1 Explanations of the Income Distribution

This section just contains some quotes to consider.

> Any city however small, is in fact divided into two, one the city of the poor, the other of the rich; these are at war with one another.
> – Plato, The Republic, 370 BC

> In a community regulated by laws of demand and supply, but protected from open violence, the persons who become rich are, generally speaking, industrious, resolute, proud, covetous, prompt, methodical, sensible, unimaginative, insensitive, and ignorant. The persons who remain poor are the entirely foolish, the entirely wise, the idle, the reckless, the humble, the thoughtful, the dull, the imaginative, the sensitive, the well-informed, the improvident, the irregularly and impulsively wicked, the clumsy knave, the open thief, the entirely merciful, just, and godly person.
> – John Ruskin in F.H. Knight *The Ethics of Competition* (New York: Harper, 1935).

> The general truth will stand that no man in this land suffers from poverty unless it be more than his fault—unless it be his *sin*.
> Henry Ward Beecher (a protestant minister) quoted in Herbert G. Gutman, "Protestantism and the American Labor Movement: The Christian Spirit in the Gilded Age," Am. Hist. Rev., Oct 1966, p.76 n.

> The fact is that most people who have no skill have had no education for the same reason- low intelligence or low ambition.
> Senator Barry Goldwater in *Time* (24 Jan 1964), opposing the war on poverty's attempts to provide skills through education to the structurally unemployed.

> The production of all first-quality goods would cease. The skill they demand would be lost and the taste they shape would be coarsened. The production of artistic and intellectual goods would be affected first and foremost. Who could buy paintings? Who even could buy books other than pulp?
> – Bertrand de Jouvenel, *The Ethics of Redistribution* (Cambridge: Cambridge University Press, 1961) p.41.

> Everyone but an idiot knows that the lower classes must be kept poor, or they will never be industrious.
> – Edmund Burke quoting Arthur Young as cited by R.H. Tawney *Equality* (London: Allen and Unwin, 1952).

> In general it is only hunger which can spur and goad them on to labour.
> – The Reverend Joseph Townsend, as cited in Polanyi, K., The Great Transformation, (Boston: Beacon Press, 1957.

> Il faut que le peuple vive, mais il faut que sa vie soit pauvre et frugale; plus il est occupé, moins il est factieux, et il est d'autant plus occuppé, qu'il a plus de peine à pouvoir à ses besoins.
> – Diderot, as cited in Ragon, M., Histoire de la Litterature Proletarienne en France, (Paris: Albin Michel, 1974).

These are some ways of explaining the existing distribution. How plausible is each?

2 Measuring Income Inequality

Suppose we have N income recipients (e.g., households) with incomes

$$x_1 \leq x_2 \leq \ldots \leq x_N \tag{1}$$

Before we can make any sense of comparisons between these income recipients, we need to know the time unit (week, year, lifetime) and income unit (per family, per household, per adult, per individual, per equivalence scale weighted household).[1]

[1] See Deaton and Muellbauer, 1980, ch.8.

One way to get at the notion of income inequality is the Lorenz curve. This is the graph of

$$L(p) = \sum_{i=1}^{Np} x_i/X \tag{2}$$

where X is total income ($X = \sum_i x_i$) and p is the proportion of the "population" that we are considering. Note that since we have ordered incomes, we are asking what proportion of total income is received by the Np lowest income recipients. Thus $L(0) = 0$, $L(1) = 1$, and the curve is everywhere below the 45 degree line.

If all incomes were equal, the Lorenz curve would lie along the 45 degree line; which is therefore called the line of perfect equality. Similarly, if one Lorenz curve lies above another, we say the first Lorenz dominates the second.

Note that the Lorenz curve is capturing relative inequality. Less Lorenz curve inequality does not mean less absolute inequality (unless total income is constant).

Lorenz curve inequality addresses only relative and not an absolute income differences. For example, if we double all incomes in a distribution we get the same Lorenz curves, although absolute differences are much larger.

For the same reason, a strictly proportional income tax will not shift the Lorenz curve. Nevertheless, post-tax Lorenz curves generally Lorenz dominate pre-tax Lorenz curves, because of the progressivity of most tax structures: higher incomes are taxed at higher rates than lower incomes.[2]

Another problem with Lorenz curves is that they offer only a partial ordering of income distributions. Empirical Lorenz curves often cross, and sometimes more than once. This can happen in cross country comparisons because, say, developing countries have more inequality at the bottom. Crossing can even result from differing statistical sources in the same country.[3]

There are two popular proposal to derive a complete inequality ordering from Lorenz curves: The Gini coefficient and the Schutz coefficient. Each is an effort to capture the "distance" of a distribution from perfect equality.

The Gini coefficient measures how much of the area below the 45 degree line (an area = 1/2) lies above the Lorenz curve.

$$G = A/(A + B) = 2A = 1 - 2B \tag{3}$$

since $A + B = 1/2$.

2.1 Income and Wealth Inequality in the US

Income-based Gini coefficients for the United States are reported by the Census Bureau. See for example `http://www.census.gov/hhes/www/income/histinc/ineqtoc.html`. As measured by the Gini coefficient, there has been a clear upward trend in income inequality, from 0.397 in 1967 to 0.469 in 2005. There are even state-level Gini coefficients at `http://www.census.gov/hhes/www/income/histinc/state/state4.html`. These display the same upward trend.

Wealth is much more unequal than income. You can find lots of information in a recent WIDER study: `http://www.wider.unu.edu/research/2006-2007/2006-2007-1/wider-wdhw-launch-5-12-2006/wider-wdhw-press-r` `htm`. This study computed a wealth Gini for the U.S. of 0.8, making it one of the most unequal in the world.

2.2 Gini Algebra

Suppose we have an *increasing* list x_i for $i = 1, \ldots, N$ and we wish to compute $A/(A + B)$.

Define $y_j = \sum_{i=1}^{j} x_i$, which is the sum of the j lowest incomes. Cumulative income relative to total income, y_j/y_N, is the height of the Lorenz curve at j/N.

[2] Note that most tax codes use some non-income characteristcs in determining tax liabilities, so that household will be reranked when moving from pre-tax to post-tax Lorenz curves. If instead we plot post-tax income shares against quantiles determined by the pre-tax income ranking, the result is called a post-tax concentration curve. In the presence of re-ranking, the post-tax concentration curve will lie above the post-tax Lorenz curve, thereby suggesting more reduction in inequality than has taken place.

[3] Lambert 1989 notes that in the UK, the different coverage of Inland Revenue's Survey of Personal Incomes and the Central Statistical Office's Economic Trends generates crossing Lorenz curves when the two data sources are compared.

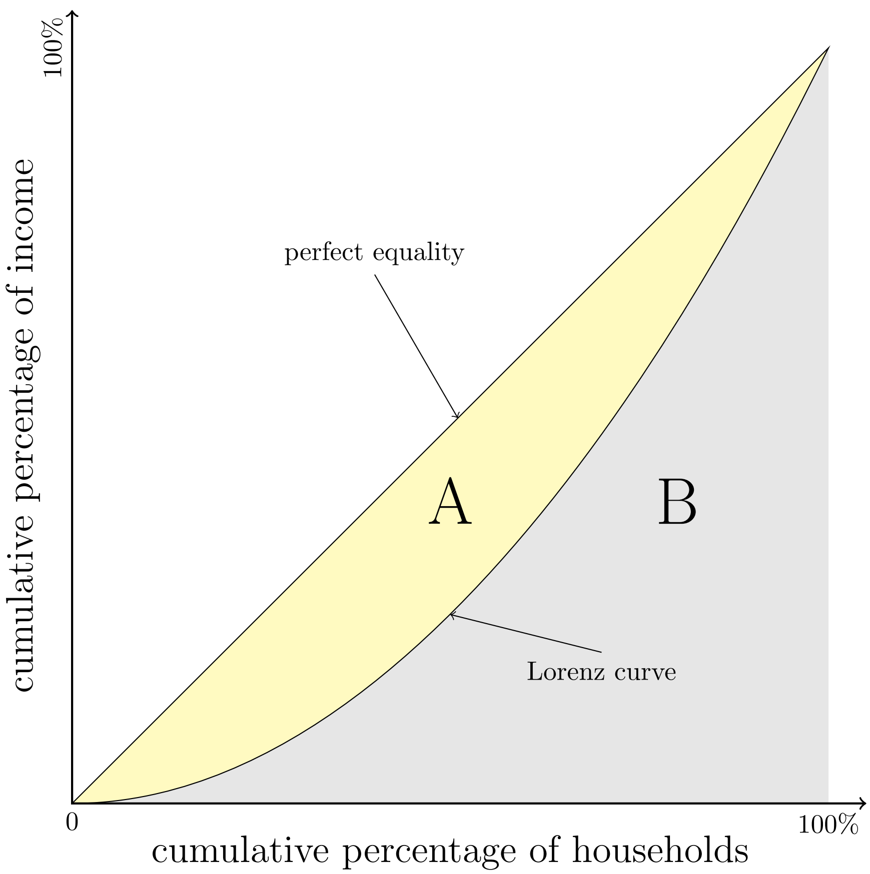

Figure 1: Gini Coefficient: $A/(A+B)$

With perfect equality, we would always have $y_j = (j/N)y_N$. In this special case we would have

$$\sum_{j=1}^{N} y_j = \frac{y_N}{N}\sum_{j=1}^{N} j = \frac{y_N}{N}\frac{N(N+1)}{2} = y_N(N+1)/2 \tag{4}$$

(Recall that $\sum_{j=1}^{N} j = N(N+1)/2$.)

Of course we do not have perfect equality. So actually

$$\sum_{j=1}^{N} y_j = \sum_{j=1}^{N}\sum_{i=1}^{j} x_i = \sum_{i=1}^{N}(N+1-i)x_i \tag{5}$$

So the difference between the sum under perfect equality and the actual sum is $y_N(N+1)/2 - \sum_{i=1}^{N}(N+1-i)x_i$

To scale this to a unit square, so that we are looking at the proportion of total income claimed by the proportion of the total population, deflate this difference by Ny_N to get

$$A = \frac{N+1}{2N} - \sum_{i=1}^{N}(N+1-i)x_i/Ny_N \tag{6}$$

The Gini G coefficient is 2A:

$$G = 2A = \frac{N+1}{N} - \frac{2}{Ny_N}\sum_{i=1}^{N}(N+1-i)x_i \tag{7}$$

We can implement this quite directly in Python:

```
def calc_gini(x):
        x = list(x)
        N = len(x)
        x.sort()  # increasing order
        G = sum( x[i] * (N-i) for i in xrange(N) )
        G = 2.0*G/(N*sum(x))
        return (1 + 1./N) - G
```

(Recall that Python uses zero-based indexing.)

An alternative representation:

$$G = \frac{N+1}{N} - \frac{2}{Ny_N}\sum_{i=1}^{N}(N+1)x_i + \frac{2}{Ny_N}\sum_{i=1}^{N} ix_i \tag{8}$$

$$G = \frac{N+1}{N} - \frac{2(N+1)}{Ny_N}\sum_{i=1}^{N} x_i + \frac{2}{Ny_N}\sum_{i=1}^{N} ix_i \tag{9}$$

$$G = \frac{N+1}{N} - 2\frac{N+1}{N} + \frac{2}{Ny_N}\sum_{i=1}^{N} ix_i \tag{10}$$

$$G = -\frac{N+1}{N} + \frac{2}{Ny_N}\sum_{i=1}^{N} ix_i \tag{11}$$

A Python implementation:

```
def calc_gini2(x):
        x = list(x)
        N = len(x)
        x.sort()  # increasing order
        G = sum(x[i] * (i+1) for i in xrange(N))
        G = 2.0*G/(N*sum(x))  #2*B
        return G - (1 + 1./N)
```

3 Just Distribution

We have focused on the actual distribution. If you want to think about the just distribution of income, start by taking a look at: Rescher, Nicholas, *Distributive Justice* (Indianapolis: Bobbs-Merrill, 1966), p.73:

> In the course of the long history of discussions on the subject, distributive justice has been held to consist, wholly or primarily, in the treatment of all people: (1) as equals (except possibly in the case of certain "negative" distributions such as punishments); (2) according to their needs; (3) according to their ability or merit or achievement; (4) according to their efforts and sacrifices; (5) according to their actual productive contribution; (6) according to the requirements of the common good, or the public interest, or the welfare of mankind, or the greater good of a greater number; (7) according to a valuation of their socially useful services . . . in the essentially economic terms of supply and demand.

As you consider these, be sure to specify exactly what is being distributed. For example, if we focus on the allocation of material goods we may reach different conclusions than if we also consider the distribution of "leisure time" which may, in turn, be a necessary adjunct to many consumption activities.