Attitude Determination Given Sensor Angles

Grotifer Team

May 2024

1 Introduction

1.1 Grotifer

Grotifer is a satellite with two boons rotating about perpendicular axes to get electric field measurements in three dimensions. In the context of attitude control, the backward solution is the matrix composed of the three orthonormal column vectors that define the new x axis, new y axis, and new z axis in given sensor readings from the inclinometer and the sun sensor. This matrix is known as the direction cosine matrix and is in reference to the global coordinate system as defined by three axes. In our case, our three axes is defined as follows: the y axis is gravity and the x axis points toward the sun.

1.2 Given Information

The two angles measured by the sun sensor are given below, where the ray is in the direction of the sun(note the x and z axes are flipped in this diagram):

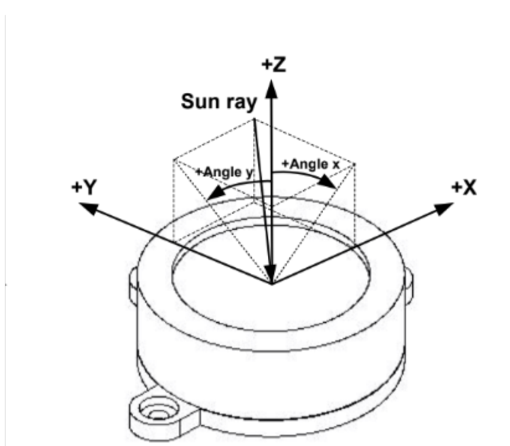

Figure 1: The angles measured by the sun sensor

The inclinometer measures two similar angles but in reference to the global y axis instead of the global x (the sun sensor measures the angle in reference to global x as its z axis is actually the x axis in our case). Given these four angle readings, we must find our direction cosine matrix.

1.3 Our Old Strategy

When we were not able to find an analytic solution, we used iterative methods to solve our problem. Given four sensor angles, how to find the three unknowns (the rotated x, y, and z vectors)? We used the Nelder-Mead method to find the x, y, and z vectors of the body fixed frame that minimized our error function, which just checked if the sensor readings of our x, y, and z vectors was close to the inputted sensor readings. The issue with this was computation time. Since Nelder-Mead was iterative, it's runtime duration was not consistent nor was it fast enough.

1.4 Our New Strategy

There is an algebraic method to find the direction cosine matrix. To use this method, we need to know two vectors and their components in the global and body fixed frame. Lets call these vectors u and v. We must find the components of these two vectors in the two different frames.

For our case, since the sun sensor measures the projection of the sun vector into the zy and xy planes, we can find the components of the sun vector in the body fixed frame. Since the x axis in the global frame is always pointing toward the sun, we can set our u to be [1, 0, 0] in the global reference frame. To find the components of u in the body fixed frame, we use the following equations:

$$\begin{aligned}
&\vec{u} = [u_1, u_2, u_3] \\
&u_1 = \frac{1}{\sqrt{1 + \tan^2\theta_y + \tan^2\theta_x}} \\
&u_2 = u_1 * \tan\theta_y \\
&u_3 = u_1 * \tan\theta_z
\end{aligned} \tag{1}$$

Two of the equations are derived from trig and geometry. However, we need three as we have three unknowns (the three components of u). The third equation, in this case the equation for u1, comes from normalizing the sun vector and enforcing the magnitude must be 1: $u_1^2 + u_2^2 + u_3^2 = 1$.

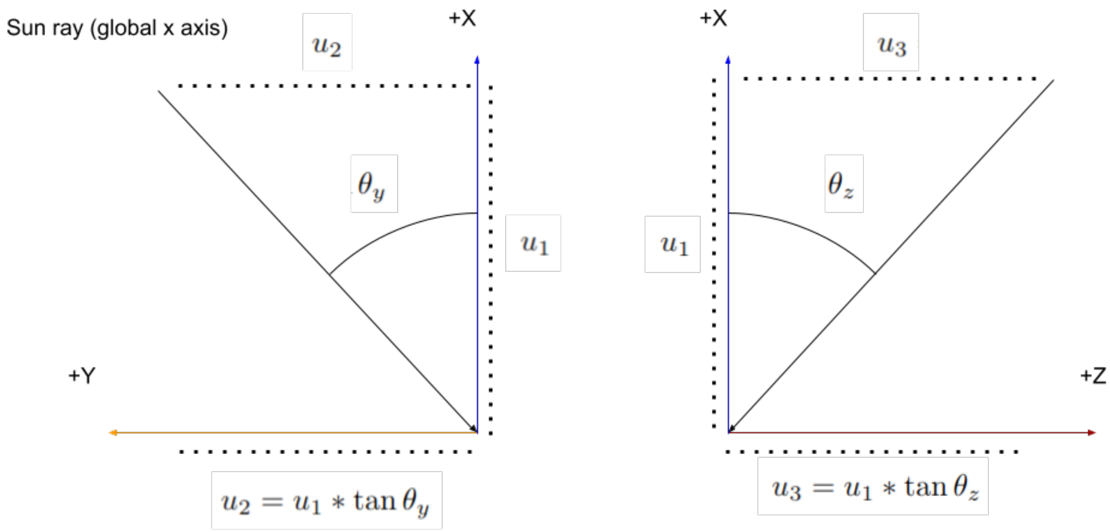

Figure 2: Geometric derivation

We now have three equations and three unknowns. Solving the equations yields the equations found in (1). The same idea and steps are used for the inclinometer to find vector v, where v is the gravity vector (global y axis), [0, 1, 0]. This works as the inclinometer uses a simlar projection model to the sun sensor.

1.5 What We Do With U and V

We now have two vectors, u and v, and we know their components in both the body fixed frame as well as the global frame (essentially just two vectors represented with two different coordinate systems). The next step of the algebraic method is to find the vectors q, r, and s for both the body fixed frame and the global reference frame as well. The equations for these are as follows:

$$\begin{aligned}\vec{q} &= \vec{u}\\ \vec{r} &= \vec{u} \times \vec{v}\\ \vec{s} &= \vec{q} \times \vec{r}\end{aligned} \tag{2}$$

To find the q, r, and s in the global reference frame, you use the vectors u and v with their global components. To find the q, r, and s in the body fixed frame, you use the vectors u and v with their body fixed components (found in 1.3).

With our two sets of q, r, and s, we can construct two matrices: the body matrix M_B and teh reference matrix M_R where

$$
\begin{aligned}
M_B &= [\vec{q_b}, \vec{r_b}, \vec{s_b}] \\
M_B &= [\vec{q_r}, \vec{r_r}, \vec{q_r}]
\end{aligned} \tag{3}
$$

Finally, the direction cosine matrix A, is given by:

$$
A = M_B M_R^T \tag{4}
$$

With our direction cosine matrix A, we now have the attitude of Grotifer.