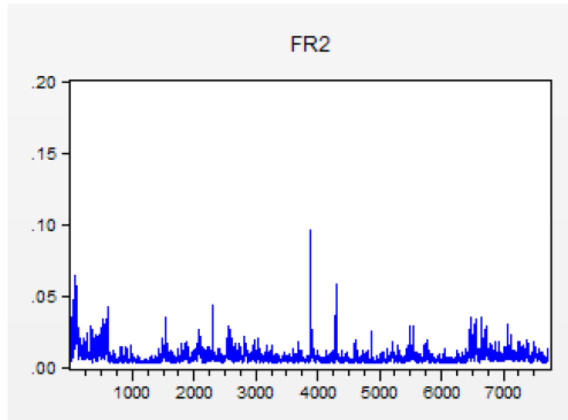
Συντελεστές υποδείγματος	**Χώρες**		
	Κορέα	**Ηνωμένο Βασίλειο**	**Η.Π.Α.**
a0	5,16E-06	1,88E-06	1,05E-06
	(0,000)	(0,000)	(0,000)
a1	0,1504	0,0932	0,0661
	(0,000)	(0,000)	(0,000)
a2	0,8478	0,8895	0,9247
	(0,000)	(0,000)	(0,000)

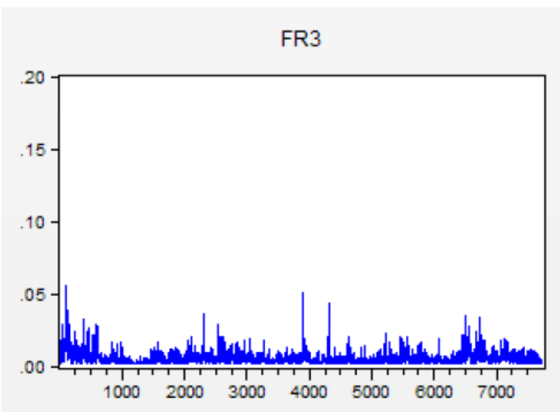
Appendix C

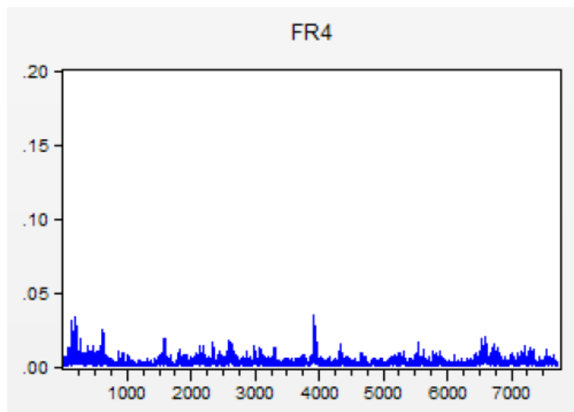
This appendix presents all the volatility charts resulting from the GARCH(1.1) analysis for all countries and their frequencies.

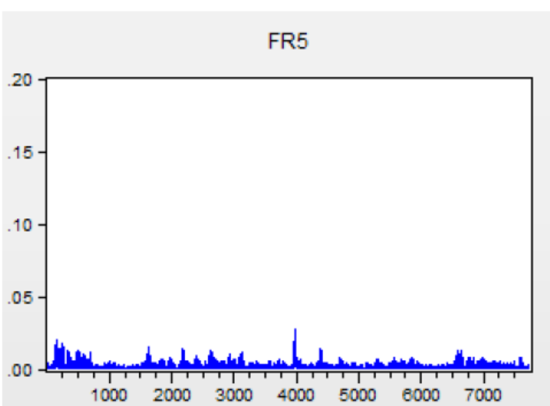
- Korea

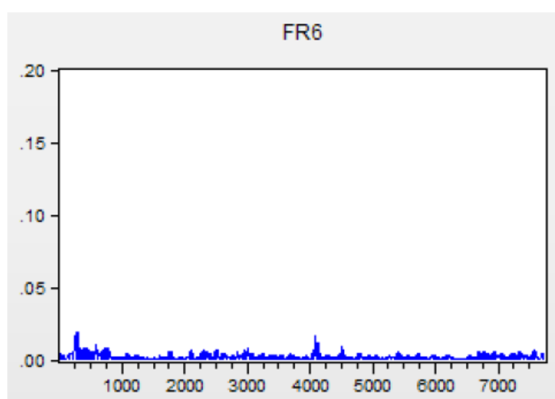


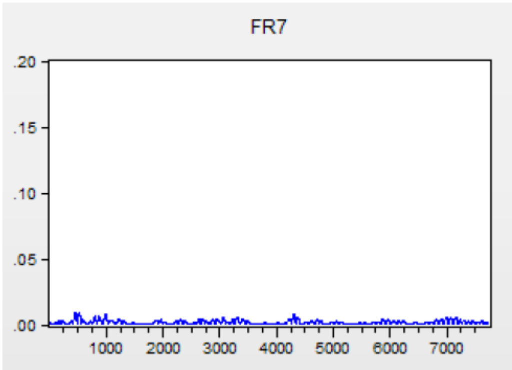








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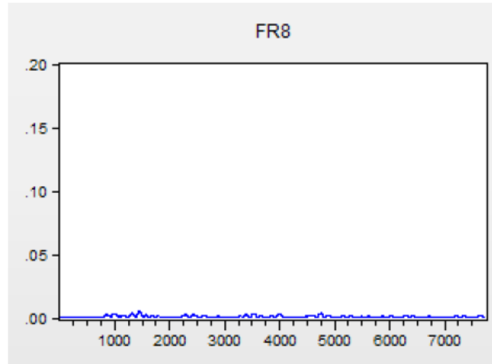
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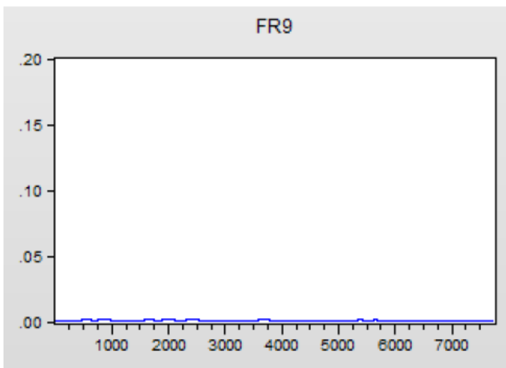
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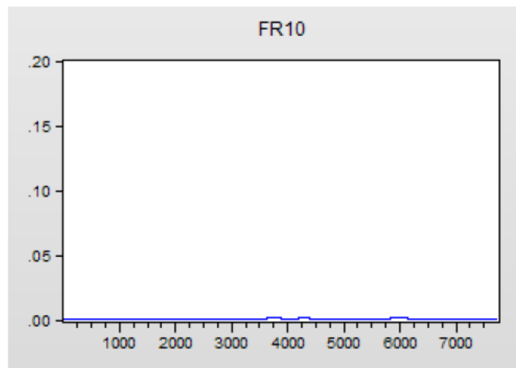
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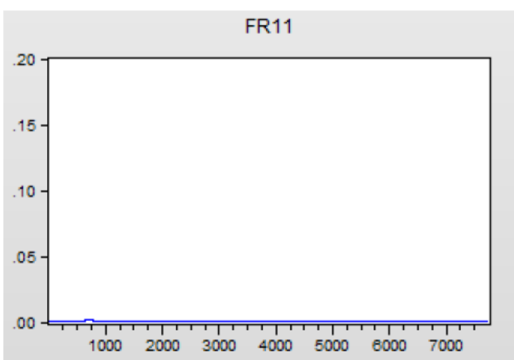
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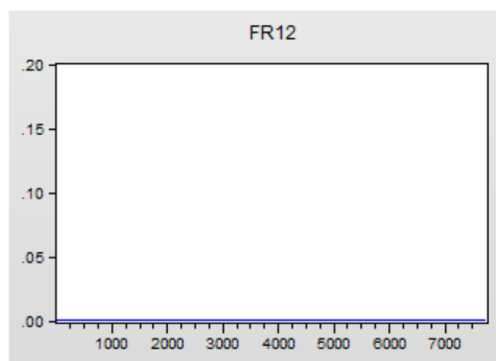
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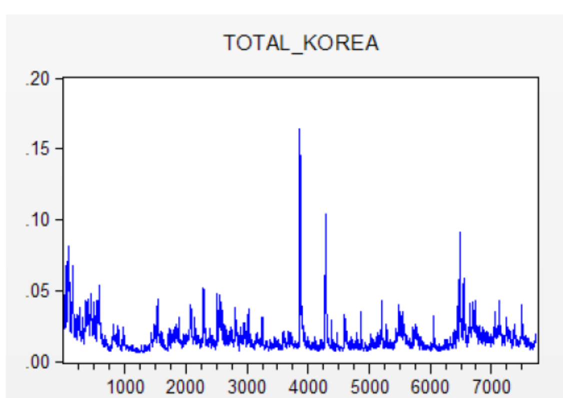
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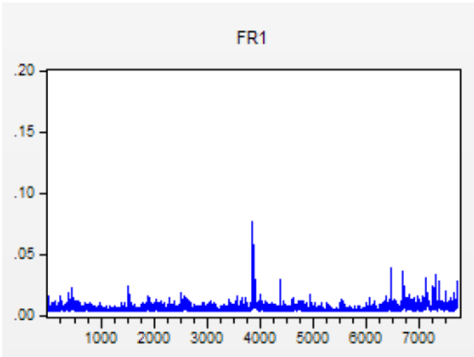
- USA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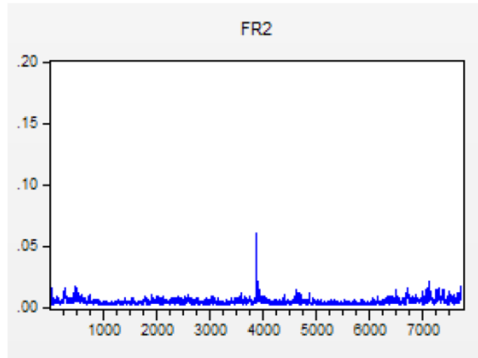
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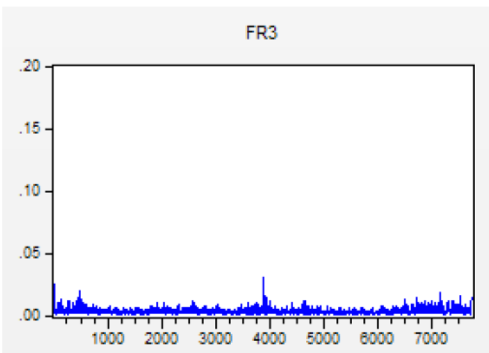
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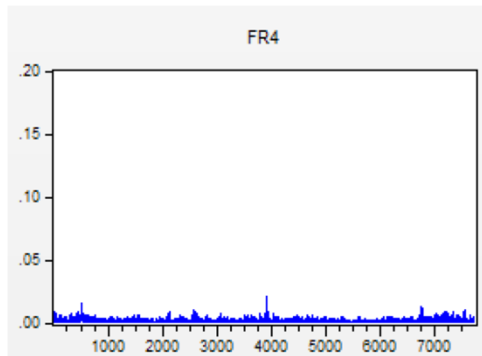
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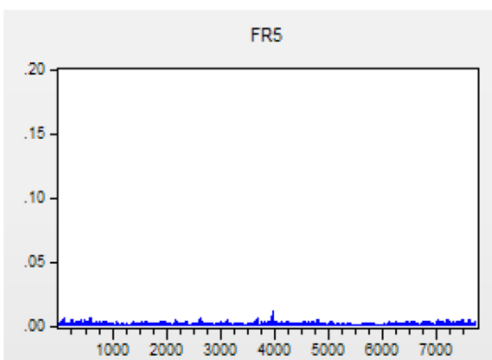
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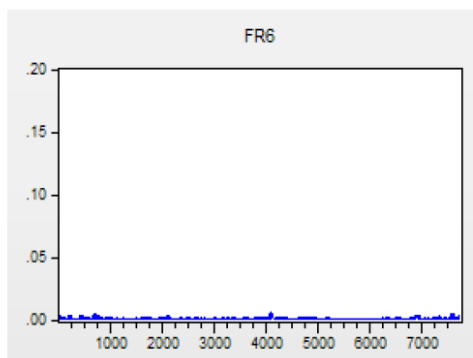
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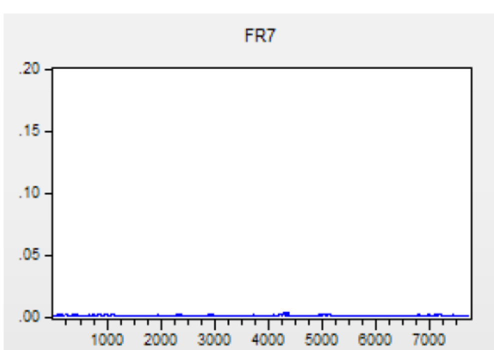
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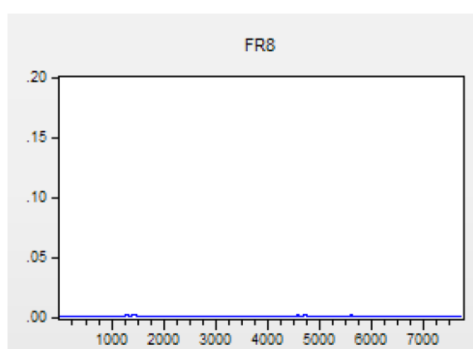
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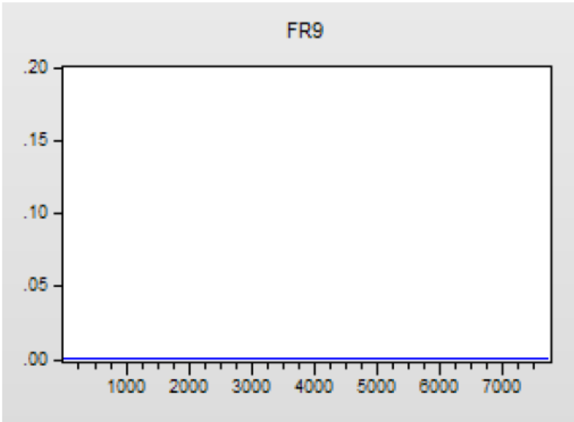
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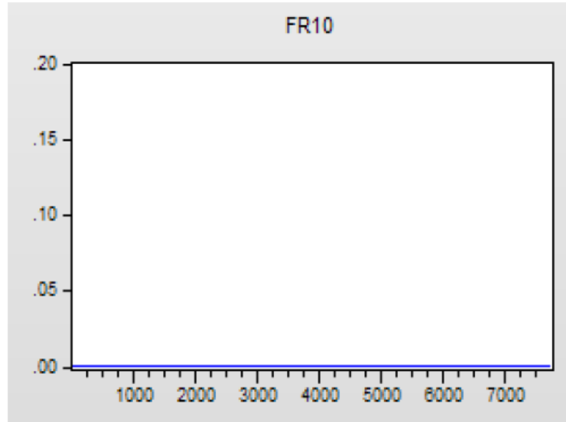
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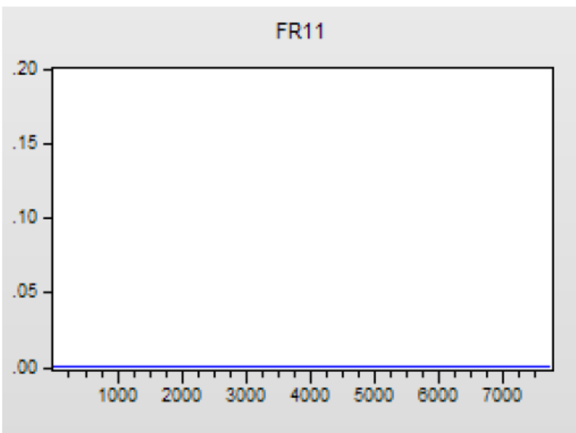
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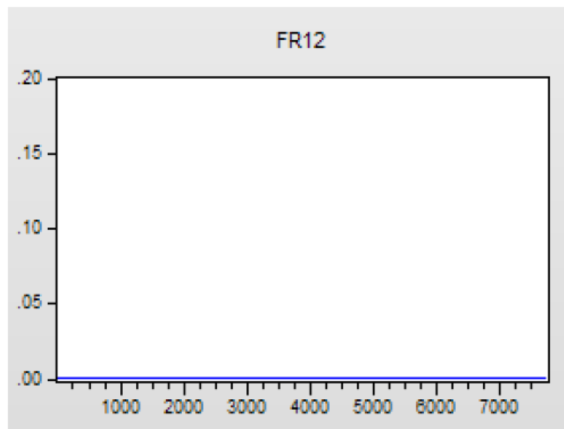
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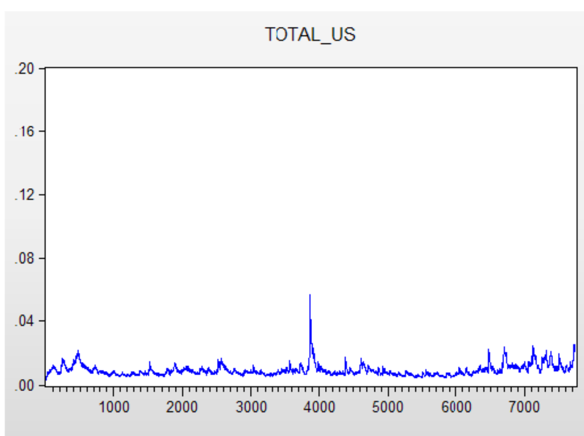
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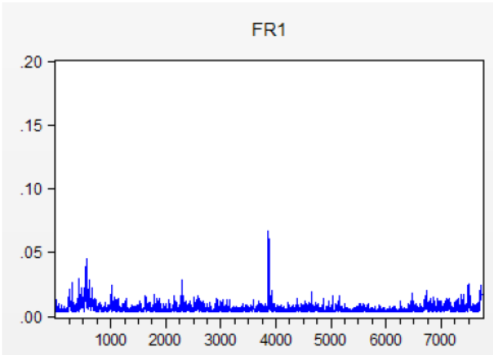
- UK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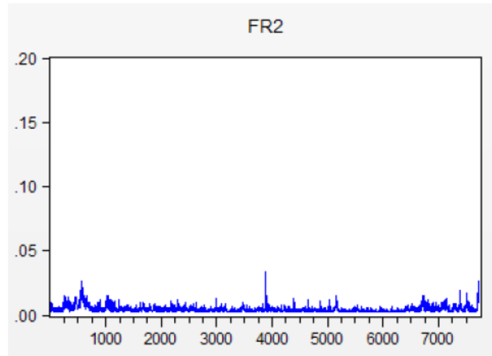
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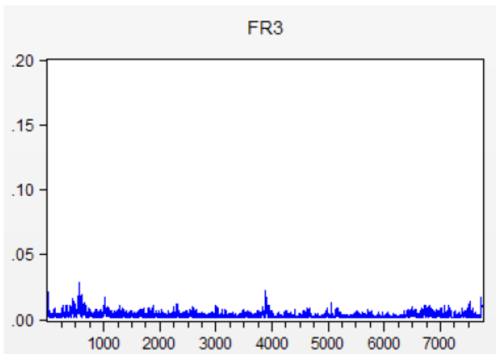
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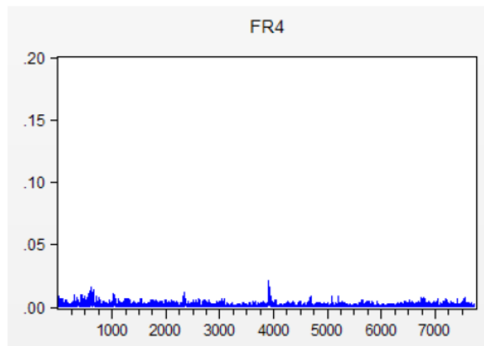
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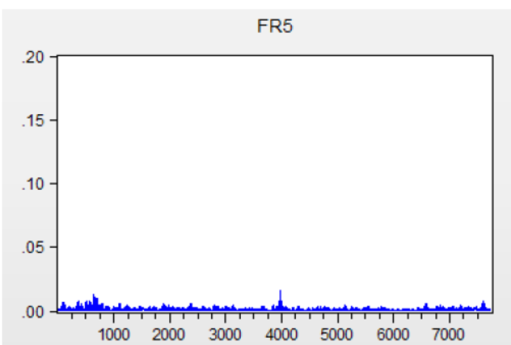
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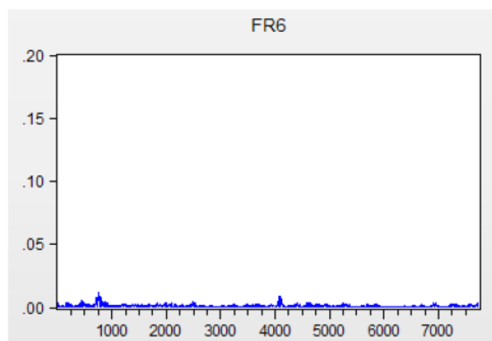
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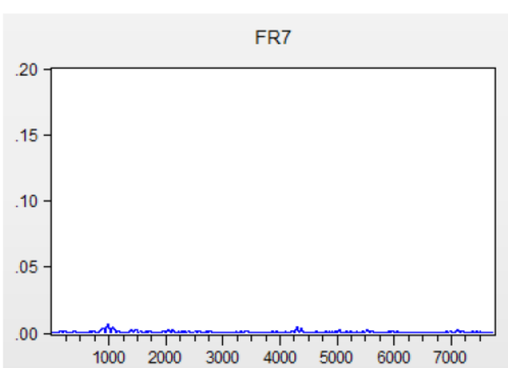
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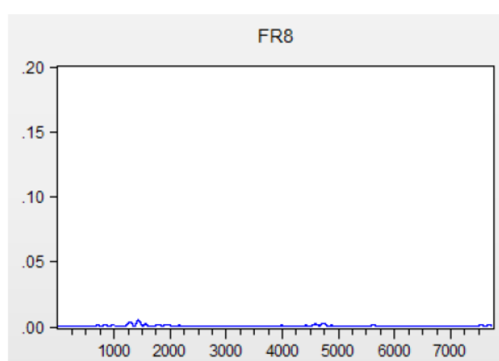
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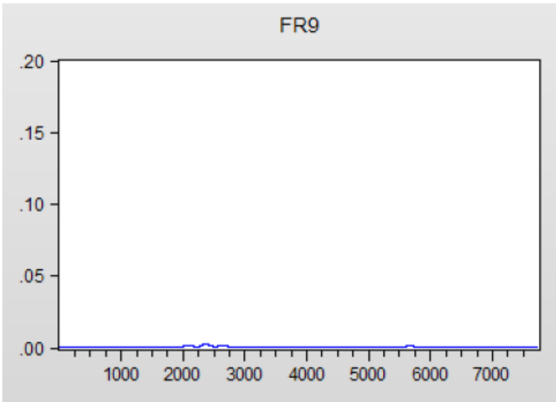
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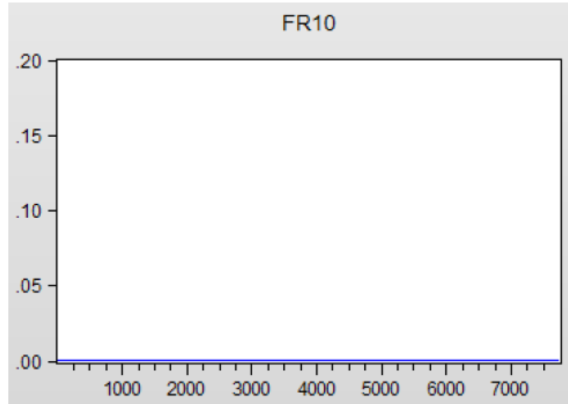
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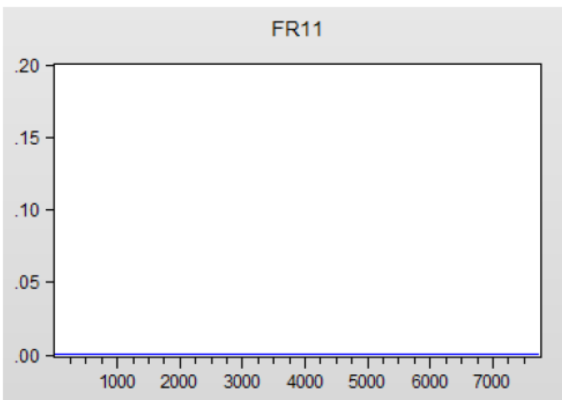
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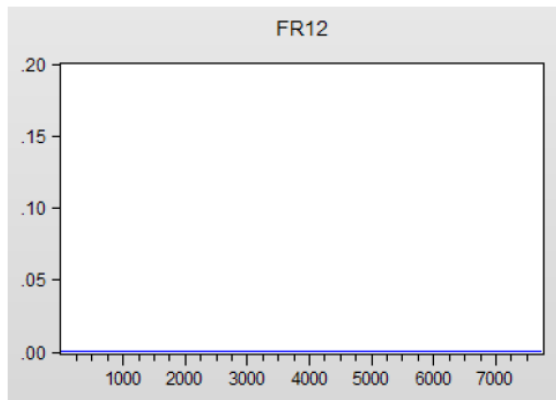
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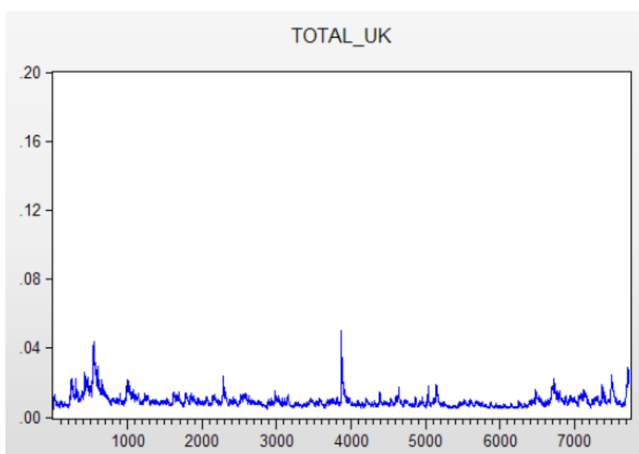
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