Natural Deduction Rules

June 20, 2017

1 Propositional Rules

(Rules cited from [1])

Rule 1 (And Introduction). $\dfrac{A \quad B}{A \wedge B}$ *or* $\dfrac{A \quad B}{B \wedge A}$

Rule 2 (And Elimination). $\dfrac{A \wedge B}{A}$ *or* $\dfrac{A \wedge B}{B}$

Rule 3 (Or Introduction). $\dfrac{A}{A \vee B}$ *or* $\dfrac{A}{B \vee A}$

Rule 4 (Or Elimination). $\dfrac{A \vdash C \quad B \vdash C \quad A \vee B}{C}$ *where A and B are assumptions*

Rule 5 (Not Introduction). $\dfrac{A \vdash \bot}{\neg A}$ *where A is an assumption*

Rule 6 (Not Elimination). $\dfrac{\neg A \quad A}{\bot}$

Rule 7 (Double Not Elimination). $\dfrac{\neg\neg A}{A}$

Rule 8 (Implies Introduction). $\dfrac{A \vdash B}{A \Rightarrow B}$ *where A is an assumption*

Rule 9 (Implies Elimination). $\dfrac{A \quad A \Rightarrow B}{B}$

Rule 10 (Iff Introduction). $\dfrac{A \Rightarrow B \quad B \Rightarrow A}{A \Leftrightarrow B}$

Rule 11 (Iff Elimination). $\dfrac{A \Leftrightarrow B}{A \Rightarrow B}$ *or* $\dfrac{A \Leftrightarrow B}{B \Rightarrow A}$

2 Predicate Rules

Rule 12 ($\forall$ Introduction). $\dfrac{P(a)}{\forall x.P(x)}$ *where a is arbitrary*

Rule 13 ($\forall$ Elimination). $\dfrac{\forall x.P(x)}{P(a)}$ *where a is arbitrary*

Rule 14 ($\exists$ Introduction). $\dfrac{\exists x.P(x) \quad \forall x.(P(x) \Rightarrow Q)}{Q}$

Rule 15 ($\exists$ Elimination). $\dfrac{P(t)}{\exists x.P(x)}$ *where t is any term*

Rule 16 (Substitution). $\dfrac{m = n \quad S(n)}{S[m/n]}$ *or* $\dfrac{m = n \quad S(m)}{S[n/m]}$

[1] J. Woodcock and M. Loomes, *Software Engineering Mathematics.* Pitman Publishing, 1989